11

Solving Equations

Thousands of years ago, people wrote coded messages on stone. You might even say this entire book is written in a code—our common alphabet!

Another language can seem like a secret code until you figure it out. During World War II, Canadian Cree people were recruited to relay messages by telephone and radio in their own language. For them, it was as easy as saying "hello" to a neighbour. However, enemy forces listening to the conversations never broke the "code" to understand the messages.

Other secret codes are made by substituting each letter with a number, another letter, or a symbol. You can even use math operations to convert letters to numbers. In this chapter, you will learn to solve equations and use them to decode messages

What You Will Learn

- ❑ to tell the difference between an expression and an equation
- ❑ to solve problems involving equations

Key Words

equation
opposite operations

MATH LINK

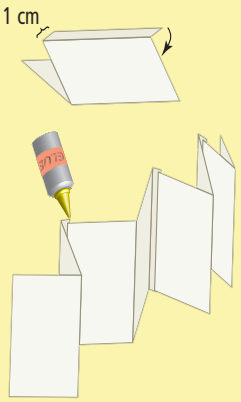
When you want to send secret written messages, how can you use mathematics to create a code?

FOLDABLES™
Study Tool

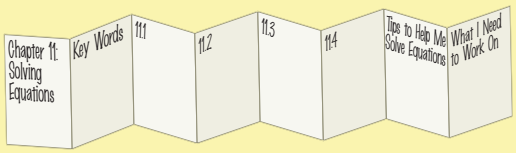
Make the following Foldable to organize what you learn in Chapter 11.

Step 1 Collect four sheets of paper. Fold each sheet of paper in half as shown.

Step 2 Fold a 1-cm tab along the edge of three of the folded sheets of paper. Glue the papers together along the tabs.



Step 3 Label the sections made by each fold.

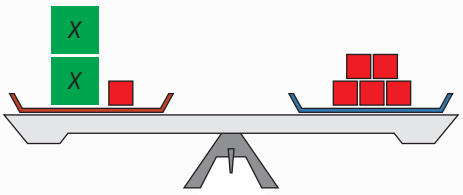


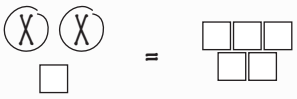
Literacy Link

As you work through Chapter 11, make notes on the appropriate fold. Include information about the key words, examples, and key ideas. Once you have completed the Chapter Review, make a list of reminders under "Tips to Help Me Solve Equations."

11.1 Expressions and Equations

You can model patterns with objects, drawings, and expressions. Three different ways to model the same mathematical statement are shown. What do the models have in common? How are they different?





$2x + 1 = 5$

Focus on...

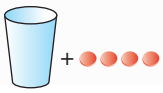
After this lesson, you will be able to...

- identify constants, numerical coefficients, and variables in expressions and equations
- describe the difference between an expression and an equation

Explore the Math

What is the difference between an expression and an equation?

1. The cup contains an unknown number of counters. Use the variable x to represent the unknown number of counters in the cup. Write an expression to model the total number of counters.



In Chapter 10, you learned about variables and expressions.

Materials

- 2 cups or containers
- 1 to 15 counters

Literacy Link

Writing Expressions

An expression can be written using a single constant, a single variable, or a combination of operations with constants, variables, or numerical coefficients. For example,

$2y - 7$ ← constant

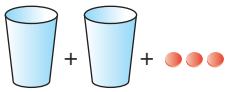
numerical coefficient, variable

2. a) Place 5 counters in the cup. What is the value of the cup now?

b) How many counters do you have in total?

c) If you let $x = 5$, what is the value of your expression in #1?

3. Each cup in this diagram contains the same number of counters.

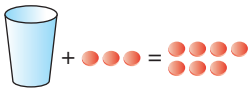


a) Use the variable x to represent the unknown number of counters in each cup. What expression does the diagram represent?

b) What is the numerical coefficient in this expression?

c) What is the constant in this expression?

4. a) Write the **equation** shown by this diagram.



b) How many counters must be in the cup to make both sides of the equation equal?

equation

- a mathematical statement with two expressions that have the same value
- $x + 2 = 3$, $y - 7 = -4$, $3a - 2 = a + 2$, and $b = 4$ are examples of equations

Reflect On Your Findings

5. a) What do expressions and equations have in common?

b) What is the difference between expressions and equations?

Example 1: Identify Expressions and Equations

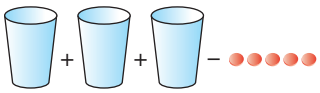
- Model each phrase using cups and counters.
- Write each phrase as an expression or an equation.
- Identify any variables, numerical coefficients, and constants used in the expression or equation.

a) three times a number minus five

b) two times a number plus four equals ten

Solution

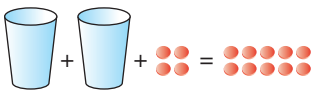
a) Let n represent the unknown number of counters in each cup.



The expression is $3n - 5$.

The variable is n, the numerical coefficient is 3, and the constant is 5.

b)



Let z represent the unknown number of counters in each cup.

The equation is $2z + 4 = 10$.

The variable is z, the numerical coefficient is 2, and the constants are 4 and 10.

Show You Know

a) Model the following phrase using cups and counters.

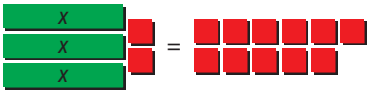
four times a number minus five equals seven

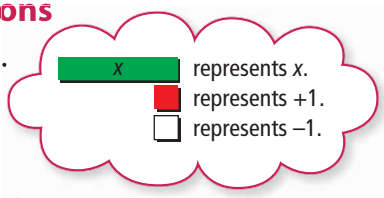
b) Write the phrase as an expression or as an equation.

c) Identify any variables, numerical coefficients, and constants.

Example 2: Write Expressions and Equations

The algebra tile diagram represents an equation.





a) What are the two expressions that make up this equation?
b) What is the equation?

Solution

a) The variable is x.
The first expression is $3x + 2$.
The second expression is 11.

b) The equation is $3x + 2 = 11$.

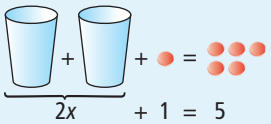
Show You Know

The diagram represents an equation.

a) What are the two expressions that make up this equation?
b) What is the equation?

Key Ideas

- An expression can be a single constant, a single variable, or a combination of operations with constants, variables, or numerical coefficients.
- An equation is made up of two expressions that are equal in value to each other.
- Expressions and equations both contain variables, numerical coefficients, and constants.
- Always identify what your variable stands for. For example, in the equation shown x represents the unknown number of counters in each cup.



Communicate the Ideas

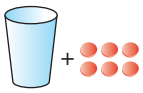
1. What is the difference between an expression and an equation?

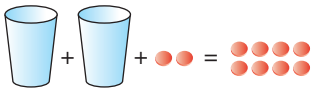
2. Show a friend how to model the equation $6x - 2 = 10$ using cups and counters.

