Operations on Decimal Numbers

2

General Outcome
• Develop number sense.

Specific Outcomes

N2 Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
2.1	✓ use estimation to check if sums are reasonable
	✓ use front-end estimation to place the decimal in a sum
	✓ use estimation to check if differences are reasonable
	✓ use front-end estimation to place the decimal in a difference
	✓ solve problems using addition of decimal numbers
	✓ solve problems using subtraction of decimal numbers
2.2	✓ use estimation to place a decimal point in a product
	✓ multiply decimal numbers with a calculator
	✓ multiply decimal numbers without a calculator
	✓ solve problems using estimation and multiplication of decimal numbers
2.3	✓ use estimation to place a decimal point in a quotient
	✓ divide decimal numbers with a calculator
	✓ divide decimal numbers without a calculator
	✓ solve problems using estimation and division of decimal numbers
2.4	✓ use the order of operations with decimal numbers
	✓ solve problems using operations on decimals to the thousandth's place

Assessment *as* Learning	Supported Learning
Use the Before column of **BLM 2–1 Chapter 2 Self-Assessment** to provide students with the big picture for this chapter and to help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• As students complete each section of the chapter or complete the Chapter 2 Review, have them review the related parts of **BLM 2–1 Chapter 2 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Chapter 2 Planning Chart

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter Opener • 25–30 minutes		BLM 2–1 Chapter 2 Self-Assessment BLM 2–2 Operations on Decimal Numbers BLM 2–3 Place Value Charts	• notebook paper • scissors • ruler • atlas or Internet access (optional) • calculator
2.1 Add and Subtract Decimal Numbers • 80–100 minutes	**Essential:** 1, 2, 4, 6, 8, 11, 15, Math Link **Typical:** 1–4, 6, 8, 11–13, 15, 16, 18, 19, 23, Math Link **Extension/Enrichment:** 1–3, 12, 14, 17–19, 22–25, Math Link	Master 10 Hundred Grids BLM 2–1 Chapter 2 Self-Assessment BLM 2–3 Place Value Charts BLM 2–4 Section 2.1 Extra Practice BLM 2–5 Section 2.1 Math Link	• base 10 blocks (optional)
2.2 Multiply Decimal Numbers • 80–100 minutes	**Essential:** 1–3, 5, 7, 10, 15, Math Link **Typical:** 1–3, 5, 7, 10, 11, 13, 15, 18, 21, Math Link **Extension/Enrichment:** 1, 2, 16, 18, 20, 21, Math Link	Master 2 Two Stars and One Wish Master 10 Hundred Grids BLM 2–1 Chapter 2 Self-Assessment BLM 2–6 Section 2.2 Extra Practice BLM 2–7 Section 2.2 Math Link	• base 10 blocks • calculator • Sudoku puzzle (optional) • calendar (optional)
2.3 Divide Decimal Numbers • 80–100 minutes	**Essential:** 1–4, 6, 8, 12, 16, Math Link **Typical:** 1–4, 6, 8, 12, 14–17, 20, Math Link **Extension/Enrichment:** 1–3, 11, 16, 18–22, Math Link	Master 4 Vertical and Horizontal Number Lines Master 10 Hundred Grids BLM 2–1 Chapter 2 Self-Assessment BLM 2–8 Section 2.3 Extra Practice BLM 2–9 Section 2.3 Math Link	• base 10 blocks • calculator
2.4 Order of Operations and Decimal Numbers • 80–100 minutes	**Essential:** 1, 4, 6, 8, 9, 11, 15 **Typical:** 1–4, 6, 9, 13, 15–17, 21 **Extension/Enrichment:** 1–3, 9, 10, 14, 16, 17, 22–24	BLM 2–1 Chapter 2 Self-Assessment BLM 2–10 Section 2.4 Extra Practice	• calculator
Chapter 2 Review • 80–100 minutes	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	BLM 2–1 Chapter 2 Self-Assessment BLM 2–4 Section 2.1 Extra Practice BLM 2–6 Section 2.2 Extra Practice BLM 2–8 Section 2.3 Extra Practice BLM 2–10 Section 2.4 Extra Practice	• calculator
Chapter 2 Practice Test • 40–50 minutes	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1, 3–6, 8, 9, 13	BLM 2–1 Chapter 2 Self-Assessment BLM 2–11 Chapter 2 Test	• calculator
Chapter 2 Wrap It Up! • 60–75 minutes		Master 1 Project Rubric BLM 2–5 Section 2.1 Math Link BLM 2–7 Section 2.2 Math Link BLM 2–9 Section 2.3 Math Link BLM 2–12 Chapter 2 Wrap It Up!	• trip planning resources (maps, menus, activity guides, transportation flyers, etc) • calculator
Chapter 2 Math Games • 40–50 minutes		BLM 2–13 Decimal Delights Game Boards	• coin • coloured pencils (optional)
Chapter 2 Challenge in Real Life • 60–75 minutes		Master 1 Project Rubric BLM 2–14 Chapter 2 *MathLinks 7* Student Resource Answers BLM 2–15 Chapter 2 BLM Answers	• calculator (optional)

Chapter 2 Assessment Planner

Assessment Options	Type of Assessment	Assessment Tool
Chapter Opener	Assessment *as* Learning (TR pages i, 43)	BLM 2–1 Chapter 2 Self-Assessment Chapter 2 Foldable
2.1 Add and Subtract Decimal Numbers	Assessment *as* Learning (TR pages 46, 48, 50) Assessment *for* Learning (TR pages 46, 47, 49, 51)	Math Learning Log (TR page 50) BLM 2–1 Chapter 2 Self-Assessment
2.2 Multiply Decimal Numbers	Assessment *as* Learning (TR pages 54, 57, 59) Assessment *for* Learning (TR pages 54, 55, 58, 59)	Master 2 Two Stars and One Wish Math Learning Log (TR page 59) BLM 2–1 Chapter 2 Self-Assessment
2.3 Divide Decimal Numbers	Assessment *as* Learning (TR pages 62, 65, 67) Assessment *for* Learning (TR pages 62, 63, 66, 67)	Math Learning Log (TR page 67) BLM 2–1 Chapter 2 Self-Assessment
2.4 Order of Operations and Decimal Numbers	Assessment *as* Learning (TR pages 69, 71, 73) Assessment *for* Learning (TR page 72)	Math Learning Log (TR page 73) BLM 2–1 Chapter 2 Self-Assessment
Chapter 2 Review	Assessment *for* Learning (TR page 74) Assessment *as* Learning (TR page 75)	Math Learning Log (TR page 75) BLM 2–1 Chapter 2 Self-Assessment
Chapter 2 Practice Test	Assessment *as* Learning (TR page 76) Assessment *of* Learning (TR page 77)	BLM 2–1 Chapter 2 Self-Assessment BLM 2–11 Chapter 2 Test
Chapter 2 Wrap It Up!	Assessment *of* Learning (TR page 76a)	Master 1 Project Rubric
Chapter 2 Math Games	Assessment *for* Learning (TR page 78)	
Chapter 2 Challenge in Real Life	Assessment *for* Learning (TR page 78a) Assessment *of* Learning (TR page 78a)	Master 1 Project Rubric

You may wish to use one or more of the following materials to help you assess student readiness for Chapter 2.

Assessment *for* Learning	Supported Learning
Method 1: Have students develop a journal to explain what they personally know about estimating decimal number operations and how they use estimation and decimal numbers in their lives. **Method 2:** Have students complete **BLM 2–2 Operations on Decimal Numbers** to check their conceptual understanding. Remind students that you are looking for the scope of their knowledge.	• Students who require reinforcement of prerequisite skills may wish to complete the Get Ready materials available in the *MathLinks 7 Workbook* and at the **www.mathlinks7.ca** book site.

Chapter Opener

What's the Math?

In this chapter, students estimate and calculate decimal number operations. They use front-end estimation and relative size to add and subtract decimal numbers. Then, students estimate, and multiply or divide decimal numbers. Students use a calculator for operations with more than two-digit multipliers or one-digit divisors. Students follow the order of operations and apply their knowledge of decimal number operations to solve problems.

Activity Planning Notes

Before starting Chapter 2, explain that the chapter is about performing operations on decimal numbers and the strategies involved in estimating and calculating operations on decimal numbers. Ask students what places use decimal numbers. Discuss situations where students might need to add, subtract, multiply, or divide decimal numbers. Ask students what strategies they may have used when performing decimal number operations. You may wish to brainstorm and develop a mind map for the topic, including how students represent decimal numbers, the operations that they can perform with decimal numbers, and the strategies that they have used.

The visual in the opener shows Cavendish beach in Prince Edward Island. Ask students whether they have travelled to other provinces or territories, or to the United States. Have students estimate the distance to some places that they have been. Discuss how to keep track of money or convert to another currency while travelling. Encourage students to talk about what they know and to give real-world examples. Try to elicit ideas from all class members.

Math Link

Move from a discussion of travelling in Canada to a discussion about a dream vacation. Ask students where they dream of going. In the chapter problem wrap-up at the end of this chapter, students will plan a week-long trip. Ask students what aspects of their trip they need to plan. Emphasize that their trip is for only one week. Discuss how this time constraint might affect their trip.

You may wish to read the Wrap It Up! on page 77. Students should begin to gather information now while they work on the rest of the chapter.

This chapter emphasizes estimation and problem solving. Discuss with students the importance of using mental math to approximate answers. Ask them when they have estimated decimal number operations in their own lives. You may wish to discuss examples of how poor estimating skills can impact peoples' wallets (e.g., being overcharged for an item, not setting enough money aside for transportation, charging a friend more than the fair share of costs).
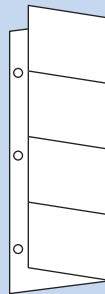
FOLDABLES™
Study Tool

Have students make the Foldable in the student resource to keep track of the information in the chapter.

You may prefer to have students keep track of Key Words using a design specifically for that purpose. Students can make the following Foldable and write vocabulary terms on the front of each tab. Have them use the space beneath the tab to write definitions and provide examples.

Step 1 Fold a sheet of notebook paper in half along the long axis with the crease to the right.

Step 2 On one side, cut every eighth line as far as the fold, creating tabs as you go. This will create 4 tabs. The top one will be larger than the rest.

Step 3 Label each tab with a math term. Write definitions and give examples underneath the tabs.

Assessment *as* Learning	Supported Learning
Chapter 2 Foldable As students work on each section in Chapter 2, have them keep track of any problems they are having under the What I Need to Work On tab in their chapter Foldable.	• As students complete each section, have them review the list of items they need to work on and then have them check off any that have been handled.

2.1 Add and Subtract Decimal Numbers

Specific Outcomes

N2 Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.

Warm-Up

1. Sketch a coordinate grid.
2. On the coordinate grid, identify the origin.
3. On the coordinate grid, identify the *x*-axis and the *y*-axis.
4. On the coordinate grid, label each quadrant.
5. Plot one point on the coordinate grid. Identify the coordinates of that point.

Mental Math

Identify the place value of the circled parts of each of the following numbers.

6. 12⑦8.21
7. 39.⓪6
8. 501.6④
9. 121.⑦2
10. 300.0⑥

Activity Planning Notes

Have students examine the map of the Yellowhead Highway on page 44. Discuss places along the highway that class members have travelled through and interesting sights along the way. Have students estimate how long it would take to drive from Winnipeg to Jasper.

Discuss the Math

Students could work individually, in pairs, or as a class on this investigation. It is important that students understand and are able to use more than one estimation strategy. The chapter emphasizes front-end estimation and relative size as two estimation strategies. Students should also be able to determine whether an estimate is larger or smaller than a calculated answer. As a class, discuss situations when it is beneficial to make either an overestimate or an underestimate.

Warm-Up

1.–4.

Coordinate Grid
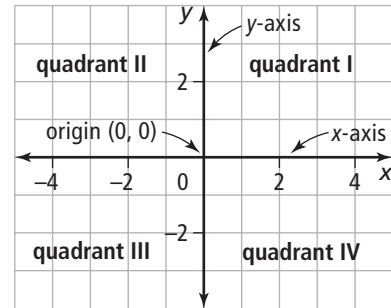
5. Answers will vary depending on the point students plot. Make sure that they list the coordinate on the *x*-axis followed by the coordinate on the *y*-axis, in round brackets.

6. tens   **7.** tenths   **8.** hundredths   **9.** tenths   **10.** hundredths

Discuss the Math

1. Marshall likely used front-end estimation. He noticed that all the place values of the leading digits represented hundreds. So, he added all the leading digits $(3 + 2 + 2 + 3 + 2 + 2)$ to get 14 hundreds or 1400.

2. Marshall's estimate is less than the actual distance (i.e., underestimate). Answers should indicate that he used front-end estimation.

3. Ashley used relative size.

4. a), b) Answers will vary.
   c) overestimate; make sure that there is enough money to cover expenses

Supported Learning

Learning Style
• Encourage students with a strong preference for oral communication to explain their answers clearly and concisely to a partner or group member.

ESL and Language
• Have students work in groups with other students who have a good understanding of the terminology.

Meeting the Needs of All Learners
• Many students travel to other communities to visit friends or family members, or celebrate cultural activities. Discuss with your class what types of family and cultural celebrations they have and where they go for them.

Show You Know: Example 1

a) 316.40

b) 14.033

Learning Style, Memory, ESL, and Language

• Make sure that students understand that adding zeroes as placeholders after a decimal point does not change the value. Point out the Literacy Link on page 46 and have them add an illustrated example to their chapter Foldable.

• Some students do not line up the decimal points when adding or subtracting decimal numbers.

R$_x$ Review place value with students. Emphasize that adding or subtracting the same place value gives the correct answer and make sure that students understand why adding or subtracting different place values is not correct. Show students examples of how (and how not) to perform these operations so that students realize the importance of lining up the decimal points.

Example 1: Use Estimation to Place the Decimal Point

Place the decimal point in the correct position in the answer to make a true statement.

a) 87.85 + 14.60 + 73.52 = 175970
b) $485.20 + $38.73 + $20 + $785.10 = $132903
c) 4189.675 − 1501.941 = 2687734

Solution

a) *Method 1: Use Front-End Estimation*
The leading digits 8, 1, and 7 all represent tens.
Think: 80 + 10 + 70 = 160
The answer closest to 160 is 175.970, or 175.97.

Method 2: Use Relative Size
The leading digits are all in the tens position, so estimate each number to the nearest ten.
87.85 is between 80 and 90, and closer to 90.
14.60 is between 10 and 20, and closer to 10.
73.52 is between 70 and 80, and closer to 70.
Think: 90 + 10 + 70 = 170
The answer closest to 170 is 175.970, or 175.97.

