Geometry and Measurement

3

General Outcomes

- Use direct or indirect measurement to solve problems.
- Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Specific Outcomes

SS2 Develop and apply a formula for determining the area of:

- triangles
- parallelograms
- circles.

SS3 Perform geometric constructions, including:

- perpendicular line segments
- parallel line segments
- perpendicular bisectors
- angle bisectors.

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
3.1	✓ identify and draw line segments that are parallel to each other
	✓ identify and draw line segments that are at right angles to each other
	✓ describe real-life examples of parallel line segments and perpendicular line segments
3.2	✓ draw a perpendicular bisector using more than one method
3.3	✓ draw an angle bisector using more than one method
3.4	✓ show how the area of a rectangle can be used to determine the area of a parallelogram
	✓ develop the formula for the area of a parallelogram
	✓ solve problems that involve calculating the area of a parallelogram
3.5	✓ show how the area of a rectangle can be used to determine the area of a triangle
	✓ develop the formula for the area of a triangle
	✓ solve problems that involve calculating the area of a triangle

Assessment *as* Learning	Supported Learning
Use the Before column of **BLM 3–1 Chapter 3 Self-Assessment** to provide students with the big picture for this chapter and to help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• As students complete each section of the chapter or complete the Chapter 3 Review, have them review the related parts of **BLM 3–1 Chapter 3 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Chapter 3 Planning Chart

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter Opener • 20–30 minutes		BLM 3–1 Chapter 3 Self-Assessment BLM 3–2 Geometry and Measurement	• ruler • stapler • notebook paper
3.1 Parallel and Perpendicular Line Segments • 80–100 minutes	**Essential:** 1 *or* 2, 3–5, 7, 9, 11, Math Link **Typical:** 1 *or* 2, 3–5, 7–13, Math Link **Extension/Enrichment:** 1 *or* 2, 3, 4, 13–16, Math Link	Master 2 Two Stars and One Wish BLM 3–1 Chapter 3 Self-Assessment BLM 3–3 Section 3.1 Extra Practice BLM 3–4 Section 3.1 Math Link	• ruler • protractor • unlined paper • right triangle (optional) • Mira (optional)
3.2 Draw Perpendicular Bisectors • 80–100 minutes	**Essential:** 1–5, 8, Math Link **Typical:** 1–4, 6–9, Math Link **Extension/Enrichment:** 1–3, 7, 9, 10, Math Link	BLM 3–1 Chapter 3 Self-Assessment BLM 3–5 Section 3.2 Extra Practice BLM 3–6 Section 3.2 Math Link	• tracing paper • ruler • protractor • compass • right triangle
3.3 Draw Angle Bisectors • 80–100 minutes	**Essential:** 2 of 1–4, 5, 6, 8, Math Link **Typical:** 2 of 1–4, 5, 6, 8, 9, 11 *or* 12, 13, Math Link **Extension/Enrichment:** 2 of 1–4, 5, 11–15, Math Link	BLM 3–1 Chapter 3 Self-Assessment BLM 3–7 Section 3.3 Extra Practice BLM 3–8 Section 3.3 Math Link	• tracing paper • ruler • protractor • compass • Mira (optional)
3.4 Area of a Parallelogram • 80–100 minutes	**Essential:** 1–3, 5, 7, 9, 11, Math Link **Typical:** 1–3, 5, 7, 9, 11–18, Math Link **Extension/Enrichment:** 1, 2, 14–16, 18–20, Math Link	Master 8 Centimetre Grid Paper BLM 3–1 Chapter 3 Self Assessment BLM 3–9 Section 3.4 Extra Practice BLM 3–10 Section 3.4 Math Link	• ruler • scissors • tape
3.5 Area of a Triangle • 80–100 minutes	**Essential:** 1 *or* 3, 2, 4, 6, 8, 10, 12, Math Link **Typical:** 1 *or* 3, 2, 4, 6, 8, 10, 11, 13–15, Math Link **Extension/Enrichment:** 1 *or* 3, 2, 14–19, Math Link	Master 5 Tangram Master 8 Centimetre Grid Paper BLM 3–1 Chapter 3 Self-Assessment BLM 3–11 Section 3.5 Extra Practice BLM 5–12 Section 3.5 Math Link	• ruler • scissors
Chapter 3 Review • 40–50 minutes	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	BLM 3–1 Chapter 3 Self-Assessment BLM 3–3 Section 3.1 Extra Practice BLM 3–5 Section 3.2 Extra Practice BLM 3–7 Section 3.3 Extra Practice BLM 3–9 Section 3.4 Extra Practice BLM 3–11 Section 3.5 Extra Practice	• ruler • right triangle • compass • protractor
Chapter 3 Practice Test • 40–50 minutes	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 3, 4, 6–11	BLM 3–1 Chapter 3 Self-Assessment BLM 3–13 Chapter 3 Test	• ruler • right triangle • compass • protractor
Chapter 3 Wrap It Up! • 40–50 minutes		Master 1 Project Rubric BLM 3–4 Section 3.1 Math Link BLM 3–6 Section 3.2 Math Link BLM 3–8 Section 3.3 Math Link BLM 3–9 Section 3.4 Math Link BLM 3–14 Chapter 3 Wrap It Up!	• ruler • right triangle • compass • protractor

Chapter 3 Planning Chart (continued)

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter 3 Math Games • 40–50 minutes		Master 8 Centimetre Grid Paper	• ruler • tracing paper
Chapter 3 Challenge in Real Life • 60–75 minutes		Master 1 Project Rubric BLM 3–15 Pool Table BLM 3–16 Chapter 3 *MathLinks 7* Student Resource Answers BLM 3–17 Chapter 3 BLM Answers	• ruler • compass or right triangle (optional) • protractor

Chapter 3 Assessment Planner

Assessment Options	Type of Assessment	Assessment Tool
Chapter Opener	Assessment *as* Learning (TR pages i, 81)	BLM 3–1 Chapter 3 Self-Assessment Chapter 3 Foldable
3.1 Parallel and Perpendicular Line Segments	Assessment *as* Learning (TR pages 84, 86, 88) Assessment *for* Learning (TR pages 84, 87, 88)	Master 2 Two Stars and One Wish Math Learning Log (TR page 88) BLM 3–1 Chapter 3 Self-Assessment
3.2 Draw Perpendicular Bisectors	Assessment *as* Learning (TR pages 90, 92, 93) Assessment *for* Learning (TR pages 91, 92, 93)	Math Learning Log (TR page 93) BLM 3–1 Chapter 3 Self-Assessment
3.3 Draw Angle Bisectors	Assessment *as* Learning (TR pages 95, 97, 99) Assessment *for* Learning (TR pages 96, 98, 99)	Math Learning Log (TR page 99) BLM 3–1 Chapter 3 Self-Assessment
3.4 Area of a Parallelogram	Assessment *as* Learning (TR pages 102, 104, 106) Assessment *for* Learning (TR pages 103, 105, 107)	Math Learning Log (TR page 106) BLM 3–1 Chapter 3 Self-Assessment
3.5 Area of a Triangle	Assessment *as* Learning (TR pages 110, 113, 115) Assessment *for* Learning (TR pages 111, 114, 115)	Math Learning Log (TR page 115) BLM 3–1 Chapter 3 Self-Assessment
Chapter 3 Review	Assessment *for* Learning (TR page 116) Assessment *as* Learning (TR page 117)	Math Learning Log (TR page 117) BLM 3–1 Chapter 3 Self-Assessment
Chapter 3 Practice Test	Assessment *as* Learning (TR page 118) Assessment *of* Learning (TR page 119)	BLM 3–1 Chapter 3 Self-Assessment BLM 3–13 Chapter 3 Test
Chapter 3 Wrap It Up!	Assessment *of* Learning (TR page 118a)	Master 1 Project Rubric
Chapter 3 Math Games	Assessment *for* Learning (TR page 120)	
Chapter 3 Challenge in Real Life	Assessment *for* Learning (TR page 120a) Assessment *of* Learning (TR page 120a)	Master 1 Project Rubric

You may wish to use one or more of the following materials to help you assess student readiness for Chapter 3.

Assessment *for* Learning	Supported Learning
Method 1: Have students develop a journal entry to explain what they personally know about measuring angles and determining area. **Method 2:** Have students complete **BLM 3–2 Geometry and Measurement** to check their conceptual understanding. Remind students that you are looking for the scope of their knowledge.	• Students who require reinforcement of prerequisite skills may wish to complete the Get Ready materials available in the *MathLinks 7 Workbook* and at the **www.mathlinks7.ca** book site.

Chapter Opener

Suggested Timing
20–30 minutes

Materials
- ruler
- stapler
- notebook paper

Blackline Masters
BLM 3–1 Chapter 3 Self-Assessment

Key Words

parallel
perpendicular
perpendicular bisector
angle bisector
parallelogram
base
height

Supported Learning

Meeting the Needs of All Learners

- As an option for the Math Link, have students research Doug J. Cardinal, a famous architect from a First Nations community in Alberta called Saddle Lake Nation. You may want students to study his designs and then create their own designs using geometric shapes. Alternatively, have a geometric scavenger hunt where students find items related to the various concepts.
- Discuss where the runway/airport is in relation to students' community and why that location was chosen. For example, the runway in Pangnirtung goes through the centre of town. Students could research the runways found in Ottawa, Winnipeg, Edmonton, and Yellowknife, which are gateway cities to the North.

WWW Web Link

For a site where students can read a biography of Douglas J. Cardinal and see examples of his designs, go to **www.mathlinks7.ca** and follow the links.

What's the Math?

In this chapter, students work with lines, angles, and areas of triangles and parallelograms. They start by identifying and drawing parallel and perpendicular line segments. Students then develop skills in constructing perpendicular bisectors and angle bisectors. Finally, students apply these skills to help them develop formulas for calculating the areas of parallelograms and triangles.

Activity Planning Notes

Begin this section by having a class discussion about the things that need to be considered when designing runways at airports (e.g., making sure that planes do not crash into each other, that there are enough runways, that there is enough room for the planes to take off safely, etc.). Discuss how when looking at runways, some lines look like they are side by side and some at right angles. Discuss with students how different kinds of lines can form shapes on an airport runway.

Have students work independently or in pairs to identify common shapes (triangles, parallelograms, rectangles, etc.) formed by runways at an airport. Ask students:
- How do you know what the shape is?
- What are the characteristics of each shape?

Math Link

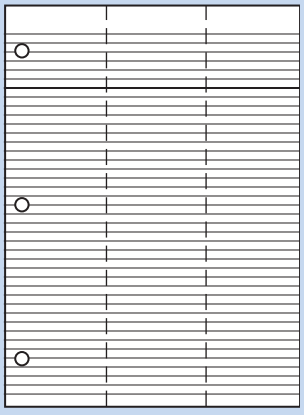
Ask students if any have flown or if they have seen aerial pictures of airports. Discuss the things they noticed about their experiences or about the pictures they have seen. Brainstorm why most municipal airports have more than one runway (e.g., volume of traffic, wind direction, etc.), and discuss some of the problems associated with runways that cross each other. Tell students that they will design their own airport runway system throughout this chapter. Note that students will need to complete the Math Links in each section of the chapter in order to do the Wrap It Up! on page 119.

FOLDABLES™
Study Tool

Have students make the Foldable in the student resource to keep track of the information in the chapter. Students could have an 8 × 10 envelope or clear plastic folder in their binder to hold these items; they could also be displayed around the room.

You may wish to have students keep track of Key Words using a design specifically for that purpose. Students can make the following Vocabulary Trifold Foldable and write vocabulary terms in the left-hand column. The right-hand column can be folded over the middle column to create a self-check study guide.

Step 1 Fold a sheet of notebook paper lengthwise into three equal parts.



Step 2 Write each key term in the left-hand column.

Step 3 Use the amount of space needed to place a definition and a visual in the centre column. If there are a lot of words, or long definitions, make a second trifold.

Step 4 Students fold the right-hand column over the middle column, read the term, tell themselves the definition, and then open the fold to see if they are correct.

Assessment *as* Learning	Supported Learning
Chapter 3 Foldable As students work on each section in Chapter 3, have them keep track of any problems they are having under the What I Need to Work On tab in their chapter Foldable.	• As students complete each section, have them review the list of items they need to work on and then have them check off any that have been handled.

Supported Learning

ESL

- Students may need to have the words *airport* and *area* explained to them.

ESL and Memory

- Students will benefit from creating the vocabulary/picture Foldable for the Key Words. Matching a picture and/or symbol with each Key Word and its definition helps students consolidate their understanding of the vocabulary.

ESL and Language

- Write new vocabulary on study cards and display them in the classroom on a math word wall.

Learning Style

- Creating the Foldable is a good activity for concrete, kinesthetic, and visual learners.
- Students who are highly visual will benefit from generous use of diagrams and visual aids in this chapter to help ensure that they have access to the information.

Common Errors

- Students may have difficulty visualizing the different shapes in the runways.

R_x You may wish to provide students with copies of pictures of airports with the different shapes highlighted.

3.1 Parallel and Perpendicular Line Segments

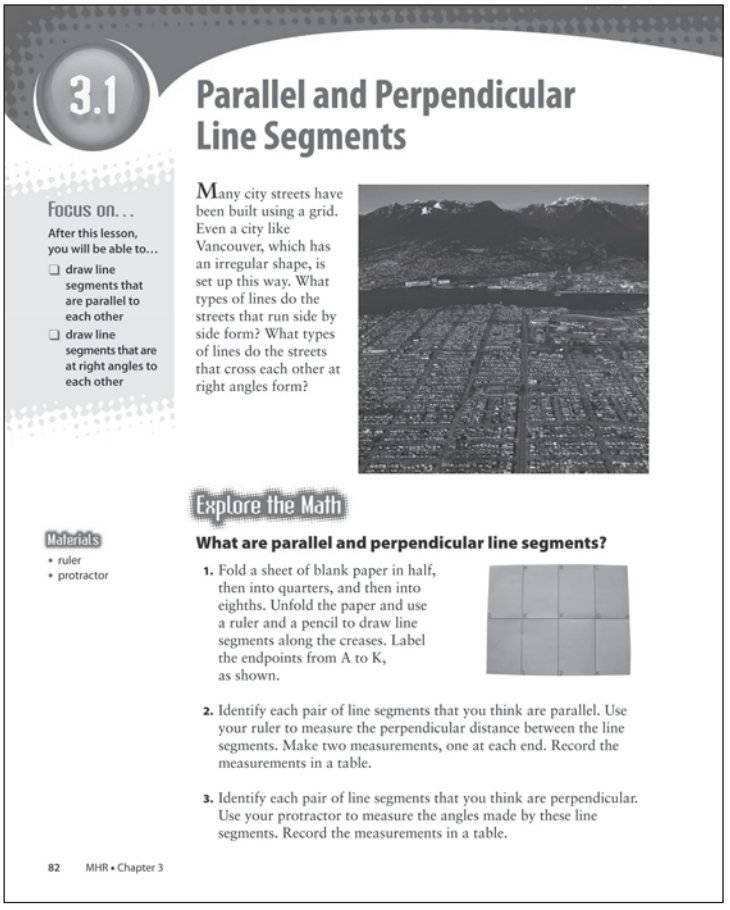
Suggested Timing
80–100 minutes

Materials
- ruler
- protractor
- unlined paper
- right triangle (optional)
- Mira (optional)

Blackline Masters
Master 2 Two Stars and One Wish
BLM 3–1 Chapter 3 Self-Assessment
BLM 3–3 Section 3.1 Extra Practice
BLM 3–4 Section 3.1 Math Link

Mathematical Processes

- [ ] Communication
- [x] Connections
- [ ] Mental Mathematics and Estimation
- [ ] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



3.1 Parallel and Perpendicular Line Segments

Focus on...
After this lesson, you will be able to...
- draw line segments that are parallel to each other
- draw line segments that are at right angles to each other

Many city streets have been built using a grid. Even a city like Vancouver, which has an irregular shape, is set up this way. What types of lines do the streets that run side by side form? What types of lines do the streets that cross each other at right angles form?

Explore the Math

Materials
- ruler
- protractor

What are parallel and perpendicular line segments?

1. Fold a sheet of blank paper in half, then into quarters, and then into eighths. Unfold the paper and use a ruler and a pencil to draw line segments along the creases. Label the endpoints from A to K, as shown.

2. Identify each pair of line segments that you think are parallel. Use your ruler to measure the perpendicular distance between the line segments. Make two measurements, one at each end. Record the measurements in a table.

3. Identify each pair of line segments that you think are perpendicular. Use your protractor to measure the angles made by these line segments. Record the measurements in a table.

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Specific Outcomes

SS3 Perform geometric constructions, including:
- perpendicular line segments
- parallel line segments
- perpendicular bisectors
- angle bisectors.

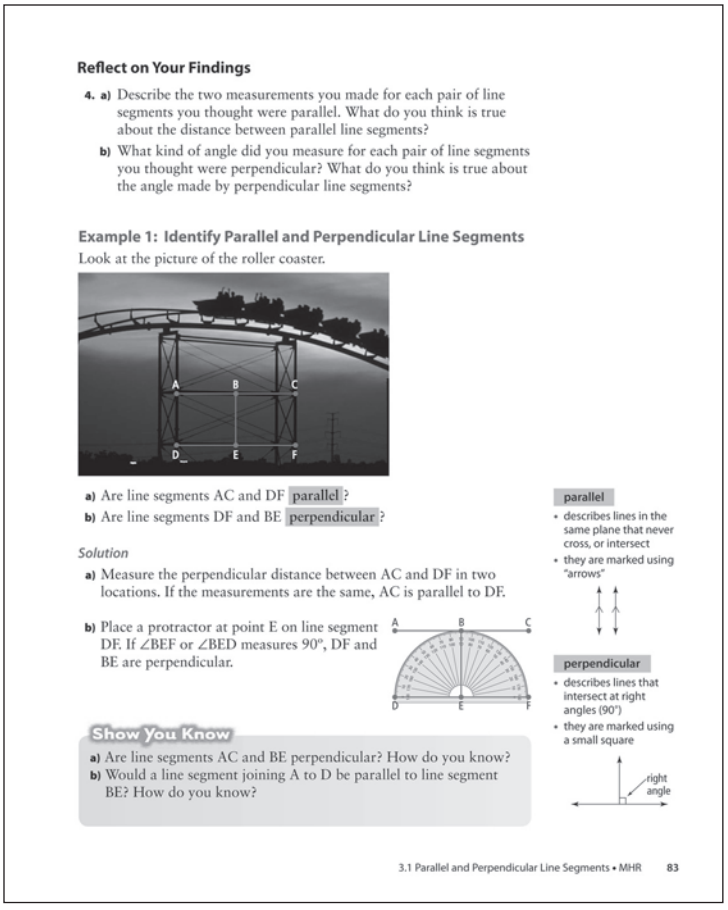
Warm-Up

1. Solve $3 \times \$4.50 + 4 \times \$0.75 + 10 \times \$0.50$.
2. Write a problem for the expression $35.7 \div 7 + 12$.
3. Add brackets to make this statement true: $8 + 16 \div 4 = 6$.
4. Estimate and then calculate $28.52 \div 3.1$.
5. Estimate and then calculate $767.75 \div 92.5$.

Mental Math

For #6 to #9, place the decimal in the correct place without calculating. Show your thinking.

6. $45.45 \div 9 = 505$
7. $6.88 \div 0.4 = 172$
8. $7.2 \times 11.5 = 828$
9. $6.7 \times 3.9 = 2613$



Reflect on Your Findings

4. a) Describe the two measurements you made for each pair of line segments you thought were parallel. What do you think is true about the distance between parallel line segments?

b) What kind of angle did you measure for each pair of line segments you thought were perpendicular? What do you think is true about the angle made by perpendicular line segments?

Example 1: Identify Parallel and Perpendicular Line Segments

Look at the picture of the roller coaster.

a) Are line segments AC and DF parallel?

b) Are line segments DF and BE perpendicular?

Solution

a) Measure the perpendicular distance between AC and DF in two locations. If the measurements are the same, AC is parallel to DF.

b) Place a protractor at point E on line segment DF. If ∠BEF or ∠BED measures 90°, DF and BE are perpendicular.

Show You Know

a) Are line segments AC and BE perpendicular? How do you know?

b) Would a line segment joining A to D be parallel to line segment BE? How do you know?

parallel
- describes lines in the same plane that never cross, or intersect
- they are marked using "arrows"

perpendicular
- describes lines that intersect at right angles (90°)
- they are marked using a small square

Activity Planning Notes

Use the illustration in the student resource to discuss how grids are used when planning cities. Ensure that students can identify parallel and perpendicular lines, as well as understand the concept of right angles.

Explore the Math

Have students work independently or in pairs to identify parallel line segments and measure the perpendicular distance between the segments, and to identify perpendicular line segments and measure the angles made by these lines. Allow students to compare their answers with a classmate and note any differences. For #2 and #3, have students complete tables such as the ones shown in the margin on this page.

Answers

Warm-Up

1. $\$13.50 + \$3.00 + \$5.00 = \$21.50$

2. Answers will vary. For example: There are seven small banners a total of 35.7 m long and a gigantic banner 12 m long. What is the length of one small banner and the gigantic one?

3. $(8 + 16) \div 4 = 6$

4. Estimate: $27 \div 3 = 9$. Calculate: 9.2

5. Estimate: $720 \div 90 = 8$; $810 \div 9 = 9$. It will be between these two numbers but closer to 8. Calculate: 8.3

6. $45 \div 9 = 5$; 5.05 **7.** $7 \div 0.5 = 14$; 17.2

8. $7 \times 11 = 77$; 82.8 **9.** $7 \times 4 = 28$; 26.13

Explore the Math

2. Answers may vary. For example:

Parallel Line Segments	Measurement 1	Measurement 2
CK and BJ	7 cm	7 cm
AI and BJ	7 cm	7 cm
AI and CK	14 cm	14 cm
AE and BF	7 cm	7 cm
BF and CG	7 cm	7 cm
EI and FJ	7 cm	7 cm
FJ and GK	7 cm	7 cm
AE and CG	14 cm	14 cm
EI and GK	14 cm	14 cm

3. Answers may vary. For example:

Perpendicular Line Segments	Measurement 1	Measurement 2
AI and DF	90°	90°
BJ and EG	90°	90°
CK and FH	90°	90°

Additional perpendicular line segments may be chosen.

4. a) The two measurements were the same for each pair. The distance between parallel line segments is the same at either end.

b) 90°. The angle is always 90°.

Supported Learning

Meeting the Needs of All Learners

- Bring in a map of your community or the gateway city used by the community in which you live. For example, the Baffin communities typically use Ottawa to reach the rest of Canada. The Kivalliq region (the area on the west side of Hudson Bay) uses Winnipeg. Have students look for the grid lines.

Answers

Show You Know: Example 1

a) Yes. ∠CBE and ∠ABE equal 90°.

b) Yes. The perpendicular distance between AB and DE is equal.

Supported Learning

Learning Style

- Kinesthetic learners will benefit from the paper folding activity.

ESL

- Assist English language learners in understanding the terms *irregular shape, right angles,* and *creases*. Explain that right angles are angles at 90°.

ESL and Language

- English language learners and students who struggle with written expression can improve their verbal skills and vocabulary through group discussion. Encourage students to orally explain the reasoning in their answers.

Memory

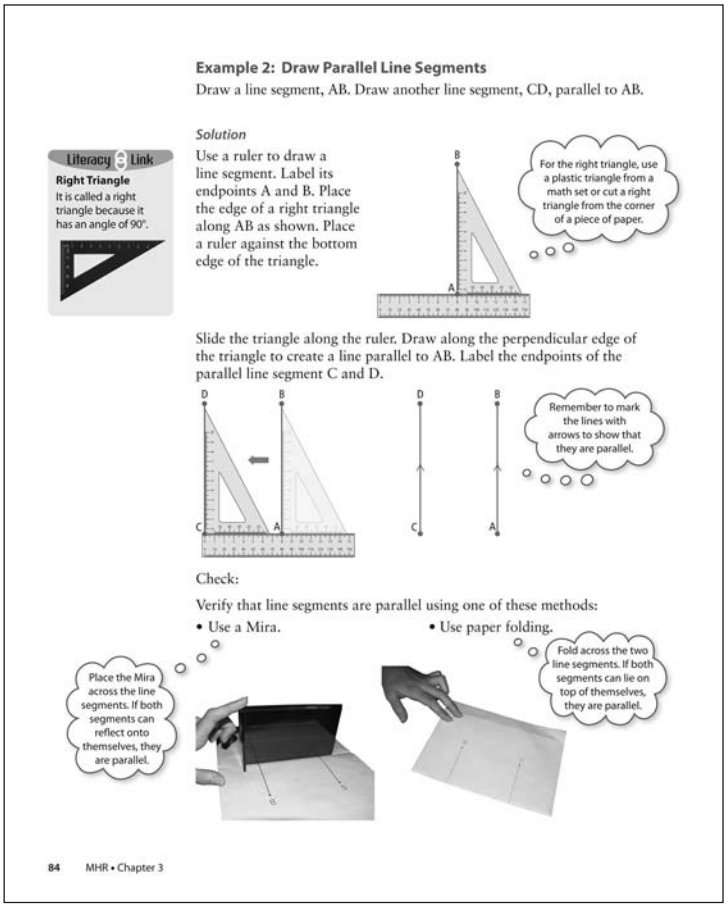
- For students who mix up parallel and perpendicular, help them with this memory trick: the word *parallel* has two *l*s that look like parallel line segments.

