9.5 Solve Problems Involving Surface Area and Volume

Strand
Measurement and Trigonometry

Student Text Pages
398–405

Suggested Timing
80–160 min

Tools
- building blocks
- calculators
- magazines
- paper
- scissors
- tape

Related Resources
BLM 9.5.1 Practice: Solve Problems Involving Surface Area and Volume
BLM 9.5.2 Investigate: Surface Area and Volume of a Composite Solid
BLM 9.5.3 Achievement Check Rubric
BLM 9.2.2 Formula Sheet
BLM G1 Grid Paper

Specific Expectations

Solving Problems Involving Surface Area and Volume, Using the Imperial and Metric Systems of Measurement

In this section, students will

MT3.01 use the imperial system when solving measurement problems (e.g., problems involving dimensions of lumber, areas of carpets, and volumes of soil or concrete)

MT3.02 perform everyday conversions between the imperial system and the metric system (e.g., millilitres to cups, centimetres to inches) and within these systems (e.g., cubic metres to cubic centimetres, square feet to square yards), as necessary to solve problems involving measurement

MT3.04 solve problems involving the surface areas of prisms, pyramids, and cylinders, and the volumes of prisms, pyramids, cylinders, cones, and spheres, including problems involving the combinations of these figures, using the metric system or the imperial system, as appropriate

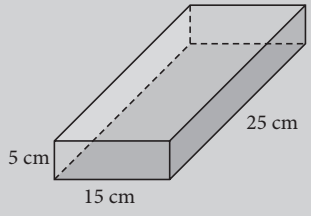
Link to Get Ready

Students will solve problems related to surface area and volume of composite solids. This requires skills learned in earlier sections, including converting measurements. Have students complete all of the Get Ready questions before proceeding with Section 9.5.

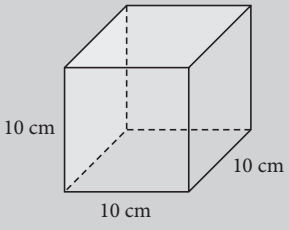
Warm-Up

1. Determine the surface area of each shape.

a)



b)



2. Find the volume of each shape in question 1.

3. A cone with a base radius of 5 cm is placed on top of a cube with a side length of 10 cm. Only a portion of the top of the cube is showing. How much of the surface of the cube is exposed?

Warm-Up Answers

1. a) 1150 cm^2 **b)** 600 cm^2

2. a) 1875 cm^3 **b)** 1000 cm^3

3. 21.5 cm^2

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class. (15–20 min)

Common Errors

- Some students may not use the correct units for area and volume in multi-step problems.

R_x Encourage students to record the correct units for each step of the problem as they go.

- Some students may forget the formulas for surface area and volume.

R_x Provide students with **BLM 9.2.2 Formula Sheet** to help them.

Ongoing Assessment

- As students work on the Investigate, circulate and observe how well students work with a partner. Consider recording each student's learning skills: group work, work habits, organization, and initiative.
- Use Chapter Problem question 6 to assess students' ability to apply their knowledge of surface area and volume of composite solids.
- Use Achievement Check question 7 to assess students' ability to calculate volume.
- Use **BLM 9.5.3 Achievement Check Rubric** for formative or self-assessment.

Accommodations

ESL—Pair ESL students with those who have stronger language skills.

Gifted and Enrichment—Challenge students to find the equation for *n* cubes for D3.
(*n* cubes: $(24 \times n) - (2n - 2) \times 4$).

Memory—Some students will have difficulty processing the steps in problems and need coaching. Begin by having them identify the shapes in the composite solid, list the information they are given, identify what they are to find out, and write the formulas needed to solve the problem. Remind students to use **BLM 9.2.2 Formula Sheet** for reference.

Perceptual—Students with perceptual difficulties may have difficulty visualizing the composite solid in the Investigate. Use a 3-D model to help them.

Visual—Provide 3-D models of the shapes in Discuss the Concepts or have students make models to help them understand the problems.

Section Opener

- Read the opening paragraph as a class and have students work with a partner to brainstorm everyday objects made up of combinations of solids. Consider providing magazines or Internet access to help them. After students have generated the list, have student pairs swap lists and list the shapes that make up each composite.

Investigate

- Have students work in small groups. Consider having students draw and label the rink using **BLM G1 Grid Paper**. Instruct them to use two grid lines for every foot as such a scale will help display the circular corners on the rink.
- Prompt students to recall question 3 in the Warm-Up, which they solve using a square of the side length that is equal to the diameter of the circle. Using this information will help students find what is missing from the full rectangle due to its rounded corners. (The four rounded corners form a circle. If the rink were a rectangle, the total area would be 20 ft $\times$ 40 ft $= 800$ ft^2.) Consider providing students who need scaffolding to complete the Investigate with **BLM 9.5.2 Investigate: Surface Area and Volume of a Composite Solid**.
- Use **BLM 9.5.1 Practice: Solve Problems Involving Surface Area and Volume** for extra practice or remediation.

