Probability

5

General Outcome

- Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Outcomes

SP4 Express probabilities as ratios, fractions and percents.
SP5 Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.
SP6 Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
5.1	✓ find the probability of an event in several different ways
	✓ give answers as probabilities from 0% to 100%
5.2	✓ explain how to identify an independent event
	✓ determine the outcomes of two independent events
	✓ organize the outcomes of two independent events using tables and tree diagrams
5.3	✓ solve probability problems involving two independent events
5.4	✓ use tree diagrams, tables, and other graphic organizers to solve probability problems
5.5	✓ conduct a probability experiment and organize the results
	✓ compare experimental probability with theoretical probability

Assessment *as* Learning	Supported Learning
Use the Before column of **BLM 5–1 Chapter 5 Self-Assessment** to provide students with the big picture for this chapter and to help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• As students complete each section of the chapter or complete the Chapter 5 Review, have them review the related parts of **BLM 5–1 Chapter 5 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Chapter 5 Planning Chart

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter Opener • 20–30 minutes		BLM 5–1 Chapter 5 Self-Assessment BLM 5–2 Probability	• paper • scissors • stapler • examples of games that use dice (optional)
5.1 Probability • 80–100 minutes	**Essential:** 1, 2, 3 *or* 4, 5 *or* 6, 7, Math Link **Typical:** 1, 2, 3 *or* 4, 5 *or* 6, 7, 8 *or* 9, 10, Math Link **Extension/Enrichment:** 1, 2, 10–12	Master 2 Two Stars and One Wish BLM 5–1 Chapter 5 Self-Assessment BLM 5–3 Section 5.1 Extra Practice BLM 5–4 Section 5.1 Math Link	• ruler • marbles or other coloured counters (optional) • bag (optional)
5.2 Organize Outcomes • 80–100 minutes	**Essential:** 1 *or* 3, 2, 4 *or* 5, 6 *or* 7, 8, Math Link **Typical:** 1 *or* 3, 2, 4 *or* 5, 6 *or* 7, 8, 9 *or* 10, 11, 12, Math Link **Extension/Enrichment:** 1 *or* 3, 2, 13, 14	BLM 5–1 Chapter 5 Self-Assessment BLM 5–5 Section 5.2 Extra Practice BLM 5–6 Section 5.2 Math Link	• coins—quarters, dimes, nickels, and pennies (optional) • ruler
5.3 Probabilities of Simple Independent Events • 80–100 minutes	**Essential:** 1–4, 6, 8, 9 **Typical:** 1–4, 6, 8–11, 12 *or* 13 **Extension/Enrichment:** 1, 2, 13–15	BLM 5–1 Chapter 5 Self-Assessment BLM 5–7 Section 5.3 Extra Practice	• ruler
5.4 Applications of Independent Events • 80–100 minutes	**Essential:** 1–3, 5, 7, Math Link **Typical:** 1–3, 5, *two of* 7, 8, 9, Math Link **Extension/Enrichment:** 1, 2, 10, 11, Math Link	BLM 5–1 Chapter 5 Self-Assessment BLM 5–8 Section 5.4 Extra Practice BLM 5–9 Crunch Time Game Board BLM 5–10 Section 5.4 Math Link	• 2 dice • ruler • integer chips or coins
5.5 Conduct Probability Experiments • 80–100 minutes	**Essential:** 1–4, 6, 8 **Typical:** 1–4, 6, 8–11 **Extension/Enrichment:** 1–3, 9, 11–13	BLM 5–1 Chapter 5 Self-Assessment BLM 5–11 Random Number Generator BLM 5–12 Section 5.5 Extra Practice	• paper clip • pencil • compass or circular object to trace around • 2 coloured pencils • ruler
Chapter 5 Review • 40–50 minutes	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	BLM 5–1 Chapter 5 Self-Assessment BLM 5–3 Section 5.1 Extra Practice BLM 5–5 Section 5.2 Extra Practice BLM 5–7 Section 5.3 Extra Practice BLM 5–8 Section 5.4 Extra Practice BLM 5–12 Section 5.5 Extra Practice	• ruler • loading-strip model • calculator
Chapter 5 Practice Test • 40–50 minutes	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1, 3, 4, 8, 9	BLM 5–1 Chapter 5 Self-Assessment BLM 5–13 Chapter 5 Test	• ruler
Chapter 5 Wrap It Up! • 40–50 minutes		Master 1 Project Rubric BLM 5–4 Section 5.1 Math Link BLM 5–6 Section 5.2 Math Link BLM 5–10 Section 5.4 Math Link BLM 5–14 Chapter 5 Wrap It Up!	• counters or coins • 2 dice • poster board or regular paper
Chapter 5 Math Games • 40–50 minutes			• paper bag • red and blue counters of the same shape and equal sizes
Chapter 5 Challenge in Real Life • 60–75 minutes		Master 1 Project Rubric BLM 5–15 System 1 Spinners BLM 5–16 Chapter 5 *MathLinks 7* Student Resource Answers BLM 5–17 Chapter 5 BLM Answers	

Chapter 5 Assessment Planner

Assessment Options	Type of Assessment	Assessment Tool
Chapter Opener	Assessment *as* Learning (TR pages i, 157)	BLM 5–1 Chapter 5 Self-Assessment Chapter 5 Foldable
5.1 Probability	Assessment *as* Learning (TR pages 160, 162, 164) Assessment *for* Learning (TR pages 160, 161, 163, 164)	Math Learning Log (TR page 164) BLM 5–1 Chapter 5 Self-Assessment
5.2 Organize Outcomes	Assessment *as* Learning (TR pages 167, 168, 170) Assessment *for* Learning (TR pages 168, 169, 170)	Master 2 Two Stars and One Wish Math Learning Log (TR page 170) BLM 5–1 Chapter 5 Self-Assessment
5.3 Probabilities of Simple Independent Events	Assessment *as* Learning (TR pages 172, 175, 176) Assessment *for* Learning (TR pages 173, 174, 175)	Math Learning Log (TR page 176) BLM 5–1 Chapter 5 Self-Assessment
5.4 Applications of Independent Events	Assessment *as* Learning (TR pages 178, 180, 182) Assessment *for* Learning (TR pages 181, 182)	Math Learning Log (TR page 182) BLM 5–1 Chapter 5 Self-Assessment
5.5 Conduct Probability Experiments	Assessment *as* Learning (TR pages 184, 186, 189) Assessment *for* Learning (TR pages 185, 186, 187)	Math Learning Log (TR page 189) BLM 5–1 Chapter 5 Self-Assessment
Chapter 5 Review	Assessment *for* Learning (TR page 190) Assessment *as* Learning (TR page 191)	Math Learning Log (TR page 191) BLM 5–1 Chapter 5 Self-Assessment
Chapter 5 Practice Test	Assessment *as* Learning (TR page 192) Assessment *of* Learning (TR page 193)	BLM 5–1 Chapter 5 Self-Assessment BLM 5–13 Chapter 5 Test
Chapter 5 Wrap It Up!	Assessment *of* Learning (TR page 192a)	Master 1 Project Rubric
Chapter 5 Math Games	Assessment *for* Learning (TR page 194)	
Chapter 5 Challenge in Real Life	Assessment *for* Learning (TR page 194a) Assessment *of* Learning (TR page 194a)	Master 1 Project Rubric

You may wish to use one or more of the following materials to help you assess student readiness for Chapter 5.

Assessment *for* Learning	Supported Learning
Method 1: Have students develop a journal to explain what they personally know about the topics and how they use fractions and percents, outcomes of events, and tally charts in their lives. **Method 2:** Have students complete **BLM 5–2 Probability** to check their conceptual understanding. Remind students that you are looking for the scope of their knowledge.	• Students who require reinforcement of prerequisite skills may wish to complete the Get Ready materials available in the *MathLinks 7 Workbook* and at the **www.mathlinks7.ca** book site.

Chapter Opener

Suggested Timing
20–30 minutes

Materials
- paper
- scissors
- stapler
- examples of games that use dice (optional)

Blackline Masters
BLM 5–1 Chapter 5 Self-Assessment

Key Words

probability
outcome
favourable outcome
independent events
sample space
tree diagram
random
experimental probability
theoretical probability

Supported Learning

Learning Style and Motor

- Create a Foldable ahead of time to use as a model.
- Creating the Foldable is a good hands-on activity for tactile and visual learners.

ESL, Language, and Memory

- Some English language learners may have a difficult time working independently on the Math Link. You might have students start the Math Link as a group brainstorm or project.
- Consider displaying the Key Words on a math word wall. Students may also create their own vocabulary/picture dictionary. Matching a visual with a definition helps students consolidate their understanding of the key terms.