> **Literacy Link**
> **Adding zeros**
> • Adding zeros after the decimal point does not change the value.
> 27.83 = 27.830
> • When there are no digits for place values before a number or after a decimal, you can add a zero as a placeholder.
> 38.73 → 038.73
> This shows there are 0 hundreds in 38.73.

b) The leading digits do not all have the same place values. Arrange the numbers vertically and align the decimal points.
In this case, the 4 and 7 represent hundreds.
Think: 400 + 700 = 1100
The answer closest to 1100 is 1329.03, or $1329.03.

$485.20
$38.73 ($20 = $20.00)
$20.00
+ $785.10

c) *Method 1: Use Front-End Estimation*
The leading digits 4 and 1 represent thousands.
Think: 4000 − 1000 = 3000
The answer closest to 3000 is 2687.734.

Method 2: Use Relative Size
4189.675 is between 4000 and 5000, and closer to 4000.
1501.941 is between 1000 and 2000, and closer to 2000.
Think: 4000 − 2000 = 2000
The answer closest to 2000 is 2687.734.

Show You Know

Without calculating the answer, place the decimal point in the correct position. Show your thinking.
a) 423.6 − 107.2 = 31640   **b)** 7.85 + 2.06 + 4.123 = 14033

46   MHR • Chapter 2

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss what they discovered during the Discuss the Math activity, or read student responses to #4 on page 45. Attempt to have students generalize the conclusions about their findings.	• Reinforce that students can use any method of estimation that provides a reasonable answer. • Encourage students to consider times in their own lives when they use, or have used, estimation. • Ask students who are having difficulty with this question to use the class responses as springboards to similar ones of their own. • Encourage students to realize that sometimes it is better to overestimate cost. This concept will come up several times in Chapter 2, including during the Wrap It Up!

Example 1 provides two methods for estimating. Method 1 will almost always provide an underestimate; Method 2 will usually provide an overestimate. Get students thinking about the fact that the calculated answer is between these two estimates.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• Have students talk through their thinking in a group. • You may wish to provide additional questions very close to those in the Show You Know for students who would benefit from them: **a)** 579.6 − 288.5 = 29110 (This answer has an unneeded 0 in the hundredths place. Students should estimate that the answer is more than 200 and place the decimal between the two 1s: 291.10.) **b)** 6.791 + 3.45 + 5.126 = 15367 (The answer should be around 14. Students should place the decimal between the 5 and the 3: 15.367.) Coach students through a) and then have them try b) on their own.

Example 2: Add and Subtract Decimal Numbers
a) Add 1.23 and 1.7.
b) Subtract 0.23 from 0.7.

Solution
a) *Method 1: Use Hundreds Grids*

1.23 means 1 and $\frac{23}{100}$.

1.7 means 1 and $\frac{7}{10}$
or 1 and $\frac{70}{100}$.

Add the grids.
$1 + 1 + 0.7 + 0.23$
$= 2 + 0.93$
$= 2.93$
So, $1.23 + 1.7 = 2.93$.

Method 2: Use Paper and Pencil
When you add decimal numbers, align the decimal points so that digits with the same place value line up.

$$\begin{array}{r} 1.23 \\ + 1.70 \\ \hline 2.93 \end{array}$$

1.7 is the same as 1.70.

b) $0.7 - 0.23 = 0.70 - 0.23$
To subtract 3 from 0, you need to regroup or change 1 tenth into 10 hundredths.

Method 1: Use a Place Value Chart

Ones		Tenths	Hundredths
0	•	6	10
0	•	2	3
0	•	4	7

$0.7 - 0.23 = 0.47$

1 tenth (0.1) has the same value as 10 hundredths (0.10)

Method 2: Use Paper and Pencil

$$\begin{array}{r} 0.\overset{6}{\cancel{7}}0 \\ - 0.23 \\ \hline 0.47 \end{array}$$

$0.7 - 0.23 = 0.47$

Show You Know
Calculate each answer.
a) $8.04 + 1.839$ **b)** $1.65 - 1.37$

Show You Know: Example 2
a) 9.879
b) 0.28

Common Errors

• Some students may have difficulty with subtractions that require regrouping.

R$_x$ Review regrouping with students using base 10 blocks and **BLM 2–3 Place Value Charts**.

$123 = 1$ hundred $+ 2$ tens $+ 3$ ones
$123 = 1$ hundred $+ 1$ ten $+ 13$ ones
$123 = 0$ hundreds $+ 12$ tens $+ 3$ ones
$123 = 1$ hundred $+ 2$ tens $+ 2$ ones $+ 10$ tenths

Example 2, part b) and Show You Know, part b) involve regrouping in order to subtract. If students are having difficulty with these questions, provide a subtraction question that does not require regrouping in order to determine whether students are having difficulty with subtracting or with regrouping.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• Students who are still at the concrete and semi-concrete stage may continue to use base 10 blocks or **Master 10 Hundred Grids** to answer these questions. Allow students to use manipulatives or representations until they feel comfortable using only numerals. • Some students will find it easier to use the place value charts on **BLM 2–3 Place Value Charts**. • You may wish to provide additional questions very close to those in the Show You Know for students who would benefit from them: **a)** $9.72 + 4.563$ (It is important for students to notice that they are dealing with hundredths and thousandths and to line up the decimal points directly underneath each other. The answer is 14.283.) **b)** $2.94 - 1.97$ (Students need to regroup in order to subtract. The answer is 0.97.) Coach students through a) and then have them try b) on their own.

Communicate the Ideas

1. Answers may vary. For example: overestimate. Taxes may need to be paid on some or all of the items.

2. No. Look for a hundreds grid or a place value chart to help explain the reasoning.

ones	.	tenths	hundredths
1	•	5	10
0	•	4	6
1	•	1	4

3. Similarities: The numbers are changed to numbers that are easier to work with.
Differences: The numbers are always truncated or rounded down.
Relative size estimation gives a more balanced result. Situations where one method might be better than the other: Front-end estimation—estimating daily distances travelled on a trip to make sure that enough time is allowed for entire trip; relative size estimation—grocery shopping (a person would want the higher estimate to be sure the groceries are affordable).

Key Ideas

- There are different ways to estimate the answer to any addition or subtraction question, including front-end estimation and relative size.

Estimate $125 + 476$.

Front-End Estimation:
$100 + 400 = 500$

Relative Size Estimation:
125 is between 100 and 200 but closer to 100.
476 is between 400 and 500 but closer to 500.
$100 + 500 = 600$

- When you add or subtract decimal numbers, align the decimal points, then add or subtract digits with the same place value.

$$41.65$$
$$9.4$$
$$+ 0.365$$
$$\overline{51.415}$$

$$24.869$$
$$- 9.570$$
$$\overline{15.299}$$

Communicate the Ideas

1. You have $50 to spend on a class party. As you place items in the grocery cart, should you overestimate or underestimate the cost of each item? Explain your thinking.

2. Is the answer to the following subtraction correct? Use hundreds grids or a place value chart to help explain your reasoning.
$1.6 - 0.46 = 1.26$

3. How is relative size estimation similar to methods you have learned in the past for rounding numbers when estimating? How is it different? Discuss which method you prefer and situations where one method or the other might work better.

Practise

For help with #4 to #7, refer to Example 1 on page 46.

4. Place a decimal point in each sum without calculating. Show your thinking.
a) $62.57 + 28.41 = 9098$
b) $75.83 + 37.9 + 28 = 14173$
c) $631.5 + 902.4 + 217.83 = 175173$

5. Show where the decimal point belongs in each answer without calculating. Explain your thinking.
a) $0.458 + 0.319 + 0.2 = 9770$
b) $\$9.14 + \$6.99 + \$0.49 = \$1662$
c) $296\ cm + 38.7\ cm + 429\ cm = 76370\ cm$
d) $324.4 + 57.5 + 126.8 = 5097$

Key Ideas

This section summarizes using front-end estimation and relative size to estimate answers to addition and subtraction questions and lining up the decimal points in a vertical format before adding or subtracting. Have students prepare their own summary of the key ideas in Section 2.1.

Communicate the Ideas

In #1, students apply their understanding of overestimating and underestimating. In #2, students review a given solution to a problem involving decimals. In #3, students compare different methods of estimation.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students complete #1 and #2. Use their responses to assess students' understanding of estimating and of subtracting decimal numbers.	• For students at the concrete stage, you may wish to provide some packages of party favours with price tags. Have students estimate the cost of each item as they place it in a basket, and then explain whether the estimation is high or low—and how they know. • Students may need to use **Master 10 Hundred Grids** or **BLM 2–3 Place Value Charts** to answer #2. • Encourage students to explain the difference between the relative size estimation method and any estimation methods they have learned in previous years. These include front-end rounding (estimate $15 + 43$ as $10 + 40$), compensation (show $97 + 18$ as less than $100 + 18$), and compatible numbers (show $75 + 33 + 25$ as $75 + 25 + 33 = 100 + 33$).

6. Place a decimal point in each answer without calculating. Show your thinking. (M E)
a) 68.4 + 26.8 = 952
b) $335.61 − $240 = $9561
c) 4.831 + 2.765 = 7596

7. Position the decimal point in the answer without calculating. Show your thinking. (M E)
a) 28.3 − 5.19 = 2311
b) $3402.50 + $4102.05 = $750455
c) 627 m − 580.9 m = 461 m

For help with #8 and #9, refer to Example 2 on page 47.

8. Calculate.
a) 46.1 + 13.27
b) 105.86 + 47.3 + 10.5
c) 87.49 − 5.13
d) 7.8 − 0.64

9. Calculate.
a) 27.689 − 15.471
b) 0.317 + 1.4 + 0.38
c) $113.99 + $25.80 + $100 + $23

10. Replace each ▉ with a number to make each of the following statements true.

a)
```
   12.03
+  ▉
-------
   15.13
```

b)
```
  $117.68
+  ▉
--------
  $120.70
```

c)
```
   1.619
−  ▉
-------
   1.407
```

d)
```
  $870.49
−  ▉
--------
  $630.20
```

Apply

11. Twila is looking at two bicycles.

$248.90

$399.99

a) Estimate how much more the blue mountain bike costs before tax.
b) Is your estimate higher or lower than the actual difference in price? How do you know?
c) How much more does the blue mountain bike cost before tax?

12. At a winter camp, Mary melted three pieces of lake ice for water. The pieces had masses of 5.76 kg, 4.86 kg, and 9.7 kg. How much ice did she melt?

13. A steel bar is cut into five pieces with lengths 37.62 cm, 49.23 cm, 21.5 cm, 76.43 cm, and 45.1 cm. If you ignore the small amount of material that is lost in cutting, how long was the bar?

14. Mechanical pencil leads have widths of 0.3 mm, 0.5 mm, 0.7 mm, or 0.9 mm. How could you lay four leads side by side in order to make a total of exactly 2.0 mm? Give at least three possible answers.

2.1 Add and Subtract Decimal Numbers • MHR   **49**

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 2, 4, 6, 8, 11, 15, Math Link
Typical	1–4, 6, 8, 11–13, 15, 16, 18, 19, 23, Math Link
Extension/Enrichment	1–3, 12, 14, 17–19, 22–25, Math Link

Practise

The following pairs of similar questions provide additional practice in positioning the decimal point with and without calculating: #4 and #5, #6 and #7, and #8 and #9. In #10, the subtraction questions are presented in a different form. Each statement requires some thought before students do the subtraction.

Assessment *for* Learning	Supported Learning
Practise Have students do #4, #6, and #8. Students who have no problems with these questions can do #10 and then go on to the Apply questions.	• Students who have difficulty with #4, #6, and #8 will need additional coaching. Have students explain their thinking on these questions. Clarify any misunderstandings. Coach students through #5a), #7a), and #9a), and then have them complete the remaining parts of each question on their own. • Have students refer back to the examples in the student resource. Check several times to make sure that students understand the concepts.

ESL

- Explain terms that some English language learners might have difficulty with such as *sum*, *difference*, *heavier*, *lighter*, and *drainage basin*. Use a river that is familiar to students to describe a drainage basin.

- Many of the Apply questions refer to sports that may be unfamiliar. For example, for #15, you might describe speed skating and the scoring system. For #16, describe each sport (i.e., high jump, 100 m sprint, downhill skiing, decathlon) and its corresponding scoring system. Ask volunteers to share what they know about each sport to help you. For example, in high jump, the highest number wins; in downhill skiing, the lowest number wins.

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15. Canadian Cindy Klassen won the 1500 m speed skating gold medal with a time of 1 min, 55.27 s. The year before, she set a world record of 1 min, 51.79 s in the World Cup competition. Which time was faster and by how much?

16. In some sports, the lowest score wins. In others, the highest score wins. For each of the following events, which person would win, and by how much? Give a reason for each answer.

Event	Person A	Person B
a) High Jump	1.92 m	1.93 m
b) 100 m Sprint	11.07 s	11.3 s
c) Downhill Skiing	1 min, 46.8 s	1 min, 46 s
d) Decathlon	8454 points	8618 points

17. What number can be added to 23.4 so that the sum is 5.67 less than 51.23?

18. Jason says that $0.75 - 0.5 - 0.25 = 0$. Do you agree? Explain.

19. All of Canada's land area drains into one of four drainage basins. The table shows the approximate sizes of the ocean drainage basins of Canada.

Ocean Drainage Basins in Canada	
Drainage Basin	**Approximate Land Area (in millions of square kilometres)**
Atlantic Ocean	1.52
Hudson Bay, James Bay, Ungava Bay	3.86
Arctic Ocean	3.58
Pacific Ocean	1.03

a) List the drainage basins in order from largest to smallest.

b) Estimate the difference in size between the largest and smallest drainage basin.

c) Estimate the total approximate land area of Canada.

d) Calculate Canada's total land area.

e) Is your estimate larger or smaller than the total? Explain why.

20. Markus purchased three balls for $4.45, $5.99, and $9.60, including tax. He has $20 to pay the bill. Without finding the total, decide whether or not Markus has enough money. Show how you know.

21. Parcel A is heavier than parcel B by 1.5 kg. Parcel C is lighter than parcel B by 2.65 kg. How heavy is parcel A if parcel C is 3.75 kg?

Extend

22. George and Tina go shopping. George has $89.25 and Tina has $96.32. Tina buys a sweater and has $48.17 left. George buys pants and has $33.02 left. Which item was more expensive, the sweater or the pants? How do you know?