Motor

- Students may find it so challenging to draw the table for #2 of the Explore the Math that it distracts them from the intended learning. Allow students to create their table using a computer.

Meeting the Needs of All Learners

- Have students use cooperative learning groups to explore the math. Allow students to use concrete examples.



Example 2: Draw Parallel Line Segments

Draw a line segment, AB. Draw another line segment, CD, parallel to AB.

Solution

Literacy Link

Right Triangle

It is called a right triangle because it has an angle of 90°.

Use a ruler to draw a line segment. Label its endpoints A and B. Place the edge of a right triangle along AB as shown. Place a ruler against the bottom edge of the triangle.

For the right triangle, use a plastic triangle from a math set or cut a right triangle from the corner of a piece of paper.

Slide the triangle along the ruler. Draw along the perpendicular edge of the triangle to create a line parallel to AB. Label the endpoints of the parallel line segment C and D.

Remember to mark the lines with arrows to show that they are parallel.

Check:

Verify that line segments are parallel using one of these methods:

- Use a Mira.
- Use paper folding.

Place the Mira across the line segments. If both segments can reflect onto themselves, they are parallel.

Fold across the two line segments. If both segments can lie on top of themselves, they are parallel.

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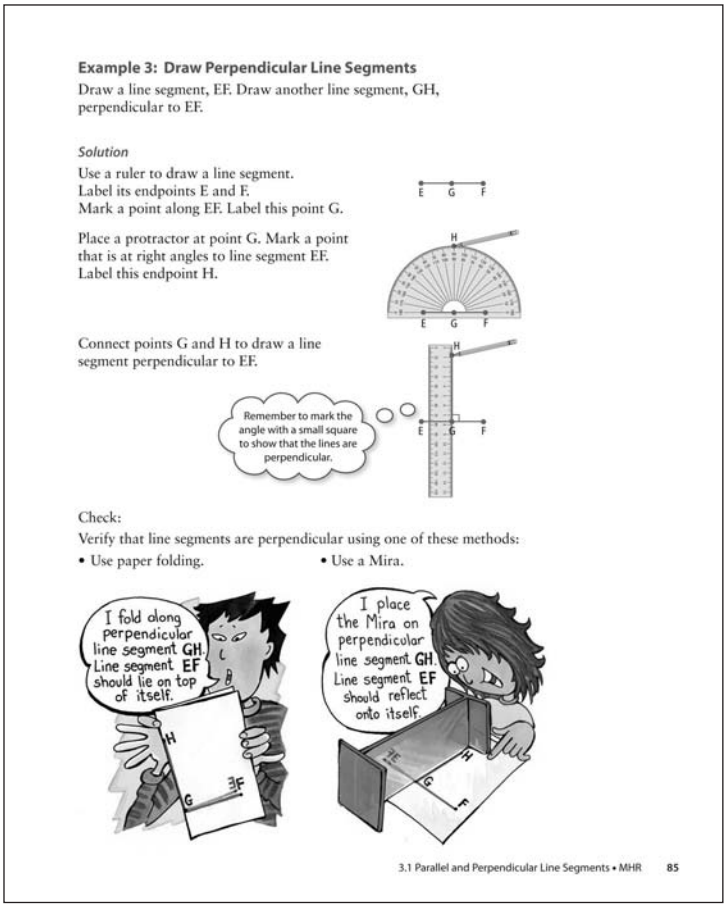
Discuss the findings as a class, and then fill in the table on the board. Students should correct their notebook copy. As a class, generalize the findings into the properties of parallel line segments (never crossing) and perpendicular line segments (at right angles to each other).

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss the features of parallel and perpendicular line segments. During this process, they are generalizing what they have learned during the Explore the Math section.	• Reinforce the differences between the two kinds of line segments, and the distinctive features of each one. • Encourage students to identify parallel and perpendicular line segments in the classroom to consolidate their understanding.

For Example 1, ensure that students use rulers and protractors appropriately to identify the line segments. Point out the definitions in the margin of student resource page 83 to emphasize how to mark parallel or perpendicular lines (i.e., arrows or small square).

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know on page 83 related to Example 1.	• You may wish to provide additional questions for students who would benefit from them. For example: **a)** Are the wall and ceiling perpendicular? How do you know? (Yes. They meet at a 90° angle.) **b)** Are the top and bottom edges of the board parallel or perpendicular? How do you know? (Parallel. They never cross.) Coach students through a). Have them try b) on their own.



Example 3: Draw Perpendicular Line Segments

Draw a line segment, EF. Draw another line segment, GH, perpendicular to EF.

Solution

Use a ruler to draw a line segment.
Label its endpoints E and F.
Mark a point along EF. Label this point G.

Place a protractor at point G. Mark a point that is at right angles to line segment EF. Label this endpoint H.

Connect points G and H to draw a line segment perpendicular to EF.

Remember to mark the angle with a small square to show that the lines are perpendicular.

Check:

Verify that line segments are perpendicular using one of these methods:

- Use paper folding.
- Use a Mira.

I fold along perpendicular line segment GH. Line segment EF should lie on top of itself.

I place the Mira on perpendicular line segment GH. Line segment EF should reflect onto itself.

3.1 Parallel and Perpendicular Line Segments • MHR 85

Supported Learning

Learning Style

- Kinesthetic learners will benefit from working through the three examples using protractors, rulers, and right triangles.

ESL

- Some English language learners may not be familiar with the word *verify*. Have them add this word to their translation dictionary.
- English language learners may require support when reading through the directions in Examples 2 and 3.

Meeting the Needs of All Learners

- Provide students with examples of parallel and perpendicular lines in the environment. Look in the classroom, but also consider sleds, siding on buildings, and vehicle tracks.

Example 2 demonstrates how to draw parallel line segments. The thought bubbles beside the solution provide support to students as they follow the example. Ensure that students understand how to use one of the two methods shown to verify the line segments are parallel. You may wish to demonstrate one or both methods in front of the class.

You may also wish to show students two quick ways to draw parallel line segments:

- Hold a ruler on a piece of paper and draw a line along both edges of the ruler.
- Use a ruler to trace over two lines on lined paper.

Example 3 demonstrates how to draw perpendicular line segments. Again, you may wish to demonstrate one or both of the verification methods shown.

Answers

Communicate the Ideas

1. a) Answers may vary. For example: highways, fingers, street poles, pencils in a box, floors of a building

b) Answers may vary. For example: street light and road, ceiling and wall, building and street, side and bottom of a book, or person and floor

2. a) Parallel. Line segments will never intersect.

b) Perpendicular. Line segments intersect at 90°.

c) Neither. Line segments intersect, but not at 90°.

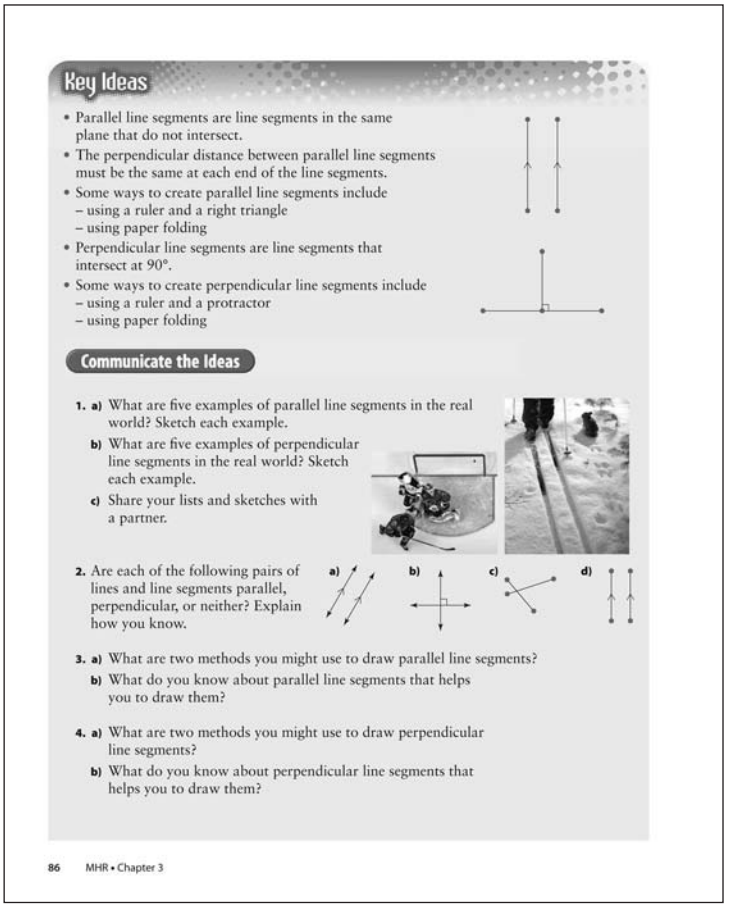
d) Parallel. Line segments will never intersect.

3. a) Answers may vary. For example: Use the two sides of a ruler or use parallel lines on graph paper.

b) Answers may vary. For example: They are the same distance apart at either end.

4. a) Answers may vary. For example: Use a ruler and protractor or use intersecting lines on graph paper.

b) Answers may vary. For example: They intersect at 90°.



Key Ideas

- Parallel line segments are line segments in the same plane that do not intersect.
- The perpendicular distance between parallel line segments must be the same at each end of the line segments.
- Some ways to create parallel line segments include
  – using a ruler and a right triangle
  – using paper folding
- Perpendicular line segments are line segments that intersect at 90°.
- Some ways to create perpendicular line segments include
  – using a ruler and a protractor
  – using paper folding

Communicate the Ideas

1. a) What are five examples of parallel line segments in the real world? Sketch each example.

b) What are five examples of perpendicular line segments in the real world? Sketch each example.

c) Share your lists and sketches with a partner.

2. Are each of the following pairs of lines and line segments parallel, perpendicular, or neither? Explain how you know.

a) b) c) d)

3. a) What are two methods you might use to draw parallel line segments?

b) What do you know about parallel line segments that helps you to draw them?

4. a) What are two methods you might use to draw perpendicular line segments?

b) What do you know about perpendicular line segments that helps you to draw them?

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Supported Learning

ESL and Language

- Students who struggle with reading and writing can communicate their understanding by using a combination of diagrams, oral explanations, and demonstrations (drawing parallel and perpendicular line segments).

ESL

- Some English language learners may need assistance with the words *intersect, sketch,* and *neither*.
- Partner English language learners with students who can provide them with good one-on-one discussion. This facilitates comprehension of vocabulary.

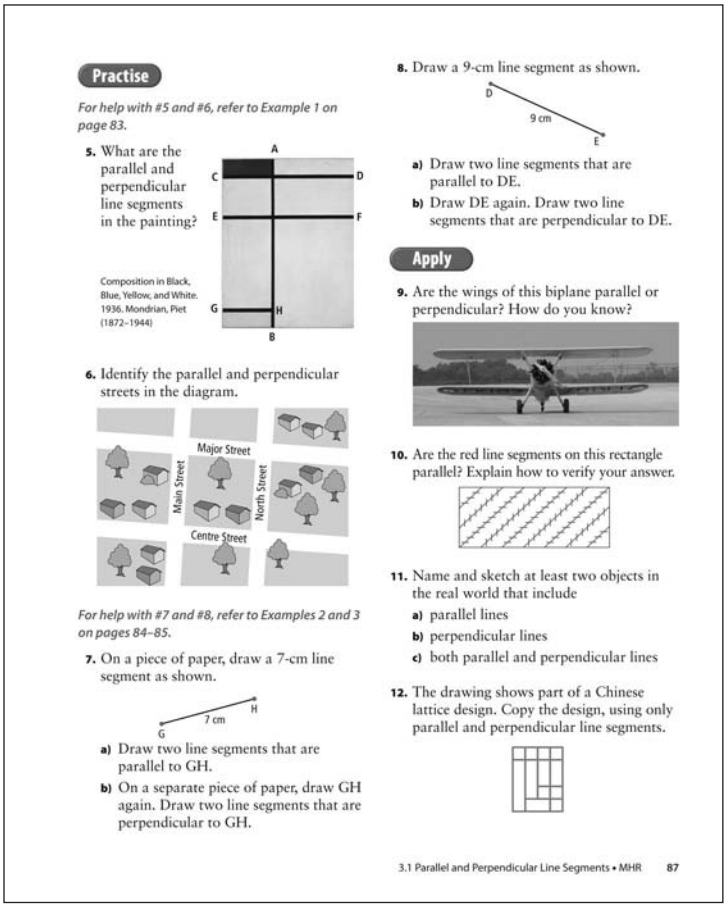
Key Ideas

Students could prepare their own list of Key Ideas and put it in their chapter Foldable. An intention of this section is to show students how to identify and draw parallel and perpendicular line segments.

Communicate the Ideas

Use students' responses to assess their understanding of parallel and perpendicular line segments. A brief discussion summarizing major points after students have attempted the questions might be a good idea.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Rather than asking students to complete all four questions, you may wish to assign #3 and #4 and have them choose one of the other questions. This could be done individually or as a class.	• This Communicate the Ideas is intended to allow students to explain their understanding of parallel and perpendicular line segments. • Have students write the real-world examples in their chapter Foldable and share their responses with the rest of the class. • Work with the class to develop criteria for judging each answer. For example, criteria for #1 might include – identifies and sketches examples of parallel line segments – identifies and sketches examples of perpendicular line segments • Use **Master 2 Two Stars and One Wish** to have students critique other students' work. This master allows them to write two things they like and one thing they would improve.



Practise

For help with #5 and #6, refer to Example 1 on page 83.

5. What are the parallel and perpendicular line segments in the painting?

Composition in Black, Blue, Yellow, and White. 1936. Mondrian, Piet (1872–1944)

6. Identify the parallel and perpendicular streets in the diagram.

Major Street, Main Street, North Street, Centre Street

For help with #7 and #8, refer to Examples 2 and 3 on pages 84–85.

7. On a piece of paper, draw a 7-cm line segment as shown.

a) Draw two line segments that are parallel to GH.

b) On a separate piece of paper, draw GH again. Draw two line segments that are perpendicular to GH.

8. Draw a 9-cm line segment as shown.

a) Draw two line segments that are parallel to DE.

b) Draw DE again. Draw two line segments that are perpendicular to DE.

Apply

9. Are the wings of this biplane parallel or perpendicular? How do you know?

10. Are the red line segments on this rectangle parallel? Explain how to verify your answer.

11. Name and sketch at least two objects in the real world that include

a) parallel lines

b) perpendicular lines

c) both parallel and perpendicular lines

12. The drawing shows part of a Chinese lattice design. Copy the design, using only parallel and perpendicular line segments.

3.1 Parallel and Perpendicular Line Segments • MHR 87

Common Errors

- Students may incorrectly measure the distance between parallel line segments. This will cause significant errors in their drawings.

R_x Encourage students to accurately measure the distance at two different places to ensure that the line segments are parallel. Remind students that they can also use a Mira or paper folding technique to verify that the line segments are parallel.

- When identifying parallel and perpendicular line segments, students may assume the distances between the line segments are equal or that the line segments form a right angle.

R_x Encourage students to use a ruler to measure the distance between the line segments to confirm that they are parallel and to use a protractor or right triangle to measure the angle to confirm it is a right angle.

Supported Learning

Learning Style

- Kinesthetic, concrete, and visual learners will appreciate the hands-on nature of actually drawing the perpendicular and parallel line segments. This activity makes the concepts more concrete.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1 *or* 2, 3–5, 7, 9, 11, Math Link
Typical	1 *or* 2, 3–5, 7–13, Math Link
Extension/Enrichment	1 *or* 2, 3, 4, 13–16, Math Link

Practise

For #7a), ensure that students measure the perpendicular distance twice (at each end of the parallel line segment). For #7b), ensure that students measure the angle accurately or use a right-angle triangle.

Assessment *for* Learning	Supported Learning
Practise Have students do #5 and #7. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #5 will need additional coaching with Example 1. Work with them to correct their answer to #5, and then assign #6. • Students who have problems with #7 will need additional coaching with Examples 2 and 3. Work with them to correct their answer to #7, and then assign #8. • Allow students to work on their own. Check back with students several times to make sure that they understand the concepts.

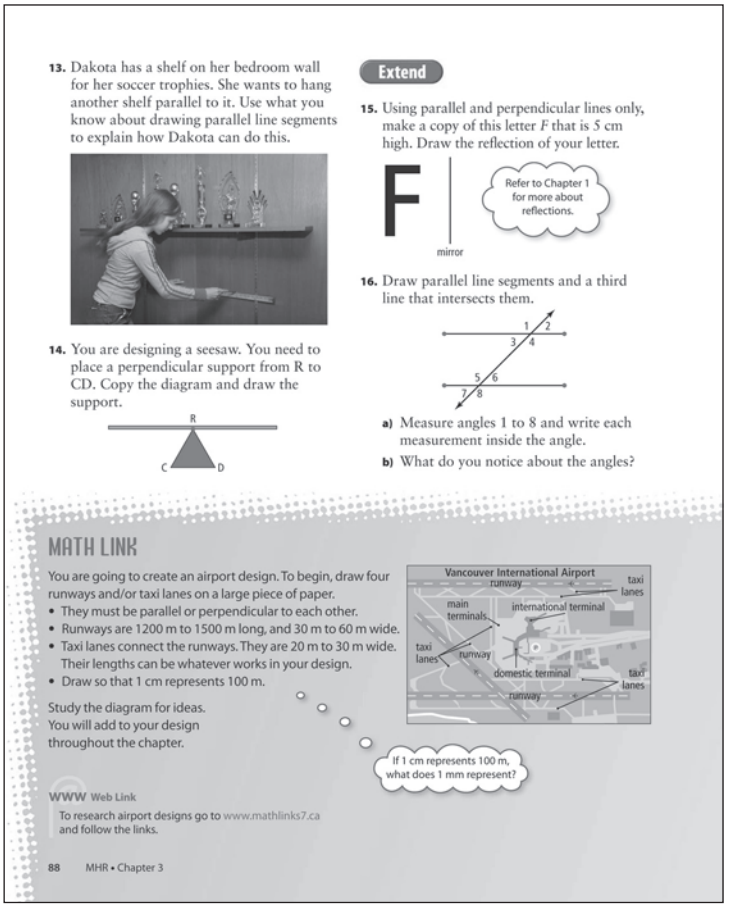
Supported Learning

- **BLM 3–3 Section 3.1 Extra Practice** provides additional reinforcement for students who need it.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • How do you know that two line segments are parallel? • How do you know that two line segments are perpendicular?	• Encourage students to recognize the importance of parallel and perpendicular line segments in everyday life. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • You may also wish to have students review the part related to section 3.1 in **BLM 3–1 Chapter 3 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Math Link

The Math Link on page 88 provides students with an opportunity to draw parallel and perpendicular lines while creating an airport runway design. Make sure students understand that they should research runway designs before they begin their own designs.



13. Dakota has a shelf on her bedroom wall for her soccer trophies. She wants to hang another shelf parallel to it. Use what you know about drawing parallel line segments to explain how Dakota can do this.

14. You are designing a seesaw. You need to place a perpendicular support from R to CD. Copy the diagram and draw the support.

Extend

15. Using parallel and perpendicular lines only, make a copy of this letter *F* that is 5 cm high. Draw the reflection of your letter.

Refer to Chapter 1 for more about reflections.

16. Draw parallel line segments and a third line that intersects them.

a) Measure angles 1 to 8 and write each measurement inside the angle.

b) What do you notice about the angles?

MATH LINK

You are going to create an airport design. To begin, draw four runways and/or taxi lanes on a large piece of paper.
- They must be parallel or perpendicular to each other.
- Runways are 1200 m to 1500 m long, and 30 m to 60 m wide.
- Taxi lanes connect the runways. They are 20 m to 30 m wide. Their lengths can be whatever works in your design.
- Draw so that 1 cm represents 100 m.

Study the diagram for ideas. You will add to your design throughout the chapter.

If 1 cm represents 100 m, what does 1 mm represent?

WWW Web Link
To research airport designs go to www.mathlinks7.ca and follow the links.

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Apply and Extend

For the Apply questions, students need to be able to apply their knowledge of parallel and perpendicular line segments to answer problems.

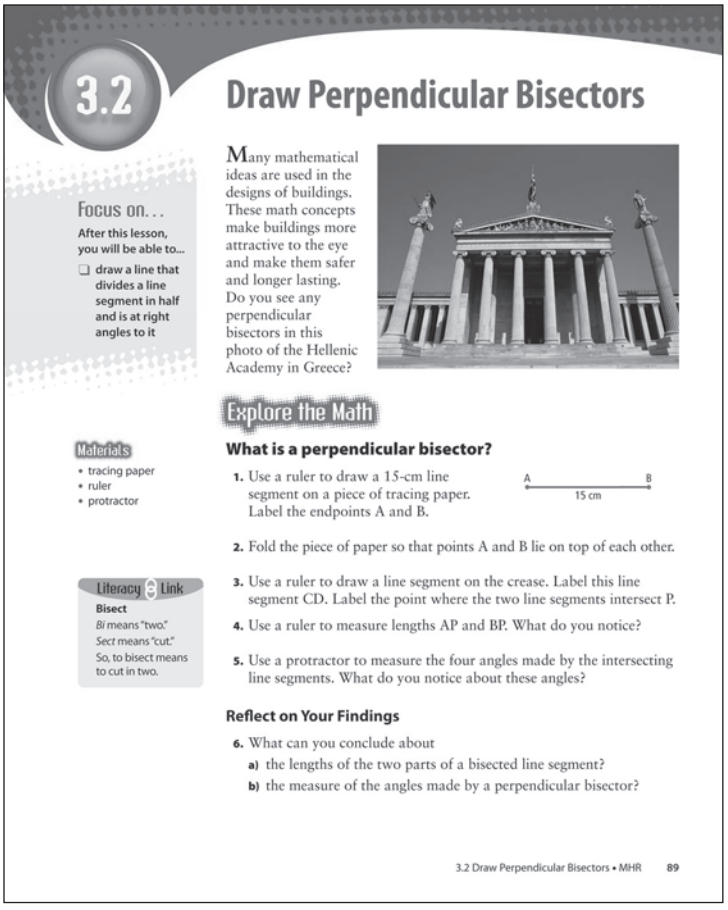
The Extend problems require significantly more analysis than those in the Apply section. For #15, refer students to Chapter 1 for more information about reflections.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 88 is intended to provide students with an opportunity to begin working on their airport runway design. This project will be developed as further Math Links are encountered in preparation for the chapter problem wrap-up titled Wrap It Up! on page 119.	• Make sure that all students do this Math Link, as they will work on the design they start here throughout the chapter and complete it during the Wrap It Up! • Have students use **BLM 3–4 Section 3.1 Math Link** to assist them with completing the activity.

WWW Web Link

When students use the Web Link, ask them to study the runway designs. Are some runways parallel and some perpendicular to others? Have them sketch what they notice. Go to **www.mathlinks7.ca** and follow the links.

3.2 Draw Perpendicular Bisectors



3.2

Draw Perpendicular Bisectors

Focus on...

After this lesson, you will be able to...

- draw a line that divides a line segment in half and is at right angles to it

Many mathematical ideas are used in the designs of buildings. These math concepts make buildings more attractive to the eye and make them safer and longer lasting. Do you see any perpendicular bisectors in this photo of the Hellenic Academy in Greece?

Explore the Math

Materials
- tracing paper
- ruler
- protractor

What is a perpendicular bisector?

1. Use a ruler to draw a 15-cm line segment on a piece of tracing paper. Label the endpoints A and B.

A — B
15 cm

2. Fold the piece of paper so that points A and B lie on top of each other.
3. Use a ruler to draw a line segment on the crease. Label this line segment CD. Label the point where the two line segments intersect P.
4. Use a ruler to measure lengths AP and BP. What do you notice?
5. Use a protractor to measure the four angles made by the intersecting line segments. What do you notice about these angles?

Literacy Link

Bisect
Bi means "two."
Sect means "cut."
So, to bisect means to cut in two.

Reflect on Your Findings

6. What can you conclude about
   a) the lengths of the two parts of a bisected line segment?
   b) the measure of the angles made by a perpendicular bisector?

3.2 Draw Perpendicular Bisectors • MHR 89

Suggested Timing
80–100 minutes

Materials
- tracing paper
- ruler
- protractor
- compass
- right triangle

Blackline Masters
BLM 3–1 Chapter 3 Self-Assessment
BLM 3–5 Section 3.2 Extra Practice
BLM 3–6 Section 3.2 Math Link

Mathematical Processes

- ☐ Communication
- ☑ Connections
- ☐ Mental Mathematics and Estimation
- ☐ Problem Solving
- ☑ Reasoning
- ☐ Technology
- ☑ Visualization

Specific Outcomes

SS3 Perform geometric constructions, including:
- perpendicular line segments
- parallel line segments
- perpendicular bisectors
- angle bisectors.