ESL

- Explain terms such as *satellites*, *underestimated*, *probability*, and *luck* to English language learners. Have students add any new terms to their dictionary.

What's the Math?

In this chapter, students explore how to calculate the probabilities of two independent events. They begin by reviewing the concept of probability. Next, students use tables and tree diagrams to organize the possible outcomes when two independent events occur. This section of the chapter is very important for student success in learning about advanced probability theory at the senior high school and post-secondary level. Students then calculate the probabilities of two independent events. At the end of the chapter, students complete an active study to compare the difference between experimental and theoretical probabilities.

Activity Planning Notes

Consider reading the introduction as a class, and then discuss other connections between weather and probability.

There are many weather phenomena in which probability plays an important role in the decisions made by affected citizens. The uncertainty which surrounds catastrophic events such as hurricanes and tornadoes, requires people to make important decisions about where they live and what type of preparations they need to make for emergencies. On a daily basis, the probability of certain types of weather helps people determine the type of clothing they wear and the type of activities in which they engage.

The picture in the student resource illustrates the eye of Hurricane Rita as it passed over different regions of the Gulf States. During such weather conditions, people in the area rely on probability models to help them make decisions regarding their well-being.

Math Link

Use the Math Link to initiate a student-centred discussion about probability in games. Students will most likely be familiar with different board games that use dice. Yahtzee™ and Monopoly™ are two examples. A variety of sums are possible when you add the numbers on the faces of two dice. Students may not necessarily realize that different sums have different probabilities of occurring.

You may wish to read the Wrap It Up! for this chapter problem, which is on page 193. Students could start to think about the game they will develop, while they work on the rest of the chapter. At the end of the chapter, students develop a board game and write a report that includes strategies for improving the chances of winning.

Have students think of other applications of probability that help keep people safe. For example, students could explore how speed limits and seat belts reduce the probability of serious injury in auto accidents.

FOLDABLES™
Study Tool

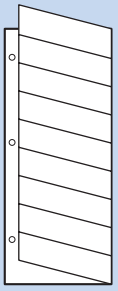
Have students make the Foldable in the student resource to keep track of the information in the chapter. If you have a large stapler, you could have students open up their folded papers and place three staples in the middle fold.

You may wish to have students keep track of Key Words using a design specifically for that purpose. Students can make the following Foldable and write vocabulary terms on the front of each tab. Have them use the space beneath the tab to write definitions and provide examples.

Step 1 Fold a sheet of notebook paper in half along the long axis with the crease to the right.

Step 2 Measure the height of the page and draw lines to divide the height into nine equal parts. Cut every part as far as the fold, creating tabs as you go. This will create nine tabs.

Step 3 Label each tab with a math term. Write definitions and give examples underneath the tabs.



Students could store Key Words Foldables in a large envelope or clear plastic folder in their binder.

Assessment *as* Learning	Supported Learning
Chapter 5 Foldable As students work on each section in Chapter 5, have them keep track of any problems they are having under the What I Need to Work On tab in their chapter Foldable.	• As students complete each section, have them review the list of items they need to work on and then have them check off any that have been handled.

Supported Learning

Meeting the Needs of All Learners

- Consider inviting a community elder to talk about games of chance. Historically, adults in some communities encouraged these games as they required the players to develop their skills. For example, many First Nation communities continue to play these games. Some games of chance played by the Plains Indians include hand games, bone dice, wooden dice, and stick games. You may also wish to have students research how probability and chance were used in the past.
- Students may find the discussion about hurricanes interesting, especially if you make connections with the probability of blizzards occurring.
- You may wish to discuss air, boat, and ATV safety in terms of probabilities. Remember that not all of the rules and infrastructure are common in some communities. Gravel roads and stop signs may be the norm.

5.1 Probability

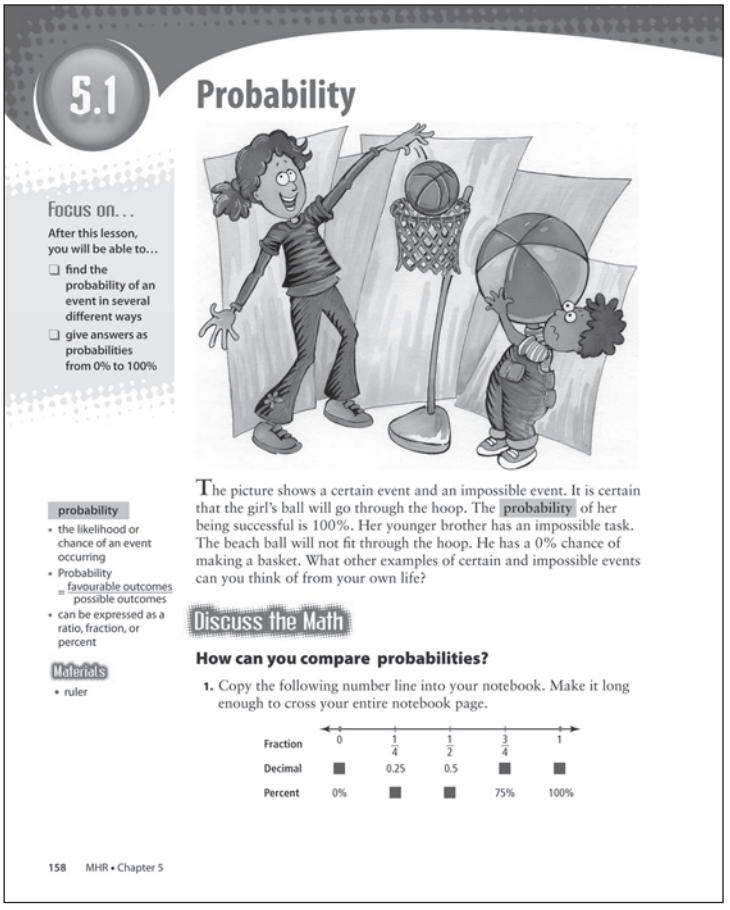
Suggested Timing
80–100 minutes

Materials
- ruler
- marbles or other coloured counters (optional)
- bag (optional)

Blackline Masters
Master 2 Two Stars and One Wish
BLM 5–1 Chapter 5 Self-Assessment
BLM 5–3 Section 5.1 Extra Practice
BLM 5–4 Section 5.1 Math Link

Mathematical Processes

- [x] Communication
- [x] Connections
- [ ] Mental Mathematics and Estimation
- [ ] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



5.1 Probability

Focus on...
After this lesson, you will be able to...
- find the probability of an event in several different ways
- give answers as probabilities from 0% to 100%

probability
- the likelihood or chance of an event occurring
- Probability = favourable outcomes / possible outcomes
- can be expressed as a ratio, fraction, or percent

Materials
- ruler

The picture shows a certain event and an impossible event. It is certain that the girl's ball will go through the hoop. The probability of her being successful is 100%. Her younger brother has an impossible task. The beach ball will not fit through the hoop. He has a 0% chance of making a basket. What other examples of certain and impossible events can you think of from your own life?

Discuss the Math

How can you compare probabilities?

1. Copy the following number line into your notebook. Make it long enough to cross your entire notebook page.

Fraction	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
Decimal	■	0.25	0.5	■	■
Percent	0%	■	■	75%	100%

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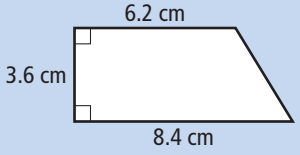
Specific Outcomes

SP4 Express probabilities as ratios, fractions and percents.

Warm-Up

1. A pair of boots regularly cost $39.99. They are on sale for 25% off. Estimate and then calculate their sale price.
2. In one store, 35 of 72 magazines are sports magazines. In another store, 48 of 94 magazines are sports magazines. Estimate which store has the greater percent of sports magazines.
3. Calculate the area of this figure.