50   MHR • Chapter 2

- In #18, some students may not agree with Jason and may calculate $0.5 - 0.25$ first because they recognize the answer as 0.25.

R$_x$ Remind students that they need to follow the order of operations when performing multiple operations. Make sure that students check their work carefully after performing each operation.

- In #20, some students may use front-end estimation to calculate the bill.

R$_x$ Remind students that front-end estimation gives an underestimate, which is not the most helpful estimate when determining whether there is enough money to buy something. Encourage students to determine which method of estimation would give them the most appropriate answer based on the problem.

Apply and Extend

For #14, students look for multiple solutions to a problem. Discuss #16 as a class. Since some students may not be familiar with the sporting events listed, discuss whether a low score or a high score wins for each event. For #21, students are required to make and justify a number of purchases not exceeding $25. For #25, consider having pairs of students with similar math ability play a game to see who finishes first. Alternatively, since the pyramid on top requires only addition, it could be assigned to a weaker student in order to increase his or her chance of winning. Students could also create their own pyramids to use with a partner.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • How is regrouping the same when adding and subtracting decimal numbers? • How is it different?	• Have students explain the process of regrouping without tying it to either addition or subtraction. • Encourage concrete and kinesthetic learners to provide examples that illustrate how to regroup. • Depending on students' learning style, have them provide oral or written answers. • You may wish to have students review the part related to Section 2.1 in **BLM 2–1 Chapter 2 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

23. a) Round the difference between 9.83 and 4.18 to the nearest whole number.

b) If you were to use front-end estimation, what would be the new answer?

c) Which answer is more accurate? Explain.

24. You have $25 to spend on school supplies. A flyer shows the following prices.

No tax sale!

Glue stick	$1.94
Coloured pencils	$7.49
Calculator	$5.77
Pencils	$0.99 per package
Art eraser	$1.87
Ruler	$0.49
3-ring notebook	$1.97
Pencil case	$3.96

a) Show two ways to spend your $25.

b) What is the difference in price between your two plans?

c) Which plan do you prefer? Why?

25. Each square block in the pyramids shown contains the sum of the two square blocks below it.

```
        2.1
   0.6  1.5  1.4  0.08
```

```
     1.7      2.4
       1.45  1.03
```

a) Copy and complete the two pyramids in your notebook.

b) Describe how you filled in the empty blocks in each pyramid. Which blocks did you do first? Which did you do last?

MATH LINK

Your grandfather has offered to take you and two of your cousins to the Québec Winter Carnival. You will leave from his home in Brandon, Manitoba.

a) What is the least expensive way to travel to Québec City?

b) What is the difference in cost between that and the most expensive way to travel?

c) What other factors would you think about when you decide how you should travel?

d) What method of transportation do you recommend? How much will it cost for four people to travel this way?

$595 return

$336.40 return

$460.90 return

2.1 Add and Subtract Decimal Numbers • MHR   51

Supported Learning

Learning Style and Memory

• **BLM 2–4 Section 2.1 Extra Practice** provides additional reinforcement for students who need it.

Motor

• If students find it difficult to draw the pyramids in #25, consider allowing them to use virtual manipulatives. Alternatively, you may wish to discuss this question as a class, draw the pyramids on the board, and have students write their answers in the appropriate box on the board.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 51 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 77.	• You may wish to have students do this Math Link to provide them with an opportunity to explain their understanding and rationale for choosing from different modes of transportation to a specific destination. It will allow them to assess their understanding in calculating and comparing costs. • Students who are having difficulty getting started could use **BLM 2–5 Section 2.1 Math Link**, which provides scaffolding for this activity. • Observe students as they work on the Math Link and have them clarify their recommendations. Encourage students to mathematically justify their recommendations and to assess whether their justification supports their recommendation.

MATH LINK

This Math Link provides students with an opportunity to calculate and compare costs of different methods of transportation.

Emphasize to students that transportation is a key aspect when planning a trip. Have students look at the three methods of transportation shown. Encourage students to think of other ways to travel and the factors involved. You may wish to have students bring in travel brochures to explore sale prices of different methods of transportation.

Suggested Timing
80–100 minutes

Materials
- base 10 blocks
- calculator
- Sudoku puzzle (optional)
- calendar (optional)

Blackline Masters
Master 2 Two Stars and One Wish

Master 10 Hundred Grids

BLM 2–1 Chapter 2 Self-Assessment

BLM 2–6 Section 2.2 Extra Practice

BLM 2–7 Section 2.2 Math Link

Mathematical Processes	
✓	Communication
✓	Connections
✓	Mental Mathematics and Estimation
✓	Problem Solving
✓	Reasoning
	Technology
✓	Visualization

2.2 Multiply Decimal Numbers

Focus on…
After this lesson, you will be able to…
- ❏ use estimation to place a decimal point in a product
- ❏ multiply decimal numbers with and without a calculator
- ❏ solve problems using estimation and multiplication of decimal numbers

Discuss the Math

How can you estimate and calculate products of decimal numbers?

As Ashley and Marshall's family travels along the Yellowhead Highway, they keep busy by solving Sudoku puzzles. During a stop, they look in a convenience store for more puzzles.

Marshall finds Sudoku puzzle books on sale for $1.69 including tax. He wants to buy five books and has $9.00. He asks Ashley to help estimate the total cost of the five puzzle books.

$1.69 \times 5 = $ ■

1. Marshall estimates the total bill as $5.00.
   a) How do you think Marshall got his estimate?
   b) Is Marshall's estimate over or under the total? How do you know?

52   MHR • Chapter 2

Specific Outcomes

N2 Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.

Warm-Up

1. Draw a coordinate grid with axes labelled from −5 to +5.

2. Plot each ordered pair on the coordinate grid from #1.
   A (4, −3)   **B** (−5, 0)   **C** (−3, −3)
   D (0, 2)   **E** (1, 5)

3.   49.7
   + 53.2

4. 589.64
   − 53.3

5. Fill in the number to make each statement true.
   a) 23.59
       + ■
        34.14

   b) $47.89
       − ■
        $12.50

Mental Math

6. Order the numbers 1.8, 2.2, and 1.9 from greatest to least.

7. **a)** Estimate 329 ÷ 6.
   b) Is your estimate high or low? Explain how you know.

8. **a)** Estimate 6399 − 5738.
   b) Is your estimate high or low? Explain how you know.

9. Calculate each answer mentally. Show your thinking.
   a) 198 + 63
   b) 198 × 6
   c) 49 + 97 + 55

2. Ashley estimates the total bill as $10.00.
   a) How do you think Ashley got her estimate?
   b) Is Ashley's estimate over or under the total? How do you know?

Reflect on Your Findings

3. Neither estimate tells for sure whether $9.00 is enough money to buy the five puzzle books.
   a) What is another way to estimate the total bill?
   b) Is $9.00 enough money?
   c) Calculate $1.69 × 5. How close was your estimate?

WWW Web Link
To learn more about Sudoku puzzles, and to generate some new puzzles to play, go to www.mathlinks7.ca and follow the links.

Did You Know?
Sudoku was invented many hundreds of years ago, and traded around the world by ancient mathematicians!

Each digit from 1 to 9 must occur in
• each row
• each column
• each 3 × 3 square.

Example 1: Use Estimation to Place the Decimal Point
Without calculating the answer, place the decimal point in the correct position.

2.2 × 1.8 = 3960

Solution
Use front-end estimation and multiplication.
Think: 2 × 1 = 2

The answer closest to 2 is 3.960.

2.2 × 1.8 = 3.960 or 3.96.

Show You Know
Without calculating the answer, place the decimal point in the correct position. Show your thinking.
a) 3.9 × 5.8 = 22620
b) 2.57 × 0.46 = 118220

2.2 Multiply Decimal Numbers • MHR   53

Activity Planning Notes

A Sudoku puzzle can be used to emphasize the mathematical processes of communication, reasoning, and visualization. As a class, read about Sudoku in the Did You Know? box. Work through an easy Sudoku, before having students try some on their own or with a partner.
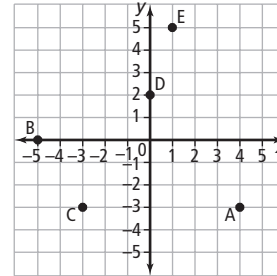
Discuss the Math

In this investigation, students estimate and then calculate products. Students could work individually, in pairs, or as a class to complete the activity, and then discuss their findings.

Show You Know: Example 1

a) 22.620

b) 1.18220

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss what they discovered during the Discuss the Math activity or read student responses to #3. Attempt to have students generalize conclusions about their estimating.	• Reinforce that students can use any method of estimation that provides a reasonable answer. • Encourage students to consider times in their own lives when they use, or have used, estimation to determine how much something costs. • Ask students who are having difficulty with this question to use the class responses as springboards to responses of their own.

Example 2: Multiply Decimals
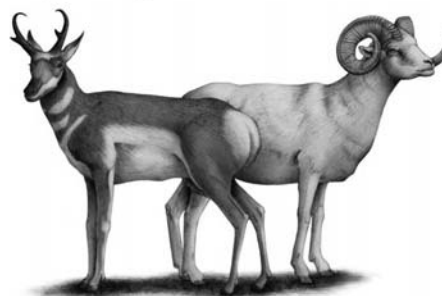
A pronghorn antelope has a mass of 58 kg. A Dall's sheep has a mass 1.5 times as great as a pronghorn.
a) Estimate the mass of the Dall's sheep.
b) Calculate the mass of the Dall's sheep.

Solution

a) *Method 1: Use Front-End Estimation*
$1 \times 50 = 50$ underestimate

Method 2: Use Relative Size
1.5 is close to 2.
58 is close to 60.

$2 \times 60 = 120$ overestimate

The mass of the Dall's sheep is between 50 kg and 120 kg.

Strategies
Model It
Refer to page xvi.

b) *Method 1: Use Hundreds Grids*
Model the mass of 1 pronghorn.

58

54   MHR • Chapter 2

Supported Learning

Learning Style

• Use base 10 blocks to help students understand why numbers get smaller when multiplied by decimals less than 1.

ESL and Language

• Encourage students who have strong oral skills as well as English language learners to share their ideas during group discussion.

• Explain that *1.5 times as great* means to *multiply*. Have students add this term to their dictionary.

In Example 1, make sure that students understand that front-end estimation gives an estimate that is smaller than the calculated answer because the numbers being multiplied are always rounded down first. You may wish to have students use base 10 blocks, along with hundreds charts to show the connections among concrete, pictorial, and symbolic representations.

| Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1 on page 53.	• Have students talk through their thinking in a group. • Encourage students to use base 10 blocks and hundreds grids from **Master 10 Hundreds Grids** to work through the example. • You may wish to provide additional questions very close to those in the Show You Know for students who would benefit from them: **a)** $4.9 \times 5.9 = 28910$ (This answer has an unneeded 0 in the thousandths place. Students should estimate that the answer is greater than 20 and less than 30, and place the decimal point between the 8 and the 9: 28.910.) **b)** $2.58 \times 0.47 = 12126$ (The answer should be around 1. Students should place the decimal point to give 1.2126.) Coach students through a), and then have them try b) on their own.

Model the mass of 0.5 pronghorns.

0.5 is the same as $\frac{1}{2}$ or dividing by 2.
50 squares divided by 2 is 25 squares.
8 squares divided by 2 is 4 squares.
Shade 25 + 4 = 29 squares.

Combine the masses of 1 pronghorn and 0.5 pronghorns.

Combine the partially shaded charts.

The mass of the Dall's sheep is 87 kg.

Method 2: Use Paper and Pencil
Multiplying decimal numbers is like multiplying whole numbers and then placing the decimal point using estimation. To multiply 58×1.5, first multiply 58×15.

$$\begin{array}{r} 58 \\ \times 15 \\ \hline 290 \\ 580 \\ \hline 870 \end{array}$$
← 58 × 5
← 58 × 10

In part a) the estimate was between 50 kg and 120 kg, so the decimal point must go between the 0 and the 7.

The mass of the Dall's sheep is 87 kg.

Show You Know
Estimate, then calculate.
a) 46×2.5
b) 64×4.5

Supported Learning

ESL
• Explain that *combine the masses* is another way to say *add*.

Common Errors

• Some students may forget to place the decimal point after calculating a product using paper and pencil.

R_x Have students estimate an answer before performing any calculations. Encourage them to compare their answer to their estimate before going ahead. Have students write a statement including units once they have solved a problem. This will help remind them of the context of the problem and they may be more likely to identify whether their answer is reasonable.

Example 2 provides two methods for estimating a mass and two methods for calculating a mass. In Method 2 part a), mention that 1.5 is halfway between 1 and 2. The number 58 is close to 60, so a close estimate would be halfway between 60 and 120, or 90. Reinforce the importance of checking the reasonableness of the answer. In part b), hundreds grids and paper and pencil are used to calculate the mass. Method 2 shows calculating a product by multiplying whole numbers and then placing the decimal point. For multiplication questions with a multiplier that has more than two digits, students are expected to use a calculator.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• Have students answer these questions using base 10 blocks or **Master 10 Hundred Grids**. Allow students to use manipulatives or representations until they feel comfortable using only numerals. • You may wish to provide additional questions similar to those in the Show You Know for students who would benefit from them: a) 56×2.7 (Students should estimate that the product is at least 50×2 or 100.) b) 54×4.5 (Students should estimate a product between 200 and 300: 243.) Coach students through a), and then have them try b) on their own.

Learning Style and Motor

- Encourage students who are at the concrete and semi-concrete stage to use base 10 blocks or **Master 10 Hundred Grids** to work through Example 3.

ESL and Language

- Allow students to discuss the Key Ideas orally.

ESL

- Some English language learners may have difficulty with terms, such as *gala music festival, tickets*, and *ribbon*. Have students add these terms to their dictionary.

- Students may be concerned about the range between their low and high estimates, especially if the range seems large.

- R_x Explain to students that the method of estimation and the numbers and operation involved can make an underestimate seem very low and an overestimate seem high.

Example 3: Multiply Decimals Using a Calculator

Tickets to a gala music festival cost $37.50 each. A total of 207 tickets were sold. How much money was collected in ticket sales?

Solution

Estimate 37.50×207.

Front-End Estimation:
$30 \times 200 = 6000$

Relative Size Estimation:
37.50 is between 30 and 40 but closer to 40.
207 is between 200 and 300 but closer to 200.
$40 \times 200 = 8000$

The calculated answer is between the estimates of 6000 and 8000. The answer is reasonable.