Warm-Up

1. Draw a line segment 5 cm long. Draw a line segment parallel to it and the same length.
2. Draw a line segment 8 cm long. Draw a line segment perpendicular to it and the same length.
3. Name an item with parallel line segments. Sketch the item.
4. Name an item with perpendicular line segments Sketch the item.
5. Estimate and then calculate 72×4.5.

Mental Math

For #6 to #9, place the decimal in the correct place without calculating. Show your thinking.

6. $183.23 \div 25.1 = 730$
7. $9.258 \div 0.6 = 1543$
8. $3.4 \times 5.8 = 1972$
9. $1.7 \times 7.8 = 1326$
10. Count by 10s from 230 to 300.

Answers

Warm-Up

1. For most students, the length of the line segments should be correct within 1 mm. Check that they are parallel.

2. For most students, the length of the line segments should be correct within 1 mm. Check that they are perpendicular.

3. Answers will vary. For example: railway tracks

4. Answers will vary. For example: the corners of a book

5. Estimate: $70 \times 4 = 280$; $70 \times 5 = 350$. The answer is between these two numbers. Calculate: 324

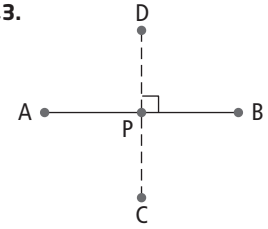
6. $200 \div 25 = 8$; 7.30 **7.** $9 \div 0.5 = 18$; 15.43

8. $3 \times 6 = 18$; 19.72 **9.** $2 \times 8 = 16$; 13.26

10. 230, 240, 250, 260, 270, 280, 290, 300

Explore the Math

2.,3.



4. AP = BP. They are both 7.5 cm.

5. They are all 90°.

6. a) They are equal to each other.
b) They are equal to 90°.

Supported Learning

Learning Style

- Kinesthetic and concrete learners will benefit from the hands-on Explore the Math activity. It is also a good activity for students who need the support of concrete materials.

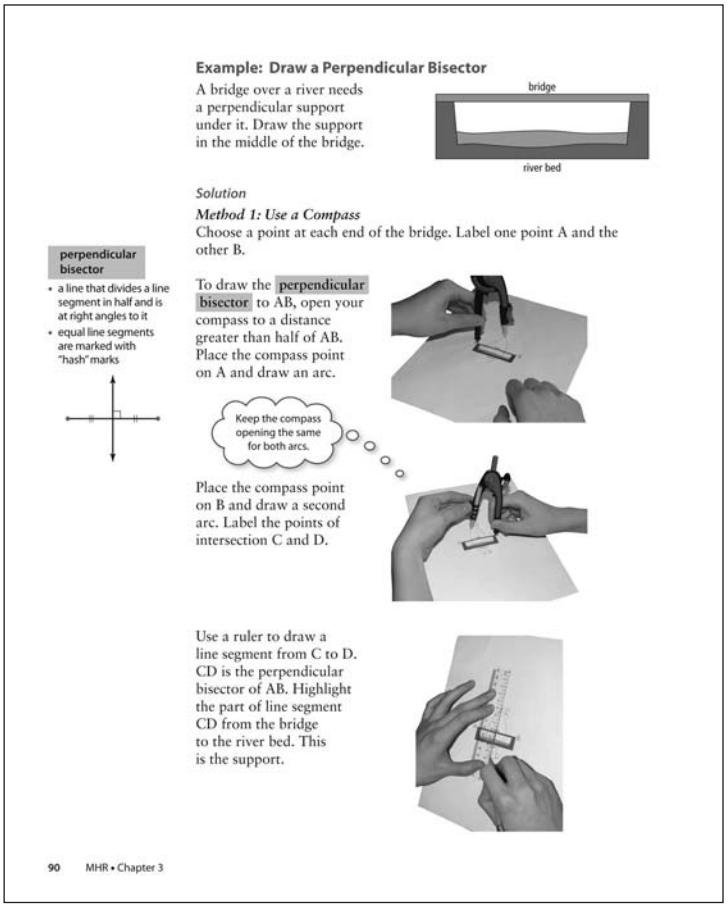
Motor

- Students with motor challenges may find it difficult to draw on tracing paper and then make the endpoints line up. Ensure that students have sufficient time to complete the activity.

Meeting the Needs of All Learners

- Use cooperative groups to explore, study, and discuss the concepts in this section.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss their conclusions. During this process, they are generalizing what they have learned during the Explore the Math section.	• Work with students to ensure that they measure the line segments and the angles accurately. • Have students discuss their conclusions. Ask them questions such as – Why do you think the lengths of the two parts of a bisected line segment are equal? – Do you think the angles made by a perpendicular bisector always measure 90°?



Example: Draw a Perpendicular Bisector

A bridge over a river needs a perpendicular support under it. Draw the support in the middle of the bridge.

bridge

river bed

Solution

Method 1: Use a Compass

Choose a point at each end of the bridge. Label one point A and the other B.

perpendicular bisector

- a line that divides a line segment in half and is at right angles to it
- equal line segments are marked with "hash" marks

To draw the perpendicular bisector to AB, open your compass to a distance greater than half of AB. Place the compass point on A and draw an arc.

Keep the compass opening the same for both arcs.

Place the compass point on B and draw a second arc. Label the points of intersection C and D.

Use a ruler to draw a line segment from C to D. CD is the perpendicular bisector of AB. Highlight the part of line segment CD from the bridge to the river bed. This is the support.

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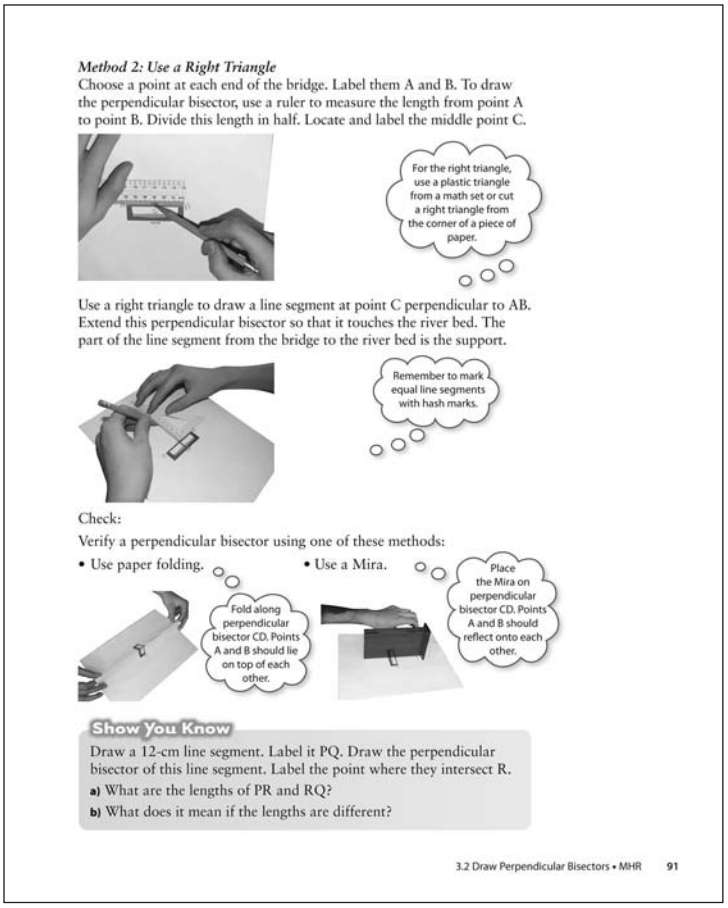
Activity Planning Notes

This section focuses on drawing perpendicular bisectors. Discuss the importance of perpendicular bisectors in architecture and building design. Review the Literacy Link in the margin on page 89, which explains the meaning of the word *bisect*. Look at the photo in the student resource and see if students can identify perpendicular bisectors in the building. You may want to mention that because of the way the photo was taken, the lines do not appear perfectly parallel or perpendicular, as they would be in real life.

Explore the Math

Students can work through the section on their own or in small groups. Walk among students and ensure that they are accurately measuring and drawing the required line segments.

After students have had a chance to work on this activity, collect line segment length and angle measurement data from the entire class and post it on the board. Have students discuss their findings. As a class, review the findings about the properties of a perpendicular bisector (i.e., a perpendicular line that divides a line segment in half and is at right angles to it).



Method 2: Use a Right Triangle

Choose a point at each end of the bridge. Label them A and B. To draw the perpendicular bisector, use a ruler to measure the length from point A to point B. Divide this length in half. Locate and label the middle point C.

For the right triangle, use a plastic triangle from a math set or cut a right triangle from the corner of a piece of paper.

Use a right triangle to draw a line segment at point C perpendicular to AB. Extend this perpendicular bisector so that it touches the river bed. The part of the line segment from the bridge to the river bed is the support.

Remember to mark equal line segments with hash marks.

Check:

Verify a perpendicular bisector using one of these methods:

- Use paper folding.
- Use a Mira.

Fold along perpendicular bisector CD. Points A and B should lie on top of each other.

Place the Mira on perpendicular bisector CD. Points A and B should reflect onto each other.

Show You Know

Draw a 12-cm line segment. Label it PQ. Draw the perpendicular bisector of this line segment. Label the point where they intersect R.

a) What are the lengths of PR and RQ?

b) What does it mean if the lengths are different?

3.2 Draw Perpendicular Bisectors • MHR 91

Answers

Show You Know: Example

a) 6 cm

b) You have not bisected the line segment.

Supported Learning

Learning Style, ESL, and Language

- Students with written or oral language difficulties often are more successful by learning from diagrams, so they should be encouraged to develop skill in this area. This section provides excellent opportunities to practise this skill.

ESL

- Some English language learners may be unfamiliar with the terms *perpendicular support, arc,* and *river bed* (with *bed* being out of context for many students).

Motor

- Students may need extra practice in using a compass. You may wish to have them use a virtual compass.

Meeting the Needs of All Learners

- For those students would benefit from it, take the time to identify each step needed to construct a perpendicular bisector. List the steps on the board or on a chart, and encourage students to follow the steps each time.

The Example demonstrates two techniques for drawing a perpendicular bisector. Ensure that students understand the importance of accurate length and angle measurements.

Assessment *for* Learning	Supported Learning
Example Have students do the Show You Know related to the Example.	• If the lengths of the final line segments are different in part b) of the Show You Know, have students use paper folding or a Mira to confirm the difference in length. Discuss that the line segment has not been bisected and problem solve to identify the error(s). • You may wish to provide additional questions for students who would benefit from them. For example: **a)** Draw a 16-cm line segment. Label it AB. Draw the perpendicular bisector of this line segment. Label the point where they intersect C. What are the lengths of AC and BC? (8 cm) **b)** Draw a 9-cm line segment. Label it ST. Draw the perpendicular bisector of this line segment. Label the point where they intersect U. What are the lengths of SU and TU? (4.5 cm) Coach students through a), and then have them do b) independently to make sure that they understand the concepts.

Common Errors

- Students may incorrectly divide the length to be bisected.

R_x When drawing the perpendicular with a right triangle, ensure that students take care that the triangle base is on the line to be bisected.

Key Ideas

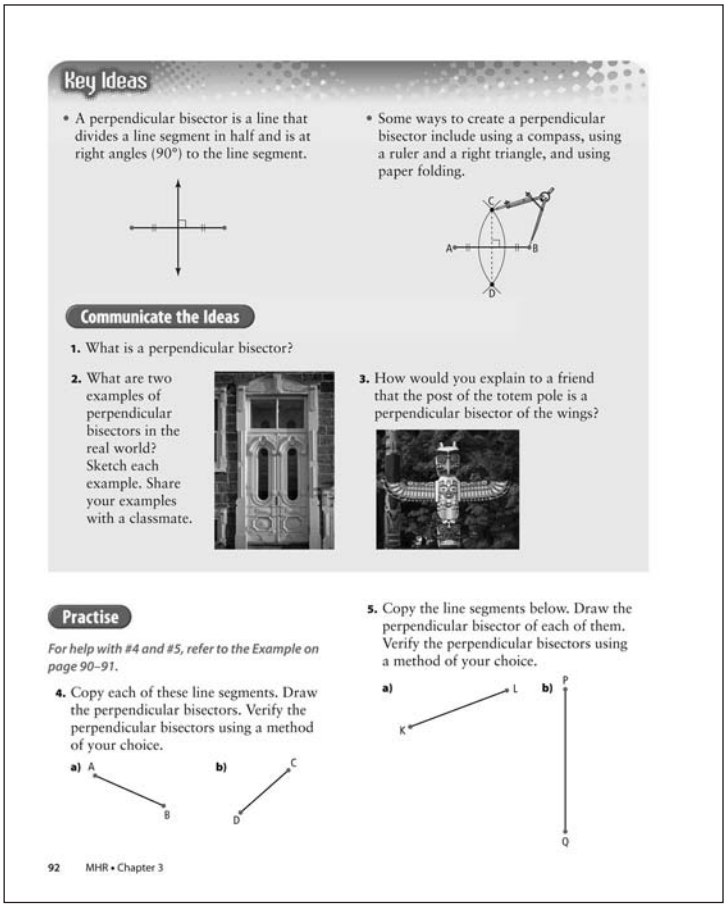
Students could prepare their own list of Key Ideas and put it in their chapter Foldable. An intention of this section is to develop the concept of perpendicular bisectors and to inform students of the different ways of creating a perpendicular bisector.

Answers

Communicate the Ideas

1. Answers may vary. For example: A line that divides a line segment in half and is at a 90° angle to it.
2. Answers may vary. For example: Cover of a booklet, a cupboard door
3. The post is at a 90° angle to the wings, and the lengths of the wings on either side of the post are equal.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–5, 8, Math Link
Typical	1–4, 6–9, Math Link
Extension/Enrichment	1–3, 7, 9, 10, Math Link



Key Ideas

- A perpendicular bisector is a line that divides a line segment in half and is at right angles (90°) to the line segment.
- Some ways to create a perpendicular bisector include using a compass, using a ruler and a right triangle, and using paper folding.

Communicate the Ideas

1. What is a perpendicular bisector?
2. What are two examples of perpendicular bisectors in the real world? Sketch each example. Share your examples with a classmate.
3. How would you explain to a friend that the post of the totem pole is a perpendicular bisector of the wings?

Practise

For help with #4 and #5, refer to the Example on page 90–91.

4. Copy each of these line segments. Draw the perpendicular bisectors. Verify the perpendicular bisectors using a method of your choice.

5. Copy the line segments below. Draw the perpendicular bisector of each of them. Verify the perpendicular bisectors using a method of your choice.

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Supported Learning

Learning Style

- Encourage students to share their understanding of the Key Ideas during group discussion.

ESL

- In #3 of Communicate the Ideas, some English language learners may have no prior knowledge of totem poles. Use the picture to show what part of the pole is the post.

ESL and Language

- For tasks that require students to communicate their ideas verbally with a partner, have these students work in appropriate groups, partnered with students who have a good understanding of the terminology.

Motor

- You may wish to have students who struggle with motor skills find pictures on the Internet to answer #2 of the Communicate the Ideas questions.

Communicate the Ideas

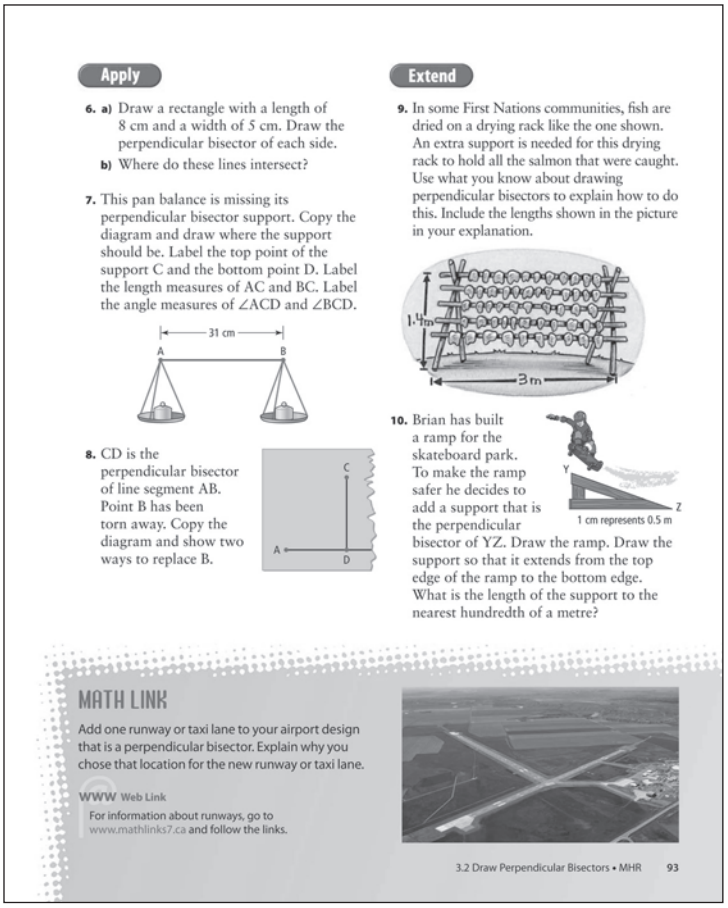
The ideas in all three questions are important. Briefly discuss the major points after students have attempted the questions.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students complete all three questions.	• In #1, students are asked to explain their understanding of perpendicular bisectors. Have students work in pairs and then compare their answers with other groups. • In #2, students are required to provide two examples of perpendicular bisectors in the real world, sketch them, and share them with other students. • In #3, students are expected to synthesize their understanding and explain how the post of the totem pole in the picture is a perpendicular bisector of the wings.

Practise

In #4 and #5, students are asked to verify their answers using a method of their choice. Encourage students to use more than one method: compass, ruler and right triangle, and/or paper folding.

Assessment *for* Learning	Supported Learning
Practise Have students do #4. Students who have no problems with this question can go on to the Apply questions.	• Students who have problems with #4 will need additional coaching with the Example. Review this material with them, coach them as they correct #4, and then have them do #5. • Check back with students several times to make sure that they understand the concepts.



Apply

6. a) Draw a rectangle with a length of 8 cm and a width of 5 cm. Draw the perpendicular bisector of each side.
b) Where do these lines intersect?

7. This pan balance is missing its perpendicular bisector support. Copy the diagram and draw where the support should be. Label the top point of the support C and the bottom point D. Label the length measures of AC and BC. Label the angle measures of ∠ACD and ∠BCD.

8. CD is the perpendicular bisector of line segment AB. Point B has been torn away. Copy the diagram and show two ways to replace B.

Extend

9. In some First Nations communities, fish are dried on a drying rack like the one shown. An extra support is needed for this drying rack to hold all the salmon that were caught. Use what you know about drawing perpendicular bisectors to explain how to do this. Include the lengths shown in the picture in your explanation.

10. Brian has built a ramp for the skateboard park. To make the ramp safer he decides to add a support that is the perpendicular bisector of YZ. Draw the ramp. Draw the support so that it extends from the top edge of the ramp to the bottom edge. What is the length of the support to the nearest hundredth of a metre?

MATH LINK

Add one runway or taxi lane to your airport design that is a perpendicular bisector. Explain why you chose that location for the new runway or taxi lane.

WWW Web Link
For information about runways, go to www.mathlinks7.ca and follow the links.

Apply and Extend

The Apply questions provide a variety of contexts in which perpendicular bisectors are used. For #6, students will see that the perpendicular bisectors of a rectangle meet at the same point (at the centre of the rectangle).

Ensure that students accurately draw the required diagram in #10, and that they understand how to use the scale provided (1 cm represents 0.5 m).

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 93 allows students to continue to explore designing airport runways, this time using perpendicular bisectors.	• Have all students complete the Math Link. • Have students use a compass, a ruler and a right triangle, and/or paper folding to create and verify their perpendicular bisector. • Students may wish to use **BLM 3–6 Section 3.2 Math Link** to assist them with completing the activity.

Supported Learning

Learning Style and Memory

- You may wish to have students complete **BLM 3–5 Section 3.2 Extra Practice** for additional reinforcement.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • How do you know that a line is the perpendicular bisector of a line segment?	• Encourage students to verbalize the process they use to verify that a line is a perpendicular bisector. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • You may also wish to have students review the part related to section 3.2 in **BLM 3–1 Chapter 3 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

MATH LINK

The Math Link on page 93 gives students an opportunity to add a perpendicular bisector to their airport runway design. Explain to students that the locations of the runways and taxi lanes are important. Remind them to research the factors that should be considered when choosing the locations.

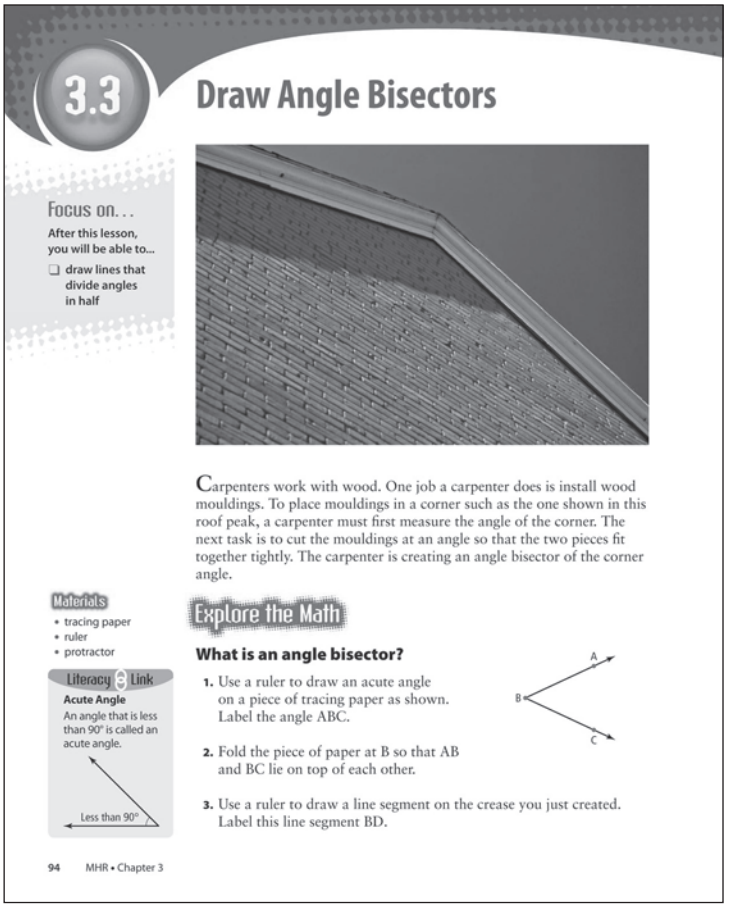
3.3 Draw Angle Bisectors

Suggested Timing
80–100 minutes

Materials
- tracing paper
- ruler
- protractor
- compass
- Mira (optional)

Blackline Masters
BLM 3–1 Chapter 3 Self-Assessment
BLM 3–7 Section 3.3 Extra Practice
BLM 3–8 Section 3.3 Math Link

Mathematical Processes
- [ ] Communication
- [x] Connections
- [ ] Mental Mathematics and Estimation
- [ ] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



3.3 Draw Angle Bisectors

Focus on...
After this lesson, you will be able to...
- draw lines that divide angles in half

Carpenters work with wood. One job a carpenter does is install wood mouldings. To place mouldings in a corner such as the one shown in this roof peak, a carpenter must first measure the angle of the corner. The next task is to cut the mouldings at an angle so that the two pieces fit together tightly. The carpenter is creating an angle bisector of the corner angle.

Materials
- tracing paper
- ruler
- protractor

Literacy Link
Acute Angle
An angle that is less than 90° is called an acute angle.
Less than 90°

Explore the Math

What is an angle bisector?

1. Use a ruler to draw an acute angle on a piece of tracing paper as shown. Label the angle ABC.
2. Fold the piece of paper at B so that AB and BC lie on top of each other.
3. Use a ruler to draw a line segment on the crease you just created. Label this line segment BD.

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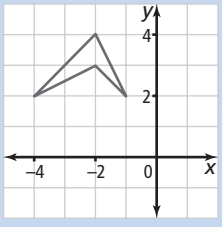
Specific Outcomes

SS3 Perform geometric constructions, including:
- perpendicular line segments
- parallel line segments
- perpendicular bisectors
- angle bisectors.