4. Add: 37.64 cm + 59.32 cm + 83.5 cm + 101.1 cm.
5. Show using bar notation: 0.1877777…

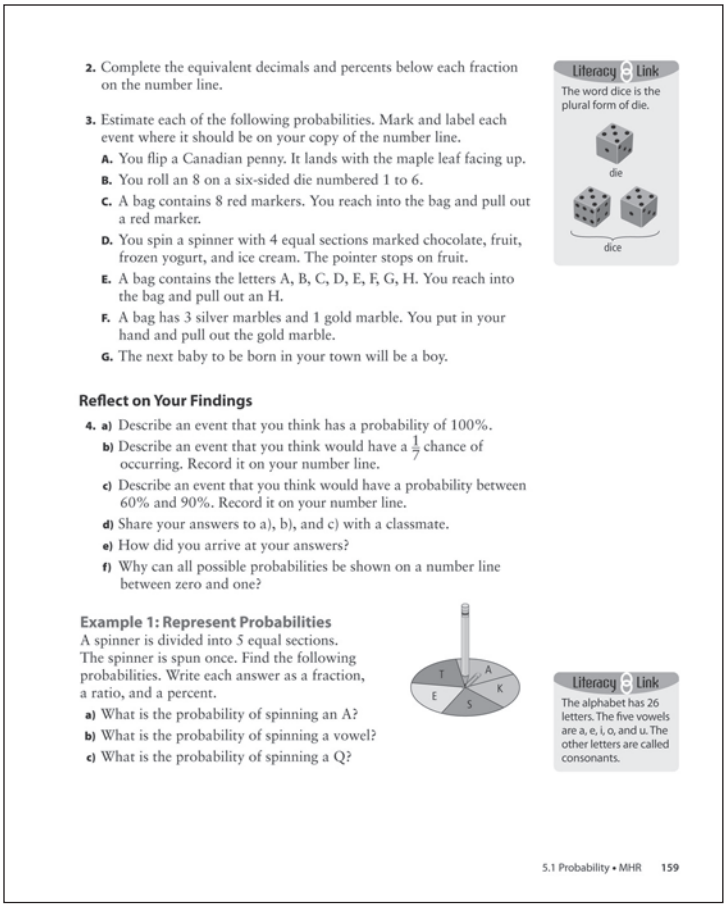
Mental Math

Show your thinking for each of #6 to #8.

6. Estimate 50% of 46.5.
7. Calculate 70% of $45.
8. Estimate the area of the following triangle: $b = 3.2$ cm, $h = 5.89$ cm

Use estimation to place the decimal point in the answers for #9 and #10.

9. $9.38 + $7.35 + $0.10 + $25.00 = $41830
10. $54.8 \times 0.8 = 4384$



2. Complete the equivalent decimals and percents below each fraction on the number line.

3. Estimate each of the following probabilities. Mark and label each event where it should be on your copy of the number line.
   - A. You flip a Canadian penny. It lands with the maple leaf facing up.
   - B. You roll an 8 on a six-sided die numbered 1 to 6.
   - C. A bag contains 8 red markers. You reach into the bag and pull out a red marker.
   - D. You spin a spinner with 4 equal sections marked chocolate, fruit, frozen yogurt, and ice cream. The pointer stops on fruit.
   - E. A bag contains the letters A, B, C, D, E, F, G, H. You reach into the bag and pull out an H.
   - F. A bag has 3 silver marbles and 1 gold marble. You put in your hand and pull out the gold marble.
   - G. The next baby to be born in your town will be a boy.

Literacy Link
The word dice is the plural form of die.
die
dice

Reflect on Your Findings

4. a) Describe an event that you think has a probability of 100%.
   b) Describe an event that you think would have a $\frac{1}{7}$ chance of occurring. Record it on your number line.
   c) Describe an event that you think would have a probability between 60% and 90%. Record it on your number line.
   d) Share your answers to a), b), and c) with a classmate.
   e) How did you arrive at your answers?
   f) Why can all possible probabilities be shown on a number line between zero and one?

Example 1: Represent Probabilities
A spinner is divided into 5 equal sections. The spinner is spun once. Find the following probabilities. Write each answer as a fraction, a ratio, and a percent.
a) What is the probability of spinning an A?
b) What is the probability of spinning a vowel?
c) What is the probability of spinning a Q?

Literacy Link
The alphabet has 26 letters. The five vowels are a, e, i, o, and u. The other letters are called consonants.

5.1 Probability • MHR 159

Activity Planning Notes

Use the illustration in the student resource to discuss the range of probabilities for an event from impossible (0% likelihood of occurring) to certain (100% likelihood of occurring).

Discuss the Math

Students use a number line to help compare probabilities of different events. Have students copy the number line in #1 into their notebook, and then work individually to answer #3. Allow students to compare their answers with a classmate and note any differences. Discuss the findings as a class and demonstrate how to place each event correctly on a number line. Have students correct their copy.

Answers

Warm-Up

1. Estimate: $25\% = \frac{1}{4}$; divide by 4. Discount is about \$10.
Sale price is about \$30.
Calculate: \$29.99

2. 35 of 72: $50\% = 36$ A little high
48 of 94: $50\% = 47$ A little low
The second store has the greater percent.

3. Area of rectangle $= 6.2 \times 3.6 = 22.32\ \text{cm}^2$
Area of triangle $= (8.4 - 6.2) \times 3.6 \div 2 = 3.96\ \text{cm}^2$
$22.32\ \text{cm}^2 + 3.96\ \text{cm}^2 = 26.28\ \text{cm}^2$

4. 281.56 cm **5.** $0.18\overline{7}$ **6.** $46 \div 2 = 23$

7. 10% of \$45 = \$4.50
$7 \times \$4.50 = \$28 + \$3.50 = \$31.50$

8. $3 \times 6 \div 2 \approx 9\ \text{cm}^2$

9. \$41.830 **10.** 43.84

Discuss the Math

2. Decimal: 0, 0.25, 0.5, 0.75, 1; Percent: 0%, 25%, 50%, 75%, 100%

3. A: $\frac{1}{2}$; B: 0; C: 1; D: $\frac{1}{4}$; E: $\frac{1}{8}$; F: $\frac{1}{4}$; G: $\frac{1}{2}$

4. **a)** Answers may vary. For example: The sun will rise in the morning.
b) Answers may vary. For example: Choosing a piece of paper with "Friday" written on it from a cup containing seven pieces of paper each with one of the seven days of the week written on it.
c) Answers may vary. For example: It will snow on December 25.
e) Answers may vary.
f) Answers may vary. Look for the idea that probabilities can't be less than 0 or more than 1.

Supported Learning

ESL and Language

- Draw students' attention to the Literacy Links on page 159 that explain *die*, *dice*, *vowels*, and *consonants*.

Meeting the Needs of All Learners

- As an alternative to the illustration on page 158, discuss the probabilities of certain and impossible events of two people jigging for fish, with one person catching a small fish and the other one catching a fish that is too large to pull out of the ice-fishing hole.

Answers

Show You Know: Example 1

a) $\frac{1}{4}$ or 1:4 or 25%

b) $\frac{3}{4}$ or 3:4 or 75%

c) 0 or 0%

Supported Learning

Learning Style

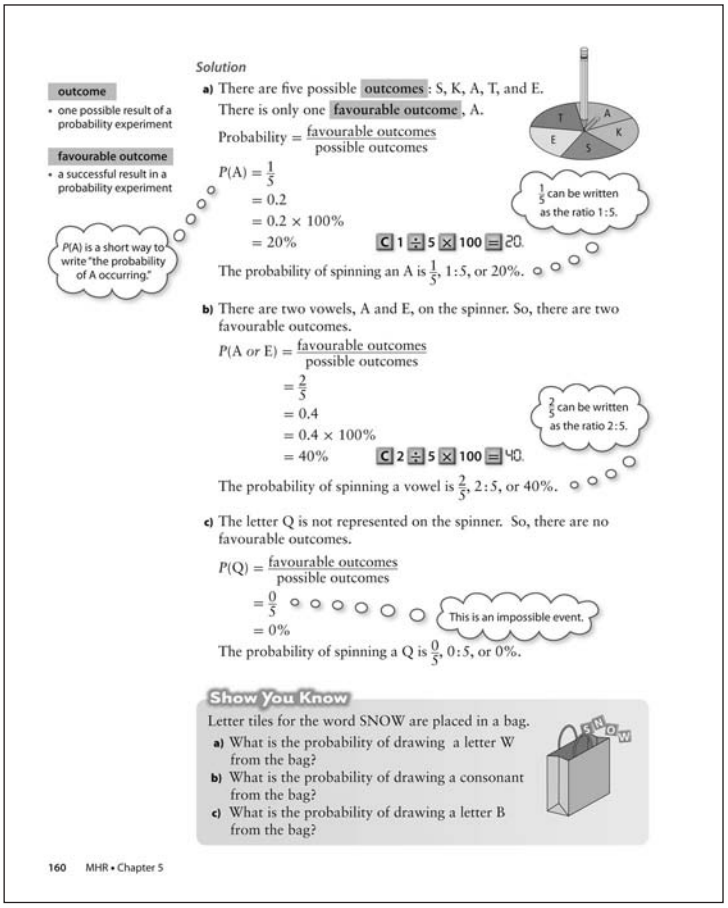
- Encourage concrete and kinesthetic learners to use a spinner, marbles (or counters), letter tiles, and a bag to carry out each of the events described in Examples 1 and 2 and the Show You Know questions.