Use a calculator.
$37.50 \times 207 = 7762.5$ **C** 37.50 ☒ 207 ▤ 7762.5

The school made $7762.50 in ticket sales.

Key Ideas

- You can use front-end estimation and relative size to estimate the answer to a multiplication question.
Estimate 2.65×3.72.

  Front-End Estimation:
  $2 \times 3 = 6$

  Relative Size Estimation:
  2.65 is between 2 and 3, but closer to 3.
  3.72 is between 3 and 4, but closer to 4.
  $3 \times 4 = 12$

- When using a calculator, estimate to make sure your answer is reasonable.
  C 2.65 ☒ 3.72 ▤ 9.85

  The estimates suggest an answer between 6 and 12. The answer 9.858 is reasonable.

- You can multiply decimal numbers the same way you multiply whole numbers and then use estimation to place the decimal point.

  Multiply 1.54×25.

  $$\begin{array}{r} 154 \\ \times\ 25 \\ \hline 770 \\ 3080 \\ \hline 3850 \end{array}$$

  $25 \times 1 = 25$
  $25 \times 2 = 50$
  The answer lies between 25 and 50. The decimal point should go between the 8 and the 5.

  The answer is 38.50.

Example 3 uses front-end estimation and relative size to estimate a product. Check that students can identify whether an estimate is an underestimate or an overestimate. Draw students' attention to the fact that the calculated answer is between the two estimates.

Key Ideas

Consider having students prepare a list of key ideas and put it in their chapter Foldable before reading the Key Ideas in the student resource. Then have students make revisions to their key ideas, as necessary.

Communicate the Ideas

These questions allow students to demonstrate different methods of estimation and to justify their thinking when estimating a product of two decimal numbers. In #1, students explain their understanding of front-end estimation and relative size involving multiplication in the context of a purchase. In #2, students identify an error in a multiplication that was incorrectly estimated.

Communicate the Ideas

1. Fancy ribbon sells for $3.20 per metre. You want to buy 2.6 m of the ribbon for a dance costume.
   a) Use front-end estimation and one other estimation technique to help find both an underestimate and an overestimate of the cost of the ribbon.
   b) Which would be a better estimate of the cost, an underestimate or an overestimate? Explain why.
   c) Show how to calculate the actual cost of the ribbon.

2. Michael was putting the decimal in the answer to a multiplication question.

   $2.5 \times 4.6 = 115$

   He placed the decimal between the two 1s: 1.15. He said that the answer should show hundredths because you are multiplying tenths by tenths. Is his answer correct? Explain your thinking.

Practise

For help with #3 and #4, refer to Example 1 on page 53.

3. Without calculating the answer, place the decimal point in the correct position. Show your thinking. **M E**
   a) $6.8 \times 12.2 = 8296$
   b) $48.6 \times 0.9 = 4374$

4. Without calculating the answer, place the decimal point in the correct position. Show your thinking. **M E**
   a) $4.7 \times 8.8 = 4136$
   b) $11.2 \times 3.4 = 3808$

For help with #5 and #6, refer to Example 2 on pages 54–55.

5. Estimate and then calculate.
   a) 1.75×3
   b) 12.8×0.2
   c) 396×1.5
   d) 13.8×2.5

6. Estimate and then calculate.
   a) 68×3.5
   b) 3.6×2.7
   c) 270×0.1
   d) 46×8.5

Communicate the Ideas

1. Answers may vary.
   a) For example: Front-end estimation: $3 \times 2 = 6$; relative size estimation: $3 \times 3 = 9$
   b) overestimate; to be sure of having enough money to make the purchase
   c)
   $$\begin{array}{r} 3.20 \\ \times\ 2.6 \\ \hline 1920 \\ 6400 \\ \hline 8320 \end{array}$$
   $\leftarrow 6 \times 320$
   $\leftarrow 20 \times 320$

   The estimate was 6, so the decimal point must lie between the 8 and the 3. $8.32

2. No. Answers may vary. For example: Underestimate gives 8. Overestimate gives 15. Answer should be 11.5.

Supported Learning

Learning Style

- For students at the concrete stage, you may wish to provide pieces of string cut into 1-m lengths to represent the ribbon. This might help students determine the cost of the ribbon being purchased by visualizing an underestimate and an overestimate of the length of ribbon.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students complete #1 and #2 on their own and then discuss the answers as a class. Offer several students the opportunity to explain their thinking.	• In #2, students' explanation of "why" should help them identify whether they understand how estimation applies to multiplication of decimals. • Work with the class to develop criteria for judging each answer. For example, criteria for #2 might include: – explains why Michael's answer is not correct – provides possible reason for Michael's confusion – mentions importance of estimating answer – provides correct answer – is logical and easy to follow • Use **Master 2 Two Stars and One Wish** to have students critique other students' writing pieces. This blackline master allows students to write two things they like about a piece and one thing they would like to improve.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 7, 10, 15, Math Link
Typical	1–3, 5, 7, 10, 11, 13, 15, 18, 21, Math Link
Extension/Enrichment	1, 2, 16, 18, 20, 21, Math Link

Learning Style and Memory

• **BLM 2–6 Section 2.2 Extra Practice** provides additional reinforcement for students who need it.

ESL

• In #12, remind students that *h* is the abbreviation for hours.

• In #15 and #16, explain the meaning of *smaller* and *larger*.

For help with #7 and #8, refer to Example 3 on page 56.

7. Estimate and then use a calculator to determine each answer.

a) 3.89×565

b) $\$13.45 \times 478$

c) 7.05×2.24

8. Estimate and then use a calculator to determine each answer.

a) $\$4.49 \times 194$

b) 2.75×2.62

c) 73.9×25.3

Apply

9. An Alaskan malamute dog has a mass of 39 kg. A Newfoundland dog has a mass 1.8 times that amount. What is the mass of the Newfoundland dog?

10. The cost of tickets for a concert was $16.75. The number of tickets sold for a performance was 468. How much money was collected on ticket sales?

11. Renata runs 5.7 km per day. How far will she run in the month of January?

12. An electrical contractor charges $65 per hour. How much does he earn when he works for 4.75 h?

13. $32 \times 86 = 2752$. Use what you know about place values to find each of the following products without multiplying.

a) $3.2 \times 86 = \blacksquare$

b) $32 \times 8.6 = \blacksquare$

c) $0.32 \times 86 = \blacksquare$

d) $0.32 \times 8.6 = \blacksquare$

e) $3.2 \times 8.6 = \blacksquare$

14. Copy and complete the following pattern. Then describe how the position of the decimal point changes.

$$3 \times 100 = \blacksquare$$
$$3 \times 10 = \blacksquare$$
$$3 \times 1 = 3$$
$$3 \times \blacksquare = 0.3$$
$$3 \times \blacksquare = 0.03$$
$$3 \times 0.001 = \blacksquare$$

15. a) Copy and complete each multiplication statement.

$$4.65 \times 10 = \blacksquare$$
$$37 \times 100 = \blacksquare$$
$$0.58 \times 1000 = \blacksquare$$

b) When multiplying by a number greater than 1, should the answer be larger or smaller than the original number?

c) Write a rule that describes how to multiply by 10, 100, or 1000.

16. a) Copy and complete each multiplication statement.

$$3.0 \times 0.1 = \blacksquare$$
$$4.5 \times 0.01 = \blacksquare$$
$$0.345 \times 0.001 = \blacksquare$$

b) When multiplying by a number less than 1, should the answer be larger or smaller than the original number?

c) Write a rule that describes how to multiply by 0.1, 0.01, or 0.001.

58   MHR • Chapter 2

Practise

The following pairs of similar questions provide additional practice: #3 and #4, #5 and #6, and #7 and #8. They could be used some time after covering this material or as a review prior to a test.

Assessment *for* Learning	Supported Learning
Practise Have students do #3, #5, and #7. Students who are successful with these questions can do #9 and then go on to the other Apply questions.	• Students who have problems with #3, #5, and #7 will need additional coaching. Have students explain their thinking on these questions. Clarify any misunderstandings. Coach students through #4a), #6a), and #8a), and then have them complete the remaining parts of each question on their own. Have students refer back to the examples in the student resource. Check back several times to make sure that they understand the concepts.

Apply and Extend

Note that #10 is very similar to Example 3. In #11, have students find how many days are in January by looking at a calendar or on the Internet. In #13 to #16, students reinforce their understanding of what happens when decimal numbers are multiplied. Some students may think that all products are larger than the multipliers.

17. What is the cost of each purchase before tax?

a) 5 large juice bottles $1.99

b) 4 kg of grapefruit $2.99 per kg

18. A certain golf ball has a mass of 45.4 g. The packaging that holds 12 balls has a mass of 57.1 g.
a) Estimate the total mass of a package with 12 balls. How did you estimate?
b) Calculate the total mass of the package and balls.

19. One baseball has a mass of 143.6 g. The empty shipping box has a mass of 89.6 g. What is the mass of a box of 8 baseballs?

143.6 g   89.6 g

Extend

20. Tamara earns $9.25 per hour at her part-time job in the grocery store. The table gives the times she worked last week.

Day	Hours Worked
Monday	3:30 p.m.–8:00 p.m.
Tuesday	3:30 p.m.–8:00 p.m.
Friday	3:00 p.m.–9:45 p.m.
Saturday	8:00 a.m.–4:00 p.m.

a) How many hours did Tamara work last week?
b) How much did she earn last week?

21. Each side of a square is 13.6 cm long. The length of a rectangle is three times as long as the side of the square. The width of the rectangle is twice as long as the side of the square.
a) Determine the perimeter of the rectangle.
b) Compare your method with a classmate's. What is the shortest way to calculate the answer?

MATH LINK

Your dance group has been asked to perform as part of a cultural festival. Twelve dancers and two dance coaches will attend. Your group will eat lunch at the community centre cafeteria.

Your group has $98 for lunches. How will you spend the money? Show at least two different plans. Include estimates and calculations of the final cost.

Special		Drinks	
Stir fry	$5.00	250 mL milk	$0.90
Sandwiches		500 mL milk	$1.75
Egg salad	$2.50	500 mL water	$1.25
Grilled cheese	$3.25	250 mL juice	$1.50
Tuna melt	$3.50	**Other**	
Roast beef	$3.45	Apple or banana	$0.75
Salads		Orange	$0.90
Garden salad	$2.15	Corn chips	$0.95
Caesar salad	$3.50	Fries	$1.95
Prices include tax.		Rice and veggies	$2.70

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • What is the purpose of estimating prior to multiplying?	• Have students explain the thinking involved in using front-end estimation. • Encourage students to think of scenarios involving money where people tend to underestimate and overestimate an amount. • Depending on students' learning style, have them provide oral or written answers. • You may wish to have students review the part related to Section 2.2 in **BLM 2–1 Chapter 2 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 59 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 77.	• You may wish to have students do this Math Link to practise estimating and calculating the cost of several items without exceeding their budget. It will allow students to assess their understanding of overestimating and whether their estimation method is effective. • Students who are having difficulty getting started could use **BLM 2–7 Section 2.2 Math Link**, which provides scaffolding for this activity. • Observe students as they work on the Math Link and have them clarify their menu selection. Encourage students to choose balanced meals.

MATH LINK

This Math Link activity gives students an opportunity to apply their estimation and decimal number multiplication skills to a real-world scenario. You may wish to have students bring in a menu from a popular restaurant and use that to plan the group's lunch.

2.3 Divide Decimal Numbers

| Mathematical Processes |
| --- |
| ✓ Communication |
| ✓ Connections |
| ✓ Mental Mathematics and Estimation |
| ✓ Problem Solving |
| ✓ Reasoning |
| ✓ Technology |
| ✓ Visualization |

2.3 Divide Decimal Numbers

Focus on...
After this lesson, you will be able to...
❑ use estimation to place a decimal point in a quotient
❑ divide decimal numbers with and without a calculator
❑ solve problems using estimation and division of decimal numbers

Scrapbooking is a popular hobby that uses coloured paper, stickers, ribbon, and other decorations to create attractive displays of photographs.

Scrapbookers often buy multiple quantities of certain decorations. How might a scrapbooker decide how many items he or she can afford to buy?
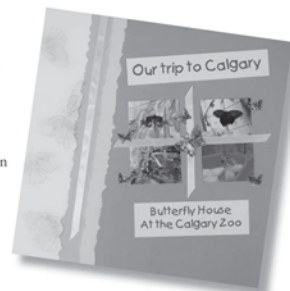
Explore the Math

How can you estimate and calculate quotients of decimal numbers?

You and your best friend have entered a scrapbooking competition and are planning what supplies to buy. Glitter pens are $0.40 each. You have $6.

Materials
• base 10 blocks or hundreds grids

60   MHR • Chapter 2

Specific Outcomes

N2 Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.

Warm-Up

1. In point form, explain how to plot points on a coordinate grid.

2. Identify the coordinates of each point shown on the coordinate grid.
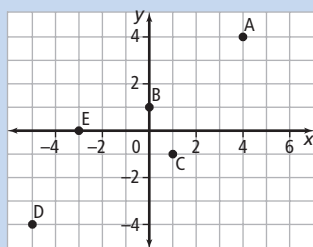
3. Predict in which quadrant each of the following points will lie. For each, explain how you know.
   a) $(6, 9)$ **b)** $(6, -9)$ **c)** $(-6, -9)$ **d)** $(-6, 9)$

4. Estimate and then calculate.
   a) 1.62×4 **b)** 2.75×2

5. You earn $5.25 per hour and work for 4 h. How much do you earn?

Mental Math

6. Arrange this set of numbers from the least to the greatest: 249.5, 250, 249.75.

7. Show how to use place value to provide the answer for each of the following:
   a) one hundred less than 365.72
   b) one tenth less than 365.72
   c) one hundredth more than 365.72
   d) ten more than 365.72

8. Show where the decimal point belongs without calculating.
   a) $93.56 + 35.92 = 12948$
   b) $345.2 + 74.89 + 316.345 = 736435$

Boxed worksheet content (left)

1. How many pens do you think you can you buy with $6?

2. Use base 10 blocks or hundreds grids to model the number of $0.40 pens in $6.

3. How many pens can you purchase?

Reflect on Your Findings

4. a) How can you use estimation to help you divide decimal numbers?
   b) How can a model help you divide decimal numbers?

Example 1: Use Estimation to Place the Decimal Point
Without calculating the answer, place the decimal point in the correct position.
a) $15.4 \div 3.6 = 427778$
b) $4.4 \div 0.42 = 1047619$

Solution
a) Use front-end estimation and division.