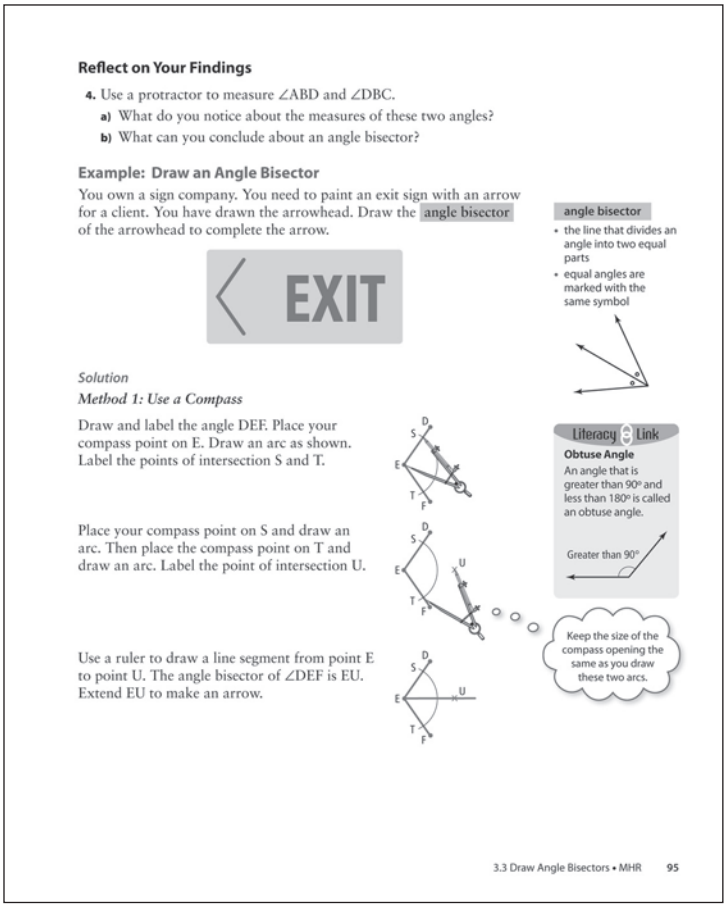
Warm-Up

1. Draw a line segment 7 cm long. Draw the perpendicular bisector of that line segment.
2. Draw a line segment parallel to the perpendicular bisector in #1.
3. Name an item with both parallel line segments and perpendicular line segments. Sketch the item.
4. Copy the figure shown onto grid paper. Translate the image 2 units right and 3 units down.
5. Estimate and then calculate 48×6.3.



Mental Math

6. Estimate 1.75×4.
7. Estimate 14.9×0.1.
8. Estimate $30.6 \div 6$.
9. Estimate $5.56 \div 0.9$.
10. Count by 2s from 1022 to 1036.



Reflect on Your Findings

4. Use a protractor to measure ∠ABD and ∠DBC.
   a) What do you notice about the measures of these two angles?
   b) What can you conclude about an angle bisector?

Example: Draw an Angle Bisector

You own a sign company. You need to paint an exit sign with an arrow for a client. You have drawn the arrowhead. Draw the angle bisector of the arrowhead to complete the arrow.

angle bisector
- the line that divides an angle into two equal parts
- equal angles are marked with the same symbol

Solution

Method 1: Use a Compass

Draw and label the angle DEF. Place your compass point on E. Draw an arc as shown. Label the points of intersection S and T.

Place your compass point on S and draw an arc. Then place the compass point on T and draw an arc. Label the point of intersection U.

Use a ruler to draw a line segment from point E to point U. The angle bisector of ∠DEF is EU. Extend EU to make an arc.

Literacy Link

Obtuse Angle

An angle that is greater than 90° and less than 180° is called an obtuse angle.

Greater than 90°

Keep the size of the compass opening the same as you draw these two arcs.

3.3 Draw Angle Bisectors • MHR 95

Activity Planning Notes

This section focuses on drawing angle bisectors. Discuss how carpenters use measurements of angles to do their work, as described in the student resource. Look in the classroom for other examples of angle bisectors (e.g., door or baseboard mouldings).

Explore the Math

Begin by ensuring that students understand how to use symbols to mark equal angles. Discuss the definitions of acute and obtuse angles. Have students work independently or in pairs to follow the procedure outlined in the Explore the Math section. Have students discuss their findings and conclusions about angle bisectors.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss their conclusions about angle bisectors.	• Work with students to ensure that they measure the angles accurately. • Have students discuss their conclusions. Ask: – Why do you think the measures of these two angles are equal? – Will it make a difference if the angle is obtuse?

Answers

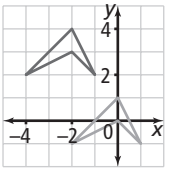
Warm-Up

1. For most students, the length of the line segment should be correct within 1 mm. Check that the perpendicular bisector is correctly drawn.

2. Check that the line segment is parallel to the perpendicular bisector, not the original line segment.

3. Answers will vary. For example: a square or rectangular box

4.



5. Estimate: 50 × 6 = 300. Calculate: 302.4

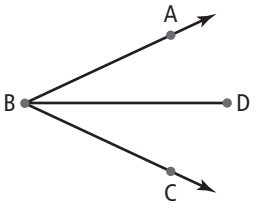
6. 2 × 4 = 8 **7.** 15 × 0.1 = 1.5

8. 30 ÷ 6 = 5 **9.** 6 ÷ 1 = 6

10. 1022, 1024, 1026, 1028, 1030, 1032, 1034, 1036

Explore the Math

3.



4. Answers may vary.
a) They are equal.
b) It divides an angle into two equal angles.

Supported Learning

ESL

- Assist English language learners with the terms *carpenters* and *mouldings*, and then have them add these words to their translation dictionaries.

ESL and Language

- Students may require support when reading through the Explore the Math section.

Motor

- Students may find it difficult to draw on tracing paper and then make the endpoints line up. Allow students to use virtual manipulatives.

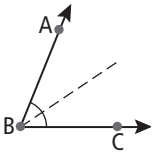
Learning Style

- For concrete and kinesthetic learners, you may wish to bring in a few pieces of wood moulding to the classroom, and have them explore how the pieces fit together in a corner.

Answers

Show You Know: Example

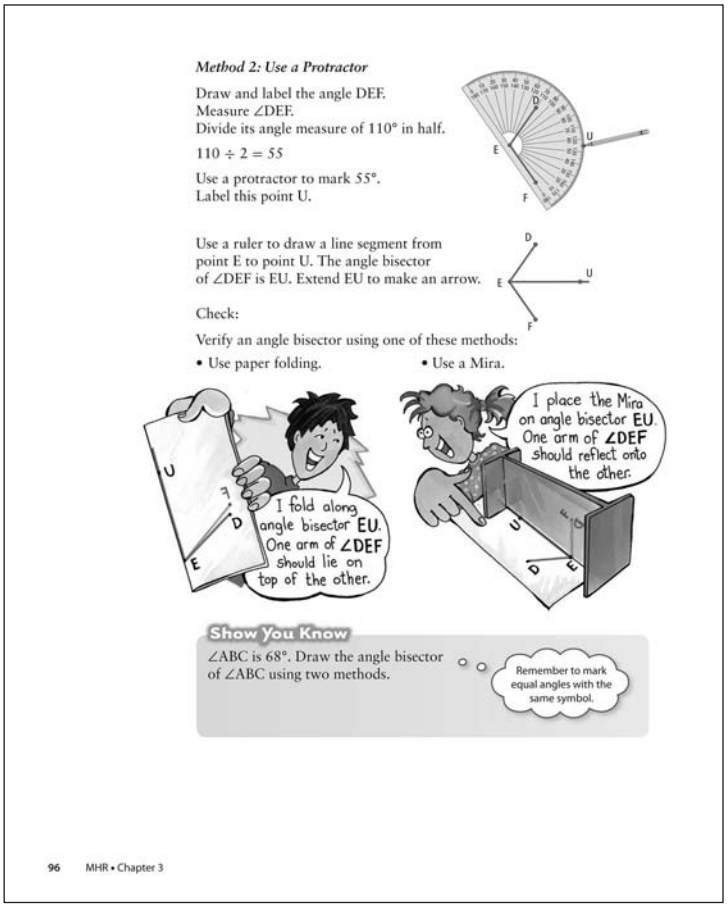
Check that two methods have been used.



Common Errors

- Students may not know how to use a protractor to draw an angle larger than 90° and smaller than 180° (i.e., an obtuse angle).

R_x When drawing an angle, students should be sure to start counting at 0° on the protractor.



Supported Learning

ESL

- Assist English language learners in understanding the word *intersection*.

ESL and Language

- Be sure to point out and explain the Literacy Links on pages 94 and 95 regarding obtuse and acute angles. Use the diagrams to ensure that students understand the differences between the two types of angles.

Motor

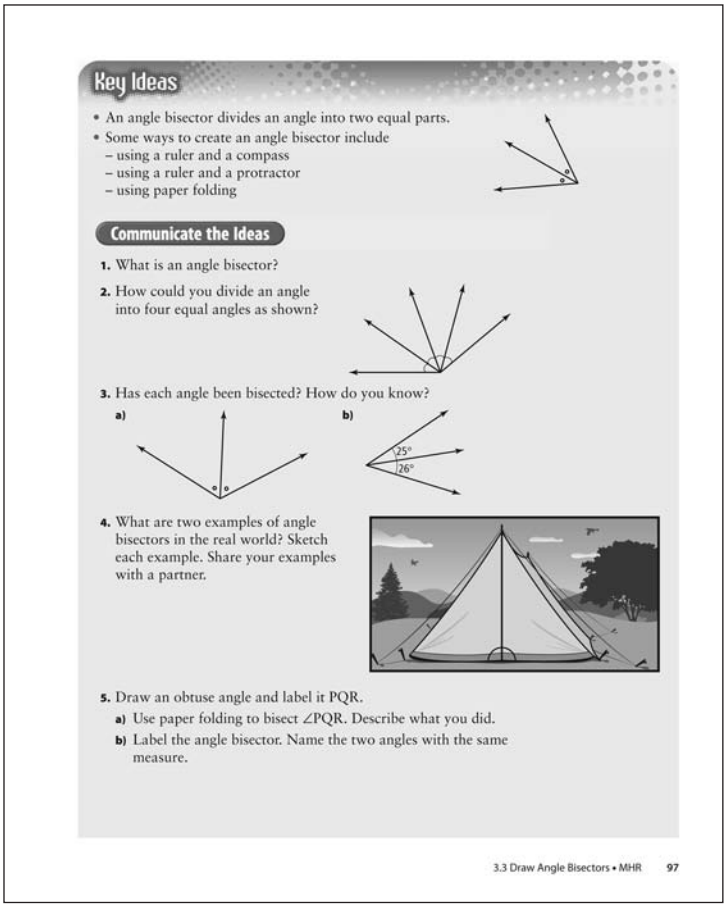
- Students may need extra practice in using a compass. Encourage them to use virtual manipulatives.

Meeting the Needs of All Learners

- Take the time to identify each step needed to construct an angle bisector. List the steps on the board or on a chart, and encourage students to follow the steps each time.

The Example demonstrates two techniques for drawing an angle bisector. Ensure that students understand the importance of using the compass and protractor accurately.

Assessment *for* Learning	Supported Learning
Example Have students do the Show You Know related to the Example.	• Encourage students to verify their work using paper folding and/or a Mira. • You may wish to provide additional questions for students who would benefit from them. For example: **a)** Draw a 120° angle. Construct the angle bisector using a compass. Measure and label the two smaller angles. (Coach students through the process for using a compass to draw an angle bisector. The two angles should each measure 60°.) **b)** Draw a 134° angle. Use a protractor to draw the angle bisector. Measure and label the two smaller angles. (Coach students through the process for using a protractor. The two angles should each measure 67°.) Coach students through a), and then have them complete b) independently to make sure that they understand the concepts.



Key Ideas

- An angle bisector divides an angle into two equal parts.
- Some ways to create an angle bisector include
  - using a ruler and a compass
  - using a ruler and a protractor
  - using paper folding

Communicate the Ideas

1. What is an angle bisector?
2. How could you divide an angle into four equal angles as shown?
3. Has each angle been bisected? How do you know?

a) b) 25° 26°

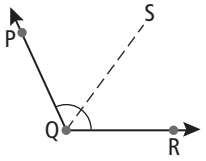
4. What are two examples of angle bisectors in the real world? Sketch each example. Share your examples with a partner.
5. Draw an obtuse angle and label it PQR.
   a) Use paper folding to bisect ∠PQR. Describe what you did.
   b) Label the angle bisector. Name the two angles with the same measure.

3.3 Draw Angle Bisectors • MHR 97

Answers

Communicate the Ideas

1. Answers may vary. For example: A line that divides an angle into two equal parts.
2. Answers may vary. For example: Bisect the two angles created by the angle bisector of the original angle.
3. **a)** Yes. The symbols show that the two angles are equal.
   b) No. The two angles are not equal.
4. Answers may vary. For example: The fuselage between the wings of a plane and the middle prong of three rake prongs. Check that sketches clearly show the angle bisectors.
5. Answers may vary. For example:



a) I drew an angle greater than 90° and less than 180° and folded it in half at point Q so PQ and RQ were lying on top of each other.
b) QS. ∠PQS = ∠RQS

Key Ideas

Students could prepare their own list of Key Ideas and put it in their chapter Foldable. The emphasis of this section is on using different methods to draw an angle bisector.

Communicate the Ideas

Have students work in groups to answer and discuss the Communicate the Ideas questions. Encourage them to explain their answers orally.

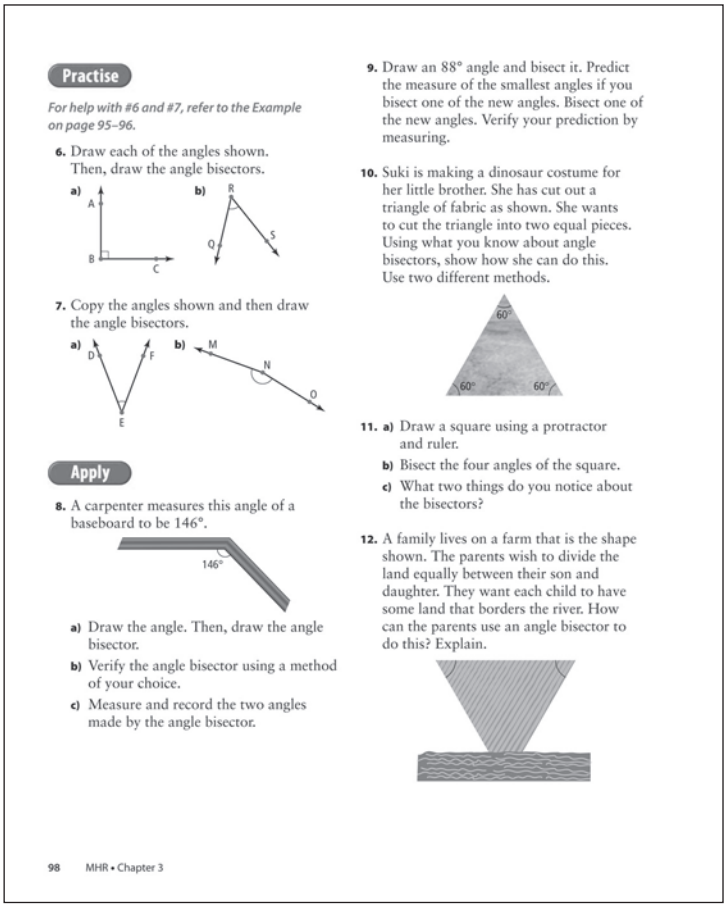
Assessment *as* Learning	Supported Learning
Communicate the Ideas Rather than asking students to complete all five questions, you may wish to assign #5 and have them choose two other questions. This could be done individually or as a class.	• The Communicate the Ideas on page 97 is intended to allow students to explain their understanding of angle bisectors. • In #1, students are asked to provide a definition of *angle bisector*. • In #2, students are instructed to describe how to divide an angle into four equal angles. • In #3, students identify whether given angles are bisected. • In #4, students are asked to provide real-world examples of angle bisectors. • In #5, students are asked to demonstrate the paper folding process for finding an angle bisector. The answers to this question could be shared with a partner. • As you circulate, listen for student explanations and assess whether they have only a basic understanding or one that provides multiple ways to solve the given problem.

Supported Learning

Learning Style, ESL, and Language

- You may wish to partner struggling students with classmates who can provide them with good one-on-one discussion. This facilitates comprehension of vocabulary.
- Encourage students to share their understanding of the Key Ideas during group discussion.
- Allow students to choose to complete the Communicate the Ideas in writing, orally, or using a combination of both methods.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	2 of 1–4, 5, 6, 8, Math Link
Typical	2 of 1–4, 5, 6, 8, 9, 11 *or* 12, 13, Math Link
Extension/Enrichment	2 of 1–4, 5, 11–15, Math Link



Practise

For help with #6 and #7, refer to the Example on page 95–96.

6. Draw each of the angles shown. Then, draw the angle bisectors.

a) b)

7. Copy the angles shown and then draw the angle bisectors.

a) b)

Apply

8. A carpenter measures this angle of a baseboard to be 146°.

146°

a) Draw the angle. Then, draw the angle bisector.

b) Verify the angle bisector using a method of your choice.

c) Measure and record the two angles made by the angle bisector.

9. Draw an 88° angle and bisect it. Predict the measure of the smallest angles if you bisect one of the new angles. Bisect one of the new angles. Verify your prediction by measuring.

10. Suki is making a dinosaur costume for her little brother. She has cut out a triangle of fabric as shown. She wants to cut the triangle into two equal pieces. Using what you know about angle bisectors, show how she can do this. Use two different methods.

60° 60° 60°

11. a) Draw a square using a protractor and ruler.

b) Bisect the four angles of the square.

c) What two things do you notice about the bisectors?

12. A family lives on a farm that is the shape shown. The parents wish to divide the land equally between their son and daughter. They want each child to have some land that borders the river. How can the parents use an angle bisector to do this? Explain.

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Practise

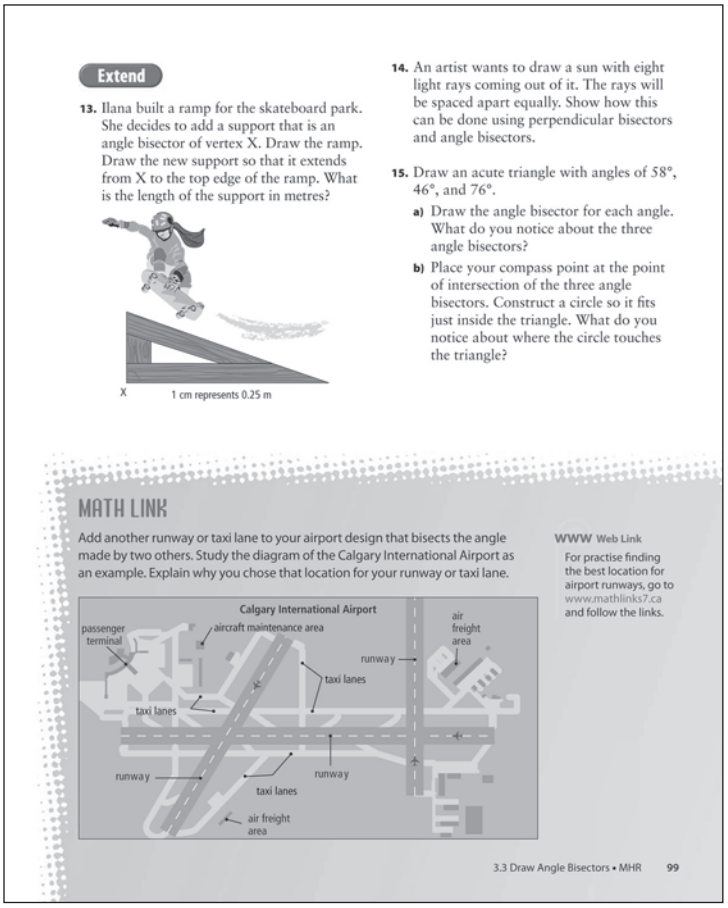
For #6 and #7, ensure that students accurately measure and draw the angles provided before starting to draw the angle bisectors.

Assessment *for* Learning	Supported Learning
Practise Have students do #6. Students who have no problems with this question can go on to the Apply questions.	• Students who have problems with #6 will need additional coaching with the Example. Coach students as they correct #6a) and then have them correct #6b) on their own. When they feel more confident, have them do #7 on their own. • Check back with students several times to make sure that they understand the concepts.

Apply and Extend

Students able to complete the Apply questions have demonstrated a good understanding of problem solving using the material in this section.

The three questions in the Extend section require students to use provided information to draw angle bisectors and perpendicular bisectors and to explore the concept of angle bisectors in more depth.



Extend

13. Ilana built a ramp for the skateboard park. She decides to add a support that is an angle bisector of vertex X. Draw the ramp. Draw the new support so that it extends from X to the top edge of the ramp. What is the length of the support in metres?

X

1 cm represents 0.25 m

14. An artist wants to draw a sun with eight light rays coming out of it. The rays will be spaced apart equally. Show how this can be done using perpendicular bisectors and angle bisectors.

15. Draw an acute triangle with angles of 58°, 46°, and 76°.
   a) Draw the angle bisector for each angle. What do you notice about the three angle bisectors?
   b) Place your compass point at the point of intersection of the three angle bisectors. Construct a circle so it fits just inside the triangle. What do you notice about where the circle touches the triangle?

MATH LINK

Add another runway or taxi lane to your airport design that bisects the angle made by two others. Study the diagram of the Calgary International Airport as an example. Explain why you chose that location for your runway or taxi lane.

WWW Web Link

For practise finding the best location for airport runways, go to www.mathlinks7.ca and follow the links.

Calgary International Airport

passenger terminal
aircraft maintenance area
air freight area
runway
taxi lanes
taxi lanes
runway
runway
taxi lanes
air freight area

3.3 Draw Angle Bisectors • MHR 99

MATH LINK

The Math Link on page 99 allows students to continue to work on their design by adding a runway that is an angle bisector. Students can use a ruler and a compass, a ruler and a protractor, and/or paper folding to construct the angle.

Assessment *for* Learning	Supported Learning
Math Link Have students complete the Math Link as they work toward the Wrap It Up! on page 119.	• Have all students do this Math Link. • Encourage students to use one or more methods to check their work . • Have students use **BLM 3–8 Section 3.3 Math Link** to assist them with completing the activity.

Supported Learning

Learning Style and Memory

- You may wish to have students complete **BLM 3–7 Section 3.3 Extra Practice** for additional reinforcement.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • How do you know that a line is the angle bisector of an angle? • What do you find most difficult about making angle bisectors? Least difficult?	• Encourage students to recognize the different ways of constructing and verifying an angle bisector (e.g., a ruler and a compass, a ruler and a protractor, a Mira, and/or paper folding). • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • You may also wish to have students review the part related to section 3.3 in **BLM 3–1 Chapter 3 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

WWW Web Link

Have students use the Web Link on page 99 to help them find the best location for their airport runways. Go to **www.mathlinks7.ca** and follow the links.

3.4 Area of a Parallelogram

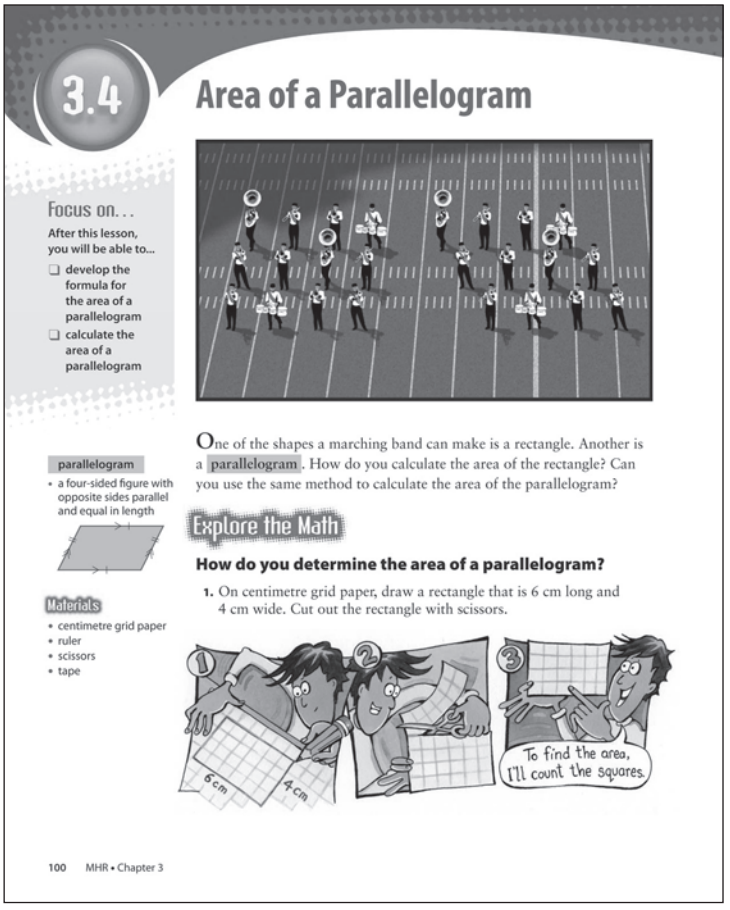
Suggested Timing
80–100 minutes

Materials
- ruler
- scissors
- tape

Blackline Masters
Master 8 Centimetre Grid Paper
BLM 3–1 Chapter 3 Self Assessment
BLM 3–9 Section 3.4 Extra Practice
BLM 3–10 Section 3.4 Math Link

Mathematical Processes

- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



3.4 Area of a Parallelogram

Focus on...
After this lesson, you will be able to...
- develop the formula for the area of a parallelogram
- calculate the area of a parallelogram

One of the shapes a marching band can make is a rectangle. Another is a parallelogram. How do you calculate the area of the rectangle? Can you use the same method to calculate the area of the parallelogram?

parallelogram
- a four-sided figure with opposite sides parallel and equal in length

Explore the Math

How do you determine the area of a parallelogram?