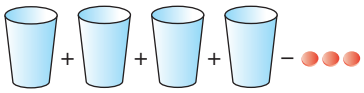
Practise

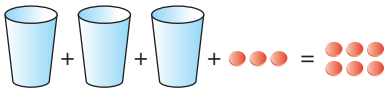
For help with #3 to #6, refer to Example 1 on page 391.

3. Identify and write each model as an expression or an equation.

a) 

b) 

c) 

d) 

4. Identify the variables, numerical coefficients, and constants in each expression or equation in #3.

5. Model each phrase using cups and counters. Write each phrase as an expression or an equation. Then, identify any constants, numerical coefficients, and variables in the expression or equation.

a) a number minus eight

b) three times a number plus two

c) a number minus two equals eight

6. Model each phrase using cups and counters. Write each phrase as an expression or an equation. Then, identify any constants, numerical coefficients, and variables in the expression or equation.

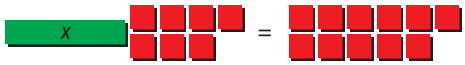
a) two times a number plus three equals seven

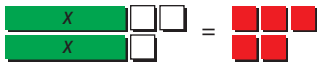
b) seven plus two times a number

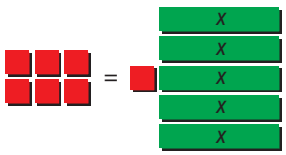
c) fifteen equals five plus two times a number

For help with #7 and #8, refer to Example 2 on page 392.

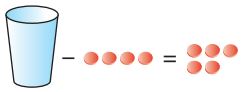
7. Write the two expressions that make up each equation. What is the equation?

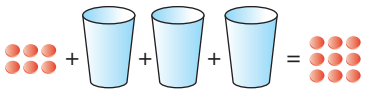
a) 

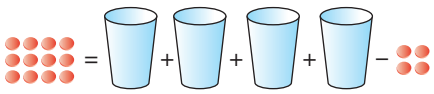
b) 

c) 

8. Write the two expressions that make up each equation. What is the equation?

a) 

b) 

c) 

Apply

9. Write an expression for each phrase.

a) twelve centimetres taller than Brady

b) five kilograms less than Tran

c) fifty-two years younger than Amanda

10. Write an expression for each phrase.

a) nine less than three times a number

b) the sum of f divided by five and four

c) eight times the result of g minus five

d) the quotient of h and eight is diminished by twelve

11. Write an equation for each phrase.

a) twice your age in years plus four years equals thirty years

b) your mass in kilograms divided by two equals twenty-five kilograms

c) four times your height in centimetres equals six hundred centimetres

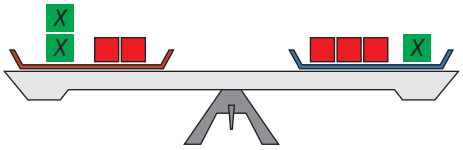
12. Write a word phrase to represent each expression.

a) $3a - 6$ **b)** $6b + 8$

c) $6(c - 3)$ **d)** $9 + 2e$

Extend

13. This scale represents an equation.



a) Identify the expressions on either side of the balance.

b) What is the equation?

14. Model the equation $12 = 4 + 2m$, where m represents a whole number.

a) What are the expressions that make up this equation?

b) What value of m would make the equation true? Show how you found the answer and how you know your answer is correct.

15. If Duncan had $7 more he could purchase a DVD that costs $23.

a) Draw a diagram to model the situation.

b) What equation could be used to model this situation?

c) What does the variable represent?

d) How much money does Duncan have? How did you determine this?

MATH LINK

One simple way to develop a code is to assign a number to each letter.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g	8 = h	9 = i	10 = j	11 = k	12 = l	13 = m
14 = n	15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u	22 = v	23 = w	24 = x	25 = y	26 = z

Using this code, H-E-L-L-O would be represented by 8 5 12 12 15.

a) Spell your first and last name with the code.

b) This code is very common. Many people can break it easily. Code writers make the code more difficult by using an expression to change the numbers that represent the letters. Then, only the people who know the correct expression can decode the message.

For example, some people use a $2n - 1$ code. To do this, they multiply each number from the chart by two, and subtract one. Using this code, the letter c is represented by $2 \times 3 - 1$, or 5. Rewrite the code chart using a $2n - 1$ code.

c) Write H-E-L-L-O using the new code.

d) Spell your first and last name with the new code.

e) Use the new code to write a message to a friend and decode a message from a friend.

11.2

Solve One-Step Equations: $x + a = b$

Focus on...

After this lesson, you will be able to...

- ❑ model problems with equations
- ❑ solve equations and record the process
- ❑ verify solutions to equations

Aaron and his mother spend $56 to take the ferry from Vancouver to Victoria. Aaron knows that the cost for each person is $11. So, the cost for two people is $2 \times \$11 = \$22$. He decides to model the situation with the equation $C + 22 = 56$, where C is the cost of the car. How could he determine the cost of the car?

Explore the Math

How do you solve one-step equations?

Hilda's grandmother gives her $5 for her birthday. Hilda puts this in her piggy bank and now has $12. How much money did she have before her birthday?

Materials

- 30 loonies
- cup (foam or lightweight plastic)

1. Model this situation using a cup and coins.

2. Remove one coin at a time from each side of the equation until only the cup remains on the left side.

a) How many coins did you remove from each side of your model?

b) How many coins remain on the right side of your model?

3. How much money did Hilda have before her grandmother gave her the birthday money?

4. What equation can be used to represent this problem? Use m as the variable in your equation. What does m represent?

Reflect on Your Findings

5. a) Draw a diagram to show the steps you took to model and solve the problem.

b) What mathematical operation did you apply to both sides of the equation?

c) Why do you have to apply the same operation to both sides of the equation?

Literacy Link

To solve by inspection means to use mental math.

Example 1: Solve by Inspection

Use mental math to solve each equation.

a) $j + 4 = 12$ **b)** $15 - y = 5$ **c)** $17 = d - 9$

Solution

a) $j + 4 = 12$
$8 + 4 = 12$
The solution is $j = 8$.

Ask yourself:
"What number added to 4 makes 12?"
or
"What number subtracted from 12 makes 4?"

b) $15 - y = 5$
$15 - 10 = 5$
The solution is $y = 10$.

Ask yourself:
"15 less what number makes 5?"
or
"15 minus 5 equals what number?"

c) $17 = d - 9$
$17 = 26 - 9$
The solution is $d = 26$.

Ask yourself:
"What number less 9 makes 17?"
or
"What number results from adding 17 and 9?

Show You Know

Solve by inspection.

a) $14 = k + 5$ **b)** $t - 3 = 11$ **c)** $10 - y = 8$

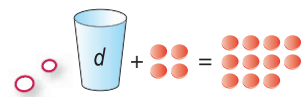
Example 2: Model and Solve a Problem

Stephanie and her sister cycle 4 km to the shopping mall, then travel farther to their mother's office. If they cycle 11 km in total, how far is it from the shopping mall to the office?