Think: $15 \div 3 = 5$

The answer closest to 5 is 4.27778.

b) Use a number line.

Think: $4 \div 0.4$

Show how many times 0.4 goes into 4.

0.4

0   1   2   3   4

It takes 10 jumps of 0.4 to reach 4.
The answer closest to 10 is 10.47619.

Literacy Link
Understanding Division
A division statement such as $6 \div 2 = 3$ means that in 6 there are 3 groups of 2.

Show You Know
Without finding the answer, place the decimal point in the correct position. Show your thinking.
a) $20.1 \div 4.7 = 42766$
b) $3.5 \div 0.213 = 164319$

2.3 Divide Decimal Numbers • MHR   61

Activity Planning Notes

As a class, discuss scrapbooking. For example, you might explore what scrapbooking is, what types of supplies are needed, whether anyone in the class is involved in scrapbooking, and how estimation is helpful in scrapbooking.

Explore the Math

The purpose of this investigation is to get students thinking about division with decimal numbers and how estimation can help. Have students work individually or in pairs to complete the activity, and then discuss their findings.

Show You Know: Example 1

a) 4.2766   b) 16.4319

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math activity, or read student responses to #4.	• Reinforce that students can use any method of estimation that provides a reasonable answer. • Encourage students to consider times in their own lives when they use, or have used, estimation to divide (e.g., paying a restaurant bill).

Supported Learning

Learning Style

• Have students work in groups. Discuss the meaning of division and explain why dividing by a decimal number less than 1 gives an answer that is larger than the number started with. You may wish to model the explanation with base 10 blocks.

Example 2: Divide Decimals

Four friends buy 1.36 L of pure orange juice and divide it equally.

a) Estimate each person's share.
b) Calculate each person's share.

Solution

a) To estimate, round 1.36 L to a number that is easier to work with.

4 divides evenly into 12 and 16. Round 1.36 to 1.2 or to 1.6.

Try 1.2.
$1.2 \div 4 = 0.3$ underestimate $12 \div 4 = 3$, so $1.2 \div 4$ must be 0.3

Try 1.6.
$1.6 \div 4 = 0.4$ overestimate $16 \div 4 = 4$, so $1.6 \div 4$ must be 0.4

The answer is between 0.3 L and 0.4 L of juice per person. The answer should be closer to 0.3 because 1.36 is closer to 1.2 than to 1.6.

Strategies Model It Refer to page xvi.

b) *Method 1: Use Base 10 Blocks or Diagrams*

Let a hundreds flat represent 1.

1.36 can be shown as

Since [   ] cannot be divided by 4, exchange it for 10 [   ].

1.36 is now represented by 13 tenths + 6 hundredths.

Common Errors

• Some students may expect a smaller answer after performing the operation of division.

R_x Emphasize that a division expression such as $10 \div 2$ means "how many groups of 2 are in 10?" Then $10 \div 0.5$ would refer to the number of halves in 10, thereby giving an answer larger than 10.

Example 1 on page 61 illustrates division with decimal numbers using front-end estimation and a number line to place the decimal point. In part b), make sure that students understand that the answer is greater because the divisor is a decimal number less than 1. You may wish to have students calculate the answers to a division pattern, such as $6 \div 2$, $6 \div 1.5$, $6 \div 1$, and $6 \div 0.5$ to help them understand this concept. Read the Literacy Link and make sure that students can interpret $6 \div 2$ as "how many groups of 2 are in 6?" Have students examine the division statement $6 \div 0.5$ and interpret it as "how many 0.5s are in 6?"

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1 on page 61.	• Have students talk through their thinking in a group. • Provide students who may benefit from representing division on a number line with **Master 4 Vertical and Horizontal Number Lines**. • Attempt to determine whether students who are having difficulty are challenged by the estimation or by the division. You may wish to provide questions similar to those in the Show You Know for students to work through using manipulatives: a) $20.2 \div 4.6 = 43913$ (Students should estimate that the answer is between 4 and 5 and place the decimal between the 4 and the 3: 4.3913.) b) $3.5 \div 0.25 = 1400$ (Students should estimate that the answer is around 15: 14.00. The divisor is a decimal number less than 1. Have students explain in their own words why the answer will be greater than 3.5.) Coach students through a), and then have them try b) on their own.

Supported Learning

Learning Style
• Provide base 10 blocks or hundred grids and encourage students to share their strategies with classmates.

Motor
• Allow students to use virtual manipulatives instead of base 10 blocks.

Example 2 provides a context for estimation and division of decimal numbers. Make sure that students understand the estimation process shown (i.e., rounding to find an underestimate and an overestimate). The answer is then calculated using base 10 blocks (Method 1) and paper and pencil (Method 2). You may wish to show the parallel development of Methods 1 and 2 side-by-side on the board. Students could set up the question using base 10 blocks while you use the diagrams from Method 1. The calculated answer is then compared with the estimate. In Method 2, you may decide to introduce the process of placing the decimal point directly above where it appears in the dividend (quantity being divided) when dividing using long division. If you do, students may want to use this technique to place the decimal point and may not want to estimate their answers.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• Students who are still at the concrete and semi-concrete stage may continue to use base 10 blocks or **Master 10 Hundred Grids** to answer these questions. Allow students to use manipulatives or representations until they feel comfortable using only numerals and/or performing long division. • Students need to see the importance of estimating an answer before calculating. It allows them to check whether their answer is reasonable and to place the decimal point in the answer. • Some students relate well to problems involving money. You may wish to have students work in small groups and discuss how they would divide a large and a small amount of money.

Learning Style

- In Example 3, allow students to use manipulatives to represent division.
- You may provide **Master 4 Vertical and Horizontal Number Lines** to enable students to show their thinking as they divide.
- In Example 3, some students may have difficulty visualizing the volumes and understanding the problem. You may wish to show how several items of a smaller volume can fit into an item with a larger volume.

Common Errors

- Some students may not be able to locate decimal numbers on a number line, and may not know which direction to count the "jumps" in.
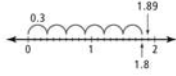- R_x Show students how to represent division on a number line, counting jumps from 0.

Example 3: Divide Decimals Using a Calculator

Fran is curious to know how many 0.295 L cans of juice would be in a large bottle containing 1.89 L. Estimate and then calculate the answer.

Solution

The division statement is $1.89 \div 0.295$.
To estimate, place 1.89 on a number line.

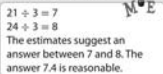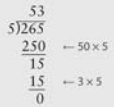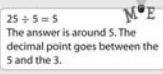0.295 is close to 0.300, or 0.3, so make jumps of 0.3.

It takes 6 jumps to get close to 1.89. The answer is close to 6 cans of juice.

To calculate, use a calculator.

$1.89 \div 0.295 \approx 6.4$ C 1.89 ÷ .295 = 6.406779661

There are approximately 6.4 cans of juice in the 1.89 L bottle of juice.

Literacy Link

Reading ≈
The symbol ≈ means "is approximately equal to."

Key Ideas

- There is more than one way to estimate the answer to a division problem.

  Estimate $4.6 \div 2.5$

  Front-End Estimation:
  $4 \div 2 = 2$

  Number Line Estimation:

  $4 \div 2 = 2$ underestimate
  $6 \div 2 = 3$ overestimate

- When using a calculator, estimate to make sure your answer is reasonable.

  C 23.68 ÷ 3.2 = 7.4

  $21 \div 3 = 7$
  $24 \div 3 = 8$
  The estimates suggest an answer between 7 and 8. The answer 7.4 is reasonable.

- You can divide decimal numbers the same way you divide whole numbers, and then use estimation to place the decimal point.

  Divide $26.5 \div 5$.

  $$\begin{array}{r} 53 \\ 5\overline{)265} \\ 250 \quad \leftarrow 50 \times 5 \\ \hline 15 \\ 15 \quad \leftarrow 3 \times 5 \\ \hline 0 \end{array}$$

  $25 \div 5 = 5$
  The answer is around 5. The decimal point goes between the 5 and the 3.

  The answer is 5.3.

64   MHR • Chapter 2

In Example 3, students estimate and divide decimals using a number line and a calculator. Read the Literacy Link on page 64 and discuss when to use this symbol. Encourage students to use the symbol when appropriate. Allow students to approximate 1.89 L as 2 L. Help them realize that there are almost three 0.3-L cans in 1L. So, 2 L must contain six 0.3-L cans. Encourage students to compare the estimate to the calculated answer. Remind students to show their calculator keying sequence each time. An example is shown in the student resource. Other keying sequences are possible depending on the student's calculator.

Key Ideas

Have students read and review the Key Ideas section. Make sure that students understand the difference between an underestimate and an overestimate. Students should write a summary of the Key Ideas in their chapter Foldable.

Answers

Communicate the Ideas

1. No. Estimates show $80 \div 8 = 10$, which is closer to 9.15 than 91.5.

2. underestimate: it takes about 16 jumps of 0.3 to get to 5; overestimate: $6 \div 0.3 = 20$

3. Answers may vary. For example: You and 4 friends decide to mow lawns to make some extra money. You plan to share the money equally. In one weekend, you and your 4 friends earned $108.50, which is $21.70 each.

Supported Learning

Motor

• The buttons on some calculators may be too small and close together for students with dexterity problems to use effectively. You may wish to provide a calculator with oversized keys.

Communicate the Ideas

These questions allow students to explain their understanding of underestimation, overestimation, and division. In #1, students review a given solution to a problem involving decimal placement. In #2, students apply their understanding of estimation. In #3, students make up a real-world problem that involves the division of decimal numbers.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students work in groups to answer and discuss the questions. You may wish to have each group present their solution to one question to the class.	• Encourage students who are having difficulty with #1 to work through the estimation and division on their own and to try to place the decimal. Ask students to explain the meaning of division in their own words and how it relates to this question. • Some students may wish to further explore #2 by rounding the divisor (0.3) up and down, and then consider how this affects their underestimate or overestimate of the answer.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8, 12, 16, Math Link
Typical	1–4, 6, 8, 12, 14–17, 20, Math Link
Extension/Enrichment	1–3, 11, 16, 18–22, Math Link

Learning Style and Memory

• **BLM 2–8 Section 2.3 Extra Practice** provides additional reinforcement for students who need it.

ESL

• Explain the terms *gravel* and *revenue* to English language learners. Have students add *revenue* to their dictionary.

• For #17, using the picture as a starting point, describe loons and their habitat, and explain what a loon sculpture is.

Apply

10. A package of 7 fish hooks costs $17.99. How much will one fish hook cost?

11. Milo wants to find how many 355 mL cans of juice are in a 2-L bottle. Hint: 355 mL is equal to 0.355 L.
   a) Show Milo how to estimate the answer.
   b) Show Milo how to calculate the answer.

12. A contractor charged $398.75 to move the gravel for a garden. The contractor charges $72.50 per hour. How long did she work? Estimate, then calculate the answer.

13. What is the cost for one item? Round each answer to the nearest cent.
   a) 5 juice boxes for $1.70

   $1.70

   b) 6 apples for $2.99

   $2.99

14. a) Copy and complete the pattern.
   $3 \div 100 = \blacksquare$
   $3 \div 10 = 0.3$
   $3 \div 1 = \blacksquare$
   $3 \div 0.1 = 30$
   $3 \div 0.01 = \blacksquare$
   $3 \div 0.001 = 3000$
   b) Describe how the position of the decimal point changes.

15. a) Copy and complete each division statement.
   $4.65 \div 10 = \blacksquare$
   $37 \div 100 = \blacksquare$
   $0.58 \div 1000 = \blacksquare$
   b) When you divide by a number greater than 1, is the answer larger or smaller than the original number?
   c) Write a rule that describes how to divide by 10, 100, or 1000.

16. a) Copy and complete each division statement.
   $40 \div 0.1 = \blacksquare$
   $1.45 \div 0.01 = \blacksquare$
   $0.524 \div 0.001 = \blacksquare$
   b) When you divide by a number less than 1, is the answer larger or smaller than the original number?
   c) Write a rule that describes how to divide by 0.1, 0.01, or 0.001.

17. A package of loon sculptures has a mass of 1.7 kg. The box and wrapping have a mass of 0.5 kg.
   a) What is the total mass of the loon sculptures without the box and wrapping?
   b) There are 12 loon sculptures in the box. What is the mass of each sculpture?

18. An package of 500 sheets of paper has a measured height of 51.5 mm.
   a) Estimate the thickness of one sheet of paper.
   b) Is the actual thickness of a sheet of paper greater or less than your estimate? Explain how you know.

66   MHR • Chapter 2

Practise

Most students will need to do only one question from each of the following pairs of similar questions: #4 and #5, #6 and #7, and #8 and #9. Remind students to expect an answer that is greater when dividing by a decimal number less than 1.

Assessment *for* Learning	Supported Learning
Practise Have students do #4, #6, and #8. Students who have no problems with these questions can begin the Apply questions.	• Have students who have difficulty with #4, #6, and #8 explain their thinking on these questions. Clarify any misunderstandings. Coach students through #5a), #7a), and #9a), and then have them complete the remaining parts of each question on their own. Have students refer back to the examples in the student resource. Check several times to make sure that they understand the concepts.

Apply and Extend

Note that #11 is very similar to Example 3. Remind students to work with the same units when dividing. In #14, students explore a pattern that is valuable for them to grasp. You may wish to discuss this question as a class, if you have not already reviewed the pattern. Students can then use this pattern to help them answer #15 and #16. Note that #17 involves subtraction of decimal numbers, and then division. You may need to remind students to line up the decimal points if subtracting using pencil and paper.

19. A box of 8 baseballs has a total mass of 1238.4 g. If the empty box has a mass of 89.6 g, what is the mass of one baseball?

1238.4 g
8 baseballs
89.6 g
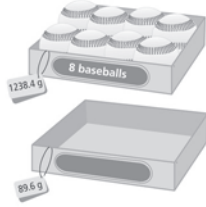
Extend

20. How many sheets of paper will it take to form a pad 2 cm thick if each sheet has a thickness of 0.08 mm?

21. The hours of business for a convenience store are displayed on the sign. The total revenue for the week is $10 585.

Store Hours	
Mon.–Tues.	9:30 a.m.–10:00 p.m.
Wed.	Closed
Thurs.–Sat.	9:00 a.m.–10:30 p.m.
Sun.	10:00 a.m.–5:30 p.m.

a) How many hours is the store open in a week?
b) What is the average revenue per hour?
c) What is the average revenue per day?