1. On centimetre grid paper, draw a rectangle that is 6 cm long and 4 cm wide. Cut out the rectangle with scissors.

Materials
- centimetre grid paper
- ruler
- scissors
- tape

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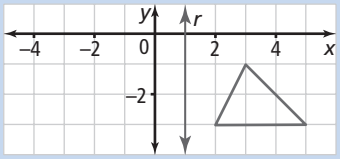
Specific Outcomes

SS2 Develop and apply a formula for determining the area of:
- triangles
- parallelograms
- circles.

Warm-Up

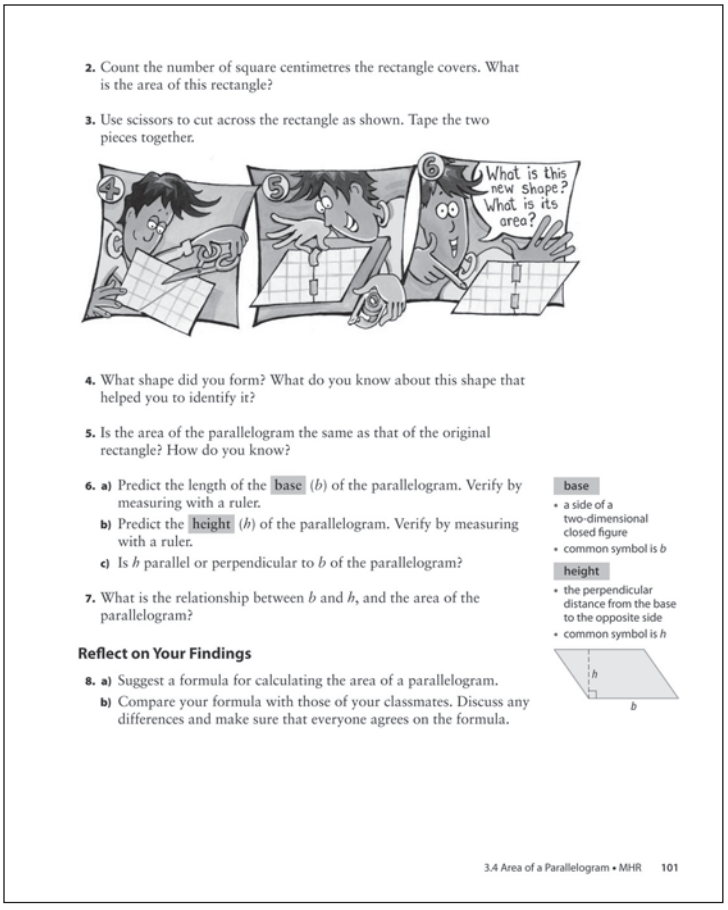
1. Draw a 60° angle. Draw a bisector of that angle.
2. How can you test that you have properly bisected an angle?
3. Copy the figure onto grid paper. Reflect this figure in line of reflection *r*.



4. Explain how to tell that one image is the reflection of another.

Mental Math

5. An angle is 79°. Estimate the size of the angle bisector.
6. An angle is 109°. Estimate the size of the angle bisector.
7. Estimate 25.1 × 3.6.
8. Estimate 135.2 × 1.2.
9. Estimate 72.8 ÷ 8.
10. Estimate 63.7 ÷ 0.9.



2. Count the number of square centimetres the rectangle covers. What is the area of this rectangle?

3. Use scissors to cut across the rectangle as shown. Tape the two pieces together.

4. What shape did you form? What do you know about this shape that helped you to identify it?

5. Is the area of the parallelogram the same as that of the original rectangle? How do you know?

6. a) Predict the length of the base (b) of the parallelogram. Verify by measuring with a ruler.
   b) Predict the height (h) of the parallelogram. Verify by measuring with a ruler.
   c) Is h parallel or perpendicular to b of the parallelogram?

7. What is the relationship between b and h, and the area of the parallelogram?

Reflect on Your Findings

8. a) Suggest a formula for calculating the area of a parallelogram.
   b) Compare your formula with those of your classmates. Discuss any differences and make sure that everyone agrees on the formula.

base
- a side of a two-dimensional closed figure
- common symbol is b

height
- the perpendicular distance from the base to the opposite side
- common symbol is h

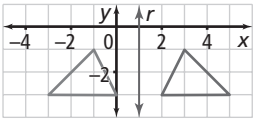
Answers

Warm-Up

1. For most students, the angle and bisector should be correct within a degree or two. Check that the correct method has been used.

2. Answers will vary. For example: Measure the angles, use a Mira, use paper folding.

3.



4. Answers will vary. For example: They should look exactly the same, but the figure is reversed.

5. About 40° **6.** About 55°

7. $25 \times 4 = 100$ **8.** $135 \times 0.1 = 13.5$

9. $72 \div 8 = 9$ **10.** $63 \div 1 = 63$

Explore the Math

2. 24 cm^2

4. Parallelogram. Answers may vary. For example: It has four sides with opposite sides parallel and equal in length.

5. Yes. The parallelogram was made from the original rectangle.

6. **a)** 6 cm **b)** 4 cm **c)** perpendicular

7. If you multiply the base by the height, you get the area.

8. **a)** $A = b \times h$ **b)** Answers will vary.

Activity Planning Notes

Begin this section by reviewing with students how to calculate the area of a rectangle (length × width). Then review the definition of a parallelogram, as shown in the margin on page 100 of the student resource. Discuss whether students believe they can use the formula for calculating the area of a rectangle to calculate the area of a parallelogram. Ask them to explain why or why not.

Explore the Math

Have students work in pairs. Students begin by calculating the area of a rectangle. They then transform the rectangle into a parallelogram, and explore the area of the parallelogram. Ensure that students understand how the length and width of a rectangle are related to the height and base of a parallelogram. When done, discuss as a class the formula students suggested for calculating the area of a parallelogram. Determine if anybody can come up with a different formula, and why it is or is not accurate.

Supported Learning

ESL

- Students may not be familiar with the terms *original* and *marching band*. Have other students share their knowledge of and experience with marching bands.

Meeting the Needs of All Learners

- Ask students to find items in the classroom and school that are in the shape of a parallelogram.

Answers

Show You Know: Example 1

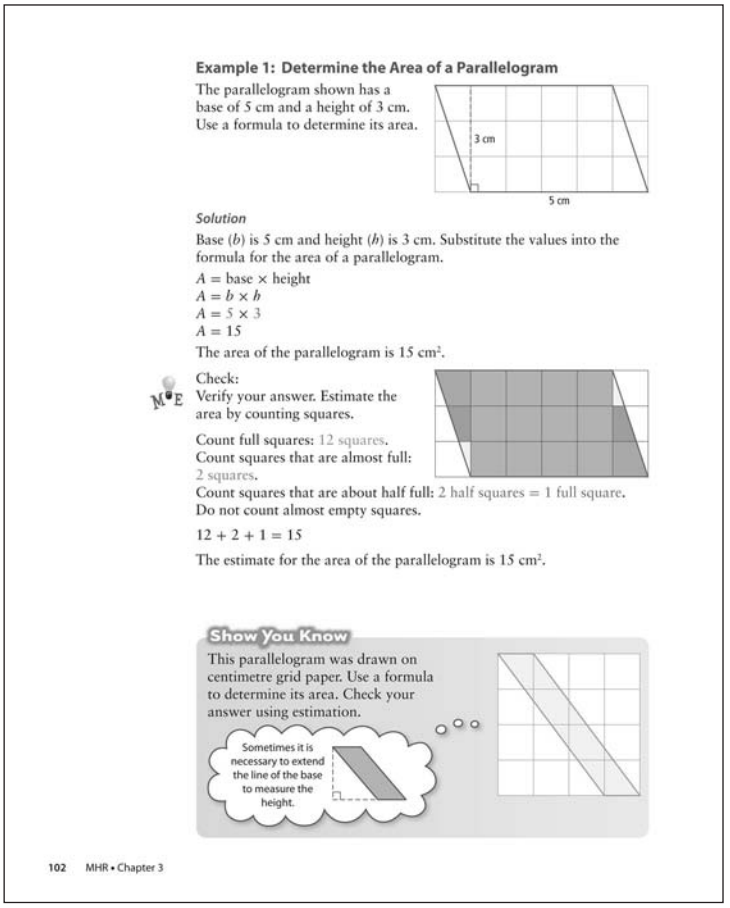
4 cm^2

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss the formula they developed. During this process, they are generalizing what they have learned during the Explore the Math.	• If students have difficulty understanding that the area of the parallelogram is equal to the area of the rectangle, you may wish to work backwards through the process. In other words, start with a parallelogram, transform it into a rectangle, and then measure the area of the rectangle. This may help reinforce that the two areas are the same.

Supported Learning

Learning Style and Motor

- Have concrete and kinesthetic learners or students with weak motor skills use geoboards. They can make the parallelogram using an elastic and estimate the area by counting squares. You might wish to have students make the parallelogram into a rectangle. This is the opposite of what they did in the Explore the Math activity and will confirm the $A = b \times h$ formula.



Example 1: Determine the Area of a Parallelogram

The parallelogram shown has a base of 5 cm and a height of 3 cm. Use a formula to determine its area.

3 cm

5 cm

Solution

Base (b) is 5 cm and height (h) is 3 cm. Substitute the values into the formula for the area of a parallelogram.

A = base × height
$A = b \times h$
$A = 5 \times 3$
$A = 15$

The area of the parallelogram is 15 cm^2.

Check:
Verify your answer. Estimate the area by counting squares.

Count full squares: 12 squares.
Count squares that are almost full: 2 squares.
Count squares that are about half full: 2 half squares = 1 full square.
Do not count almost empty squares.

$12 + 2 + 1 = 15$

The estimate for the area of the parallelogram is 15 cm^2.

Show You Know

This parallelogram was drawn on centimetre grid paper. Use a formula to determine its area. Check your answer using estimation.

Sometimes it is necessary to extend the line of the base to measure the height.

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Common Errors

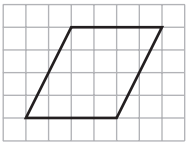
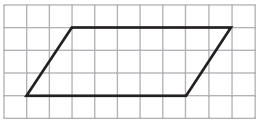
- Students may confuse perimeter and area.

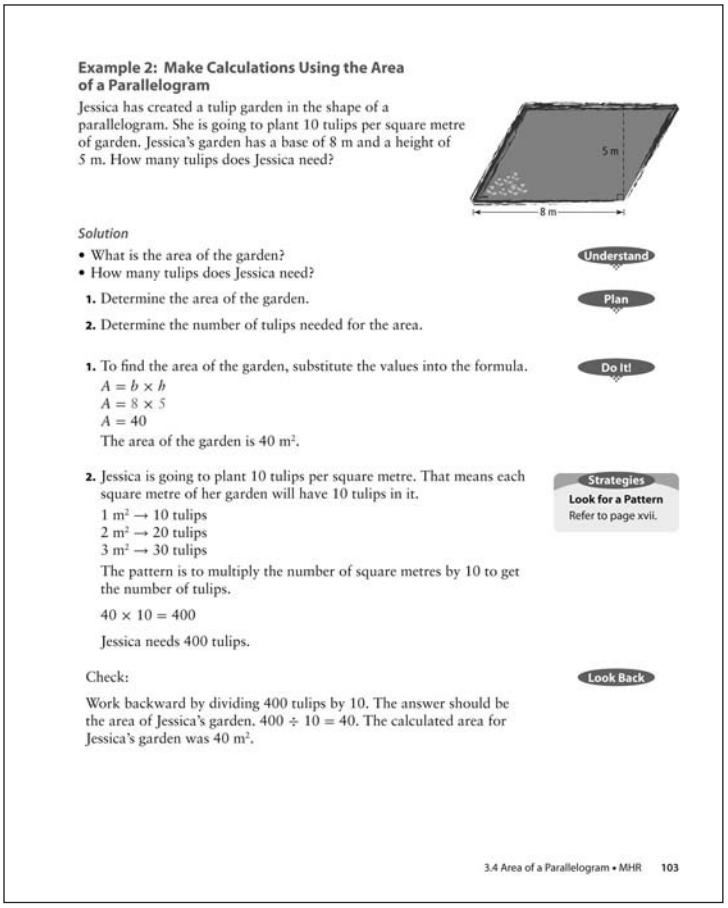
R_x Remind students that perimeter is the length around the outside of a shape, while area is the space enclosed by the sides of a shape.

- Students may measure and use the diagonal side length rather than the height when calculating the area.

R_x Remind students that when determining the height of a parallelogram, they should measure perpendicular to the base. Encourage students to extend the base, if necessary, to ensure that they accurately measure the height.

Example 1 asks students to explore how to determine the area of a parallelogram. Encourage students to extend the base of the parallelogram to help them calculate the height, if required.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• Ensure that students take the time to count the squares to estimate the area and verify their answer. • You may wish to provide additional questions for students who would benefit from them. For example: Use a formula to determine the area of the following parallelograms, which have been drawn on centimetre grid paper. Check your answer using estimation. **a)**  ($A = b \times h$ $A = 4 \times 4$ $A = 16$ cm^2) **b)**  ($A = b \times h$ $A = 7 \times 3$ $A = 21$ cm^2) Coach students through a), and then have them work through b) independently.



Example 2: Make Calculations Using the Area of a Parallelogram

Jessica has created a tulip garden in the shape of a parallelogram. She is going to plant 10 tulips per square metre of garden. Jessica's garden has a base of 8 m and a height of 5 m. How many tulips does Jessica need?

Solution

Understand

- What is the area of the garden?
- How many tulips does Jessica need?

Plan

1. Determine the area of the garden.
2. Determine the number of tulips needed for the area.

Do It!

1. To find the area of the garden, substitute the values into the formula.

$A = b \times h$
$A = 8 \times 5$
$A = 40$

The area of the garden is 40 m².

2. Jessica is going to plant 10 tulips per square metre. That means each square metre of her garden will have 10 tulips in it.

1 m² → 10 tulips
2 m² → 20 tulips
3 m² → 30 tulips

The pattern is to multiply the number of square metres by 10 to get the number of tulips.

$40 \times 10 = 400$

Jessica needs 400 tulips.

Strategies
Look for a Pattern
Refer to page xvii.

Look Back

Check:

Work backward by dividing 400 tulips by 10. The answer should be the area of Jessica's garden. 400 ÷ 10 = 40. The calculated area for Jessica's garden was 40 m².

3.4 Area of a Parallelogram • MHR 103

Supported Learning

ESL and Language

- Some students may require support when reading through the steps in the examples.

ESL

- Assist English language learners in understanding the word *tulip* by showing them a picture of the flower.

Example 2 shows students how to use the area of a parallelogram to solve problems. Ensure that students understand how to verify their answers by working backward and dividing. You may also wish to review the problem solving method with students. This method is outlined starting on page xiv in the front matter of the student resource.

Example 3 asks students to determine the height of a parallelogram, given the base measurement and area of the parallelogram.

Assessment *for* Learning	Supported Learning
Example 3 Have students do the Show You Know on page 104 related to Example 3.	• You may wish to provide additional questions for students who would benefit from them. For example: Determine the height of the following parallelograms: **a)** base = 15 cm; area = 75 cm² ($A = b \times h$ $75 = 15 \times h$ $h = 5$ cm) **b)** base = 45 m; area = 360 m² ($A = b \times h$ $360 = 45 \times h$ $h = 8$ m) Coach students through a), and then have them work through b) independently.

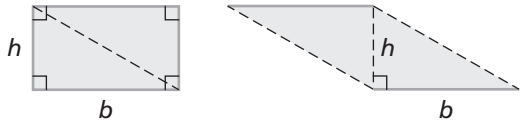
Answers

Show You Know: Example 3

5 m

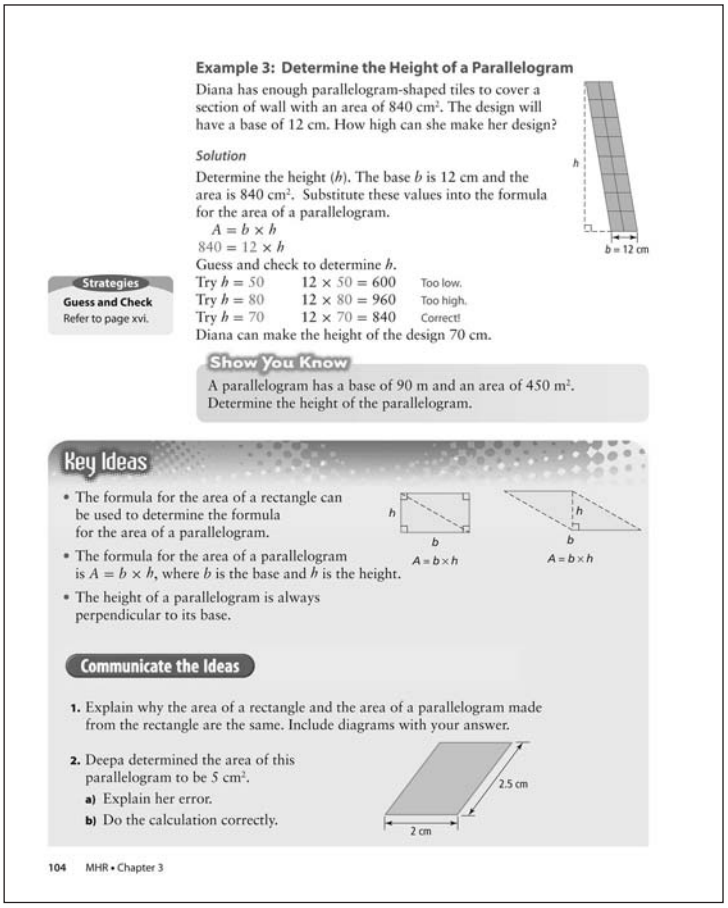
Communicate the Ideas

1. They have the same base and height. Answers may vary. For example:



2. a) She used the slant height, not the height that is perpendicular to the base.

b) 2 cm × 2 cm = 4 m^2



Example 3: Determine the Height of a Parallelogram

Diana has enough parallelogram-shaped tiles to cover a section of wall with an area of 840 cm^2. The design will have a base of 12 cm. How high can she make her design?

Solution

Determine the height (h). The base b is 12 cm and the area is 840 cm^2. Substitute these values into the formula for the area of a parallelogram.

$A = b \times h$

$840 = 12 \times h$

Guess and check to determine h.

Try $h = 50$ $12 \times 50 = 600$ Too low.

Try $h = 80$ $12 \times 80 = 960$ Too high.

Try $h = 70$ $12 \times 70 = 840$ Correct!

Diana can make the height of the design 70 cm.

h

b = 12 cm

Strategies

Guess and Check

Refer to page xvi.

Show You Know

A parallelogram has a base of 90 m and an area of 450 m^2. Determine the height of the parallelogram.

Key Ideas

- The formula for the area of a rectangle can be used to determine the formula for the area of a parallelogram.
- The formula for the area of a parallelogram is $A = b \times h$, where b is the base and h is the height.
- The height of a parallelogram is always perpendicular to its base.

h b A = b × h

h b A = b × h

Communicate the Ideas

1. Explain why the area of a rectangle and the area of a parallelogram made from the rectangle are the same. Include diagrams with your answer.

2. Deepa determined the area of this parallelogram to be 5 cm^2.

a) Explain her error.

b) Do the calculation correctly.

2.5 cm

2 cm

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Supported Learning

Learning Style and Language

- Encourage students to share their understanding of the Key Ideas during group discussion.
- Allow students to choose to answer the Communicate the Ideas questions either in writing, orally, or using a combination of both.

ESL

- Have English language learners work in appropriate groups to complete the questions in Communicate the Ideas. Ensure that they are partnered with students who have a good understanding of the terminology.

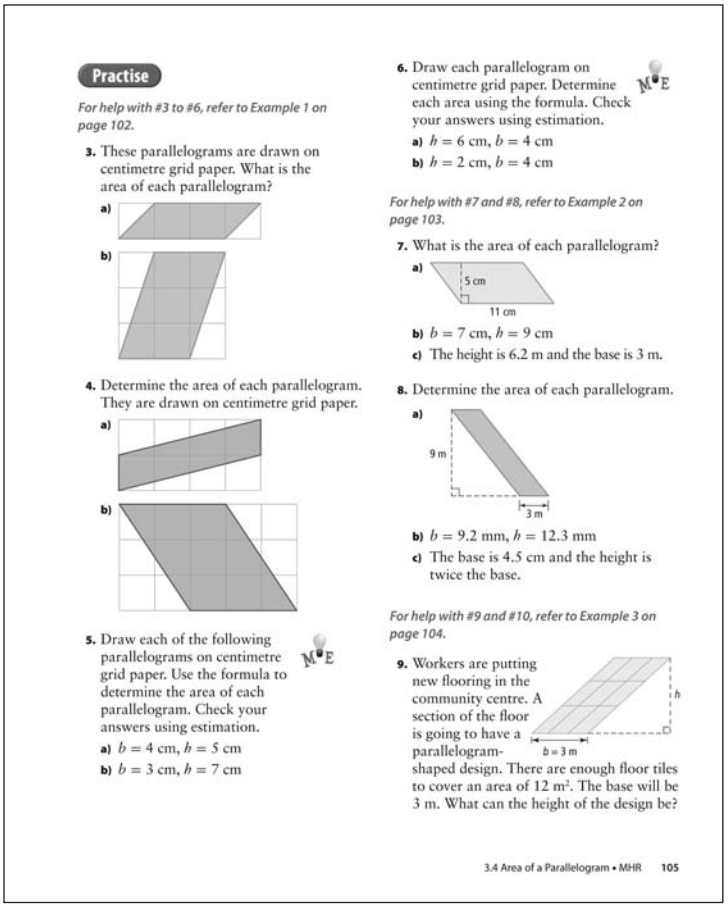
Key Ideas

Students could prepare their own list of Key Ideas and put it in their chapter Foldable. An intention of this section is to show students how to use the area of a rectangle to determine the formula for the area of a parallelogram. Another focus for the section is to ensure that students understand that the height of a parallelogram is measured perpendicular to the base of the shape.

Communicate the Ideas

Have students work in groups or pairs to answer and discuss the questions. Encourage them to explain their answers orally. Have a brief discussion summarizing major points after students have attempted the questions.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students do both questions. These will provide review of the key points of the section. Encourage them to use this communication to generalize what they have learned in the Explore the Math and the examples.	• The Communicate the Ideas gives students an opportunity to explain how the area of a rectangle and the area of a parallelogram made from the rectangle are the same and to explain how to calculate the area of a parallelogram. • Encourage students to share their answers with another student. Students should be able to justify their reasoning with diagrams and words.



Practise

For help with #3 to #6, refer to Example 1 on page 102.

3. These parallelograms are drawn on centimetre grid paper. What is the area of each parallelogram?

a)

b)

4. Determine the area of each parallelogram. They are drawn on centimetre grid paper.

a)

b)

5. Draw each of the following parallelograms on centimetre grid paper. Use the formula to determine the area of each parallelogram. Check your answers using estimation.

a) $b = 4$ cm, $h = 5$ cm

b) $b = 3$ cm, $h = 7$ cm

6. Draw each parallelogram on centimetre grid paper. Determine each area using the formula. Check your answers using estimation.

a) $h = 6$ cm, $b = 4$ cm

b) $h = 2$ cm, $b = 4$ cm

For help with #7 and #8, refer to Example 2 on page 103.

7. What is the area of each parallelogram?

a) 5 cm, 11 cm

b) $b = 7$ cm, $h = 9$ cm

c) The height is 6.2 m and the base is 3 m.

8. Determine the area of each parallelogram.

a) 9 m, 3 m

b) $b = 9.2$ mm, $h = 12.3$ mm

c) The base is 4.5 cm and the height is twice the base.

For help with #9 and #10, refer to Example 3 on page 104.