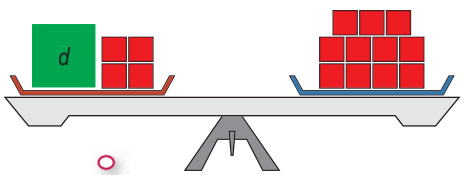
a) Model the situation. **b)** Solve by inspection.

Solution

a) Let d represent the distance from the shopping mall to the office. Model the situation using cup and counters or a sketch of a balance.



How many counters should be placed in the cup on the left side to equal the number on the right side?



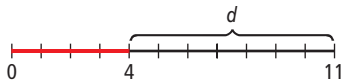
What mass must d represent if the scale is balanced as shown?

The situation can be modelled by the equation $d + 4 = 11$.

b) $d + 4 = 11$
$7 + 4 = 11$

By inspection, the answer is 7.
The distance from the shopping mall to the office is 7 km.

Since this is a distance problem, you could use a line diagram as a model.



Example 3: Apply the Opposite Operation

Aaron needs to solve the equation $C + 22 = 56$ to find the cost, C, of taking the car on the ferry from Vancouver to Victoria. What is the cost? Check your answer.

Solution

The equation $C + 22 = 56$ could be modelled using a cup and counters or a sketch of a balance, but with numbers this large it is difficult.

Aaron needs to get C by itself on one side of the equation. This process is called "isolating the variable." Since C has 22 added to it, he applies the **opposite operation**. To keep the equation balanced, he subtracts 22 from both sides.

$$\begin{aligned} C + 22 &= 56 \\ C + 22 - 22 &= 56 - 22 \quad \text{Subtract to undo addition.} \\ C &= 34 \end{aligned}$$

The cost of taking the car on the ferry is \$34.

Check:

Check by substituting 34 for C in the equation.

$$\begin{aligned} \text{Left Side} &= C + 22 \qquad \text{Right Side} = 56 \\ &= 34 + 22 \\ &= 56 \\ \text{Left Side} &= \text{Right Side} \end{aligned}$$

The answer is correct.

Show You Know

Solve by applying the opposite operation. Check your answer.

a) $n + 7 = 26$

b) $d - 3 = -5$

Did You Know?

Tourists and residents took more than 22 million ferry trips between the islands and the west coast of British Columbia in 2005.

opposite operation

- an operation that "undoes" another operation
- subtraction and addition are opposite operations
- multiplication and division are opposite operations

Literacy Link

You may sometimes hear opposite operations called "inverse operations."

Key Ideas

- Equations can be solved in several ways. You can:
  - solve by inspection, using mental math

$x - 5 = 16$
The answer is $x = 21$.

  - model the equation to help balance it

  - isolate the variable by performing the opposite operation on both sides of the equal sign

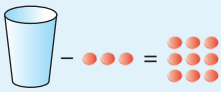
$$y - 14 = 17$$
$$y - 14 + 14 = 17 + 14$$
$$y = 31$$

- To check your solution, substitute your answer into the equation. Compare the left side of the equation to the right side. If the solution is correct, both sides should have the same value.

Left Side $= 31 - 14$ Right Side $= 17$
$= 17$
Left Side = Right Side

Communicate the Ideas

1. Wes is solving a problem. He models it to start.



a) What equation can be used to model this situation?

b) What do you think will be his next step?

2. What are two different ways you could use inspection to solve the equation $s + 12 = 20$?

3. Gavin and Rebecca are trying to explain how to solve the equation $d - 27 = 15$. Who is correct? Show how you know.

Gavin:

You need to subtract 27 from both sides.

Rebecca:

No, you need to add 27 to both sides.

Practise

For help with #4 and #5, refer to Example 1 on page 396.

4. Use mental math to solve each equation. Explain your thought process.

a) $z + 7 = 4$ **b)** $g - 2 = 5$

c) $n - 4 = 8$ **d)** $9 = k + 6$

5. Solve by inspection.

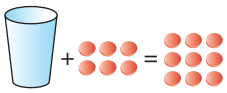
a) $b + 11 = 14$ **b)** $30 = r - 50$

c) $w - 7 = 5$ **d)** $10 - h = 8$

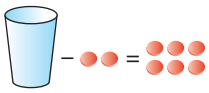
For help with #6 to #8, refer to Example 2 on pages 396–397.

6. What is the number of counters needed in each cup to make each equation true?

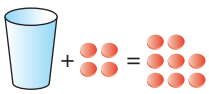
a)



b)

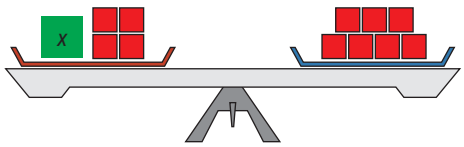


c)

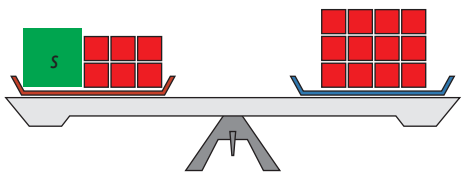


7. What value must the variable have in each model to keep the scale balanced?

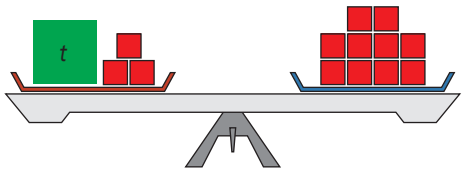
a)



b)



c)



8. Ryan has a bag of oranges. His friends eat ten oranges. If there are two oranges left in the bag, how many oranges were there to start with?

a) Model the situation using cups and counters or a sketch of a balance.

b) Write an equation to represent your model.

c) Solve by inspection.

For help with #9 to #11, refer to Example 3 on page 397.

9. Solve each equation using the opposite operation. Show your work. Check your answer.

a) $g + 7 = 13$

b) $w + 5 = 5$

c) $k - 8 = 8$

d) $p - 9 = 16$

10. Solve each equation using the opposite operation. Show your work. Check your answer.

a) $6 = 4 + m$

b) $k - 3 = -8$

c) $14 = p - 10$

d) $16 - x = 15$

11. If Charles had \$6 more in his pocket, he could buy a \$22 DVD. An equation to model this situation is $x + 6 = 22$.

a) What does the variable x represent? How do you know?

b) How much money does Charles have?

12. Show whether or not $x = 5$ is the solution to each equation.

a) $x + 10 = 15$

b) $10 - x = 15$

c) $x - 7 = -2$

d) $42 = 37 - x$

Apply

13. **a)** Draw a balance to show the equation $12 = 3 + m$, if m represents an unknown mass.

b) What total mass should be on each side of the balance?

c) Solve the equation to determine the unknown mass.