22. Kyle is entered in a snowmobile race. He has to do 5 laps on a 6.37-km course. His time for each lap is 1.35 min, 1.27 min, 1.23 min, 1.37 min, and 1.22 min.
a) How long does it take him to do the 5 laps?
b) What is his average speed? Round your answers to the nearest hundredth.

MATH LINK

You have saved $70 from work you did for a neighbour and plan to spend it doing some of the following things.

Attractive Offers!

Horseback riding:	$25 per hour
River rafting:	$36 per hour
Canoeing:	$13 per hour
Trail biking:	$10 per hour

Outline how you plan to spend your $70. Use addition, subtraction, multiplication, and division of decimals to show your plan. Show at least three different ways to spend the money.

2.3 Divide Decimal Numbers • MHR   67

Although #17 and #19 are similar, some students may benefit from completing both of these questions.

In #18 and #20, students solve problems related to thickness of paper. In #20, students need to convert between millimetres and centimetres. This Extend question is intended to provide all students an opportunity for success.

In #21, some students may not know the meaning of the word *revenue*, although this may not necessarily affect their ability to answer the question correctly.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 67 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 77.	• You may wish to have students do this Math Link to provide them with an opportunity to apply their understanding of decimal operations to a personal plan to spend a set amount of money. Have students compare their estimates and calculations for reasonableness in each of the three activity plans. • Students who are having difficulty getting started could use **BLM 2–9 Section 2.3 Math Link**, which provides scaffolding for this activity. • Observe students as they work on the Math Link and have them mathematically justify their chosen activities.

Math Link

Answers will vary. For example:

Plan 1
Horseback riding 25
Horseback riding 25
Trail Biking 10
Trail Biking 10
$2(25) + 2(10) = 70$

Total = $70

Plan 2
Horseback riding 25
River rafting 36
Canoeing 13
Trail Biking 10
84
− Canoeing − 13
71
6 min less of − 1
Trail Biking
= (0.1 × 10)

Total = $70

Plan 3
Horseback riding $0.5 × 25 = 12.50$
River rafting $0.5 × 36 = 18.00$
Canoeing $2.5 × 13 = 32.50$
63
+ Trail Biking (0.7 h × 10) + 7

Total = $70

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • If you divide a whole number by a number between 0 and 1 is the answer going to be larger or smaller than the original whole number? Explain your thinking.	• Have students explain the Literacy Link on page 61 in their own words and provide an example. • Encourage concrete and kinesthetic learners to provide examples that illustrate how to divide. • Have students provide oral or written answers. • You may wish to have students review the part related to Section 2.3 in **BLM 2–1 Chapter 2 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

MATH LINK

This Math Link gives students an opportunity to apply operations on decimal numbers in a real-world context.

Emphasize to students that activities do not need to be planned for a full hour and that students do not need to attempt to do every activity.

2.4 Order of Operations and Decimal Numbers

Suggested Timing
80–100 minutes

Materials
• calculator

Blackline Masters
BLM 2–1 Chapter 2 Self-Assessment
BLM 2–10 Section 2.4 Extra Practice

Mathematical Processes

✓	Communication
✓	Connections
✓	Mental Mathematics and Estimation
✓	Problem Solving
✓	Reasoning
✓	Technology
✓	Visualization

Specific Outcomes

N2 Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.

Warm-Up

1. Identify the coordinates of the vertices of the shape.
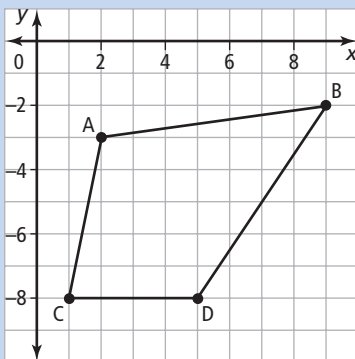
2. Estimate and then calculate.
   a) $22.5 \div 6$ **b)** $3.7 \div 0.5$

3. Five juice boxes cost $1.75. What is the cost of one box?

4. Calculate.
   a) $\begin{array}{r} 27.689 \\ + \ 15.471 \\ \hline \end{array}$ **b)** $\begin{array}{r} \$870.49 \\ - \ \$630.20 \\ \hline \end{array}$

Mental Math

5. Use estimation to place the decimal point.
   a) $87.85 + 14.60 + 73.52 = 175970$
   b) $4189.675 - 1501.941 = 2687734$

6. Estimate $\$37.59 \div 4$.

7. Estimate 28.6×5.2.

Activity Planning Notes

As a class, discuss the section opener. Encourage students to describe experiences in which they or someone that they know had to answer a skill-testing question.

Discuss the Math

This investigation emphasizes the importance of using the order of operations to solve problems. It alerts students to the fact that some calculators do not follow the correct order of operations. Each student should try to rewrite the question using brackets to show how to get answers of 158 and 156.4. Students should then work individually or in pairs to complete the activity and discuss their findings.

During the class discussion, read the Literacy Link that refers to parentheses. You may wish to show students different styles of parentheses (e.g., curved, square). Emphasize the importance of using the same style for each bracket in a pair.

Communicate the Ideas

1. No. Using the order of operations gives an answer of 4.7. Solving in order from left to right (not following the accepted rules) gives an answer of 3.85.

2. $3 \times (2.8 + 6.4 \div 4) = 13.2$. Division by 4 reduces the value, so to minimize this, the division should be done first, then addition, and then multiplication.

3. Answers may vary. For example: Something done on Saturday, Sunday, and the weekdays for the times shown, or to earn the money shown.

Supported Learning

Learning Style, ESL, and Language

• In Example 1, some students may have difficulty connecting the information on the table with the mathematical expression. Consider displaying the items in the table horizontally and then perform the calculations.

• Have students who need more practice complete #4 of the Practise questions. It is very similar to Example 1.

• For Example 2 part 1), students at the semi-concrete level may wish to use pieces of paper or blocks to represent operations and move them around as they try different combinations.

• For Example 2 part b), allow students to present their problem orally or in written form.

Learning Style and Memory

• Some students may benefit from using BODMAS to help remember the order of operations.

B – brackets, then

O – order

D – division and $\left.\vphantom{\begin{array}{c}a\\b\end{array}}\right\}$ in order from

M – multiplication $\left.\vphantom{\begin{array}{c}a\\b\end{array}}\right\}$ left to right

A – addition and $\left.\vphantom{\begin{array}{c}a\\b\end{array}}\right\}$ in order from

S – subtraction $\left.\vphantom{\begin{array}{c}a\\b\end{array}}\right\}$ left to right

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Key Ideas

• The order of operations is used with operations that involve decimals.

The order of operations is as follows:
– Do the work in brackets first.
– Multiply and divide in order from left to right.
– Add and subtract in order from left to right.

$$(0.75 - 0.5) \times (4.2 \div 0.6) + 7.3 - 1.2 \quad \text{Brackets.}$$
$$= 0.25 \times 7 + 7.3 - 1.2 \quad \text{Multiply.}$$
$$= 1.75 + 7.3 - 1.2 \quad \text{Add.}$$
$$= 9.05 - 1.2 \quad \text{Subtract.}$$
$$= 7.85$$

• Brackets can be used to change the order of operations.

$$9.1 \times 2 + 7.5 \div 2.5 \quad \text{Multiply and divide} \qquad 9.1 \times (2 + 7.5) \div 2.5 \quad \text{Brackets.}$$
$$= 18.2 + 3 \quad \text{Add.} \qquad\qquad = 9.1 \times 9.5 \div 2.5 \quad \text{Multiply and divide.}$$
$$= 21.2 \qquad\qquad\qquad = 86.45 \div 2.5$$
$$\qquad\qquad\qquad\qquad = 34.58$$

Communicate the Ideas

1. Annie has been asked to calculate $1.7 + 6 \div 2$. She claims the answer is 3.85. Do you agree? Explain why or why not.

2. Put brackets in the following expression to get the largest value possible. What problem solving strategy did you use?
$3 \times 2.8 + 6.4 \div 4$

3. Create a problem that could be solved using the following expression.
$2.5 + 1.25 + 5 \times 1.6$

70   MHR • Chapter 2

Example 1 on page 69 demonstrates how to calculate the total cost of multiple items. Two methods are shown: a table displaying the total cost for each item and a calculator keying sequence. The memory keys on a calculator could also be used to find the answer.

$\boxed{\text{C}}$ 74.2 $\boxed{\times}$ 1.129 $\boxed{\text{M+}}$ 4 $\boxed{\times}$ 1.69 $\boxed{\text{M+}}$

2 $\boxed{\times}$ 1.39 $\boxed{\text{M+}}$ $\boxed{\text{MR}}$

The answer 93.3118 should appear when you press $\boxed{\text{MR}}$ or Memory Recall.

Have students enter the calculator keying sequence in the example or make up their own expression to help them determine if the order of operations is built into their calculator.

Example 2 part a) uses the problem solving strategy of Guess and Check. Encourage students to write out and solve each possible combination of operations. In part b), students create a word problem that can be modelled and solved with the mathematical statement given. You may wish to review the scenario in the student resource to be sure all students connect the equation to the word statements. Then, ask students to work individually and make up their own story.

Key Ideas

Have students read the Key Ideas. Make sure that students understand the importance of an agreed upon order of operations. They should know the order and be able to use it to make calculations appropriately.

For help with #4 and #5, refer to Example 1 on page 69.

4. Jens wanted to go fishing. He went to the store and purchased the following items.

> 3 fishing lures at $4.49 each
> 4 floats at 79¢ each
> 12 leaders at 23¢ each

a) What was the total bill before tax?

b) Write a mathematical expression to show how to calculate the total bill before tax. Use a calculator find the value of your expression.

5. Sara bought a fruit smoothie each day of the week for one week. She bought a medium smoothie on Tuesday and a large one on Saturday. On each of the other days, she bought small smoothies.

> **Modern CAFE**
> 16 Flavours of Smoothies!
>
> | Small | Medium | Large |
> |-------|--------|-------|
> | $1.09 | $1.49 | $1.89 |

a) How much did she spend on smoothies during the week?

b) Write a mathematical expression that shows how to find the total cost of her smoothies for the week. Use a calculator to solve your expression.

For help with #6 and #7, refer to Example 2 on page 69.

6. Where should the two operations shown in square brackets be placed to make each statement true? Rewrite each statement with the correct operations.

a) $6 \blacksquare 2.5 \blacksquare 0.1 \times 3 = 14.7$ $[-, \times]$

b) $(4 \blacksquare 1.79) \blacksquare 3 + 1.5 = 3.43$ $[\div, +]$

c) $(8.1 \blacksquare 3.2) \blacksquare 2 = 22.6$ $[+, \times]$

d) $4.2 \blacksquare 2 \blacksquare 0.5 = 1.6$ $[-, \div]$

7. Where should the two operations shown in square brackets be placed to make each statement true? Rewrite each statement with the correct operations.

a) $12.4 \blacksquare 3.1 \blacksquare 1.7 = 2.3$ $[-, \div]$

b) $(4.5 \blacksquare 1.1) \blacksquare 6.7 = 22.78$ $[-, \times]$

c) $23.5 \blacksquare 6.3 - 7.6 \blacksquare 2.5 = 10.8$ $[+, \times]$

d) $4.1 \blacksquare (3.6 \div 0.9) \blacksquare 12.4 = 28.8$ $[+, \times]$

8. a) Make up a problem that could be solved using the following expression.

$3 \times 1.5 - 2 \times 1.25$

b) What is the value of the expression?

9. What are the missing numbers?

a) $\blacksquare + 4.8 \times 41 = 200$

b) $4.5 \div 5 + \blacksquare = 3$

c) $4 \times \blacksquare - 0.6 \div 2 = 2.5$

10. Fill in the numbers 0.5, 0.1, 1, and 5 to make the statement true. Use each number only once.

$\blacksquare + \blacksquare - \blacksquare \times \blacksquare = 1$

11. Ruben wants to earn $155 this week. His part-time job pays $7.75 per hour. How many hours must he work?

Communicate the Ideas

These questions allow students to explain their understanding of the order of operations as it applies to operations on decimal numbers.

In #1, students review a solution that involves the order of operations.

In #2, students explore values of expressions depending on the location of brackets. Allow students to insert brackets around two operations, but ensure that they follow the order of operations within each pair of brackets.

If students have difficulty creating a story in #3, get them started thinking about what one of the numbers might represent. For example, the 5 could represent the weekdays. The other two terms could each represent one weekend day.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students do #1, #2, and #3.	• Ask students what answer they would get if they did not follow the order of operations in #1. Have students show how to change the expression if this were the intended answer (e.g., insertion of brackets). • Have students share their story problems for #3 with a partner and observe their thinking and explanations for situations where students do not agree with each other's solution.

Learning Style

- Give examples that reinforce the need for a set order of operations. Provide students with many opportunities to practise using the order of operations with decimal numbers.

ESL

- For #4, some English language learners may not be familiar with fishing terms. Explain how *lures*, *floats*, and *leaders* are used.

- The Apply word problems require that students understand what mathematical operation is being asked for. For example, in #14, students may not understand *total admission collected*. Consider assigning fewer of these questions for English language learners.

- For #16, refer to the Did You Know? on page 72 to help explain *orbit*, which in this case means to go around the Sun.

- For #20, explain that *increased* means *more*.

Memory

- Students may wish to create their own acronym for the order of operations. For example, **B**rackets, **M**ultiply, **D**ivide, **A**dd, **S**ubtract could become **B**ecause **M**y **D**ear **A**unt **S**ays.

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12. Joanne travels a total of 5.8 km going to and from school each day. She goes to school for 189 days per year. What is the total distance she travels back and forth to school each year?

13. Charlene bought two items for a total of $56.89 before tax. One of the items cost $21.94. What was the cost of the other item?

14. A tournament volleyball game is sold out. Ticket prices are shown.

> **Volleyball Admission**
> Adults: $3.25
> Students: $1.50

a) If 80 adults and 120 students attend the game, what is the total admission collected?

b) Show your calculator key sequence for finding the total admission.

c) The expenses for the volleyball game include 3 game officials at $50 each and 2 security guards at $65 each. What are the total expenses for the game?

d) How much profit will the school make?

15. A local store is having a sale on art supplies. Sam bought 4 sheets of posterboard, 3 erasers, and 5 pencils.

> Posterboard $1.87
> Eraser $0.69
> Pencil $0.36

a) Estimate Sam's total cost before tax. Is your estimate high or low? Explain how you know.

b) What was the total cost before tax?