Investigate Answers (page 398)

a) The ice surface is a rectangle with rounded corners. It is necessary to subtract the area missing from the rectangle due to its rounded corners. The four rounded corners form a circle. The missing part is the material removed from a square of a side length equal to the diameter of the circle.
Total area of ice surface without rounded corners would be $= 20 \times 40 = 800$
Area of a square of length $5 = 25$
Area of a circle of radius $2.5 = 19.6$ ft^2
Area missing from the rectangle $25 - 19.6 = 5.4$
Area of the ice surface $= 800 - 5.4 = 794.6$ ft^2

b) Surface area of top and bottom of bladder $= 2 \times 794.6 = 1589.2$ ft^2

Surface area of side of bladder $= l \times w$

Length = perimeter of ice surface

Perimeter $= 2 \times 35$ (subtract circle at each end from the length) $+ 2 \times 15$ (subtract circle at each end from the length) $+ 2 \times 3.14 \times 2.5$ (for full circle that four corners add up to) $= 115.7$ ft

Width $= 4$ in. $= \frac{1}{3}$ ft

Surface area of side of bladder $= l \times w = 115.7 \times \frac{1}{3} = 38.6$ ft^2

Total surface area of plastic $= 1589.2 + 38.6 = 1627.8$ ft^2

c) Volume of bladder = Area of base $\times$ height

$794.6 \times \frac{1}{3} = 264.9$ ft^3

1 ft$^3 = 7.48$ gal

$\frac{264.9 \text{ ft}^3}{7.48} = 35.4$ gal

At a rate of 3 gal/min, the bladder would take 11.8 min to fill.

d) $1627.8 \times 0.27 = \$439.50$
The plastic would cost $439.50.

Examples

- As a class, work through each of the Examples to ensure that students understand surface area and volume of composite solids. These problems are perhaps the most complex that students have seen.
- Consider displaying an overhead of each composite solid and have students identify each of the shapes in the solid. You might have students develop a flow diagram that explains the steps (including the formulas) needed to solve each part of the problem. You may need to review the formulas for each shape from earlier sections in the student text.

Key Concepts

- Ensure that students understand the Key Concepts before assigning the Practise the Concepts questions.

Discuss the Concepts

- Have students work with a partner to complete the questions. Provide 3-D models of the shapes in D1 and D3. If time permits for D1, have students make a net and then cut out and tape each shape to form a composite solid. If blocks are available, have students use blocks for D3 in order to establish a pattern.
- In a class discussion, be prepared to spend time to cover the difficult concepts that some students may struggle with.

Discuss the Concepts Suggested Answers (page 402)

D1. The surface area of the base of the pyramid and the surface area of the part of the prism that the pyramid covers are not part of the exposed surface area of the toy. The total surface area of the toy comprises the surface areas of the prism and the pyramid minus twice the area of the base of the pyramid (once for the pyramid base and once for the surface of the prism that the base covers).

D2. Yes. Since there is no overlap in the volumes, the volumes can be added together.

D3. Each time that a cube is stacked on another cube, two sides are eliminated from view (one from the cube being added and one for the side that the added cube covers).
For two cubes: $(24 \times 2) - (2 \times 4) = 40 \text{ cm}^3$
For three cubes: $(24 \times 3) - (4 \times 4) = 56 \text{ cm}^3$
For four cubes: $(24 \times 4) - (6 \times 4) = 72 \text{ cm}^3$

Practise the Concepts (A)

- Encourage students to refer back to the Examples before asking for assistance.
- Refer students to the MathConnects on page 385 and on page 402 to help them understand the context of question 1.

Apply the Concepts (B)

- Question 6 is a Chapter Problem. Consider having students assemble the composite solid using nets they create for each shape. If so, provide paper, scissors, and tape. You might extend the question by having design composite shape packaging. Once completed, they can share their designs with the class. Remind students to keep the solution handy to help with the Chapter Problem Wrap-Up.
- Question 7 is an Achievement Check. It can be used as a form of diagnostic or formative assessment, or assigned as a small summative assessment piece. This provides an opportunity for formative or self-assessment, using **BLM 9.5.3 Achievement Check Rubric**.

- Question 10 is a Literacy Connect. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics. This supports general Think Literacy strategies. For more information, visit http://www.edu.gov.on.ca/eng/studentsuccess/thinkliteracy.
- For question 10b), discuss concepts of appearance, structural strength, and ergonomics with students. Mention that it is difficult to fit furniture into a round building with no corners, since most furniture are modelled on rectangles not circles. If time permits, have students use the Internet to research modern building designs, such as the Petronas Twin Towers in Kuala Lumpur, Malaysia, the Turning Torso apartment building and office tower in Malmo, Sweden, and, closer to home, city halls in Newmarket, Ontario, and Mississauga, Ontario.

Achievement Check Answers (page 404)

7. Volume of Roof in Plan A:

V = base area $\times$ height

$= \frac{1}{2}(8)(3) \times 10$

$= 120$

Greenhouse Plan A has 120 m^3 of space in the roof area.

Volume of Roof in Plan B:

V = base area $\times$ height

$= \frac{1}{2}(6)(4) \times 11$

$= 132$

Greenhouse Plan B has 132 m^3 of space in the roof area.

Arthur should choose Greenhouse Plan B.

Extend the Concepts (C)

- Assign the Extend the Concepts questions to students who are not being challenged by questions in Apply the Concepts.
- Extend the Concepts questions can be used as a diagnostic assessment for those students considering a university-level course in grade 11.