Motor

- Encourage students to use a ruler to draw the number line. Consider modelling how to draw the line and space the intervals.

Meeting the Needs of All Learners

- Have students work with manipulatives where possible. Have them start by predicting the correct answer, and then check their prediction using the manipulatives.



Solution

outcome
- one possible result of a probability experiment

favourable outcome
- a successful result in a probability experiment

a) There are five possible outcomes: S, K, A, T, and E.
There is only one favourable outcome, A.

$\text{Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$

$P(\text{A}) = \frac{1}{5}$
$= 0.2$
$= 0.2 \times 100\%$
$= 20\%$

C 1 ÷ 5 × 100 = 20.

$\frac{1}{5}$ can be written as the ratio 1:5.

P(A) is a short way to write "the probability of A occurring."

The probability of spinning an A is $\frac{1}{5}$, 1:5, or 20%.

b) There are two vowels, A and E, on the spinner. So, there are two favourable outcomes.

$P(\text{A or E}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$
$= \frac{2}{5}$
$= 0.4$
$= 0.4 \times 100\%$
$= 40\%$

C 2 ÷ 5 × 100 = 40.

$\frac{2}{5}$ can be written as the ratio 2:5.

The probability of spinning a vowel is $\frac{2}{5}$, 2:5, or 40%.

c) The letter Q is not represented on the spinner. So, there are no favourable outcomes.

$P(\text{Q}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$
$= \frac{0}{5}$
$= 0\%$

This is an impossible event.

The probability of spinning a Q is $\frac{0}{5}$, 0:5, or 0%.

Show You Know

Letter tiles for the word SNOW are placed in a bag.
- **a)** What is the probability of drawing a letter W from the bag?
- **b)** What is the probability of drawing a consonant from the bag?
- **c)** What is the probability of drawing a letter B from the bag?

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss and demonstrate each event to a classmate. During this process, they are generalizing what they learned during Discuss the Math.	• Some students may need to use the events and answers from #3 as models to help them answer #4. • For a $\frac{1}{7}$ chance, encourage students to consider an event that happens once a week.

Example 1 encourages students to represent probabilities using fractions, ratios, and percents. Ensure students understand the meaning of the terms *outcome* and *favourable outcome*, as described in the student resource. Draw students' attention to the thought bubbles, which provide additional information to help answer the questions.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• You may wish to provide additional questions for students who would benefit from them: Letter tiles for the word HOUSE are placed in a bag. **a)** What is the probability of drawing a letter *H* from the bag? ($P(H)$ = 1:5 or 20%. Since there is only one *H*, students count the number of letters in the word.) **b)** What is the probability of drawing a vowel from the bag? (P(vowel) = 3:5 or 60%. The vowels include *O*, *U*, and *E*.) Coach students through a), and then have them try b) on their own.

Example 2: Determine Probabilities

A bag contains 10 marbles. Show the probability of the following events as a fraction, a ratio, and a percent.

a) selecting a red marble
b) selecting a red or blue or green marble
c) *not* selecting a red marble

Solution

a) There are 10 possible outcomes. There are six red marbles, so there are six favourable outcomes.

$$P(\text{red}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$
$$= \frac{6}{10}$$
$$= 60\%$$

The probability of selecting a red marble is $\frac{6}{10}$, 6:10, or 60%.

b) $P(\text{red or blue or green}) = \frac{10}{10}$
$= 1$
$= 100\%$

This is a certain event.

The probability of selecting a red, blue, or green marble is $\frac{10}{10}$, 10:10, or 100%.

c) The probability of *not* selecting a red marble is the same as the probability of selecting a blue or green marble.

$P(\textit{not}\ \text{red}) = P(\text{blue or green})$
$= \frac{4}{10}$
$= 40\%$

The probability of selecting a marble that is not red is $\frac{4}{10}$, 4:10, or 40%.

Show You Know

Letter tiles for the word MOUNTAIN are placed in a bag.

a) What is the probability of drawing a letter N from the bag?
b) What is the probability of drawing a consonant from the bag?
c) What is the probability of drawing a letter from the bag?

Answers

Show You Know: Example 2

a) $\frac{2}{8}$ or 2:8 or 25%

b) $\frac{4}{8}$ or 4:8 or 50%

c) 1 or 100%

Example 2 provides an opportunity for students to actually choose items from a bag and explore the concept of probability. You may wish to demonstrate this example using marbles (or other coloured counters) in a bag. Have students take turns choosing an item to demonstrate how probability works.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• Consider having students use the loading-strip model from Chapter 4 if they need help converting fractions to percent. • You may wish to provide additional questions for students who would benefit from them: Letter tiles for the word CANADIAN are placed in a bag. **a)** What is the probability of drawing a letter *A* from the bag? (P(A) = 3:8 or 37.5%. Students show the ratio of the number of *A*s to the number of letters. They could show the percent to one decimal place or round up to 38%.) **b)** What is the probability of drawing a consonant from the bag? (P(consonant) = 4:8 or 50%. Consonants include letters that are not *A, E, I, O,* or *U*. Encourage students to memorize the percent equivalent for the common fractions that show one half.) Coach students through a), and then have them try b) on their own.

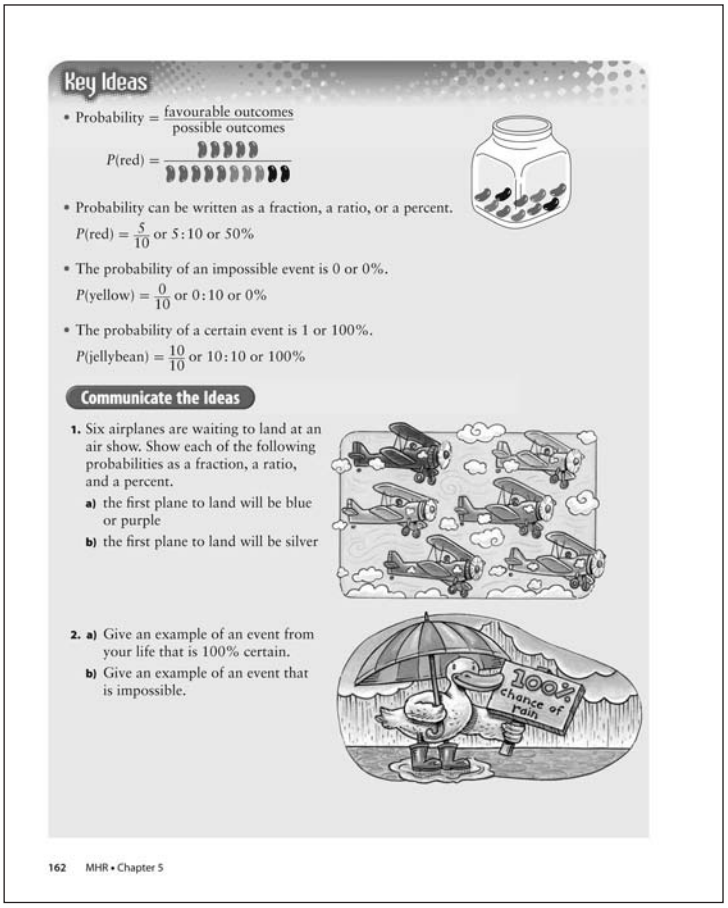
Answers

Communicate the Ideas

1. a) $\frac{2}{6}$ or 2:6 or 33.3% **b)** $\frac{1}{6}$ or 1:6 or 16.7%

2. a) Answers may vary. For example: January 1 will come after December 31.

b) Answers may vary. For example: There will be two Mondays this week.



Key Ideas

- Probability = $\frac{\text{favourable outcomes}}{\text{possible outcomes}}$

$P(\text{red}) = \frac{\text{●●●●●}}{\text{●●●●●●●●●●}}$

- Probability can be written as a fraction, a ratio, or a percent.

$P(\text{red}) = \frac{5}{10}$ or 5:10 or 50%

- The probability of an impossible event is 0 or 0%.

$P(\text{yellow}) = \frac{0}{10}$ or 0:10 or 0%

- The probability of a certain event is 1 or 100%.