16. The table shows how long it takes each planet to orbit the sun.

Planet	Approximate Time to Orbit the Sun
Mercury	0.241 years
Venus	0.616 years
Earth	1.0 year or 365.25 days
Mars	687 days
Jupiter	4332 days
Saturn	29.5 years

a) Which planet takes about 2 years to orbit the sun? Show how you know.

b) How many of our years does it take Jupiter to orbit the sun? Give your answer to the nearest hundredth.

c) How many times does Mercury orbit the sun in a year? Give your answer to the nearest thousandth.

d) Describe how you could estimate the answer in c).

Did You Know?

Earth orbits the sun in approximately 365.25 days. This gives an extra day, February 29, every 4 years. A 366-day year is called a "leap year."

17. In 2005 Bill Gates was the richest man for the eleventh year in a row, with an estimated $46.5 billion U.S. In that same year, Canada's three wealthiest people were:

Kenneth Thompson $17.2 billion U.S.
Galen Weston $7.7 billion U.S.
Jeff Skoll $6.6 billion U.S.

a) What is the sum of the estimated wealth of the 3 top-ranked Canadians?

b) What is the difference in the estimated wealth of Bill Gates and the total of the 3 top Canadians?

72   MHR • Chapter 2

- Some students may have difficulty remembering the correct order of operations.

- **R$_x$** The acronym BODMAS may help some students.

- Students may have difficulty knowing which arithmetic operation(s) to use in word problems.

- **R$_x$** Suggest that students read the problem carefully and then break up the problem into parts. Have students estimate the answer to each part.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 4, 6, 8, 9, 11, 15
Typical	1–4, 6, 9, 13, 15–17, 21
Extension/Enrichment	1–3, 9, 10, 14, 16, 17, 22–24

Practise

In #4 and #5, students calculate the total price of several items and write an expression representing the total price. Some students may not be familiar with the fishing tackle mentioned in #4 or smoothies mentioned in #5.

Since #6 and #7 are similar, some students will need to do only #6.

Assessment *for* Learning	Supported Learning
Practise Have students complete #4 and #6. Students can then work on #8 and the other Apply questions.	• Students who have difficulty with #4 or #6 will need additional coaching. Have students explain their thinking on these questions. Clarify any misunderstandings. Have students review the examples on page 69. Have students correct #4 and then try #5 on their own. Coach students through #7a), and then have them complete the remaining parts of the question on their own. Check several times to make sure that they understand the concepts.

c) About how many times as great as Jeff Skoll's is Bill Gates' estimated wealth? Round your answer to the nearest hundredth.

Did You Know?
Jeff Skoll is the Canadian-born co-founder of eBay.

18. Add brackets to make each statement true.
a) $7 + 30 \times 0.5 = 18.5$
b) $6 + 3 \times 0.2 + 0.4 \div 2 - 1 = 2.2$

19. Add brackets to the expression $80 \div 0.4 + 6 \times 0.3$ to get the following answers.
a) 201.8   b) 3.75

20. The Eagle Health Club increased its membership this year by 89 people. There were 567 members last year.
a) How many members are there this year?
b) If a membership costs $189.95 per year, estimate this year's revenue from membership fees.

21. Cecil, Kent, and Laura go to the Quickstop for lunch. Cecil orders a chicken burger and a salad. Kent and Laura order small pizzas. Kent also has a salad and Laura has an ice cream.

Chicken burger	$5.49
Side salad	$3.50
Small pizza	$4.59
Ice cream	$1.50

a) How much does it cost each person for lunch?
b) How much does lunch cost altogether?
c) What is the shortest way to determine the total cost?

Extend

22. After a 2005 earthquake, 11 410 kg of rice were distributed in 3260 bags to families in the damaged area.
a) How many kilograms of rice were in each bag, if the bags were the same size?
b) If a family used 0.25 kg per day, how many days would 1 bag of rice last?

23. A small car rental company in Saskatoon has four employees: Jeanne, Alice, Fatek, and Larry. The employees are paid by the hour. Each employee is paid a different hourly rate: $7.75, $10.50, $15.25, and $17. This week they worked 50 h, 45 h, 42 h, and 18 h. Use the following clues to match each employee with their hourly wage and the number of hours worked this week. Then, determine each employee's weekly wage.
- This week Larry worked the greatest number of hours.
- Fatek earns the least amount of money per hour.
- Alice worked less than 45 h this week.
- Jeanne earns less than $17 per hour.
- Fatek had the smallest amount of pay for the week.
- The employee who worked 50 h this week worked 59 h last week. Last week, that employee earned $137.25 more than he/she earned this week.

24. Three books stand on a bookshelf as shown. A bookworm starts at page 1 of the first book and chews a straight path to the last page of the third book. If the thickness of each book cover is 3.2 mm and the thickness of each book's pages is 4.5 cm, how far does the bookworm travel?

A-G   H-P   Q-Z

2.4 Order of Operations and Decimal Numbers • MHR   73

Apply and Extend

In #9, students solve an equation for an unknown. Students will use different strategies for solving these equations (e.g., Guess and Check). In #10, encourage students to write out the possible equations (e.g., using a tree diagram). Then, students can solve each expression until they find a true statement. This will avoid students becoming frustrated by randomly trying different combinations of numbers and not knowing what to try next.

In #18 and #19, students insert brackets to make an equation true. Some students may find this frustrating rather than challenging. Encourage students to determine whether they are trying to get the expression to have a greater or a lesser value, and to try to insert the brackets accordingly. Alternatively, students may wish to systematically place brackets until they reach the desired value for the expression.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • How do you remember the correct order of operations in a series of decimal number calculations?	• Have students explain why it is important to have a set order of operations. • Have students explain and give examples showing how brackets can be used to change the value of an expression. • Depending on students' learning styles, allow them to provide oral or written answers. • You may wish to have students review the part related to Section 2.4 in **BLM 2–1 Chapter 2 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Suggested Timing
80–100 minutes

Materials
• calculator

Blackline Masters
BLM 2–1 Chapter 2 Self-Assessment

BLM 2–4 Section 2.1 Extra Practice

BLM 2–6 Section 2.2 Extra Practice

BLM 2–8 Section 2.3 Extra Practice

BLM 2–10 Section 2.4 Extra Practice

Supported Learning

Learning Style and Memory
• Refer students to the blackline master that provides additional reinforcement for each section. See **BLM 2–4 Section 2.1 Extra Practice, BLM 2–6 Section 2.2 Extra Practice, BLM 2–8 Section 2.3 Extra Practice**, and **BLM 2–10 Section 2.4 Extra Practice**.

Learning Style
• Allow students to complete the chapter review using any combination of verbal description, diagrams for explanation, and written answers.

ESL, Language, and Memory
• Encourage students to use their chapter Foldable during the chapter review, and to add any notes into the relevant sections.

Gifted and Enrichment
• Some students may already be familiar with the skills handled in this review. To provide additional questions, go to **www.mathlinks7.ca** and follow the links.

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2 Chapter Review

Key Words

For #1 to #4, choose the letter representing the term that best matches each statement.

1. The accepted order for finding the value of math expressions

   A estimate

2. An estimate that is larger than the answer

   B overestimate

3. An estimate that is smaller than the answer

   C underestimate

   D order of operations

4. To approximate an answer

   E calculate

2.1 Add and Subtract Decimal Numbers, pages 44–51

5. Without finding the answer, place the decimal point in the correct position. Show your thinking.
   a) $0.8 + 8.8 + 0.88 + 88 = 9848$
   b) $368.2 − 89.57 = 27863$
   c) $29.563 + 13.2 − 8.69 = 34073$

6. Estimate, then calculate.
   a) $4.6 + 2.35$ b) $8.6 − 3.9$
   c) $7.5 + 1.36$ d) $9.12 − 3.5$
   e) $2.5 + 6.8 − 4.5 − 2.7$
   f) $6.5 − 3.6 − 0.123$

7. Replace each ▬ with a number to make the statement true.

   a) $\begin{array}{r} 75.86 \\ -\ \rule{1cm}{0.8mm} \\ \hline 36.91 \end{array}$ b) $\begin{array}{r} \rule{1cm}{0.8mm} \\ +\ 235.79 \\ \hline 983.245 \end{array}$ c) $\begin{array}{r} 435.6 \\ 32.8 \\ +\ 179.04 \\ \hline 703.52 \end{array}$

8. Jordan has three dogs. Max has a mass of 28.2 kg. Sam is 3.15 kg heavier than Max. The smallest dog, Lucy, is 1.8 kg lighter than Max. What is the total mass of the three dogs?

2.2 Multiply Decimal Numbers, pages 52–59

9. Without finding the answer, place the decimal point in the correct position.
   a) $4.5 \times 1.5 = 675$
   b) $4.23 \times 1.9 = 80370$
   c) $29.6 \times 63.8 = 188848$

10. Estimate the answer to 7.56×0.7. Explain how you found your estimate. Then, find the answer correct to 1 decimal place.

11. A kitten has a mass of 1.5 kg. A large cat has a mass 3.8 times that amount. What is the mass of the large cat?

74   MHR • Chapter 2

Activity Planning Notes

Have students work independently to complete the review questions. If students encounter difficulties, they could discuss strategies with other students. Encourage students to refer to the information in their chapter Foldable and then to the specific section in the student resource and/or their notebooks.

Assessment *for* Learning	Supported Learning
Chapter 2 Review The chapter review provides an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On tab of their chapter Foldable and do at least one question related to any concept, skill, or process that has been giving them trouble. • Have students revisit any section they are having difficulty with prior to working on the Chapter 2 Practice Test.

12. Ken shipped 125.4 kg of ice skates to his brother in Nunavut. The shipping company charged $4.25 per kilogram to ship the skates. How much did Ken have to pay in shipping costs?

2.3 Divide Decimal Numbers, pages 60–67

13. Without calculating the answer, place the decimal point in the correct position. Show your thinking.
a) $28.4 \div 0.4 = 7100$
b) $39.75 \div 7.5 = 530$
c) $251.472 \div 9.3 = 2704$

14. Estimate each answer, and then use a calculator to determine each answer.
a) $174.24 \div 3.2$
b) $656.82 \div 17.8$
c) $199.92 \div 8.4$

15. Isobel bought a ball of string. She found that she could cut it into either 5 or 9 equal pieces without a remainder. If she cuts it into 5 equal pieces, each piece is 3.69 m long. What is the length of each piece if she cuts the string into 9 equal pieces?

16. Eight equally spaced holes are to be drilled in a board as shown in the diagram. What is the distance centre-to-centre between adjacent holes? Express your answer to the nearest tenth of a millimetre.

|← ————— 212.6 mm ————— →|

2.4 Order of Operations and Decimal Numbers, pages 68–73

17. What is the value of each expression?
a) $2.4 - 0.6 \div 2 + 0.3 \times 2$
b) $7.64 - 7.15 \div (1.3 \times 5) + 28.67$
c) $85 \div (1.3 + 7.2) + 4.1 \times 3$

18. Where should the two operations shown in square brackets be placed to make each statment true? Rewrite each statement with the correct operations.
a) $3.6 \blacksquare 8.2 \blacksquare 4 = 5.65$ $[+, \div]$
b) $4.9 \blacksquare 7.2 \blacksquare 0.1 = 4.18$ $[-, \times]$
c) $62.32 \blacksquare (10.1 \blacksquare 2.5) = 8.2$ $[-, \div]$

19. Rewrite each statement using brackets to make a true statement.
a) $7.5 + 8.6 \times 9.1 = 146.51$
b) $45.15 \div 0.8 + 1.7 \times 2.2 = 39.732$
c) $12.6 - 3.3 \div 3 + 11.4 = 14.5$

20. A local theatre group is putting on a production of *The Lord of the Rings*. Ticket prices are shown.

Admission	
Adults:	$12.50
Seniors:	$8.25
Students:	$6.25

a) 80 adults, 30 seniors, and 50 students attend the first performance. How much admission is collected?
b) Show your calculator key sequence for finding the total.

Common Errors

• In #8, some students may determine the mass of each dog and forget to find the sum of the masses. In #15, some students may find the total length of the piece of string and neglect to divide the string into 9 pieces. Other students may divide a 3.69-m piece into 9 smaller pieces.

R$_x$ Have students read each question carefully and make sure that they understand what they are being asked to find.

• In #16, some students may not determine an answer to the nearest tenth of a millimetre.

R$_x$ Have students review the meanings of the place values. Ensure that students are comfortable with the vocabulary.

Assessment *as* Learning	Supported Learning
Math Learning Log Once students have completed the Chapter 2 Review and prior to the Chapter 2 Test, have them reflect on their progress and complete a journal entry for each statement: – I am comfortable with the following parts of the chapter … – The form of estimation I prefer is … – I am having difficulty with … – Here's how I plan to address the areas I am having difficulty with …	• Have students refer back to the What I Need to Work On tab of their chapter Foldable and answer these questions from the contents of that section. • You may wish to have students refer to **BLM 2–1 Chapter 2 Self-Assessment** when they report on what they are comfortable with, what they continue to have difficulty with, and what they plan to do about it. • There are many different ways to estimate. Encourage students to use the one that they feel most comfortable with, but to try other methods. They might find that some methods work better for some types of questions and others for other types.

Suggested Timing
40–50 minutes

Materials
• calculator

Blackline Masters
BLM 2–1 Chapter 2 Self-Assessment

BLM 2–11 Chapter 2 Test

Assessment *as* Learning	Supported Learning
Chapter 2 Self-Assessment Have students review their earlier responses on **BLM 2–1 Chapter 2 Self-Assessment**.	• Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. • Have students use their responses on the practice test and work they completed earlier in the chapter to complete the After column of this self-assessment. • Before students do the **BLM 2–11 Chapter 2 Test**, coach them in the areas in which they are having problems.

2 Practice Test

For #1 to #5, choose the best answer.

1. Calculate $4.85 + 0.5 - 3.2$.
   A 215   **B** 21.5   **C** 2.15   **D** 0.215

2. Calculate $98.2 - 4.8$.
   A 93.4   **B** 103   **C** 9.34   **D** 934

3. Estimate 64.12×18.1 by front-end estimation. What is the best estimate?
   A 600   **B** 650   **C** 1200   **D** 1400

4. Calculate $(39 - 1.7) \div 10$.
   A 0.0373   **B** 0.373   **C** 3.73   **D** 37.3

5. What is the product of 8.5×0.7?
   A 59.5   **B** 12.14   **C** 5.95   **D** 1.214

Complete the statements in #6 and #7.