14. A 2003 Calgary Flames hockey team card set sells for $12. This is $8 more than the 2003 Vancouver Canucks set.

a) Draw a model to represent the problem.

b) Write an equation to model this situation.

c) What is the cost of the Vancouver Canucks card set?

15. The blue whale is the largest animal on Earth. It is also a very fast swimmer, able to swim at a speed of up to 48 km/h when in danger. The orca (killer whale) is the fastest species of whale. It has been timed swimming at a speed 12 km/h more than the fastest speed of the blue whale.

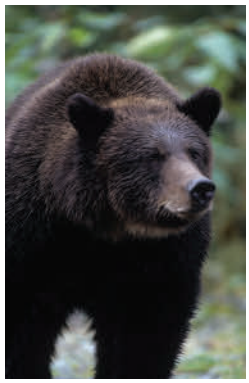
a) Write an equation that could be used to model the speed of a killer whale, k, given the speed of a blue whale.

b) What is the speed of the killer whale in this question?

16. The average life span of a grizzly bear is 25 years. This is 15 years more than the average life span of a cougar.

a) What equation will model this situation?

b) What is the average life span of a cougar?



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17. At the Commonwealth Games in Australia, Canada won 86 medals. This was 24 fewer medals than England won.

a) Write an equation to model this situation.

b) How many medals did England win?

18. Shawn received $5 change from $20 when he bought some binders. How many binders did he buy if each binder costs $3.00? Write an equation, then show how you solve it.

Extend

19. The sum of 3 and a number is -11.

a) Model this situation.

b) Write an equation.

c) What is the unknown number? Check your answer.

d) Why is a balance scale not a good method to use to solve this equation?

20. The formula $a + e = 85$ is often used to determine when an employee can retire. The variable a represents the age of the employee. The variable e represents the number of years of employment.

a) Richard is 52 years old. He has been a plumber for 21 years. Can he retire this year? Explain.

b) Joan has been working for 31 years. How old is she if she is eligible to retire this year?

21. A blue whale is the loudest animal on Earth. The call of a blue whale can reach sound levels of 188 decibels. This is 48 decibels louder than a jet engine.

a) What equation will model this situation?

b) What is the sound level in decibels for a jet engine?

c) A human whisper has a sound level about $\frac{1}{10}$ that of a jet engine. Write an equation to compare the sound of a jet engine to a whisper. Use your answer from part b) to determine the sound level in decibels for a whisper.

22. It costs \$3.00 to enter a city parking lot and then \$1.00 per hour after that.

a) What equation will model this situation?

b) What is the cost of parking in the lot for 4 h?

c) How long can you park in the lot if you have only \$5?

MATH LINK

Jim has created a code and wants to see if his friend can break it. The following is the coded message Jim wrote:

10 2 2 17 / 10 2 / −2 17 / 16 17 2 19 2 ' 16

(Spaces separate letters and the symbol "/" separates words.)

Use the following information to help crack Jim's code:

- The letter E is the most common letter in English, so code crackers often start by seeing obvious places an E might be, and solving for that number in the code.
- The next most common letters in the English language are, in order, T, A, O, I, N, S, H, R, D, and L.
- Jim uses an equation of the form $x + a = b$ to change the letters in the message to the numbers in the coded version.
- Many codes start by using the following numbers to represent letters.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g	8 = h	9 = i	10 = j	11 = k	12 = l	13 = m
14 = n	15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u	22 = v	23 = w	24 = x	25 = y	26 = z

a) What equation did Jim use as his code?

b) Rewrite the code chart using Jim's equation to determine the number that represents each letter.

c) What is the message?

11.3

Solve One-Step Equations: $ax = b, \frac{x}{a} = b$

Focus on...

After this lesson, you will be able to...

- ❑ model problems with equations
- ❑ solve equations and record the process

Canadian sprinter Donovan Bailey broke the world record by sprinting 50 m in 5.56 s. His speed was about 9 m/s.

The distance travelled in a certain amount of time at a speed of 9 m/s can be modelled by the formula $d = 9t$, where d represents distance, in metres, and t represents time, in seconds. At this speed, how long would it take to travel 900 m? How can you find out?

Explore the Math

Materials

- cups and counters

How do you solve one-step equations of the form $ax = b$ and $\frac{x}{a} = b$?

Kayla has $24 saved up and decides to buy some paperback books at a garage sale. If each book costs $3, how many books can Kayla purchase?

1. Use cups and counters to model this situation.

2. a) How can you use the cups and counters to solve the problem?
   b) How many books can Kayla purchase with $24?

Reflect on Your Findings

3. a) What is the relationship between the price per book, the number of books, and the total amount of money?
   b) Write an equation to represent the situation.
   c) What operation do you apply to the left side of the equation to isolate the variable?
   d) What operation do you apply to the right side of the equation to make it balance?

Example 1: Solve by Inspection

Use mental math to solve each equation.

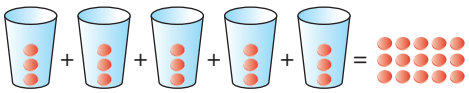
a) $5g = 15$

b) $\frac{x}{4} = 5$

Solution

a) $5g = 15$

You can model this situation using cups and counters. If you divide the counters up evenly into the 5 cups, there are 3 counters in each cup.



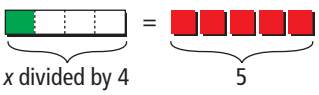
$5 \times 3 = 15$

The solution is $g = 3$.

Ask yourself: "5 times what number gives 15?"

b) $\frac{x}{4} = 5$

You can use algebra tiles to model this situation.



x represents x.
represents +1

Since you need 4 green squares to fill the rectangle, you need to multiply the number of red "ones" by 4.

$5 \times 4 = 20$

$\frac{20}{4} = 5$

The solution is $x = 20$.

Ask yourself: "What number divided by 4 is 5?"

You could start with a bunch of counters. Then, divide the counters into 4 groups of 5 counters. It takes 20 counters to make the 4 piles.

Show You Know

Solve by inspection.

a) $4k = 36$ **b)** $\frac{m}{2} = 7$

Did You Know?

A cheetah can run as fast as 112 km/h for up to 400 m!

Example 2: Divide to Apply the Opposite Operation

Suppose that Donovan Bailey could run at a constant speed of 9 m/s. The distance travelled is modelled by the formula $d = 9t$, where d represents distance, in metres, and t represents time, in seconds. How long would it take him to run 900 m?

Solution

Since the distance d is 900 m, substitute 900 into the formula $d = 9t$. Then, solve the equation.

$$900 = 9t$$
$$\frac{900}{9} = \frac{9t}{9}$$
$$100 = t$$

$9t$ means $9 \times t$. The opposite operation is $\div 9$.

It would take Donovan Bailey 100 s to run 900 m.

Check:

Left Side $= 900$ Right Side $= 9t$
$= 9(100)$
$= 900$

Left Side = Right Side

The answer is correct.