$P(\text{jellybean}) = \frac{10}{10}$ or 10:10 or 100%

Communicate the Ideas

1. Six airplanes are waiting to land at an air show. Show each of the following probabilities as a fraction, a ratio, and a percent.
   a) the first plane to land will be blue or purple
   b) the first plane to land will be silver

2. a) Give an example of an event from your life that is 100% certain.
   b) Give an example of an event that is impossible.

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Supported Learning

ESL and Language

- Encourage English language learners and students with language difficulties who have stronger oral than written skills to share the Key Ideas with a partner or during group discussion.

WWW Web Link

For a site in which students use flash media to find the probability of events like tossing a coin, spinning a spinner, and picking items out of a bag, go to **www.mathlinks7.ca** and follow the links.

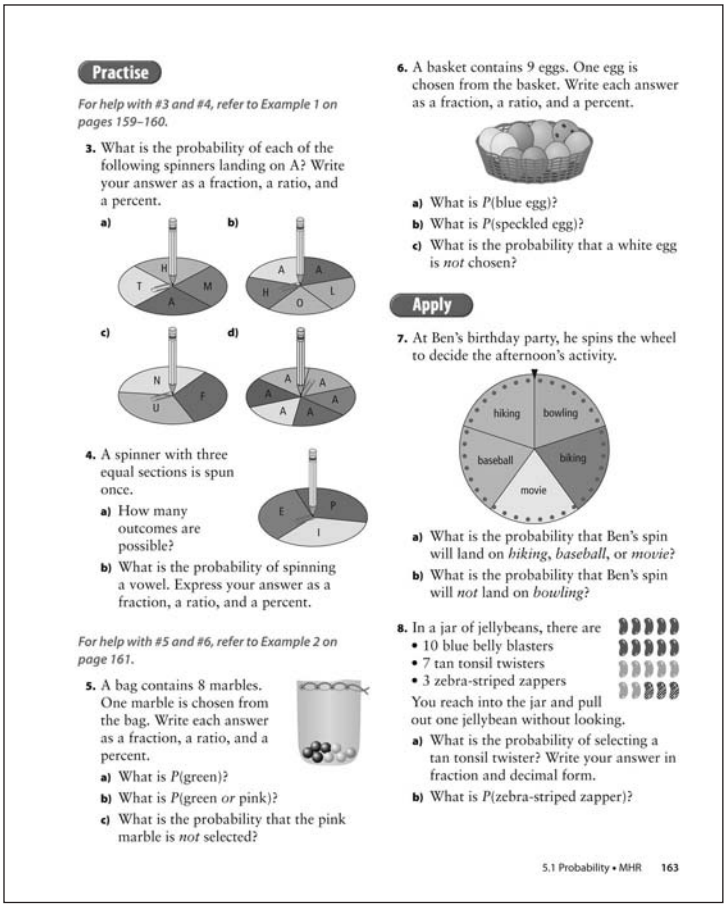
Key Ideas

The Key Ideas emphasize how to write a probability as a percent, a ratio, or a fraction, and that probability ranges from 0 to 1. Students could prepare their own list of Key Ideas and put it in their chapter Foldable. It is quite acceptable to have more or less than what is included in the student resource.

Communicate the Ideas

Both questions are important and allow students to show their understanding of writing probability as a fraction, a ratio, and a percent, and certain and impossible events.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students complete the questions individually or as a class, before summarizing the main points in a class discussion.	• Have students record the real-world examples for #2 in their chapter Foldable. • Work with the class to develop criteria for judging each answer. For example, criteria for #1 might include: – expresses probability as a fraction, a ratio, and a percent – shows how a fraction, ratio, and percent are related • Use **Master 2 Two Stars and One Wish** to have students critique other students' writing pieces. This master allows them to write two things they like about a piece and one thing they would like to see improved.



Practise

For help with #3 and #4, refer to Example 1 on pages 159–160.

3. What is the probability of each of the following spinners landing on A? Write your answer as a fraction, a ratio, and a percent.

a) **b)** **c)** **d)**

4. A spinner with three equal sections is spun once.

a) How many outcomes are possible?

b) What is the probability of spinning a vowel. Express your answer as a fraction, a ratio, and a percent.

For help with #5 and #6, refer to Example 2 on page 161.

5. A bag contains 8 marbles. One marble is chosen from the bag. Write each answer as a fraction, a ratio, and a percent.

a) What is *P*(green)?

b) What is *P*(green *or* pink)?

c) What is the probability that the pink marble is *not* selected?

6. A basket contains 9 eggs. One egg is chosen from the basket. Write each answer as a fraction, a ratio, and a percent.

a) What is *P*(blue egg)?

b) What is *P*(speckled egg)?

c) What is the probability that a white egg is *not* chosen?

Apply

7. At Ben's birthday party, he spins the wheel to decide the afternoon's activity.

hiking, bowling, biking, baseball, movie

a) What is the probability that Ben's spin will land on *biking*, *baseball*, or *movie*?

b) What is the probability that Ben's spin will *not* land on *bowling*?

8. In a jar of jellybeans, there are
- 10 blue belly blasters
- 7 tan tonsil twisters
- 3 zebra-striped zappers

You reach into the jar and pull out one jellybean without looking.

a) What is the probability of selecting a tan tonsil twister? Write your answer in fraction and decimal form.

b) What is *P*(zebra-striped zapper)?

5.1 Probability • MHR 163

Common Errors

- Students may be confused about the concept of the likelihood of an event not occurring, such as the pink marble not being selected in #5c).

R_x Coach students to find the number of favourable outcomes by counting those events that are not *P*(pink).

Supported Learning

Learning Style and Memory

- Provide **BLM 5–3 Section 5.1 Extra Practice** to students who require extra practice.

Learning Style

- Encourage students to use spinners and counters to help them answer the questions.

Meeting the Needs of All Learners

- Encourage students who struggle with reading and writing to communicate their understanding by using a combination of diagrams, oral explanations, and actual demonstrations.
- Partner English language learners with students who can provide them with good one-on-one discussion. This facilitates comprehension of vocabulary.
- Invite a community elder to teach students how to play hand games. These games are very enjoyable and are common for Plains Indians and many other First Nation communities.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 2, 3 *or* 4, 5 *or* 6, 7, Math Link
Typical	1, 2, 3 *or* 4, 5 *or* 6, 7, 8 *or* 9, 10, Math Link
Extension/Enrichment	1, 2, 10–12

Web Link

For a site that describes how to play games of the Plains Cree, go to **www.mathlinks7.ca** and follow the links.

Practise

Ensure students understand that #5, #6, and #7 refer to the probability of an event not occurring. Clarify that all other events are considered favourable.

Assessment *for* Learning	Supported Learning
Practise Have students do #3 and #5. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #3 will need additional coaching with Example 1. Work with them to correct their answers to #3, and then assign #4. • Students who have problems with #5 will need additional coaching with Example 2. Work with them to correct their answers to #5, and then assign #6. • Check back with them several times to make sure that they understand the concepts.

Answers

Math Link

a) Answers will vary.

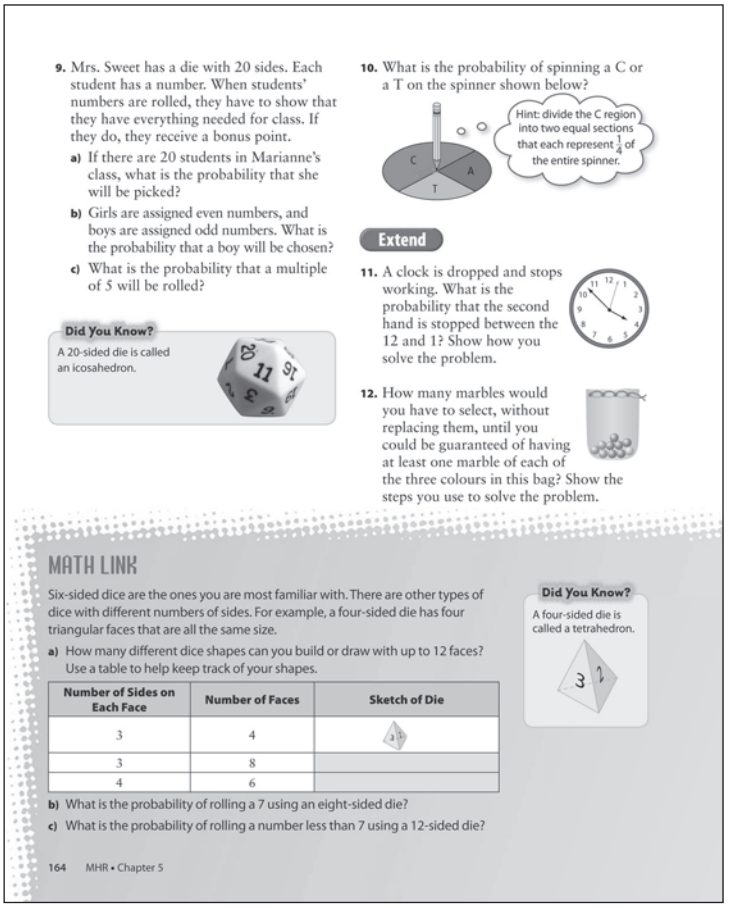
b) $\frac{1}{8}$ or 0.125 or 12.5%

c) $\frac{6}{12}$ or 0.5 or 50%

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • What is the purpose of learning how to calculate probability? • How might understanding probability affect your life as a teenager? as an adult?	• Encourage students to recognize the importance of probability in everyday life. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • You may wish to have students review the part related to Section 5.1 in **BLM 5–1 Chapter 5 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Math Link