6. A 6.65-kg package of clay is divided evenly among 19 campers. Each camper gets ▮ of clay.

7. The sum of 65 hundredths and 7 tenths is ▮.

Short Answer

8. Without finding the answer, place the decimal point in the correct position. Show your thinking.
   a) $0.458 + 0.319 + 0.2 = 9770$
   b) $48.31 - 27.65 = 2066$
   c) $5.8 \div 0.32 = 18125$
   d) $24 \times 0.083 = 19920$

9. What is the value of each expression?
   a) $1.45 + 8.9 \times 4 + 2$
   b) $3.12 \times 4 + 12 \div 1.5 \times 2$

10. Rewrite each statement using brackets to make a true statement.
   a) $4.5 + 7.2 \times 3.1 = 36.27$
   b) $1.3 \times 4.5 - 0.9 + 6.2 = 10.88$
   c) $7.1 + 3.7 \times 2 \div 0.3 = 72$
   d) $7.2 \div 0.8 + 5.6 \times 3.9 = 4.3875$

11. A Yorkshire terrier has a mass of 3.2 kg. A Saint Bernard has a mass 28 times that amount. What is the mass of the Saint Bernard?

12. A tennis ball has a mass of 57.3 g. The packaging that holds three balls has a mass of 85 g. What is the total mass of an unopened package containing three of these tennis balls?

13. Leanne buys 6.2 kg of beef steak at $18/kg, 0.8 kg of shrimp at $36/kg, vegetables for a total of $6.74, and two loaves of French bread for $2.39 each. How much does she spend altogether?

76   MHR • Chapter 2

Study Guide

Question(s)	Section(s)	Refer to	I can …
1, 2, 7	2.1	Example 2	✓ use estimation to check if sums are reasonable ✓ solve problems using addition of decimal numbers
3, 5, 11	2.2	Example 2	✓ solve problems using estimation and multiplication of decimal numbers
4, 9	2.4	Example 1	✓ use the order operations with decimal numbers
6	2.3	Example 2	✓ solve problems using estimation and division of decimal numbers
8	2.1 2.2 2.3	Example 1 Example 1 Example 1	✓ use front-end estimation to place the decimal in a sum and a difference ✓ use front-end estimation to place the decimal in a product ✓ use front-end estimation to place the decimal in a sum and a quotient
10, 15	2.4	Examples 1, 2	✓ use the order of operations with decimal numbers ✓ solve problems using order of operations on decimals to the thousandth's place
12, 13, 14	2.1 2.2 2.4	Example 2 Examples 2, 3 Example 1	✓ use estimation to check if sums are reasonable ✓ solve problems using addition of decimal numbers ✓ solve problems using estimation and multiplication or division of decimal numbers ✓ use the order of operations with decimal numbers (optional)

Extended Response

14. Tom and Michael have collected $22 to fill a shoebox with gifts for a child overseas. Since this is for a charity, the store has agreed not to charge tax. Tom and Michael bring the following items to the cashier.

bag of marbles	$1.69
glider	$3.99
2 toy cars	$0.97 each
bag of candy	$1.79
toothbrush	$1.99
toothpaste	$2.29
facecloth	$1.49
3 fancy pencils	$0.35 each
glow-in-the-dark pen	$2.39
small notebook	$1.98
magnifying glass	$2.25

a) Estimate the total. Do you think they have enough money?

b) Calculate the total. Do they have enough money?

c) Either calculate what change they should get or explain what they need to put back and why.

15. Mia claims that the answer to the following skill-testing question is 72.6. Wendy says the answer is 60.8.

$$8.2 + 16.4 \div 0.41 + 12.6$$

a) Who is correct? Show how you know.

b) What mistake did the other student make?

c) Add brackets to the expression to show how the other answer could be found.

d) Make up a skill-testing question of your own that uses decimal numbers and at least 3 different operations. What is the correct answer to your question?

e) How can you add brackets to your question to produce different answers?

WRAP IT UP!

Plan your own one-week dream vacation. You must plan out your itinerary and budget how much money you will need.

- Where will you go?
- Who will go with you?
- Where will you stay? Research two possible places and compare actual prices.
- What activities will you do? What is the cost of each?
- Where and what will you eat?

Create a travel report with all of your plans. Include daily activities, accommodations, and a budget for your trip. Make sure you outline the mathematics that is needed. Try to include realistic costs and distances travelled. Use pictures, maps, and charts to make your report interesting and realistic.

Practice Test • MHR 77

Learning Style

- You may wish to differentiate for some students the questions you feel that they *must* know, the questions that they *should* know, and the questions that are *nice* to know.

Activity Planning Notes

This practice test can be assigned as an in-class or take-home assignment. If it is used to assist you in evaluating student progress, choose which questions will help you make your assessment. These are the minimum questions that will meet the related curriculum outcomes: #1, #3–#6, #8, #9, and #13.

Answers to the Chapter 2 Practice Test are provided on **BLM 2–14** *Math Links 7* **Student Resource Answers**.

Assessment *of* Learning	Supported Learning
Chapter 2 Test After students complete the practice test, you may wish to use **BLM 2–11 Chapter 2 Test** as a summative assessment.	• Consider allowing students to use their chapter Foldable and/or a calculator. • Consider using the Math Games on page 78 or the Challenge in Real Life on page 79 to assess the knowledge and skills of students who have difficulty with tests.

Wrap It Up!

WRAP IT UP!

Plan your own one-week dream vacation. You must plan out your itinerary and budget how much money you will need.

• Where will you go?
• Who will go with you?
• Where will you stay? Research two possible places and compare actual prices.
• What activities will you do? What is the cost of each?
• Where and what will you eat?

Create a travel report with all of your plans. Include daily activities, accommodations, and a budget for your trip. Make sure you outline the mathematics that is needed. Try to include realistic costs and distances travelled. Use pictures, maps, and charts to make your report interesting and realistic.

Supported Learning

Learning Style
• Allow students to work in small groups. Suggest that students may want to focus individual research on a specific aspect of planning the trip. Even if students research and work with a group, they should prepare separate reports.

Common Errors

• Some students may plan their days unrealistically (e.g., having lunch in a restaurant while on a boat trip).

R_x Encourage students to envision what each day will be like, where they are spending it, and how much time they will have between activities. Remind students not to plan an unrealistic number of activities for one day. They may need to account for travelling time from one place to another as well as time spent travelling to their destination.

• Some students may only underestimate or only overestimate prices.

R_x Have students research actual prices of items, food, and activities at their destination. Suggest to students that if they underestimate certain items and overestimate other items, their total budget for the trip will be closer to the actual amount needed.

Specific Outcomes

N2 Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.

Activity Planning Notes

Introduce the problem and clarify the assessment criteria. Make this Wrap It Up! as real as possible by having students research actual travel destinations, accommodations, and activities.

Assessment *of* Learning	Supported Learning
Wrap It Up! This chapter problem wrap-up allows students to apply their knowledge of estimation and operations on decimal numbers by designing and planning the itinerary and budget for a trip. It is important for students to present their plan with mathematical justification in each of the areas of the problem. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this Wrap It Up! Page 77a provides notes on how to use this rubric for this Wrap It Up!	• Students may need some guidance in what a reasonable one-week trip may cost. • You may wish to have students review the work they have completed in the Math Links in Sections 2.1, 2.2, and 2.3 before they begin. • If students have not completed the Math Links earlier in the chapter, you may wish to provide them with **BLM 2–5 Section 2.1 Math Link**, **BLM 2–7 Section 2.2 Math Link**, and **BLM 2–9 Section 2.3 Math Link**. • You may wish to have students use **BLM 2–12 Chapter 2 Wrap It Up!**, which provides scaffolding for the chapter problem wrap-up. If using this blackline master, some students may need more room to record their activities. Have them continue the chart on the back of the worksheet. • Observe how accurately students design, explain, and justify the plan they have created.

The chart below shows **Master 1 Project Rubric** for tasks such as that in the Wrap It Up! and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete response that may contain a minor error that does not affect the final conclusion or outcome • communicates clearly and supports the conclusions
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete response that has minor mathematical omissions *or* • provides a complete response with one of the five areas not completely addressed *or* • provides a complete response with no comparison
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/ connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• chooses two different places to stay and provides basic information for each • provides a response that has only basic mathematical calculations to support the conclusions • lacks calculations to support the comparison • provides some relevant details and attempts to present material in a creative way
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/ connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• addresses only one place to stay, with no comparisons • omits most mathematical justification • fails to address most of the required areas • communicates weakly
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• identifies travel destination, attendees, and activities with little mathematical support *or* • makes a start but does not carry through to complete the calculations and conclusions for the trip

Math Games

Suggested Timing
40–50 minutes

Materials
• coin
• coloured pencils (optional)

Blackline Masters
BLM 2–13 Decimal Delights Game Boards

Supported Learning

Gifted and Enrichment
• This game is rich in opportunity for further development. You might have students develop a new game board to add and subtract fractions.

Common Errors

• Some students' estimates may be incorrect.

R_x Have students check their answers using paper and pencil.

Assessment *for* Learning	Supported Learning
Decimal Delights Have students play this game with a partner of similar math ability.	• After students have played the game, brainstorm winning strategies, such as using estimation to select numbers, which is better than choosing numbers at random.

Specific Outcomes

N2 Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.

Activity Planning Notes

Explain the rules of the game by reading through the game with students. Draw the scoring line from the instructions on the board. Make sure that students realize how to get the maximum number of points when it is their turn.

As a class, decide what to do if the sum or product is one of the endpoints of the scoring ranges (e.g., for the addition game, will a sum of 40 be awarded 2 points or 3 points?). Students may have to make other decisions regarding rules (e.g., what to do if the answer is wrong).

Point out that the range of scores for the multiplication game is different from that for the addition game.

Challenge in Real Life

Rounding Digits and High-Tech Crime

Suppose you are employed by a bank to search for high-tech crime. One day you are examining the software used to calculate the interest paid to customers. Millions of dollars of interest are paid out every day to these customers. You notice a line of programming that rounds money amounts to the nearest cent. This is normal.

But then, you notice something strange. Someone has programmed the rounded down amounts (tenths and hundredths of a cent, for example) to be deposited in a secret account. You check on that account and find that someone has deposited, and then withdrawn, several million dollars over the past year.

a) Suppose you collect 1 cent on 20 transactions per day. If you work 20 days a month, how much money would you have at the end of three months? at the end of a year?

b) How would it be possible to collect millions of dollars from depositing only tenths and hundredths of a cent?

c) Why might a customer not realize what was happening?

d) Why would the bank not realize what was happening?

e) Suppose the programmer was caught and put on trial. Work with a partner to develop a case for the prosecution. You may wish to create a table and prepare a presentation for the jury that would show the effect of rounding. What would you say to the jury, who might not know as much about math as you do?

Suggested Timing
60–75 minutes

Materials
• calculator (optional)

Blackline Masters
Master 1 Project Rubric

Mathematical Processes
✔ Communication
✔ Connections
✔ Mental Mathematics and Estimation
✔ Problem Solving
✔ Reasoning
✔ Technology
✔ Visualization

Specific Outcomes

N2 Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.

Activity Planning Notes

You may wish to use the following steps to introduce and complete this challenge:

1. Read through Rounding Digits and High-Tech Crime with students. Encourage students to share any stories that they may have of high-tech crime, including any stories in the current media. Discuss how easy it is to steal large amounts if you take a small amount from a very large number of people.

Supported Learning

ESL and Language

• Take time to discuss the meaning of *finances*. Students may find it easier to remember if they realize that the term comes from an old French word that means "to settle a dispute or debt" or "to bargain for." Discuss how the modern world of money might be related to the earlier meaning of the term.

• Use television programs that students watch to review the meaning of the terms *prosecution* and *jury*. Students might wish to role-play their case in front of a jury of their peers.

Gifted and Enrichment

• Students may wish to research an actual case of high-tech crime and consider how math was used.

Chapter 2 • **MHR** **79**

Challenge in Real Life

a) $0.20 per day × 20 days = $4
After 3 months = $12
After 1 year = $48

b) Look for an understanding that small amounts taken from a large number of accounts add up. For example: $.001 from 1 000 000 accounts = $1000

c) Customers wouldn't realize when fractions of a cent have been removed because they might expect banks to round less than half a cent down to the lower cent.

d) Banks might expect the money to go into some form of "slush" fund and would not expect a programmer to move it to a personal account. Often, bank management might need a computer expert to notice such things.

e) Look for:
– table, spreadsheet, or list showing how small amounts can add up
– estimation and calculation of actual amounts

2. To make this concrete, work through with students how much someone might get in 1 month if they took 1 penny per day from 100 people. How much would that amount to in 1 year? What if they did the same thing, but from 1000 times that number of people? What if they took half a cent per day from 1 million accounts? How much would that amount to in 1 month?

3. Have students work individually on parts a) to d). Students should work with a partner or in a small group for part e).

4. Clarify that the task is to
• calculate how much someone might get if they collected 1 cent per transaction on 20 transactions over a period of time
• predict how it might be possible to collect large amounts by taking tiny amounts from many accounts
• consider why such theft is not easily discovered
• prepare an explanation about how such theft works

5. Review **Master 1 Project Rubric** with students so that they will know what is expected.

This challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment *for* Learning	Supported Learning
Rounding Digits and High-Tech Crime Discuss the challenge with the class. Have students work together to develop a response, and then provide individual reports.	• Review with students how to multiply and divide by multiples of 10 by moving the decimal point to the right or left. • Allow students to present their report either in written form or orally. • For a second challenge, complete with teaching notes and student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

Assessment *of* Learning	Supported Learning
Rounding Digits and High-Tech Crime Discuss the challenge with the class. Have students work together to develop a response, and then provide individual reports.	• Use **Master 1 Project Rubric** to assist you in assessing student work. Page 79a provides notes on how to use this rubric for this challenge. • To view student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows **Master 1 Project Rubric** for tasks such as the Challenge in Real Life and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 (Standard of Excellence)	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct response with possibly minor communication errors
4 (Above Acceptable)	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete response with a weak/incorrect justification in parts d) or e)
3 (Meets Acceptable)	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides a correct response to parts a), b), and c), with weak or clear communication
2 (Below Acceptable)	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a correct response to part a) with a start to part b) *or* • completes parts a) and b) with some calculation errors *or* • provides a correct response to parts a) and b) with no justification
1 (Beginning)	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides a complete response for part a) *or* • provides a partial response for part a)