9. Workers are putting new flooring in the community centre. A section of the floor is going to have a parallelogram-shaped design. There are enough floor tiles to cover an area of 12 m². The base will be 3 m. What can the height of the design be?

h, $b = 3$ m

3.4 Area of a Parallelogram • MHR 105

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 7, 9, 11, Math Link
Typical	1–3, 5, 7, 9, 11–18, Math Link
Extension/Enrichment	1, 2, 14–16, 18–20, Math Link

Practise

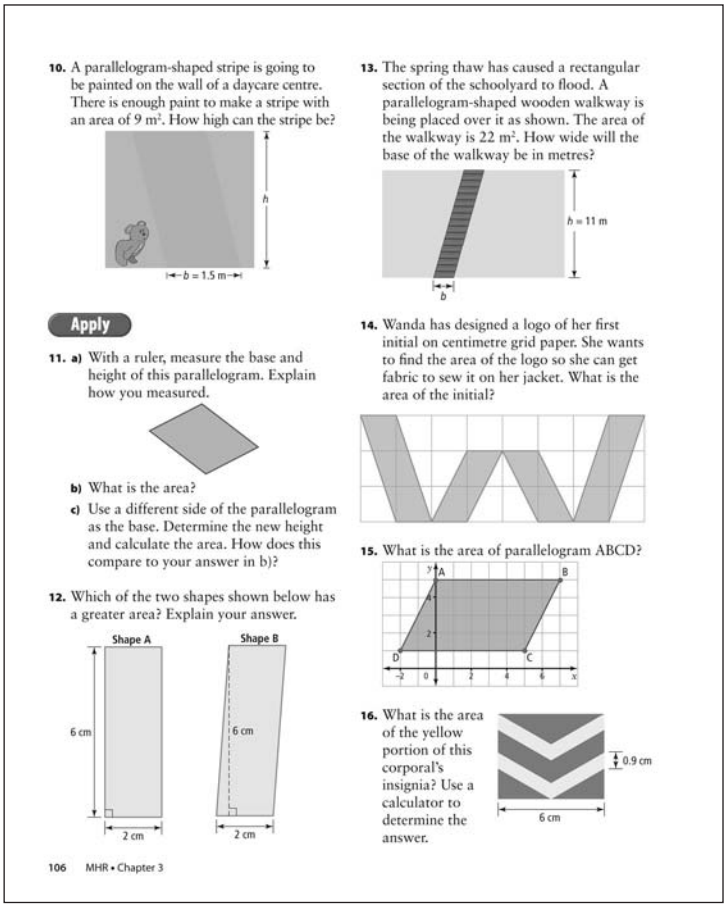
In #5 and #6, students are asked to draw the specified parallelograms on centimetre grid paper and use the formula to determine the area of each one. Ensure that students use estimation to check their answers.

Assessment *for* Learning	Supported Learning
Practise Have students do #3, #5, #7, and #9. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #3 will need additional coaching with Example 1. Coach them as they correct #3a), have them correct #3b) on their own, and assign #4. • Students who have problems with #5 will need additional coaching with Example 1. Coach them as they correct #5a), and then have them correct #5b) on their own and do #6. • Students who have problems with #7 will need additional coaching with Example 2. Coach them as they correct #7a), and then have them correct the rest of the question and do #8 on their own. • Students who have problems with #9 will need additional coaching with Example 3. Coach them as they correct #9, and then have them do #10 on their own. • Check back with students several times to make sure that they understand the concepts.

Supported Learning

ESL

- For #10, you may need to explain to students the concept of a stripe being painted on the wall of a daycare centre.



10. A parallelogram-shaped stripe is going to be painted on the wall of a daycare centre. There is enough paint to make a stripe with an area of 9 m². How high can the stripe be?

13. The spring thaw has caused a rectangular section of the schoolyard to flood. A parallelogram-shaped wooden walkway is being placed over it as shown. The area of the walkway is 22 m². How wide will the base of the walkway be in metres?

Apply

11. a) With a ruler, measure the base and height of this parallelogram. Explain how you measured.

b) What is the area?

c) Use a different side of the parallelogram as the base. Determine the new height and calculate the area. How does this compare to your answer in b)?

12. Which of the two shapes shown below has a greater area? Explain your answer.

14. Wanda has designed a logo of her first initial on centimetre grid paper. She wants to find the area of the logo so she can get fabric to sew it on her jacket. What is the area of the initial?

15. What is the area of parallelogram ABCD?

16. What is the area of the yellow portion of this corporal's insignia? Use a calculator to determine the answer.

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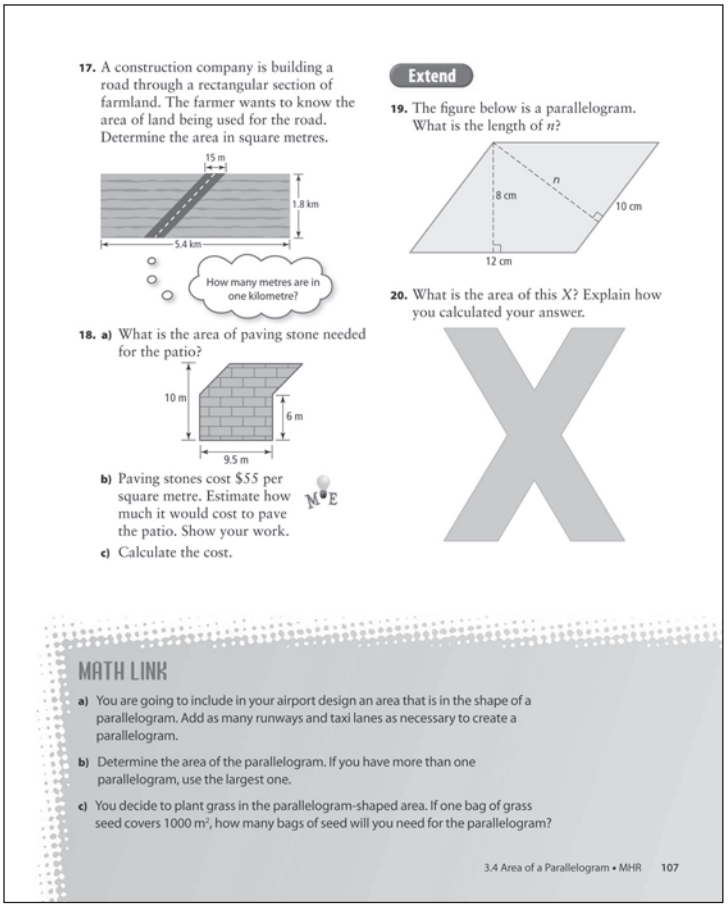
Apply and Extend

The Apply questions provide a variety of fairly straightforward contexts in which the area of parallelograms is used.

The Extend problems require significantly more analysis than those in the Apply section.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students respond to the following prompts: • What is the difference between the formula for the area of a rectangle and the formula for the area of a parallelogram? • Explain your method for remembering each formula.	• Encourage students to recognize the similarities and differences between the formula for the area of a rectangle and the formula for the area of a parallelogram (e.g., they are the same basic calculation, but they use different labels: length and width, and base and height). • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • You may also wish to have students review the part related to section 3.4 in **BLM 3–1 Chapter 3 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.



17. A construction company is building a road through a rectangular section of farmland. The farmer wants to know the area of land being used for the road. Determine the area in square metres.

18. a) What is the area of paving stone needed for the patio?

b) Paving stones cost $55 per square metre. Estimate how much it would cost to pave the patio. Show your work.

c) Calculate the cost.

Extend

19. The figure below is a parallelogram. What is the length of n?

20. What is the area of this X? Explain how you calculated your answer.

MATH LINK

a) You are going to include in your airport design an area that is in the shape of a parallelogram. Add as many runways and taxi lanes as necessary to create a parallelogram.

b) Determine the area of the parallelogram. If you have more than one parallelogram, use the largest one.

c) You decide to plant grass in the parallelogram-shaped area. If one bag of grass seed covers 1000 m², how many bags of seed will you need for the parallelogram?

Supported Learning

Learning Style and Memory

- You may wish to have students complete **BLM 3–19 Section 3.4 Extra Practice** for additional reinforcement.

MATH LINK

The Math Link on page 107 is intended to provide students with an opportunity to add the shape of a parallelogram to their runway design and then to calculate the area of the shape. Students are then asked to determine the number of bags of grass seed required to cover the parallelogram-shaped area in their design.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 107 is intended to allow students to do additional work on their airport runway design as they work toward the Wrap It Up! activity on page 119.	• Have all students complete this Math Link. • Have students use **BLM 3–10 Section 3.4 Math Link** to assist them with completing the activity.

3.5 Area of a Triangle

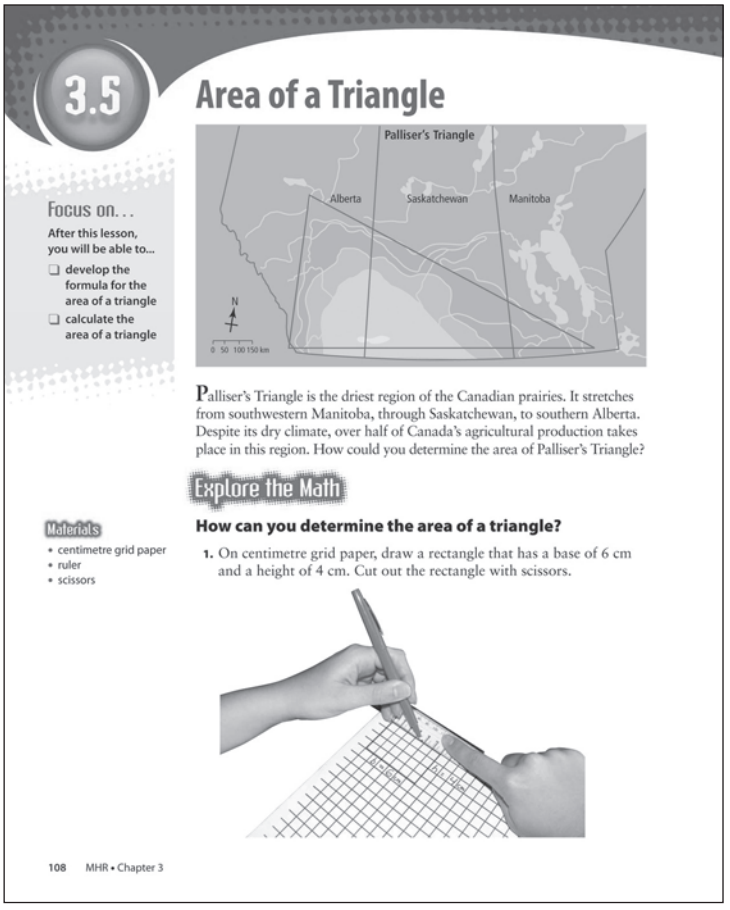
Suggested Timing
80–100 minutes

Materials
- ruler
- scissors

Blackline Masters
Master 5 Tangram
Master 8 Centimetre Grid Paper
BLM 3–1 Chapter 3 Self-Assessment
BLM 3–11 Section 3.5 Extra Practice
BLM 5–12 Section 3.5 Math Link

Mathematical Processes

- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



3.5 Area of a Triangle

Focus on...
After this lesson, you will be able to...
- develop the formula for the area of a triangle
- calculate the area of a triangle

Palliser's Triangle is the driest region of the Canadian prairies. It stretches from southwestern Manitoba, through Saskatchewan, to southern Alberta. Despite its dry climate, over half of Canada's agricultural production takes place in this region. How could you determine the area of Palliser's Triangle?

Explore the Math

Materials
- centimetre grid paper
- ruler
- scissors

How can you determine the area of a triangle?

1. On centimetre grid paper, draw a rectangle that has a base of 6 cm and a height of 4 cm. Cut out the rectangle with scissors.

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Specific Outcomes

SS2 Develop and apply a formula for determining the area of:
- triangles
- parallelograms
- circles.

Warm-Up

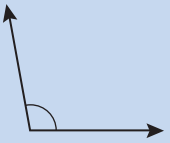
Use this visual to answer #1 to #3.

1. What is the base of the parallelogram in the visual?
2. What is the height of the parallelogram in the visual?
3. Calculate the area of the parallelogram.
4. Estimate and then calculate 100.82 ÷ 14.2.
5. Estimate and then calculate the cost of 5 m of ribbon at $3.99/m.

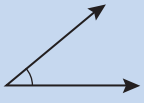
Mental Math

For #6 to #10, answer the question and then explain your thinking.

6. An angle is 302°. Estimate the size of the angle bisector.
7. Without measuring, estimate the size of this angle.



8. Without measuring, estimate the size of this angle.

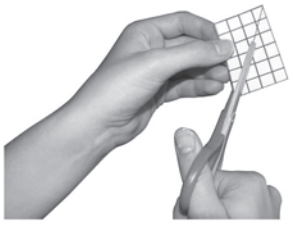


9. Without measuring, estimate the size of this angle.

10. Sketch a 45° angle.

2. Count the number of square centimetres. What is the area of this rectangle?

3. Draw a diagonal line from one corner of the rectangle to the opposite corner. Use scissors to cut along the diagonal line as shown.



4. What are the two new shapes you created?

5. Place these two shapes on top of each other. How do they compare?

6. What is the relationship between the area of one of the triangles and the area of the original rectangle? Explain how you know.

7. a) Predict the length of the base (b) of each triangle. Use a ruler to measure it.
   b) Predict the height (h) of each triangle. Use a ruler to measure it.

8. Is h parallel or perpendicular to b of the triangle?

9. What is the relationship between b and h, and the area of the triangle?

Reflect on Your Findings

10. a) Suggest a formula for calculating the area of a triangle.
    b) Compare your formula with those of your classmates. Discuss any differences and make sure that everyone agrees on the formula.

Answers

Warm-Up

1. base = 3 units **2.** height = 1 unit

3. Area = 3 units^2

4. Estimate: 100 ÷ 10 = 10; 100 ÷ 20 = 5. The answer is between 5 and 10. Calculate: 7.1

5. Estimate: 5 × 4 = $20
Calculate: 5 × 3.99 = $19.95
Note: You may wish to show students this method for calculating the cost of items that are close to a dollar. Alternatively, they can multiply it out.

6. About 150°. This is about half the size of the angle.

7. About 90°. This is close to the angle at the corner of a book.

8. About 45°. This is close to half of the corner of a book.

9. About 180°. This is almost a straight line.

10. Look for something close to 45°. This is an angle bisector of 90°.

Explore the Math

2. 24 cm^2 **4.** triangles

5. They are the same shape and equal in area.

6. The triangle is one half of the area of the rectangle. Together, the triangles have the same area as the rectangle, and each triangle has the same area as the other.

7. a) 6 cm **b)** 4 cm

8. perpendicular

9. If you multiply the base by the height and then divide the product in half, you get the area of the triangle.

10. a) $A = b \times h \div 2$ or $A = \frac{b \times h}{2}$
b) Answers will vary.

Activity Planning Notes

Read through the introduction with students. Discuss students' knowledge of Palliser's Triangle. Ask students:

- Why do you think it is called a triangle?
- What are some of the ways you could determine its area?

Have students estimate the area of Palliser's Triangle in square kilometres, and record the estimates on the board. Discuss how triangles are related to rectangles. Also discuss how they are related to parallelograms in general, which students studied in section 3.4. Then have students predict how the area of a triangle might be related to the area of a rectangle. Again, generalize this to how the area of a triangle might be related to the area of a parallelogram.

Explore the Math

Have students work in pairs to answer the questions in this section. Students begin by drawing a rectangle, cutting it out, and calculating the area. They then transform the rectangle into triangles and explore the area of the triangles. You may wish to have students also perform these steps using a parallelogram to demonstrate that the relationship to triangles applies to all parallelograms.

Supported Learning

Learning Style and Motor

- Have concrete and kinesthetic learners or students with dexterity problems use geoboards instead of cutting out the grid paper. They can make a 6 cm × 4 cm rectangle with one elastic and then use a different-coloured elastic to cut the rectangle in half along the diagonal. Students may wish to check their conclusion using rectangles of various sizes. Ask students: "Does the relationship always apply?" They might also explore this relationship using parallelograms.

Answers

Show You Know: Example 1

4.5 cm^2

Supported Learning

ESL

- You may need to explain to English language learners the meaning of the terms *driest region* and *agricultural production*.
- Students need to understand the word *diagonal*. Have them make a diagonal line across their desks with their finger. Then have students add this word to their translation dictionary.
- English language learners may struggle with the word *relationship*. Ensure that students understand what #6 of Explore the Math is asking.

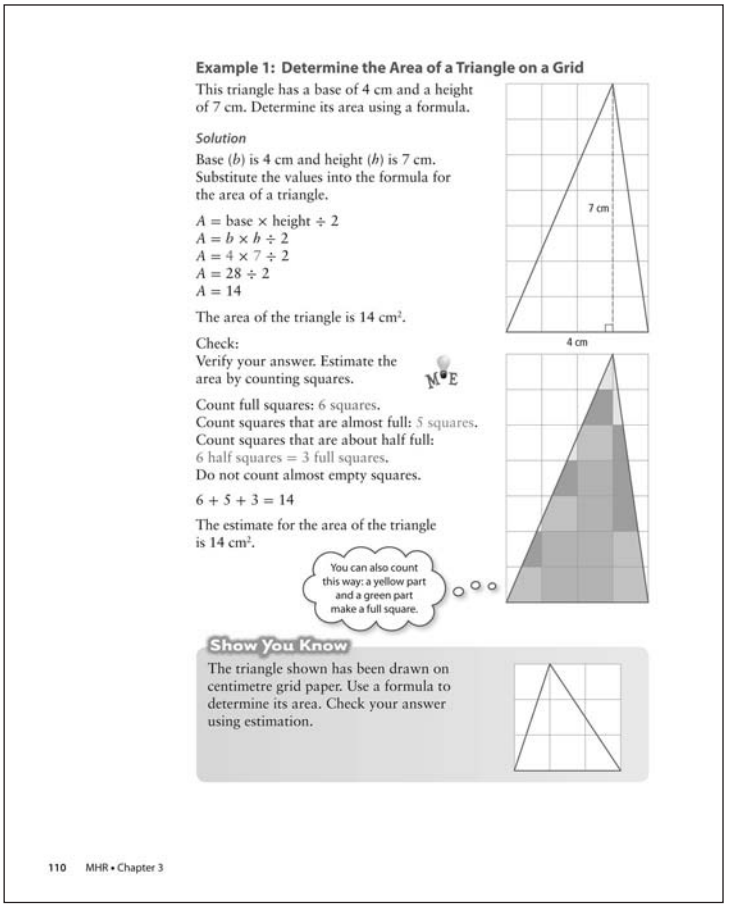
Meeting the Needs of All Learners

- Show students a number of examples of triangles.
- Allow students to work in cooperative groups for the Explore the Math.
- Use a variety of visuals and hands-on activities to ensure that students understand the concepts.

Common Errors

- Students may not recognize the base and height of a triangle.

R_x Remind students that the height of a triangle is perpendicular to the base.



Example 1: Determine the Area of a Triangle on a Grid

This triangle has a base of 4 cm and a height of 7 cm. Determine its area using a formula.

Solution

Base (b) is 4 cm and height (h) is 7 cm. Substitute the values into the formula for the area of a triangle.

A = base × height ÷ 2
$A = b \times h \div 2$
$A = 4 \times 7 \div 2$
$A = 28 \div 2$
$A = 14$

The area of the triangle is 14 cm^2.

Check:
Verify your answer. Estimate the area by counting squares.

Count full squares: 6 squares.
Count squares that are almost full: 5 squares.
Count squares that are about half full: 6 half squares = 3 full squares.
Do not count almost empty squares.

6 + 5 + 3 = 14

The estimate for the area of the triangle is 14 cm^2.

You can also count this way: a yellow part and a green part make a full square.

Show You Know

The triangle shown has been drawn on centimetre grid paper. Use a formula to determine its area. Check your answer using estimation.

7 cm
4 cm

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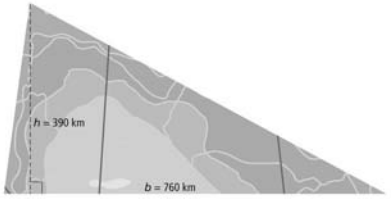
Ensure that students understand how the base and height of the triangles are related to the base and height of the rectangle. It is important that students understand that this relationship exists for all triangles, not just right triangles, which is why it is suggested you also explore how triangles relate to parallelograms. When done, discuss as a class the formula students suggested for calculating the area of a triangle. Determine if anybody came up with a different formula, and discuss whether or not it is correct.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss the formula they developed. During this process, they are generalizing what they have learned during the Explore the Math.	• If students have difficulty understanding that the area of the rectangle is equal to twice the area of the triangle, you may wish to work backward through the process. In other words, start with a triangle, transform it into a rectangle, and then measure the area of the rectangle. This may help reinforce the relationship between the two areas. Consider working backwards using parallelograms also. • You may wish to have students work with the tangram pieces on **Master 5 Tangram** to reinforce what they learned here. The two largest tangram pieces make a square. Have students calculate the area of this square, and then consider that each right triangle that makes up the square is half of the square. Each triangle must therefore have half the area. Students could repeat these steps with the small and medium triangles in the tangram set.

Example 1 asks students to explore how to determine the area of a triangle on a grid.

Example 2: Determine the Area of a Triangle

Palliser's Triangle has a base of approximately 760 km and a height of approximately 390 km. What is the area of Palliser's Triangle?



Solution

$b = 760$ km
$h = 390$ km

Substitute the values into the formula for the area of a triangle.

$A = \frac{b \times h}{2}$

$A = \frac{760 \times 390}{2}$ Substitute the values.

$A = \frac{296\,400}{2}$

$A = 148\,200$

The area of Palliser's Triangle is 148 200 km^2.

Literacy Link

There is more than one way to write the formula for the area of a triangle: $\frac{b \times h}{2}$ means the same as $b \times h \div 2$.

Show You Know

What is the area of the triangle?

3.5 Area of a Triangle • MHR 111

Example 2 shows students how to calculate the area of a triangle to solve a problem. Ensure that students understand that there are different ways to write the formula for the area of a triangle, as indicated in the Literacy Link.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• You may wish to provide additional questions for students who would benefit from them. For example: Determine the area of the following triangles: **a)** base = 15 cm, height = 5 cm ($A = \frac{b \times h}{2}$ $A = \frac{15 \times 5}{2}$ $A = \frac{75}{2}$ $A = 37.5$ cm^2) **b)** base = 45 m, height = 8 m ($A = \frac{b \times h}{2}$ $A = \frac{45 \times 8}{2}$ $A = \frac{360}{2}$ $A = 180$ cm^2) Coach students through a), and then have them work through b) independently. Note that students may prefer to work with this version of the formula: Area = base × height ÷ 2.

Answers

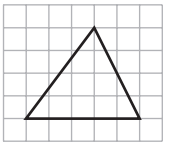
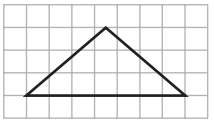
Show You Know: Example 2

13 cm^2

Supported Learning

ESL and Language

- Some English language learners may require support when reading through the steps of Example 2.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know on page 110 related to Example 1.	• Ensure that students take the time to count the squares to estimate the area and verify their answer. • You may wish to provide additional questions for students who would benefit from them. For example: Use a formula to determine the area of the following triangles, which have been drawn on centimetre grid paper. Check your answer using estimation. **a)**  ($A = b \times h \div 2$ $A = 5 \times 4 \div 2$ $A = 10$ cm^2) **b)**  ($A = b \times h \div 2$ $A = 7 \times 3 \div 2$ $A = 10.5$ cm^2) Coach students through a), and then have them work through b) independently.

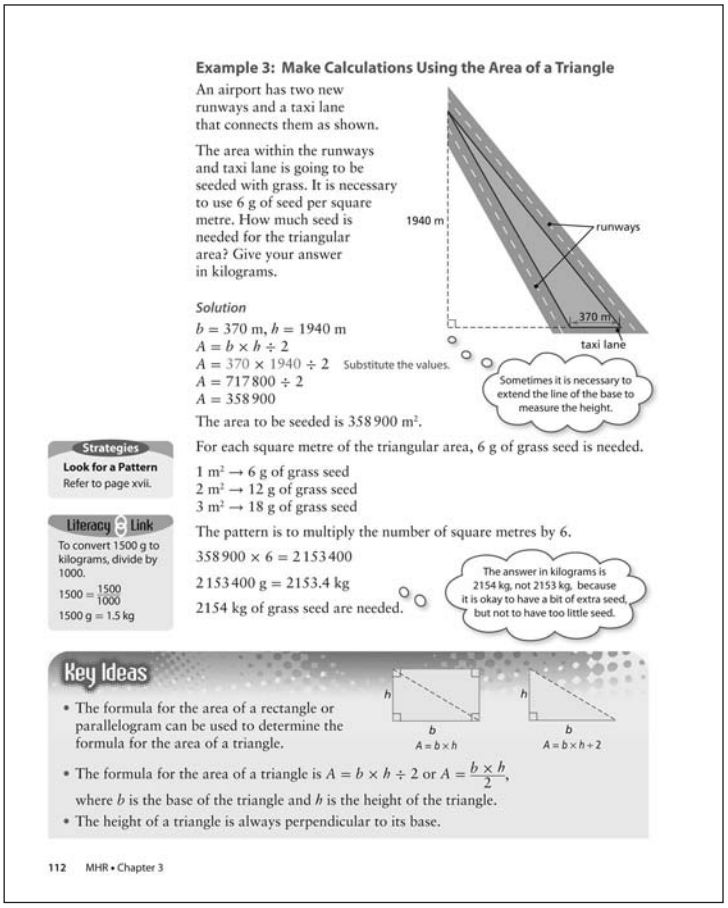
Supported Learning

ESL

- You may need to assist some English language learners in understanding the word *seed*.