The Math Link provides students with an opportunity to explore probabilities and different dice shapes. Use the examples of an icosahedron and a tetrahedron on page 164 to spark students' interest. Consider collecting dice with different numbers of sides to help students complete the activity.



9. Mrs. Sweet has a die with 20 sides. Each student has a number. When students' numbers are rolled, they have to show that they have everything needed for class. If they do, they receive a bonus point.
   a) If there are 20 students in Marianne's class, what is the probability that she will be picked?
   b) Girls are assigned even numbers, and boys are assigned odd numbers. What is the probability that a boy will be chosen?
   c) What is the probability that a multiple of 5 will be rolled?

Did You Know?
A 20-sided die is called an icosahedron.

10. What is the probability of spinning a C or a T on the spinner shown below?

Hint: divide the C region into two equal sections that each represent $\frac{1}{4}$ of the entire spinner.

Extend

11. A clock is dropped and stops working. What is the probability that the second hand is stopped between the 12 and 1? Show how you solve the problem.

12. How many marbles would you have to select, without replacing them, until you could be guaranteed of having at least one marble of each of the three colours in this bag? Show the steps you use to solve the problem.

MATH LINK

Six-sided dice are the ones you are most familiar with. There are other types of dice with different numbers of sides. For example, a four-sided die has four triangular faces that are all the same size.

a) How many different dice shapes can you build or draw with up to 12 faces? Use a table to help keep track of your shapes.

Number of Sides on Each Face	Number of Faces	Sketch of Die
3	4	
3	8	
4	6	

b) What is the probability of rolling a 7 using an eight-sided die?

c) What is the probability of rolling a number less than 7 using a 12-sided die?

Did You Know?
A four-sided die is called a tetrahedron.

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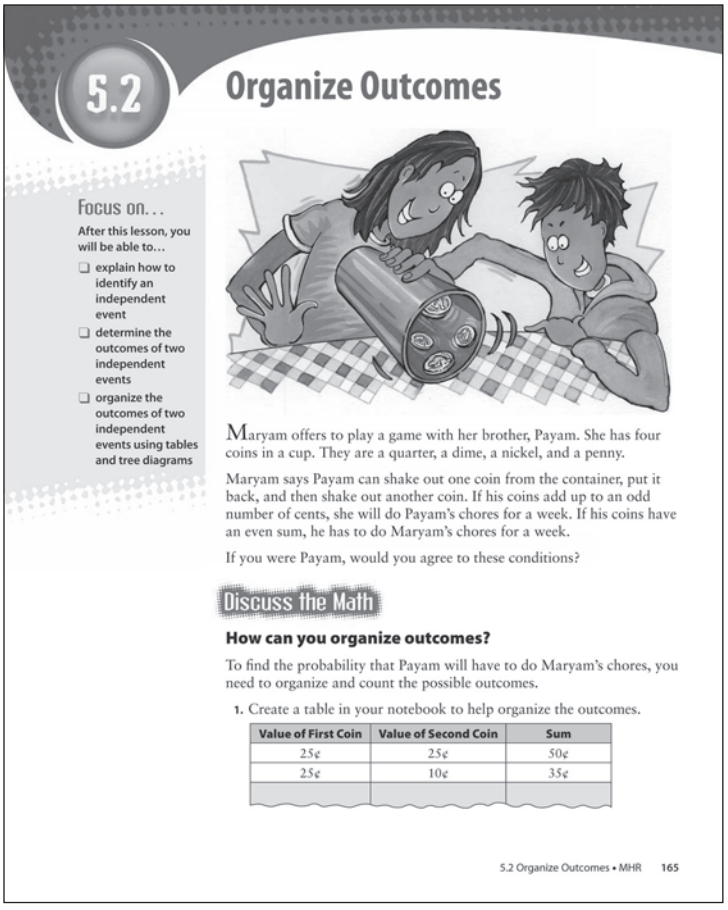
Apply and Extend

The Extend problems require significantly more analysis than those in the Apply section, and generally involve multiple steps to solve. For #11, ensure students understand that the word *between* has to be used carefully in mathematics. If someone is asked to choose a number between 1 and 10, it is assumed that 1 and 10 are included. In addition, ensure students understand that the area between 12 and 1 is the same as the area between any two consecutive numbers on a 12-hour traditional clock.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 164 is intended to help students work toward the chapter problem titled Wrap It Up! on page 193.	• You may wish to have students do this Math Link in order to provide them with additional practice with probability. • Students who are having difficulty getting started could use **BLM 5–4 Section 5.1 Math Link**, which provides scaffolding for this activity.

5.2 Organize Outcomes



5.2 Organize Outcomes

Focus on...

After this lesson, you will be able to...

- explain how to identify an independent event
- determine the outcomes of two independent events
- organize the outcomes of two independent events using tables and tree diagrams

Maryam offers to play a game with her brother, Payam. She has four coins in a cup. They are a quarter, a dime, a nickel, and a penny.

Maryam says Payam can shake out one coin from the container, put it back, and then shake out another coin. If his coins add up to an odd number of cents, she will do Payam's chores for a week. If his coins have an even sum, he has to do Maryam's chores for a week.

If you were Payam, would you agree to these conditions?

Discuss the Math

How can you organize outcomes?

To find the probability that Payam will have to do Maryam's chores, you need to organize and count the possible outcomes.

1. Create a table in your notebook to help organize the outcomes.

Value of First Coin	Value of Second Coin	Sum
25¢	25¢	50¢
25¢	10¢	35¢

5.2 Organize Outcomes • MHR 165

Suggested Timing
80–100 minutes

Materials
- coins—quarters, dimes, nickels, and pennies (optional)
- ruler

Blackline Masters
BLM 5–1 Chapter 5 Self-Assessment
BLM 5–5 Section 5.2 Extra Practice
BLM 5–6 Section 5.2 Math Link

Mathematical Processes

- [x] Communication
- [ ] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [x] Visualization

Specific Outcomes

SP4 Express probabilities as ratios, fractions and percents.
SP5 Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.

Warm-Up

The following letters are placed in a bag: S, C, H, O, O, L. *Use the letters to answer #1 to #4.*

1. What is the probability of drawing a letter S from the bag?

2. What is the probability of drawing a letter O from the bag?

3. What is the probability of drawing a consonant from the bag?

4. What is the probability of drawing a letter from the bag?

5. Draw a coordinate grid. Draw a square on your grid, using more than one quadrant. Identify the coordinates of each corner of the square.

Mental Math

6. Use estimation to decide which is greater: 75 out of 164 or 82 out of 192.

7. Mentally calculate 90% of $123.

8. Estimate the area of the following parallelogram: $b = 12.9$ m, $h = 20.2$ m

Use estimation to place the decimal point in the answers for #9 and #10.

9. $\$135.89 - \$25.94 = \$10995$

10. $2440.68 \div 12.9 = 18920$

Answers

Warm-Up

1. $P(S) = 1{:}6, \frac{1}{6}$, or $0.1\overline{6}\%$

2. $P(O) = 2{:}6, \frac{2}{6}$ *or* $\frac{1}{3}$, or $0.3\overline{3}\%$

3. $P(\text{consonant}) = 4{:}6, \frac{4}{6}$ *or* $\frac{2}{3}$, or $0.6\overline{6}\%$

4. $P(\text{letter}) = 6{:}6, \frac{6}{6}$, or 100%

5. Answers will vary. Ensure that students' coordinates are correct.

6. 75 out of 164: 50% = 82; 10% = 16.4; 5% = 8.2
50% = 82 A little high
45% = 73.8 A little low
Between 45% and 50%, but closer to 45%.
82 out of 192: 50% = 96; 10% = 19.2; 5% = 9.6
40% = 76.8 A little low
45% = 86.4 A little high
Between 40% and 45%, but closer to 45%.
75 out of 164 is greater.