Show You Know

Solve by applying the opposite operation.

a) $3t = 18$ **b)** $72 = 9t$

Example 3: Multiply to Apply the Opposite Operation

Sylvie and Murray earn money delivering groceries. Last weekend, Murray earned \$29. This was one third of the amount Sylvie earned. How much money did Sylvie earn?

Solution

Let g represent the amount of money Sylvie earned.

Murray earned one third of the amount Sylvie earned, or $\frac{g}{3}$.

$$\frac{g}{3} = 29$$
$$\frac{g}{3} \times 3 = 29 \times 3$$
$$g = 87$$

$\frac{g}{3}$ means $g \div 3$. The opposite operation is $\times 3$.

$g \div 3 \times 3 = g$

Sylvie earned \$87.

Check:

Left Side $= \frac{g}{3}$ Right Side $= 29$

$= \frac{87}{3}$

$= 29$

Left Side = Right Side

The answer is correct.

Show You Know

Solve by applying the opposite operation. Check your answer.

a) $7 = \frac{d}{5}$ **b)** $\frac{x}{3} = 8$

Key Ideas

- Equations can be solved in several ways. You can
  - solve by inspection, using mental math
  - model the equation and then balance it
  - perform the opposite operation on both sides of the equal sign
- To check your solution, substitute your answer into the equation. Then, compare the left side of the equation to the right side. If the solution is correct, both sides will have the same value.

Communicate the Ideas

1. Show the steps you would use to solve the equation $5y = 75$. Explain each step.

2. Explain why it is necessary to use the opposite operation when solving equations. With a friend, solve two equations using opposite operations. Show your friend what would happen if you did not use the opposite operation.

3. Sandra is solving the equation $\frac{w}{8} = 6$. Her solution is shown.

$$\frac{w}{8} = 6$$
$$\frac{w}{8} - 8 = 6 - 8$$
$$w = -2$$

a) Is her solution correct or incorrect? Why?

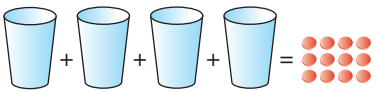
b) If you think the solution is incorrect, what would you change to solve the equation?

Practise

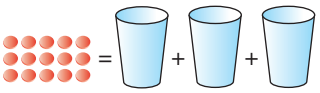
For help with #4 to #7, refer to Example 1 on page 403.

4. Solve using mental math. How many counters will be in each cup?

a)

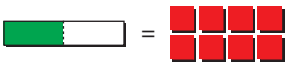


b)

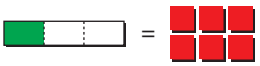


5. Use mental math to solve each equation modelled by the algebra tiles.

a)



b)



6. Solve by inspection.

a) $6r = 18$ **b)** $9g = 72$

c) $4d = 12$ **d)** $12 = 6f$

7. Solve by inspection.

a) $3 = \frac{p}{7}$ **b)** $\frac{v}{5} = 5$

c) $12 = \frac{c}{3}$ **d)** $\frac{x}{2} = 14$

For help with #8 to #10, refer to Example 2 on page 404.

8. By what number would you divide both sides of the equation to solve it?

a) $6x = 12$ **b)** $3n = 9$

c) $11t = 22$ **d)** $36 = 9k$

9. Solve each equation using the opposite operation. Check your answer.

a) $2r = 18$ **b)** $5j = 125$

c) $12g = 144$ **d)** $63 = 21t$

10. The distance a polar bear can swim is modelled by the formula $d = 6t$, where d represents distance, in kilometres, and t represents time, in hours. How long would it take a polar bear to swim 42 km? Check your answer.

For help with #11 to #13, refer to Example 3 on pages 404–405.

11. By what number would you multiply both sides of the equation to solve it?

a) $9 = \frac{m}{6}$ **b)** $\frac{h}{4} = 21$

c) $7 = \frac{q}{11}$ **d)** $\frac{x}{4} = 5$

12. Solve each equation using the opposite operation. Check your answer.

a) $\frac{u}{4} = 11$ **b)** $13 = \frac{c}{12}$

c) $\frac{w}{9} = 12$ **d)** $0 = \frac{x}{2}$

13. Paula and Kirsten work at the same restaurant, but Paula works one quarter the hours that Kirsten does. If Paula works 9 h each week, how many hours does Kirsten work? Check your answer.

Apply

14. Show whether or not $x = 3$ is the solution to each equation.

a) $8x = 24$ **b)** $10x = 30$

c) $7x = 35$ **d)** $48 = 12x$

15. Show whether or not $y = 8$ is the solution to each equation.

a) $1 = \frac{y}{8}$ **b)** $\frac{y}{4} = 16$

c) $4 = \frac{y}{2}$ **d)** $\frac{y}{2} = 16$

16. Jag rides his bike to school, which is 6 km (6000 m) from his home. Jag's speed on his bike averages 300 m/min.

a) What equation could be used to model this situation?

b) How long will it take Jag to ride his bike to school?

17. Marla's age is one half Brent's age. Marla is 21.

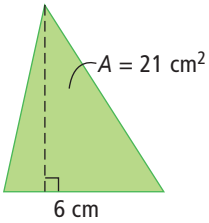
a) Write an equation to model this situation.

b) How old is Brent?

18. Derek is saving for a ski trip that costs $495. He needs to triple his savings before he has enough money for the trip. How much money has Derek saved so far?

Extend

19. The formula for the area of a triangle is $A = (b \times h) \div 2$. Find h for a triangle with base 6 cm and area 21 cm^2.



20. The perimeter of a rectangular playing field is 240 m. The length is double the width.

a) Model this situation with an equation. Use one variable only.

b) How could you simplify this equation?

c) What is the length and width of the playing field?

21. If a number of pencils are shared equally among ten girls, each will get eight. How many pencils will each girl get if the total number of pencils is the same, but there are 6 more girls?

MATH LINK

Since you were able to crack Jim's code in the previous section, he has decided to make a new one that is more complicated. Jim has written a new message to his friend using the new code:

60 24 27 57 / 9 45 12 15 / 27 57 / 63 42 6 54 15 3 33 3 6 36 15

(Spaces separate letters and the symbol "/" separates words.)

Use the following information to help you crack Jim's new code:

- Jim's new code uses an equation of the form $ax = b$ or $\frac{x}{a} = b$ to change the letters in the message to the numbers in the coded version.
- The most common letters in the English language are, in order, E, T, A, O, I, N, S, H, R, D, and L.
- Jim starts by using the following numbers to represent letters.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g	8 = h	9 = i	10 = j	11 = k	12 = l	13 = m
14 = n	15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u	22 = v	23 = w	24 = x	25 = y	26 = z

a) What equation did Jim use as his code?

b) Rewrite the code chart using Jim's equation to determine the number that represents each letter.

c) Decode the message.

11.4

Solve Two-Step Equations: $ax + b = c$

Focus on...