Learning Style and Language

- Encourage students to share their understanding of the Key Ideas during group discussion.



Example 3: Make Calculations Using the Area of a Triangle

An airport has two new runways and a taxi lane that connects them as shown.

The area within the runways and taxi lane is going to be seeded with grass. It is necessary to use 6 g of seed per square metre. How much seed is needed for the triangular area? Give your answer in kilograms.

Solution

$b = 370$ m, $h = 1940$ m
$A = b \times h \div 2$
$A = 370 \times 1940 \div 2$ Substitute the values.
$A = 717\,800 \div 2$
$A = 358\,900$

Sometimes it is necessary to extend the line of the base to measure the height.

The area to be seeded is 358 900 m².

Strategies
Look for a Pattern
Refer to page xvii.

For each square metre of the triangular area, 6 g of grass seed is needed.

1 m² → 6 g of grass seed
2 m² → 12 g of grass seed
3 m² → 18 g of grass seed

Literacy Link
To convert 1500 g to kilograms, divide by 1000.
$1500 = \frac{1500}{1000}$
1500 g = 1.5 kg

The pattern is to multiply the number of square metres by 6.

$358\,900 \times 6 = 2\,153\,400$
$2\,153\,400$ g = 2153.4 kg
2154 kg of grass seed are needed.

The answer in kilograms is 2154 kg, not 2153 kg, because it is okay to have a bit of extra seed, but not to have too little seed.

Key Ideas

- The formula for the area of a rectangle or parallelogram can be used to determine the formula for the area of a triangle.

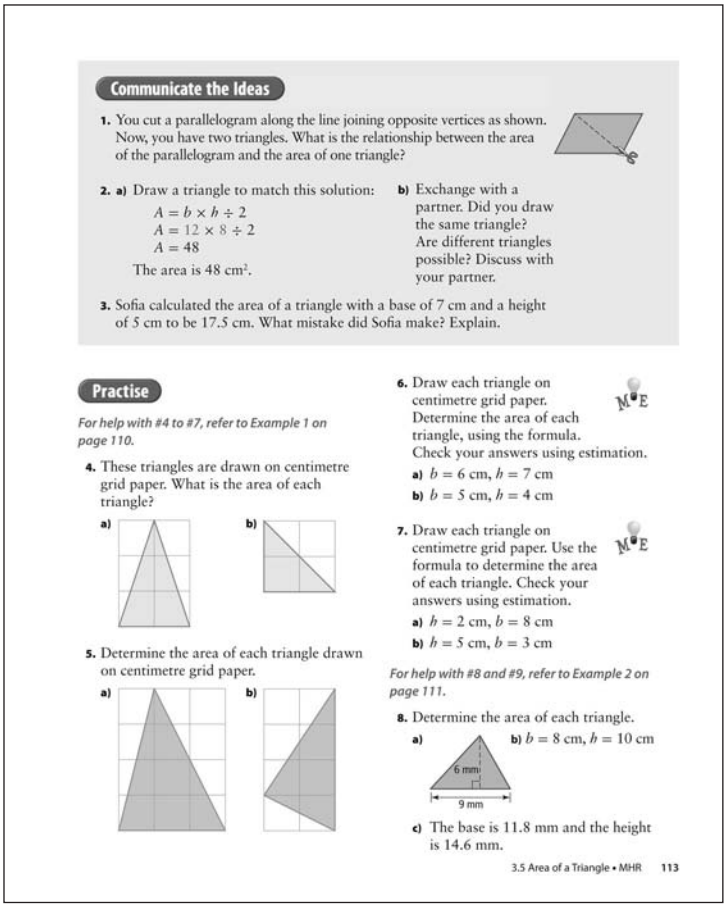
- The formula for the area of a triangle is $A = b \times h \div 2$ or $A = \frac{b \times h}{2}$, where b is the base of the triangle and h is the height of the triangle.
- The height of a triangle is always perpendicular to its base.

112 MHR • Chapter 3

Example 3 shows students how to make calculations using the area of a triangle. Ensure that students understand that it may be necessary to extend the base of a triangle in order to determine the height of a triangle. Remind students that the height refers to perpendicular height. In addition, point out to students the Literacy Link on page 112 to make sure that their answer has the correct units. (Note that students at this level are not expected to do these conversions without guidance.)

Key Ideas

Students could prepare their own list of Key Ideas and put it in their chapter Foldable. An intention of this section is to compare the formula for the area of a rectangle or parallelogram and the formula for the area of a triangle, and to determine the formula for the area of the triangle. Remind students that the height of a triangle is always perpendicular to its base.



Communicate the Ideas

1. You cut a parallelogram along the line joining opposite vertices as shown. Now, you have two triangles. What is the relationship between the area of the parallelogram and the area of one triangle?

2. a) Draw a triangle to match this solution:

$A = b \times h \div 2$
$A = 12 \times 8 \div 2$
$A = 48$
The area is 48 cm².

b) Exchange with a partner. Did you draw the same triangle? Are different triangles possible? Discuss with your partner.

3. Sofia calculated the area of a triangle with a base of 7 cm and a height of 5 cm to be 17.5 cm. What mistake did Sofia make? Explain.

Practise

For help with #4 to #7, refer to Example 1 on page 110.

4. These triangles are drawn on centimetre grid paper. What is the area of each triangle?

a) b)

5. Determine the area of each triangle drawn on centimetre grid paper.

a) b)

6. Draw each triangle on centimetre grid paper. Determine the area of each triangle, using the formula. Check your answers using estimation.
a) $b = 6$ cm, $h = 7$ cm
b) $b = 5$ cm, $h = 4$ cm

7. Draw each triangle on centimetre grid paper. Use the formula to determine the area of each triangle. Check your answers using estimation.
a) $b = 2$ cm, $b = 8$ cm
b) $b = 5$ cm, $b = 3$ cm

For help with #8 and #9, refer to Example 2 on page 111.

8. Determine the area of each triangle.

a) 6 mm, 9 mm b) $b = 8$ cm, $h = 10$ cm

c) The base is 11.8 mm and the height is 14.6 mm.

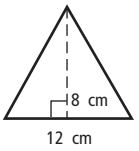
3.5 Area of a Triangle • MHR 113

Answers

Communicate the Ideas

1. The area of the parallelogram is twice the area of one triangle.

2. a) Answers will vary. For example:



b) Answers will vary, but the base and height should be the same. Yes, different triangles are possible.

3. She did not give her answer in square centimetres.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1 *or* 3, 2, 4, 6, 8, 10, 12, Math Link
Typical	1 *or* 3, 2, 4, 6, 8, 10, 11, 13–15, Math Link
Extension/Enrichment	1 *or* 3, 2, 14–19, Math Link

Communicate the Ideas

Have students work in groups or pairs to answer and discuss the questions. Encourage them to explain their answers orally. Have a brief discussion summarizing major points after students have attempted the questions.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Instead of having students answer all three questions, you may wish to have them answer #2 and either #1 *or* #3.	• In #1, students are asked to explain their understanding of the connection between the area of a parallelogram and the area of a triangle. • Students who have a clear understanding will be able to justify their thinking in #2b). Those with a basic understanding may have difficulty explaining why different triangles are possible. • Encourage students to share their answers with another student. Students should be able to justify their reasoning with diagrams and words.

Practise

In #6 and #7, students are asked to draw the specified triangles on centimetre grid paper and use the formula to determine the area of each one. Ensure that students use estimation to check their answers.

Supported Learning

ESL and Language

- Allow students to choose to answer the Communicate the Ideas questions in writing, orally, or using a combination of both methods.

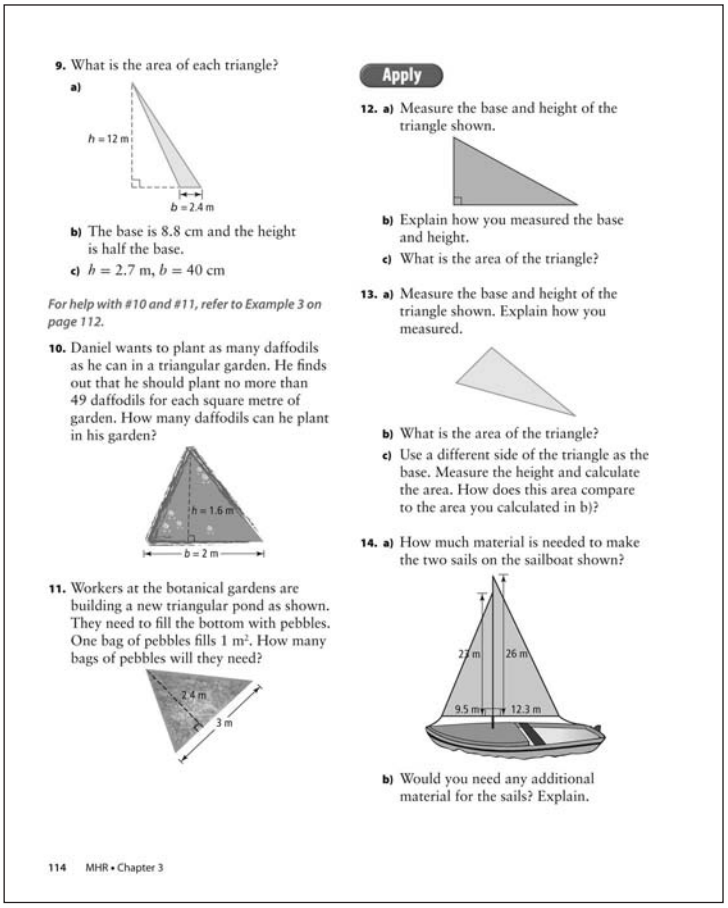
Supported Learning

ESL

- You may need to assist English language learners with the terms *daffodils, botanical gardens, pond,* and *pebbles*. Use pictures to help them with the meanings.

Learning Style and Memory

- You may wish to have students complete **BLM 3–11 Section 3.5 Extra Practice** for additional reinforcement.



9. What is the area of each triangle?

a)

b) The base is 8.8 cm and the height is half the base.

c) $h = 2.7$ m, $b = 40$ cm

For help with #10 and #11, refer to Example 3 on page 112.

10. Daniel wants to plant as many daffodils as he can in a triangular garden. He finds out that he should plant no more than 49 daffodils for each square metre of garden. How many daffodils can he plant in his garden?

11. Workers at the botanical gardens are building a new triangular pond as shown. They need to fill the bottom with pebbles. One bag of pebbles fills 1 m². How many bags of pebbles will they need?

Apply

12. a) Measure the base and height of the triangle shown.

b) Explain how you measured the base and height.

c) What is the area of the triangle?

13. a) Measure the base and height of the triangle shown. Explain how you measured.

b) What is the area of the triangle?

c) Use a different side of the triangle as the base. Measure the height and calculate the area. How does this area compare to the area you calculated in b)?

14. a) How much material is needed to make the two sails on the sailboat shown?

b) Would you need any additional material for the sails? Explain.

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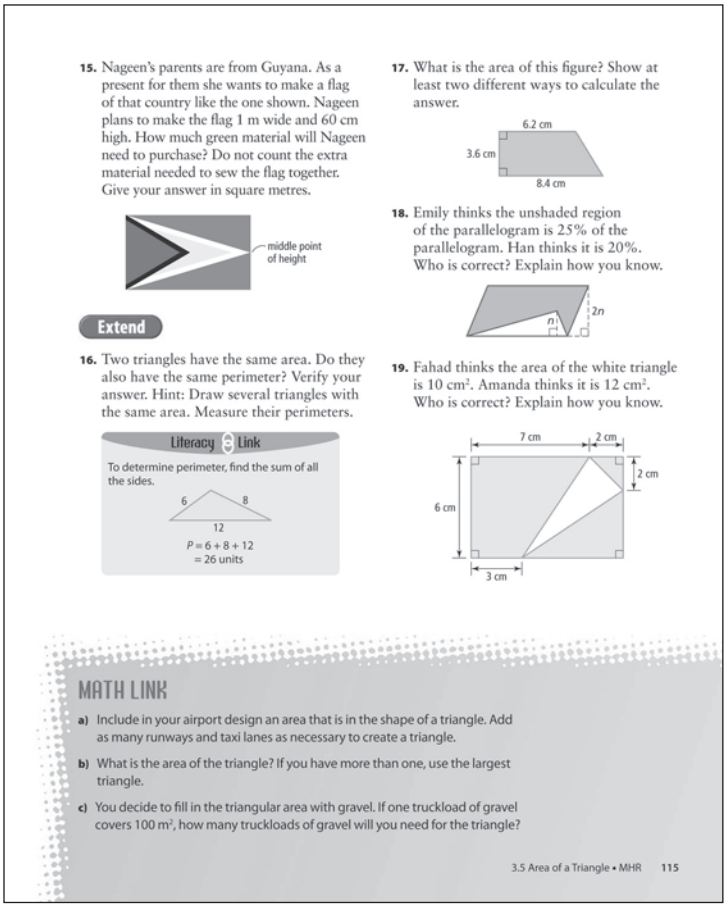
Assessment *for* Learning	Supported Learning
Practise Have students do #4, #6, #8, and #10. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #4 or #6 will need additional coaching with Example 1. Review this material with them, coach them through corrections to their answers, and then have them complete #5 and #7 on their own. • Students who have problems with #8 will need additional coaching with Example 2. Review this material with them, coach them through corrections to their answers, and then have them complete #9 on their own. • Students who have problems with #10 will need additional coaching with Example 3. Review this material with them, coach them through corrections to their answers, and then have them complete #11 on their own. • Check back with students several times to make sure that they understand the concepts.

Apply and Extend

The Apply questions provide a variety of fairly straightforward contexts in which the area of triangles is used.

The Extend problems require significantly more analysis than those in the Apply section.



15. Nageen's parents are from Guyana. As a present for them she wants to make a flag of that country like the one shown. Nageen plans to make the flag 1 m wide and 60 cm high. How much green material will Nageen need to purchase? Do not count the extra material needed to sew the flag together. Give your answer in square metres.

Extend

16. Two triangles have the same area. Do they also have the same perimeter? Verify your answer. Hint: Draw several triangles with the same area. Measure their perimeters.

Literacy Link

To determine perimeter, find the sum of all the sides.

$P = 6 + 8 + 12$
$= 26$ units

17. What is the area of this figure? Show at least two different ways to calculate the answer.

18. Emily thinks the unshaded region of the parallelogram is 25% of the parallelogram. Han thinks it is 20%. Who is correct? Explain how you know.

19. Fahad thinks the area of the white triangle is 10 cm². Amanda thinks it is 12 cm². Who is correct? Explain how you know.

MATH LINK

a) Include in your airport design an area that is in the shape of a triangle. Add as many runways and taxi lanes as necessary to create a triangle.

b) What is the area of the triangle? If you have more than one, use the largest triangle.

c) You decide to fill in the triangular area with gravel. If one truckload of gravel covers 100 m², how many truckloads of gravel will you need for the triangle?

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • How is the formula for the area of a rectangle (or a parallelogram) related to the formula for the area of a triangle? • How can you remember the difference between the two formulas?	• Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • You may also wish to have students review the part related to section 3.5 in **BLM 3–1 Chapter 3 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

MATH LINK

The Math Link on page 115 is intended to provide students with an opportunity to add an area in the shape of a triangle to their runway design by drawing runways and taxi lanes. Students calculate the area of the triangle and then determine the amount of gravel required to cover the triangle-shaped area in their design.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 115 is intended to allow students to do additional work on their airport runway design as they work toward the Wrap It Up! on page 119.	• Have all students complete this Math Link. • Have students use **BLM 3–12 Section 3.5 Math Link** to assist them with completing the activity.

3 Chapter Review

Suggested Timing
40–50 minutes

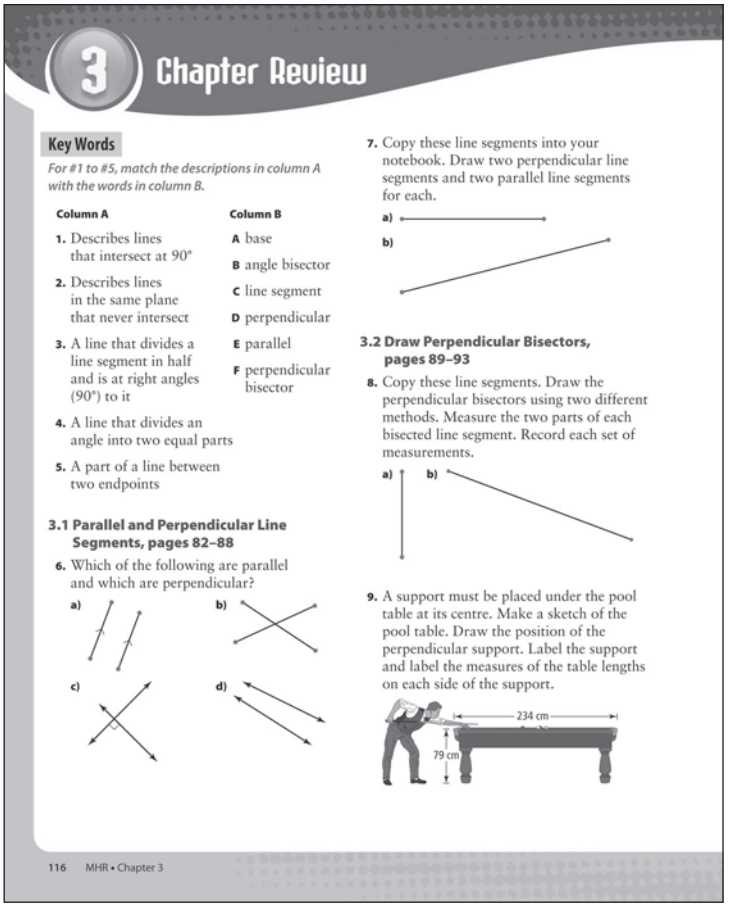
Materials
- ruler
- right triangle
- compass
- protractor

Blackline Masters
BLM 3–1 Chapter 3 Self-Assessment
BLM 3–3 Section 3.1 Extra Practice
BLM 3–5 Section 3.2 Extra Practice
BLM 3–7 Section 3.3 Extra Practice
BLM 3–9 Section 3.4 Extra Practice
BLM 3–11 Section 3.5 Extra Practice

Supported Learning

Learning Style and Memory

- Students who require more practice on a particular topic may refer to **BLM 3–3 Section 3.1 Extra Practice**, **BLM 3–5 Section 3.2 Extra Practice**, **BLM 3–7 Section 3.3 Extra Practice**, **BLM 3–9 Section 3.4 Extra Practice**, and **BLM 3–11 Section 3.5 Extra Practice**.



3 Chapter Review

Key Words

For #1 to #5, match the descriptions in column A with the words in column B.

Column A	Column B
1. Describes lines that intersect at 90°	A base
2. Describes lines in the same plane that never intersect	B angle bisector
3. A line that divides a line segment in half and is at right angles (90°) to it	C line segment
4. A line that divides an angle into two equal parts	D perpendicular
5. A part of a line between two endpoints	E parallel
	F perpendicular bisector

3.1 Parallel and Perpendicular Line Segments, pages 82–88

6. Which of the following are parallel and which are perpendicular?

a) b) c) d)

7. Copy these line segments into your notebook. Draw two perpendicular line segments and two parallel line segments for each.

a) b)

3.2 Draw Perpendicular Bisectors, pages 89–93

8. Copy these line segments. Draw the perpendicular bisectors using two different methods. Measure the two parts of each bisected line segment. Record each set of measurements.

a) b)

9. A support must be placed under the pool table at its centre. Make a sketch of the pool table. Draw the position of the perpendicular support. Label the support and label the measures of the table lengths on each side of the support.

234 cm
79 cm

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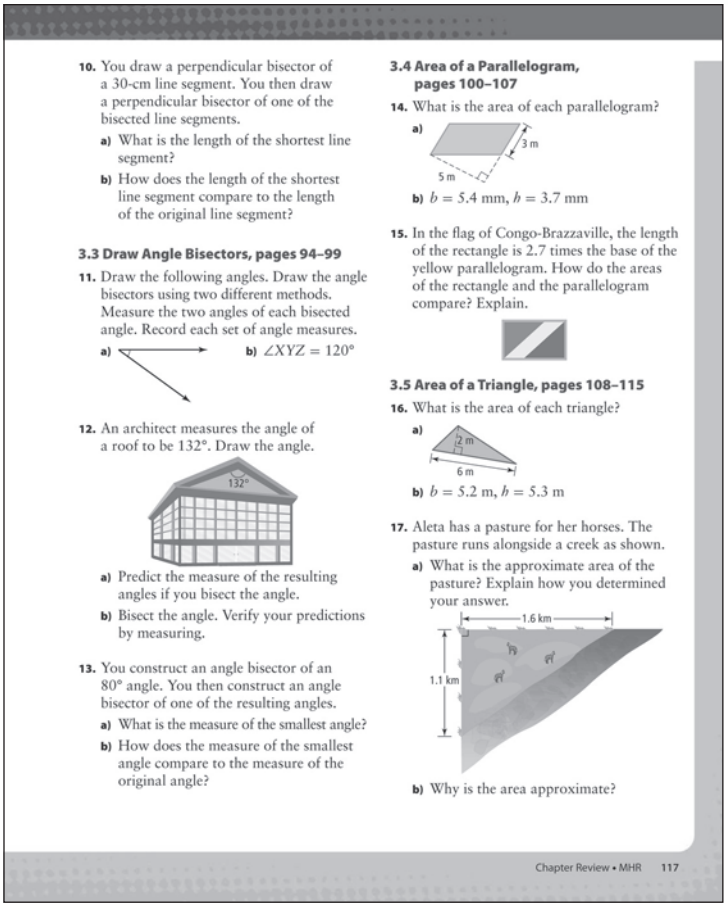
Activity Planning Notes

Have students write the numbers from 6 to 17 in two columns in their notebooks. Have students circle questions they may need a little help with, a lot of help with, or no help with using the colours they used on **BLM 3–1 Chapter 3 Self-Assessment**.

Students could work independently or in pairs on the chapter review. Have students check their answers and then revisit any question with which they had difficulty. Provide an opportunity for students to discuss questions, consider alternative methods of solving, and ask about questions they found difficult.

Assessment *for* Learning	Supported Learning
Chapter 3 Review The Chapter 3 Review provides an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On tab of their chapter Foldable. Have students do at least one question related to any concept, skill, or process that has been giving them trouble. • Have students revisit any section they are having difficulty with prior to working on the Chapter 3 Test.



10. You draw a perpendicular bisector of a 30-cm line segment. You then draw a perpendicular bisector of one of the bisected line segments.
   a) What is the length of the shortest line segment?
   b) How does the length of the shortest line segment compare to the length of the original line segment?

3.3 Draw Angle Bisectors, pages 94–99

11. Draw the following angles. Draw the angle bisectors using two different methods. Measure the two angles of each bisected angle. Record each set of angle measures.
   a)
   b) $\angle XYZ = 120°$

12. An architect measures the angle of a roof to be 132°. Draw the angle.

   a) Predict the measure of the resulting angles if you bisect the angle.
   b) Bisect the angle. Verify your predictions by measuring.

13. You construct an angle bisector of an 80° angle. You then construct an angle bisector of one of the resulting angles.
   a) What is the measure of the smallest angle?
   b) How does the measure of the smallest angle compare to the measure of the original angle?

3.4 Area of a Parallelogram, pages 100–107

14. What is the area of each parallelogram?
   a)

   b) $b = 5.4$ mm, $h = 3.7$ mm

15. In the flag of Congo-Brazzaville, the length of the rectangle is 2.7 times the base of the yellow parallelogram. How do the areas of the rectangle and the parallelogram compare? Explain.

3.5 Area of a Triangle, pages 108–115

16. What is the area of each triangle?
   a)

   b) $b = 5.2$ m, $h = 5.3$ m

17. Aleta has a pasture for her horses. The pasture runs alongside a creek as shown.
   a) What is the approximate area of the pasture? Explain how you determined your answer.

   b) Why is the area approximate?

Supported Learning

Learning Style, ESL, Language, and Memory

- Review the Key Words that are used in this chapter. Have students locate the words in the student resource and explain their meaning to a classmate.
- Allow students to practise the vocabulary terms using flash cards. Have students work together to quiz each other on the Key Words for the chapter.
- Allow students to complete the chapter review using any combination of oral description, diagrams for explanation, and written answers.

Gifted and Enrichment

- Some students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge for gifted students, go to **www.mathlinks7.ca** and follow the links.

Assessment *as* Learning	Supported Learning
Math Learning Log Once students have completed the Chapter 3 Review and prior to the Chapter 3 Test, have them reflect on their progress and write a journal entry that completes the following statements: – I am comfortable with the following parts of the chapter ... – I am having difficulty with ... – Here's how I worked on some of the areas I originally had difficulty with ... – Here's how I plan to address the areas I am still having difficulty with ...	• Have students use the What I Need to Work On tab of their chapter Foldable to provide information about what they continue to have problems with and what problems they had that have now been resolved. • You may wish to have students refer to **BLM 3–1 Chapter 3 Self-Assessment** when they report on what they are comfortable with, what they continue to have difficulty with, and what they plan to do about it.