7. 10% of \$123 = \$12.30
90% = \$123 − \$12.30 = \$123 − \$10 − \$2 − \$0.30
= \$110.70

8. $13 \times 20 = 260\ m^2$ **9.** \$109.95 **10.** 189.20

Discuss the Math

1.

Value of First Coin	Value of Second Coin	Sum
25¢	25¢	50¢
25¢	10¢	35¢
25¢	5¢	30¢
25¢	1¢	26¢
10¢	25¢	35¢
10¢	10¢	20¢
10¢	5¢	15¢
10¢	1¢	11¢
5¢	25¢	30¢
5¢	10¢	15¢
5¢	5¢	10¢
5¢	1¢	6¢
1¢	25¢	26¢
1¢	10¢	11¢
1¢	5¢	6¢
1¢	1¢	2¢

2. a) 16 **b)** 10 **c)** 6

3. a) $\frac{10}{16}$ or 0.625 or 62.5% **b)** $\frac{6}{16}$ or 0.375 or 37.5%

4. a)–c) Answers will vary.

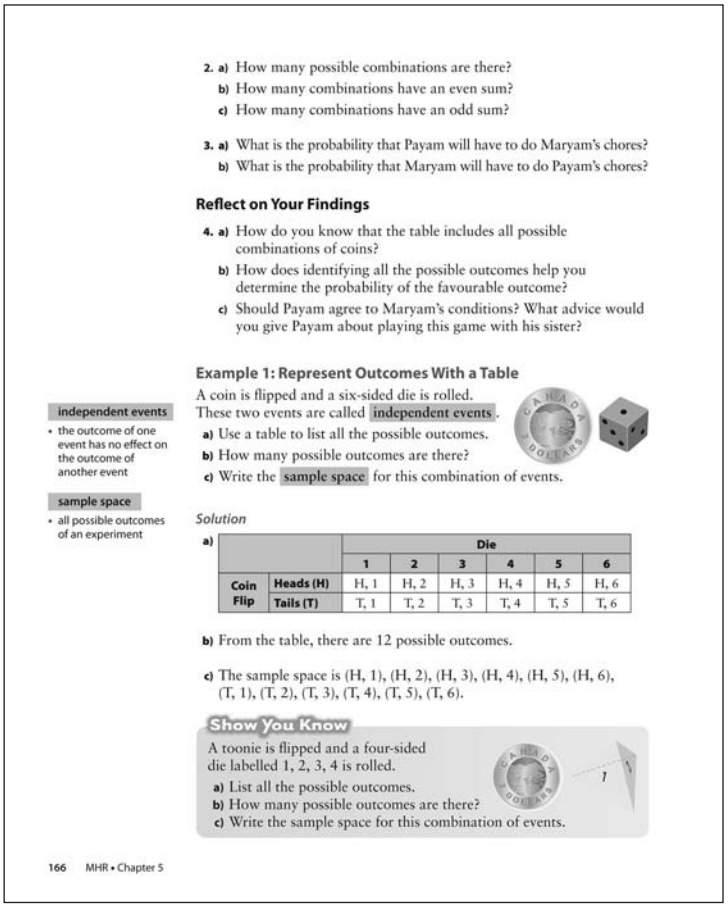
Show You Know: Example 1

a)

		Die			
		1	2	3	4
Coin Flip	Heads (H)	H, 1	H, 2	H, 3	H, 4
	Tails (T)	T, 1	T, 2	T, 3	T, 4

b) 8

c) (H, 1), (H, 2), (H, 3), (H, 4), (T, 1), (T, 2), (T, 3), (T, 4)



2. a) How many possible combinations are there?
b) How many combinations have an even sum?
c) How many combinations have an odd sum?

3. a) What is the probability that Payam will have to do Maryam's chores?
b) What is the probability that Maryam will have to do Payam's chores?

Reflect on Your Findings

4. a) How do you know that the table includes all possible combinations of coins?
b) How does identifying all the possible outcomes help you determine the probability of the favourable outcome?
c) Should Payam agree to Maryam's conditions? What advice would you give Payam about playing this game with his sister?

Example 1: Represent Outcomes With a Table

independent events
- the outcome of one event has no effect on the outcome of another event

sample space
- all possible outcomes of an experiment

A coin is flipped and a six-sided die is rolled.
These two events are called independent events.
a) Use a table to list all the possible outcomes.
b) How many possible outcomes are there?
c) Write the sample space for this combination of events.

Solution

a)

		Die					
		1	2	3	4	5	6
Coin Flip	Heads (H)	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
	Tails (T)	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

b) From the table, there are 12 possible outcomes.

c) The sample space is (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6).

Show You Know

A toonie is flipped and a four-sided die labelled 1, 2, 3, 4 is rolled.
a) List all the possible outcomes.
b) How many possible outcomes are there?
c) Write the sample space for this combination of events.

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Activity Planning Notes

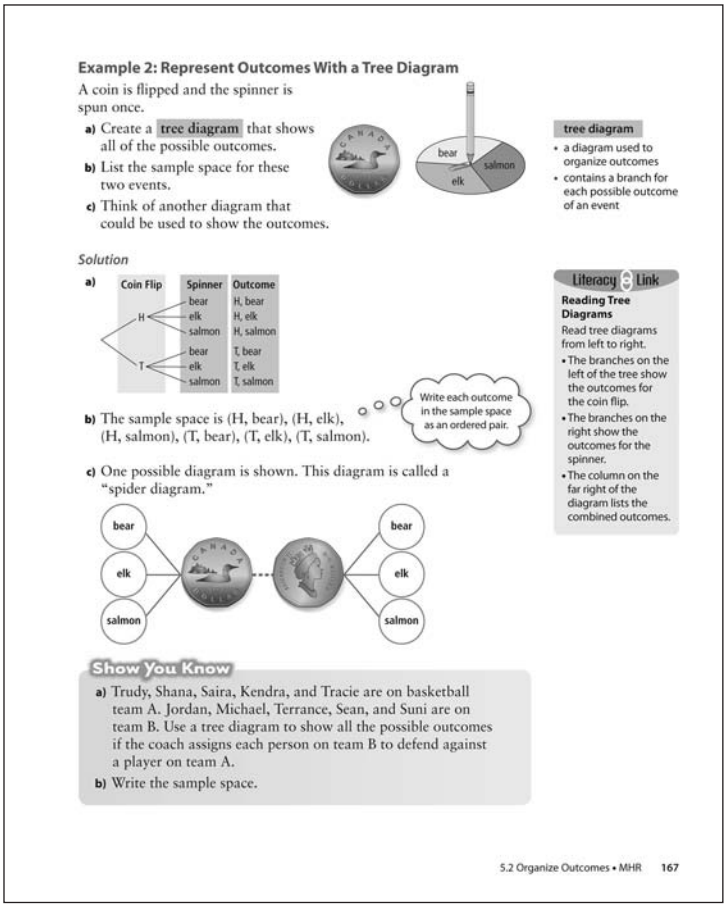
This section focuses on organizing outcomes to solve probability problems. Consider asking students to figure out the largest and smallest amounts that could result from the game described on page 165 (35¢ and 6¢). Point out that one amount is even and one is odd. Ask students to predict how many combinations of two coins will be even and how many will be odd.

Discuss the Math

Have students work through the questions individually or in small groups. For #1, consider guiding students about the number of lines needed for the table by discussing the number of possible combinations of coins.

As a class, review the completed table. Ask how students know that the table is complete. Point out the importance of filling in the table in a systematic way to ensure that all possible outcomes are listed.

Example 1 demonstrates representing outcomes with a table. Ensure students understand the meaning of *independent events* and *sample space*. Encourage students to try this problem using actual coins.



Example 2: Represent Outcomes With a Tree Diagram

A coin is flipped and the spinner is spun once.

a) Create a tree diagram that shows all of the possible outcomes.
b) List the sample space for these two events.
c) Think of another diagram that could be used to show the outcomes.

tree diagram
- a diagram used to organize outcomes
- contains a branch for each possible outcome of an event

Solution

a)

Coin Flip	Spinner	Outcome
H	bear	H, bear
	elk	H, elk
	salmon	H, salmon
T	bear	T, bear
	elk	T, elk
	salmon	T, salmon

Write each outcome in the sample space as an ordered pair.

b) The sample space is (H, bear), (H, elk), (H, salmon), (T, bear), (T, elk), (T, salmon).

c) One possible diagram is shown. This diagram is called a "spider diagram."