After this lesson, you will be able to...

- model problems with two-step equations
- solve two-step equations and record the process

When you shut down a computer, you follow a procedure. First, you should save your file, then close the program, and then shut the computer down. Doing these steps in a different order may cause a problem!

You must also follow the proper procedure when solving math equations.

Explore the Math

Materials

- cups and counters
- algebra tiles

How do you solve two-step equations of the form $ax + c = b$?

A clothing store is having a sale. Jake pays $19 for two T-shirts and a pair of sunglasses. How much does Jake pay for each T-shirt?

1. What equation can be used to represent this situation?
2. Use cups and counters, algebra tiles, or a balanced scale to model your equation.
3. Use your model to help you solve the equation.
   a) Undo the addition on the left side of the equation. What must you do to the right side of the equation to keep the equation balanced?
   b) What equation does your model represent now?
   c) What do you need to do to solve the equation now?
4. What is the cost of a single T-shirt?

Reflect on Your Findings

5. a) What two mathematical operations did you use to solve the equation?

b) Why did you use these operations when you did?

c) What is the relationship between the order of operations used to evaluate an expression and the order you used to solve the equation?

Example 1: Model Equations

Maurie saw this sign advertising T-shirts and socks. He pays \$30 for two T-shirts and four pairs of socks. What is the price of one T-shirt?

Solution

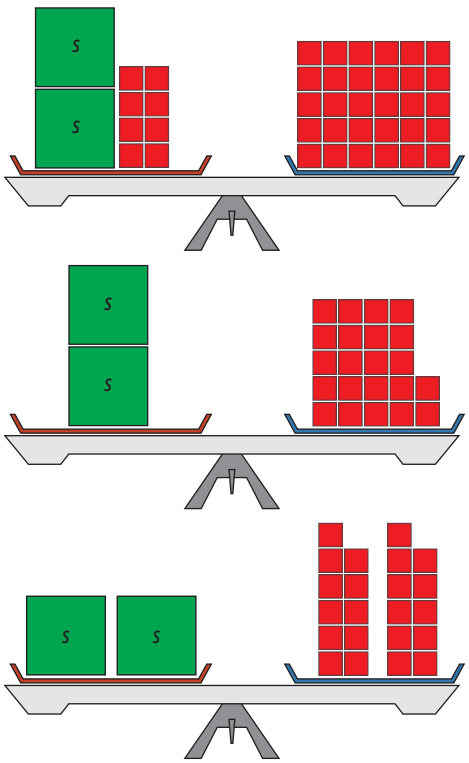
Let s represent the cost of one T-shirt.

The cost of the socks is $4 \times \$2$, or \$8.

The equation that represents this situation is $2s + 8 = 30$.

To isolate the variable, first remove the eight blocks on the left side of the scale. To keep the scale balanced you must remove the same number of blocks from the right side of the scale.

There are two s-variable blocks on the left side of the scale. There are 22 blocks on the right side of the scale. For the scale to balance, each s-variable block must have a mass of 11 blocks.



The cost of one T-shirt is \$11.

Check:

Left Side $= 2s + 8$ Right Side $= 30$

$= 2(11) + 8$

$= 22 + 8$

$= 30$

Left Side = Right Side

The answer is correct.

Strategies

Model It

Refer to page xvi.

Show You Know

Solve by modelling the equation.

a) $3w + 3 = 6$

b) $2p + 4 = 10$

Example 2: Apply the Reverse Order of Operations

The formula $R = 9T - 70$ models the chirping rate of a cricket at various temperatures. The variable R represents the number of chirps per minute, and T represents the temperature, in degrees Celsius. When the rate is 20 chirps per min, what is the approximate temperature?

Solution

Substitute 20 for R in the formula. Then, isolate the variable T, to solve the equation.

To solve, apply the reverse order of operations. Add or subtract first, then multiply or divide.

$$\begin{aligned} 20 &= 9T - 70 \\ 20 + 70 &= 9T - 70 + 70 \quad \text{Add 70 to both sides of the equation.} \\ 90 &= 9T \\ \frac{90}{9} &= \frac{9T}{9} \quad \text{Divide both sides of the equation by 9.} \\ 10 &= T \end{aligned}$$

The approximate temperature is 10°C.

Check:

$$\begin{aligned} \text{Left Side} &= 20 \\ \text{Right Side} &= 9T - 70 \\ &= 9(10) - 70 \\ &= 90 - 70 \\ &= 20 \end{aligned}$$

Left Side = Right Side

The answer is correct.

Show You Know

Solve by applying the reverse order of operations.

a) $5n + 7 = 32$

b) $53 = 4r - 11$

Key Ideas

- To solve an equation, isolate the variable on one side of the equal sign.
- When undoing the operations performed on the variable, follow the reverse order of operations:
  – subtract and/or add
  – multiply and/or divide

Communicate the Ideas

1. Show the steps you would use to solve the equation $34 = 11x + 12$. Explain each step.

2. Describe a situation that can be modelled with the equation $2c + 8 = 14$.

3. Henri and Anne are solving the equation $12r + 3 = 39$. Whose strategy is correct? Explain.

Henri:

First I divide both sides by 12.

Anne:

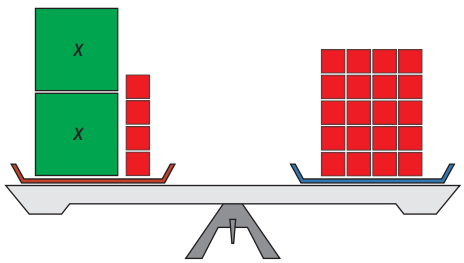
I start by subtracting 3 from both sides.

Practise

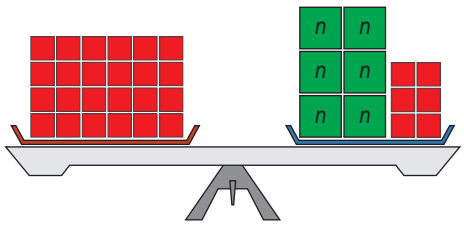
For help with #4 to #8, refer to Example 1 on page 409.

4. Solve the equation modelled by each diagram. Check your solution.

a)

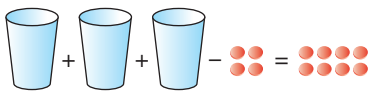


b)

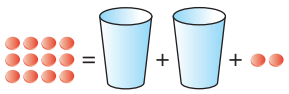


5. Solve the equation modelled by each diagram. Check your answer.

a)

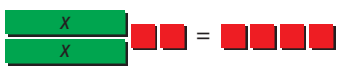


b)



6. Solve each equation modelled by the algebra tiles. Check your answer.

a)



b)

7. Model and solve each equation. Check your answer.

a) $3s + 1 = 7$ **b)** $4k - 4 = 8$

c) $2 + 5n = 12$ **d)** $15 = 2w + 7$

8. Matt pays $10 for two boxes of cereal and two 1-L cartons of milk. What is the price of one box of cereal?

For help with #9 to #13, refer to Example 2 on page 410.