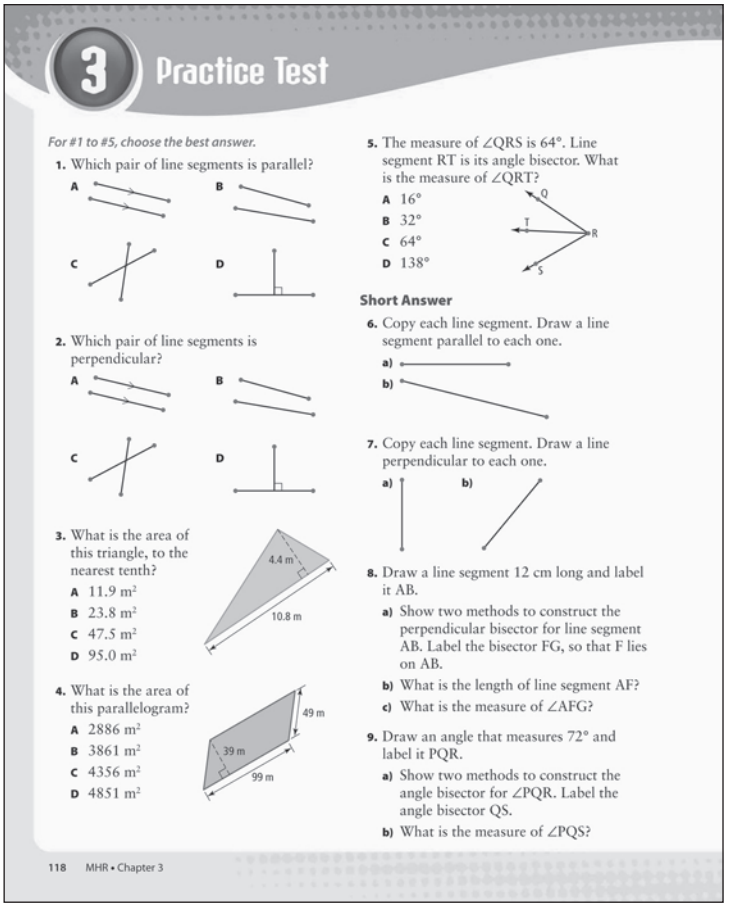
3 Practice Test

Suggested Timing
40–50 minutes

Materials
- ruler
- right triangle
- compass
- protractor

Blackline Masters
BLM 3–1 Chapter 3 Self-Assessment
BLM 3 BLM 3–13 Chapter 3 Test

Assessment *as* Learning	Supported Learning
Chapter 3 Self-Assessment Have students review their earlier responses on **BLM 3–1 Chapter 3 Self-Assessment**.	• Have students use their responses on the Chapter 3 Practice Test and work they completed earlier in the chapter to complete the After column of this self-assessment. Before they do the **BLM 3–13 Chapter 3 Test,** coach students in the areas in which they are having problems.



3 Practice Test

For #1 to #5, choose the best answer.

1. Which pair of line segments is parallel?

A B C D

2. Which pair of line segments is perpendicular?

A B C D

3. What is the area of this triangle, to the nearest tenth?
A 11.9 m²
B 23.8 m²
C 47.5 m²
D 95.0 m²

4.4 m, 10.8 m

4. What is the area of this parallelogram?
A 2886 m²
B 3861 m²
C 4356 m²
D 4851 m²

49 m, 39 m, 99 m

5. The measure of ∠QRS is 64°. Line segment RT is its angle bisector. What is the measure of ∠QRT?
A 16°
B 32°
C 64°
D 138°

Q, T, R, S

Short Answer

6. Copy each line segment. Draw a line segment parallel to each one.
a)
b)

7. Copy each line segment. Draw a line perpendicular to each one.
a) b)

8. Draw a line segment 12 cm long and label it AB.
a) Show two methods to construct the perpendicular bisector for line segment AB. Label the bisector FG, so that F lies on AB.
b) What is the length of line segment AF?
c) What is the measure of ∠AFG?

9. Draw an angle that measures 72° and label it PQR.
a) Show two methods to construct the angle bisector for ∠PQR. Label the angle bisector QS.
b) What is the measure of ∠PQS?

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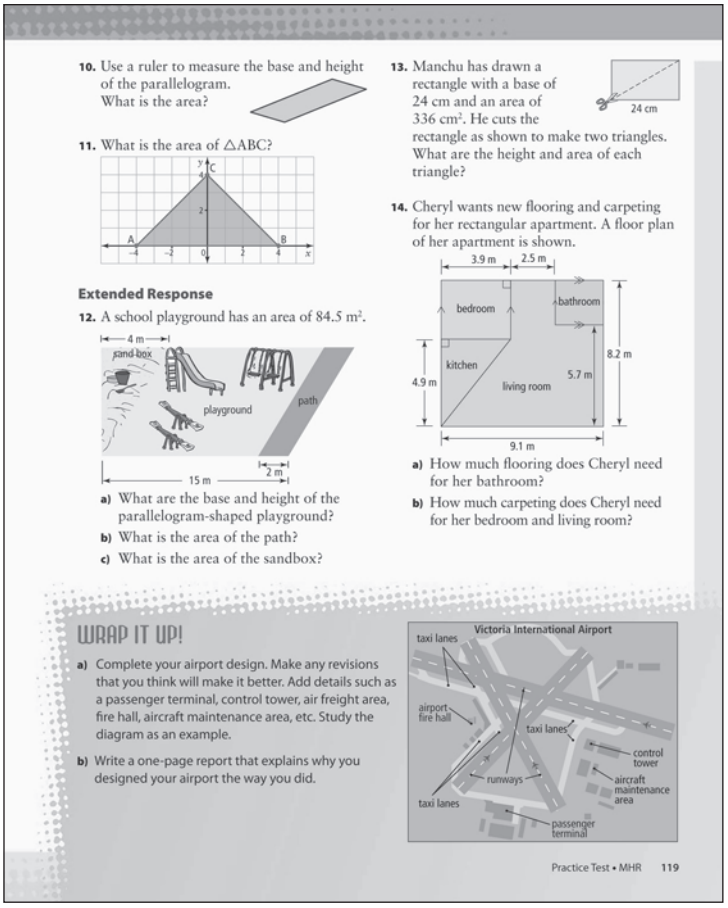
Study Guide

Question(s)	Section(s)	Refer to	I can …
1, 6	3.1	Examples 1, 2	✓ identify and draw parallel line segments
2, 7	3.1	Examples 1, 3	✓ identify and draw perpendicular line segments
3, 11	3.5	Examples 1, 2	✓ calculate the area of a triangle
4, 10	3.4	Examples 1, 2	✓ calculate the area of a parallelogram
5, 9	3.3	Explore the Math Example	✓ draw an angle bisector
8	3.2	Example	✓ draw a perpendicular bisector
12	3.4 3.5	Examples 1, 3 Example 2	✓ solve problems that involve calculating the area of a parallelogram ✓ solve problems that involve calculating the area of a triangle
13	3.5	Explore the Math Example 2	✓ develop the formula for the area of a triangle ✓ solve problems that involve calculating the area of a triangle



10. Use a ruler to measure the base and height of the parallelogram. What is the area?

11. What is the area of △ABC?

Extended Response

12. A school playground has an area of 84.5 m².

a) What are the base and height of the parallelogram-shaped playground?

b) What is the area of the path?

c) What is the area of the sandbox?

13. Manchu has drawn a rectangle with a base of 24 cm and an area of 336 cm². He cuts the rectangle as shown to make two triangles. What are the height and area of each triangle?

14. Cheryl wants new flooring and carpeting for her rectangular apartment. A floor plan of her apartment is shown.

a) How much flooring does Cheryl need for her bathroom?

b) How much carpeting does Cheryl need for her bedroom and living room?

WRAP IT UP!

a) Complete your airport design. Make any revisions that you think will make it better. Add details such as a passenger terminal, control tower, air freight area, fire hall, aircraft maintenance area, etc. Study the diagram as an example.

b) Write a one-page report that explains why you designed your airport the way you did.

Victoria International Airport

Practice Test • MHR 119

Supported Learning

Learning Style and Memory

- For students who respond well to project work and struggle with more formal tests, consider using the completed report from the Wrap It Up! activity as summative evidence in place of a chapter test.

Activity Planning Notes

This practice test can be assigned as an in-class or take-home assignment. These are the minimum questions that will meet the related curriculum outcomes: #3, #4, #6–#11.

Answers to the Chapter 3 Practice Test are provided on **BLM 3-16 Chapter 3 *MathLinks 7* Student Resource Answers**.

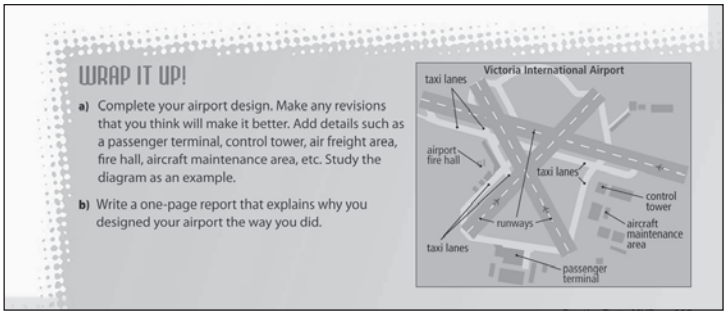
Assessment *of* Learning	Supported Learning
Chapter 3 Test After students complete the practice test, you may wish to use **BLM 3–13 Chapter 3 Test** as a summative assessment.	• Consider allowing students to use their chapter Foldable. • Consider using the Math Games on page 120 or the Challenge in Real Life on page 121 to assess the knowledge and skills of students who have difficulty with tests.

Wrap It Up!

Suggested Timing
40–50 minutes

Materials
- ruler
- right triangle
- compass
- protractor

Blackline Masters
Master 1 Project Rubric
BLM 3–4 Section 3.1 Math Link
BLM 3–6 Section 3.2 Math Link
BLM 3–8 Section 3.3 Math Link
BLM 3–9 Section 3.4 Math Link
BLM 3–14 Chapter 3 Wrap It Up!



Specific Outcomes

SS2 Develop and apply a formula for determining the area of:
- triangles
- parallelograms
- circles.

SS3 Perform geometric constructions, including:
- perpendicular line segments
- parallel line segments
- perpendicular bisectors
- angle bisectors.

Supported Learning

Learning Style and Language
- Students who struggle with written reports could do an oral presentation for the Wrap It Up! or they could complete their design without writing a report.

Motor
- Consider allowing students to use a computer to write their report for the Wrap It Up!

Activity Planning Notes

Introduce the problem and clarify the assessment criteria. Make the activity as real as possible by discussing the effects of wind on runway orientation. You may wish to provide students with additional examples of airport designs. Discuss the purposes of the passenger terminal, control tower, air freight area, fire hall, and aircraft maintenance area.

Assessment *of* Learning	Supported Learning
Wrap It Up! This chapter Wrap It Up! is intended to allow students to design an airport runway system. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this Wrap It Up! Page 119a provides notes on how to use the rubric for this activity.	• You may wish students to review the Math Links they completed earlier by referring to **BLM 3–4 Section 3.1 Math Link**, **BLM 3–6 Section 3.2 Math Link**, **BLM 3–8 Section 3.3 Math Link**, **BLM 3–9 Section 3.4 Math Link**, and **BLM 3–14 Section 3.5 Math Link**. • If students have not completed the Math Links earlier in the chapter, they will need to do them before completing this Wrap It Up! • Encourage students to include as many details as possible on their design. • In their reports, students should include – why they drew the runways and taxi lanes the way they did – why they placed the buildings where they did – the other factors they considered when drawing their design • You may wish to have students use **BLM 3–14 Chapter 3 Wrap It Up!**, which provides scaffolding for the chapter problem wrap-up.

The chart below shows **Master 1 Project Rubric** for tasks such as the Wrap It Up! and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete solution, which may have a minor mathematical, drawing, or calculation error that does not affect the final conclusion
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete airport design, including extra details, but with a few flaws or a minor omission *or* • provides a complete solution with weak communication; the question of "why" is not addressed or justified
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides an airport diagram that meets the basic details outlined in the question • includes some taxi lanes or runways that do not meet the requirements, or omits some of them • includes some calculation errors in the grass or gravel calculations • includes an explanation that addresses some of the design
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a design that has many of the requirements missing • includes many mathematical calculation errors or omissions • includes an explanation that addresses only partial pieces of the design
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• starts the design but the mathematical calculations are incorrect • includes an explanation that is weak, or the explanation is absent

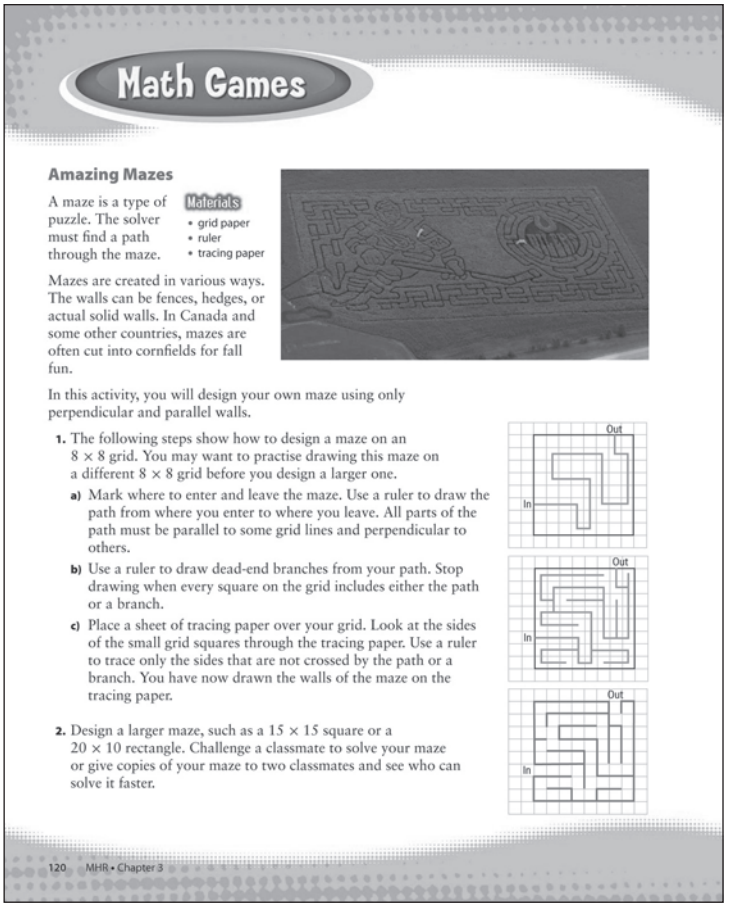
Math Games

Suggested Timing
40–50 minutes

Materials
- ruler
- tracing paper

Blackline Masters
Master 8 Centimetre Grid Paper

Assessment *for* Learning	Supported Learning
Amazing Mazes Have students create a maze and then have classmates solve it.	• After students have solved several mazes, brainstorm winning strategies, such as always taking the left branch. • Ensure that students understand that when drawing their maze, they should not draw the path or any dead ends on top of any of the grid lines. The maze lines should pass through the centre of the squares, not along the edges of the squares. Only when drawing the finished product will students draw lines along the edges of the untouched squares.



Math Games

Amazing Mazes

A maze is a type of puzzle. The solver must find a path through the maze.

Materials
- grid paper
- ruler
- tracing paper

Mazes are created in various ways. The walls can be fences, hedges, or actual solid walls. In Canada and some other countries, mazes are often cut into cornfields for fall fun.

In this activity, you will design your own maze using only perpendicular and parallel walls.

1. The following steps show how to design a maze on an 8 × 8 grid. You may want to practise drawing this maze on a different 8 × 8 grid before you design a larger one.
   a) Mark where to enter and leave the maze. Use a ruler to draw the path from where you enter to where you leave. All parts of the path must be parallel to some grid lines and perpendicular to others.
   b) Use a ruler to draw dead-end branches from your path. Stop drawing when every square on the grid includes either the path or a branch.
   c) Place a sheet of tracing paper over your grid. Look at the sides of the small grid squares through the tracing paper. Use a ruler to trace only the sides that are not crossed by the path or a branch. You have now drawn the walls of the maze on the tracing paper.
2. Design a larger maze, such as a 15 × 15 square or a 20 × 10 rectangle. Challenge a classmate to solve your maze or give copies of your maze to two classmates and see who can solve it faster.

In Out

In Out

In Out

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Common Errors

- Students may make their maze too simple.

R_x Make sure that students include a number of turns and dead-end branches to keep the maze interesting.

Supported Learning

Gifted and Enrichment

- Have students create a much larger grid, possibly using a computer.
- Have students research possible links to the real world, such as game design, drafting, etc.

Specific Outcomes

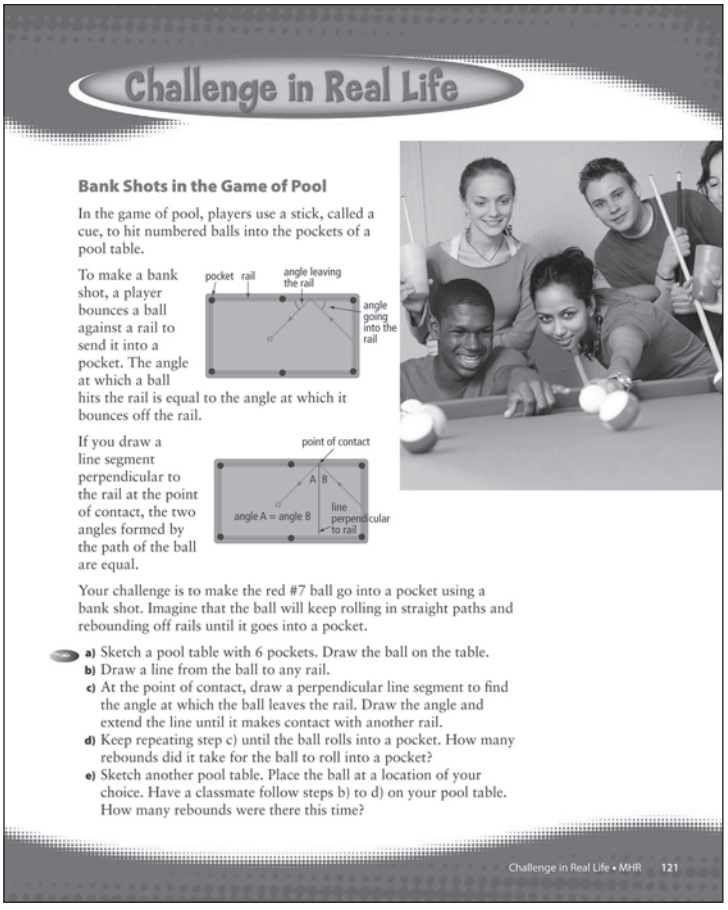
SS3 Perform geometric constructions, including:
- perpendicular line segments
- perpendicular bisectors
- parallel line segments
- angle bisectors.

Activity Planning Notes

There are two parts to this activity. Students will first create a maze puzzle, and then their classmates will solve the maze or two classmates will see who can solve the maze faster.

Discuss with students mazes in different contexts. In *Harry Potter and the Goblet of Fire* by J.K. Rowling, Harry Potter has to find his way through a maze as one part of the Triwizard Tournament. Other references in literature often associate mazes with searching for a treasure. The mathematician Leonhard Euler was one of the first to analyse plane mazes mathematically, and in doing so, founded the science of topology. Have students use the Internet to research mazes and strategies for solving them.

Challenge in Real Life



Challenge in Real Life

Bank Shots in the Game of Pool

In the game of pool, players use a stick, called a cue, to hit numbered balls into the pockets of a pool table.

To make a bank shot, a player bounces a ball against a rail to send it into a pocket. The angle at which a ball hits the rail is equal to the angle at which it bounces off the rail.

If you draw a line segment perpendicular to the rail at the point of contact, the two angles formed by the path of the ball are equal.

Your challenge is to make the red #7 ball go into a pocket using a bank shot. Imagine that the ball will keep rolling in straight paths and rebounding off rails until it goes into a pocket.

a) Sketch a pool table with 6 pockets. Draw the ball on the table.
b) Draw a line from the ball to any rail.
c) At the point of contact, draw a perpendicular line segment to find the angle at which the ball leaves the rail. Draw the angle and extend the line until it makes contact with another rail.
d) Keep repeating step c) until the ball rolls into a pocket. How many rebounds did it take for the ball to roll into a pocket?
e) Sketch another pool table. Place the ball at a location of your choice. Have a classmate follow steps b) to d) on your pool table. How many rebounds were there this time?

Challenge in Real Life • MHR 121

Suggested Timing
60–75 minutes

Materials
- ruler
- compass or right triangle (optional)
- protractor

Blackline Masters
Master 1 Project Rubric

BLM 3–15 Pool Table

Mathematical Processes

- [ ] Communication
- [x] Connections
- [ ] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [x] Technology
- [x] Visualization

Specific Outcomes

SS3 Perform geometric constructions, including:
- perpendicular line segments
- parallel line segments
- perpendicular bisectors
- angle bisectors.

Activity Planning Notes

You may wish to use the following steps to introduce and complete this challenge:

1. Read the first paragraph of Bank Shots in the Game of Pool and have students examine the visual of the pool table in the student resource. Have students share their knowledge of the game of pool. Discuss how the game is played so that all students understand the object of the game.
2. Read the second paragraph and have students look at the illustrations of the bank shot. Discuss which angles are equal. Demonstrate, or have a student demonstrate, how to draw the perpendicular line segment, how to measure the resulting angle, and how to draw the angle at which the ball leaves the rail.

Supported Learning

ESL and Language

- Provide students with guidance and visuals to help them understand the game of pool and the related vocabulary (e.g., *rail, bank shot, rebound, pocket*).

Gifted and Enrichment

- Encourage students to investigate whether it is possible that a ball might never go into a pocket. Have them determine a bank shot angle that will have this result (e.g., a shot perpendicular to the rail).
- Ask students to investigate how to choose the angle of the first shot so that the ball will go into a pocket with a minimum number of rebounds.
- Have students take into consideration how friction and the spin on the ball affect the distance and the angle at which the ball travels.

Supported Learning

Learning Style

- Kinesthetic learners could draw the perpendicular line segment using paper folding.

3. Review different methods of drawing a perpendicular line segment (e.g., using a protractor, a right triangle, a compass, or paper folding).
4. Clarify that the task is to
   - draw the first line from the centre of the ball to a rail of their choice
   - draw a line segment perpendicular to the rail at the point of contact (encourage students to notice that this line forms an angle bisector)
   - measure the angle of the ball going into the rail
   - draw an equal angle for the ball leaving the rail
   - extend this line until it touches the rail
   - continue these steps until the ball goes into a pocket
   - count how many rebounds it took for the ball to go into the pocket
   - create a problem for a classmate by placing a ball at a location of their choice on a drawing of a pool table
   - solve the problem given to them by a classmate and count how many rebounds it took for the ball to go into the pocket
5. Review the **Master 1 Project Rubric** with students so that they will know what is expected.

This challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment *for* Learning	Supported Learning
Bank Shots in the Game of Pool Discuss the challenge with the class. Discuss the rules for the game of pool and how angles can be used to make accurate shots. Have students work individually to draw how a ball will roll in straight paths and rebound off rails until it goes into a pocket.	• You may wish to provide students with **BLM 3-15 Pool Table**. • Review with students how to draw a perpendicular line segment to create an angle bisector. • As a class or a small group, discuss the factors students must consider as they do their drawing. • For a second challenge, complete with teaching notes and student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

Assessment *of* Learning	Supported Learning
Bank Shots in the Game of Pool Discuss the challenge with the class. Discuss the rules for the game of pool and how angles can be used to make accurate shots. Have students work individually to draw how a ball will roll in straight paths and rebound off rails until it goes into a pocket.	• You may wish to provide students with **BLM 3-15 Pool Table**. • Use **Master 1 Project Rubric** to assist you in assessing student work. Page 121a provides notes on how to use this rubric for this challenge. • To view student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as the Challenge in Real Life and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution with mathematical justifications present for all calculations (one minor error may be allowed) • includes a problem that is well outlined and diagrammed for sharing with a partner
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete solution that may contain a couple of calculation errors (maximum of two) *or* • provides a complete solution but the path of the ball does not take an efficient route to the pocket
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• draws the initial line from the centre of the ball to the rail and subsequent lines are drawn but angle measurements may contain errors resulting in the ball not always going to the pocket • provides a design that is somewhat functional and meets the basic understanding of the problem but the path may not be efficient • creates a problem that has errors
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• draws lines that do not meet the requirements of the question • makes several errors in bisecting angles and drawing perpendicular line segments • provides a drawing that does not reflect a successful path for the ball • shows weak communication skills
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• attempts to begin a ball path but numerous drawing or angle calculation errors are evident • makes a correct attempt at any part of the question