9. What is the first operation you should apply to solve each equation?

a) $6t - 2 = 16$ **b)** $3 + 3n = 9$

c) $22 = 10 + 2x$ **d)** $40 = 9k - 5$

10. What is the second operation you should apply to solve the equations in #9?

11. Solve using the reverse order of operations. Check your answer.

a) $6r - 6 = 18$ **b)** $4m + 8 = 12$

c) $4 + 9g = 22$ **d)** $37 = 6f - 5$

12. Solve using the reverse order of operations. Check your answer.

a) $19 = 4k + 3$ **b)** $6x + 7 = 25$

c) $29 = 12n + 5$ **d)** $14 = 4n - 2$

13. Brian has DVDs and CDs. The number of CDs he has can be modelled with the formula $C = 2D + 11$, where C represents the number of CDs and D represents the number DVDs. If he has 41 CDs, how many DVDs does he have?

14. Show whether or not $x = 6$ is the solution to each equation.

a) $8x + 8 = 25$ **b)** $3 + 7x = 45$

c) $58 = 10x - 1$ **d)** $48 = 3x + 12$

Apply

15. Solve each equation. Check your answer.

a) $3r - 7 = 20$ **b)** $6y + 5 = 125$

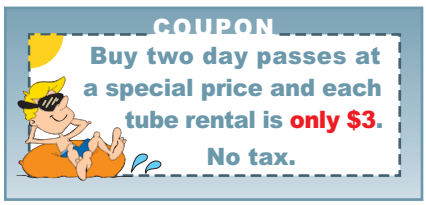
c) $12 + 9g = 93$ **d)** $130 = 25p - 20$

16. A camp charges $75 per day to use the camp plus $15 per day for food and supplies for each student. The cost for one day can be modelled using the equation $C = 15s + 75$.

a) What do the variables C and s represent?

b) A school raised $375 for a one-day trip. How many students can go?

17. Tylena has a coupon for Water World Park.



She pays $54 for two day passes and two tube rentals.

a) What equation models this situation?

b) What is the cost of each day pass?

18. Sofia has 3 more rose quartz stones than twice the number of white quartz stones in her collection. If she has 15 rose quartz stones, how many white quartz stones does she have?

Extend

19. **a)** Create an equation in the form $ax + b = c$.

b) Solve the equation using the correct order of operations.

c) Solve the equation using an incorrect order of operations.

d) Compare your answers. Are they the same? Why or why not?

e) How could you prove which answer is correct and which one is not?

20. The food energy credits required by a steeplechase racer in a video game can be modelled with the formula $E = 3520 + 25T$, where E represents the amount of food energy, in calories, and T represents the simulated air temperature, in degrees Celsius. At what temperature does the racer require 4095 Cal of food energy?

21. Lacey drops a stone off a cliff. The speed of the stone changes as it falls. The speed is modelled using the formula $v = 10t + 15$, where v represents the speed of the stone, in metres per second, and t represents the time, in seconds.

a) What is the speed of the stone one second after it is dropped?

b) At what time is the speed of the stone 45 m/s?

MATH LINK

Once again you were able to crack Jim's code. He has now decided to make a new code that is as complicated as he can make it. Jim's final message to his friend is
43 19 21 41 / 21 41 / 29 53 / 7 13 41 43 / 9 33 11 13 / 53 13 43
(Spaces separate letters and "/" separates words.)

Use the following information to help you crack Jim's new code:

- Jim's new code uses an equation of the form $ax + b = c$ to change the letters in the message to the numbers in the coded version.
- The most common letters in the English language are, in order, E, T, A, O, I, N, S, H, R, D, and L.
- Jim starts by using the following numbers to represent letters.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g	8 = h	9 = i	10 = j	11 = k	12 = l	13 = m
14 = n	15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u	22 = v	23 = w	24 = x	25 = y	26 = z

a) What equation did Jim used as his code?

b) Rewrite the code chart using Jim's equation to determine the number that represents each letter.

c) Decode the message.

11 Chapter Review

Key Words

Copy and complete each statement in #1 and #2.

1. Examples of opposite operations are
- subtract and _____
- _____ and divide

2. An _____ is made up of two expressions that are equal in value to each other.

11.1 Expressions and Equations, pages 390–394

3. Draw a diagram to represent each expression or equation.

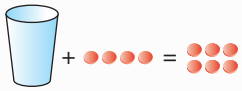
a) $x + 3$

b) $2r - 3 = 9$

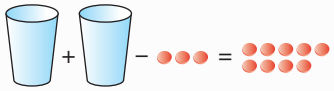
4. For each expression or equation in #3, identify any variables, numerical coefficients, and constants.

5. Write the two expressions that make up each equation. What is the equation?

a)



b)



6. Write an equation for each phrase.

a) Three times Kyra's age less one year equals twenty-two years.

b) Sean's height in centimetres divided by two equals seventy-five centimetres.

7. a) Use cups and counters to model the expression $8 + 2p$.

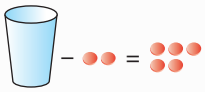
b) If each cup contains 3 counters, how many counters are represented by the expression?

11.2 Solve One-Step Equations: $x + a = b$, pages 395–401

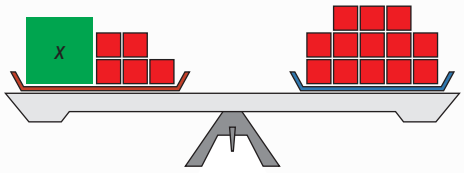
8. What is the number of counters needed in each cup to make each equation true?

a)

b)



9. What value for x will keep the scale balanced?



10. Solve by inspection.

a) $w + 12 = 14$

b) $f - 3 = 6$

c) $8 = g - 12$

d) $11 - b = 5$

11. Solve each equation. Check your answer.

a) $t + 7 = 35$

b) $y - 8 = -8$

c) $16 + x = 21$

d) $21 = 4 + p$

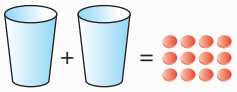
12. At the Winter Olympics in Torino, Italy, Canada won 10 more medals than Sweden. Canada won 24 medals.

a) Write an equation to represent this situation.

b) How many medals did Sweden win?

11.3 Solve One-Step Equations: $ax = b$, $\frac{x}{a} = b$, pages 402–407

13. Solve using mental math. How many counters will be in each cup?



14. Solve the equation $\frac{h}{2} = 6$ using mental math.

15. a) What equation is being modelled by the algebra tiles.

b) Solve the equation modelled by the algebra tiles. Check your answer.

16. Solve by inspection.

a) $3r = 18$ **b)** $\frac{p}{8} = 4$

c) $35 = 5w$ **d)** $11 = \frac{c}{6}$

17. By what number would you divide both sides of the equation to solve it? Solve each equation. Check your answer.

a) $3x = 12$ **b)** $4n = 16$

18. By what number would you multiply both sides of the equation to solve it? Solve each equation. Check your answer.

a) $\frac{v}{5} = 7$

b) $12 = \frac{t}{11}$

19. Sophie's age is one third Ryan's age. Sophie is 21.

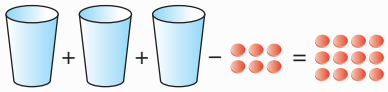
a) Write an equation to represent this situation.

b) How old is Ryan?

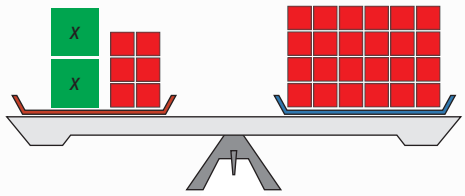
11.4 Solve Two-Step Equations: $ax + b = c$, pages 408–413

20. Solve the equation modelled in each diagram. Check your solution.

a)



b)



21. Solve each equation. Check your answer.

a) $2g + 8 = 32$

b) $3x - 5 = 85$

c) $18 + 9h = 81$

d) $34 = 6p - 8$

22. Jeremy collects hockey cards and baseball cards. The number of hockey cards he has is 21 more than twice the number of baseball cards. If he has 75 hockey cards, how many baseball cards does he have? Use an equation to help solve this problem.

11 Practice Test

For #1 to #6, choose the best answer.

1. What is the solution to $x + 4 = 8$?

A $x = -12$

B $x = -4$

C $x = 4$

D $x = 12$

2. Catarina and her brother Jaime have the same birthday. When Catarina was 8 years old, Jaime was 2. Which equation shows the relationship between Catarina's age, c, and Jaime's age, j, at all times during their lives?

A $j = 6 + c$

B $c = 6 + j$

C $c = 4j$

D $j = 4c$

3. What is the solution to $\frac{x}{3} = 12$?

A $x = 4$

B $x = 9$

C $x = 15$

D $x = 36$

4. Which of these equations has the solution $r = 8$?

A $r + 9 = 22$

B $26 - r = 20$

C $2 = r + 9$

D $5r = 40$

5. Which of the following describes the correct way to solve $12 + 3x = 18$?

A Subtract 12 from both sides, then divide both sides by 3.

B Add $3x$ to both sides, then divide both sides by 12.

C Subtract $3x$ from both sides, then multiply both sides by 12.

D Add 12 to both sides, then multiply both sides by 3.

Short Answer

6. Holly is solving an equation. She models one of her steps.

a) What operation is Holly undoing?

b) What will her next step be?

7. a) Draw a diagram that models the equation $11 + c = 19$, where c represents a number.

b) What is the missing number in this equation?

8. Solve each equation.

a) $b + 7 = 12$

b) $4 = x - 12$

c) $x - 25 = -7$

d) $25 = -13 + h$

9. Solve $\frac{x}{3} = 8$.

10. Solve $2x + 4 = 12$. Check your answer.

11. In a recent basketball game Eva scored by making foul shots and 2-point baskets. She made five foul shots, which are each worth one point. In total she scored 33 points. How many 2-point baskets did she make?

Extended Response

12. John and Cody are planning a hiking trip. Kutluk's Outfitters charges $400 a day for the equipment and a guide for a wilderness hike. There is an extra charge of $200 per day per person for meals and accommodation. The cost for one day can be modelled using the equation $C = 200p + 400$.

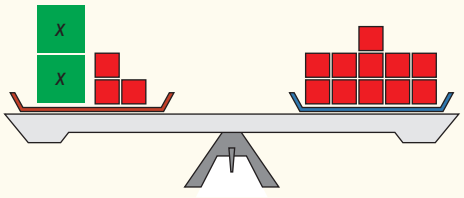
a) What do the variables C and p represent?

b) How much will it cost John and Cody for one day of hiking?

c) Write an equation that John and Cody could use to calculate the cost for more than one day of hiking.

d) John and Cody have saved $3200 for this trip. How many days can they afford?

13. a) What equation does this balance represent?



b) Solve the equation. Explain your steps.

c) Draw a diagram to represent the equation $3x + 4 = 25$.

d) Solve the equation in part c). Explain your steps.

14. Elizabeth is sewing two types of trim on a new parka. The length of leather trim is 20 cm more than 5 times the length of ribbon trim. She uses 245 cm of leather trim. How much ribbon does she use?

a) Write an equation to represent the situation.

b) Solve the equation and check your answer.

WRAP IT UP!

Create your own code.

Start with this base.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g	8 = h	9 = i	10 = j	11 = k	12 = l	13 = m
14 = n	15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u	22 = v	23 = w	24 = x	25 = y	26 = z

Work in small groups to develop your own code.

- Your code must be based on using numbers for letters.
- It must use one of the equation types you studied in this chapter to code the numbers.

a) Use your code to write a message.

b) See if another group can crack it. You may need to give them a hint about the type of equation used.

Math Games

Equation Puzzles

1. In the following message, each number stands for a letter. The "/" symbol marks a space between words. The message will tell you how the dentist described her dinner.

7 3 8 1 / 9 2 6 6 2 4 5

a) Solve the equations to find the value of each variable.

$4e = 12$ $\quad \frac{f}{3} = 3$

$g + 5 = 10$ $\quad 6i = 12$

$l - 4 = 2$ $\quad 2n - 1 = 7$

$\frac{r}{2} = 4$ $\quad v + 2 = 9$

$2y + 3 = 5$

b) Replace each number in the message by the variable with this value. How did the dentist describe her dinner?

2. a) As a class or in a group, brainstorm how you would go about writing a puzzle like the one in #1.

b) Write a puzzle of your own.
It must include
- a short message made with whole numbers
- a set of equations that can be solved to determine the letters that will replace the numbers in the message.

c) Check that your equations give your intended message.

d) Have a classmate solve your puzzle.

Challenge in Real Life

Wrapping Gifts

Your school council is having a fundraiser. To raise money, volunteers will gift-wrap artwork being sold at an arts festival in your community.

You be the fundraiser coordinator! You have developed the following table:

Number of Gifts	Number of Rolls of Paper Needed (61 cm × 244 cm rolls)	Length of Ribbon Needed (m)
4	1	8
8	2	16
12	3	24

a) About 600 pieces of artwork will be for sale. Usually about 80% of the artwork is sold. How many gifts do you expect to wrap? Show your calculations.

b) Use the numbers in the table to develop two equations showing the relationship between the number of gifts and
- how much paper you will need
- how much ribbon you will need

c) Use your equations to calculate how much paper and ribbon you will need, based on your estimate in a). Show your calculations.

d) Research the cost of paper and ribbon at local stores in your community. How much does the school council need to invest in paper and ribbon?

e) Given these costs, how much would you charge to wrap each gift? Explain.