Literacy Link

Reading Tree Diagrams

Read tree diagrams from left to right.
- The branches on the left of the tree show the outcomes for the coin flip.
- The branches on the right show the outcomes for the spinner.
- The column on the far right of the diagram lists the combined outcomes.

Show You Know

a) Trudy, Shana, Saira, Kendra, and Tracie are on basketball team A. Jordan, Michael, Terrance, Sean, and Suni are on team B. Use a tree diagram to show all the possible outcomes if the coach assigns each person on team B to defend against a player on team A.
b) Write the sample space.

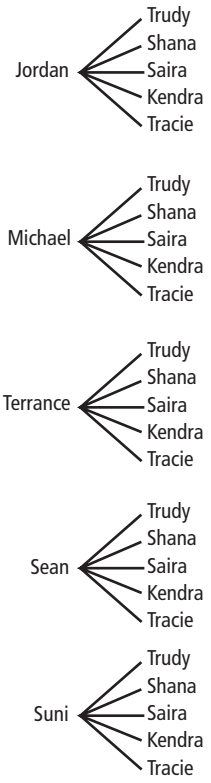
Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss and demonstrate the questions. During this process, they are generalizing what they have learned during Discuss the Math.	• Develop a method for organizing outcomes with students to ensure that all possible combinations of coins are included in the table (e.g., use manipulatives for the coins). Help them organize the first set of outcomes: – quarter first + quarter second = \$0.50 Even – quarter first + dime second = \$0.35 Odd – quarter first + nickel second = \$0.30 Even – quarter first + penny second = \$0.26 Even Then have them list the possible combinations for a dime. • Ask students what the probability is that Payam will end up doing chores ($\frac{10}{16}$ or 0.625 or 62.5%). Would they like to take a chance like that? • Have students consider how Payam might revise this offer to either make it fair (50% probability) or to his advantage.

Example 2 introduces representing outcomes with a tree diagram. Refer to the lines that go from each outcome in the first event to each outcome in the second event as *branches*. In the case of the spider diagram, each connection between possible outcomes is a *leg* of the spider. As a class, read the Literacy Link about reading a tree diagram.

Answers

Show You Know: Example 2

a) Team B **Team A**



b) (Jordan, Trudy), (Jordan, Shana), (Jordan, Saira), (Jordan, Kendra), (Jordan, Tracie), (Michael, Trudy), (Michael, Shana), (Michael, Saira), (Michael, Kendra), (Michael, Tracie), (Terrance, Trudy), (Terrance, Shana), (Terrance, Saira), (Terrance, Kendra), (Terrance, Tracie), (Sean, Trudy), (Sean, Shana), (Sean, Saira), (Sean, Kendra), (Sean, Tracie), (Suni, Trudy), (Suni, Shana), (Suni, Saira), (Suni, Kendra), (Suni, Tracie)

Supported Learning

Learning Style

- Have students use concrete materials such as coins to help them organize and count the possible outcomes.

ESL

- Check that English language learners understand the terms *odd* and *even*.

Motor

- Make sure that students use a ruler to draw the table in their notebook. Alternatively, consider allowing students to use a computer to create the table.

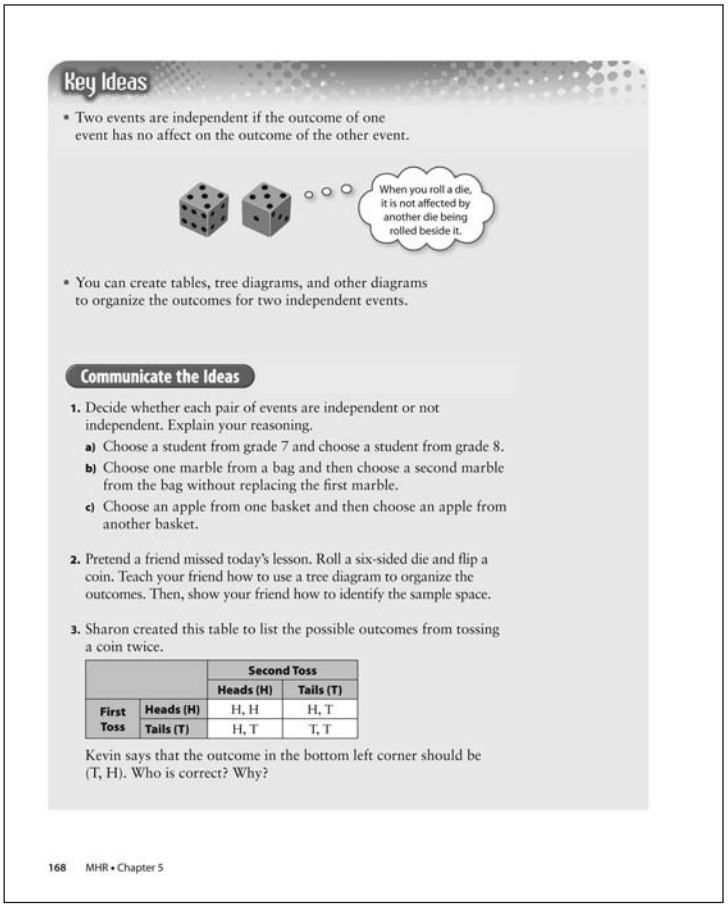
Common Errors

- Students may struggle with using a systematic approach to list the sample space.

R_x Help students develop these skills by modelling. For example, if a coin is flipped and a spinner with numbers 1, 2, 3, and 4 is spun, list all of the spins with heads first, and then all of the spins with tails: (H, 1), (H, 2), (H, 3), (H, 4), (T, 1), (T, 2), (T, 3), (T, 4).

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1 on page 166.	• Have students create a table to help answer the questions. • You may wish to provide additional questions to students who would benefit from them: **a)** A loonie is flipped and a six-sided die labelled 1, 2, 3, 4, 5, 6 is rolled. Answer questions a)–c) from Show You Know. (The table will look exactly like the one in Example 1 part a). There are 12 possible outcomes. The sample space is the same as the one in Example 1 part c). **b)** A quarter is flipped and an eight-sided die labelled 1, 2, 3, 4, 5, 6, 7, 8 is rolled. Answer questions a)–c) from Show You Know. (The table will be similar to the one in Example 1 part a) except for the addition of headings 7 and 8, and 4 cells: (H, 7), (H, 8), (T, 7), (T, 8). There are 16 possible outcomes. The sample space is (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (H, 7), (H, 8), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6), (T, 7), (T, 8).) Coach students through a), and then have them try b) on their own.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2 on page 167.	• Have students talk through their thinking with a partner. • You may wish to provide additional questions to students who would benefit from them: **a)** A coin is flipped and a four-sided die is rolled. Use a tree diagram to show all the possible outcomes. Write the sample space. (Look for a tree diagram set up similarly to the one in Example 2 part a) except for replacing the second column head with Die and using numbers 1, 2, 3, 4. The sample space is (H, 1), (H, 2), (H, 3), (H, 4), (T, 1), (T, 2), (T, 3), (T, 4).) **b)** Think of another diagram that could be used to show the outcomes. (Students might draw a spider diagram.) Coach students through a), and then have them try b) on their own. Encourage them to use the organizer that they prefer.



Key Ideas

- Two events are independent if the outcome of one event has no affect on the outcome of the other event.

When you roll a die, it is not affected by another die being rolled beside it.

- You can create tables, tree diagrams, and other diagrams to organize the outcomes for two independent events.

Communicate the Ideas

1. Decide whether each pair of events are independent or not independent. Explain your reasoning.
   a) Choose a student from grade 7 and choose a student from grade 8.
   b) Choose one marble from a bag and then choose a second marble from the bag without replacing the first marble.
   c) Choose an apple from one basket and then choose an apple from another basket.
2. Pretend a friend missed today's lesson. Roll a six-sided die and flip a coin. Teach your friend how to use a tree diagram to organize the outcomes. Then, show your friend how to identify the sample space.
3. Sharon created this table to list the possible outcomes from tossing a coin twice.

		Second Toss	
		Heads (H)	Tails (T)
First Toss	Heads (H)	H, H	H, T
	Tails (T)	H, T	T, T

Kevin says that the outcome in the bottom left corner should be (T, H). Who is correct? Why?

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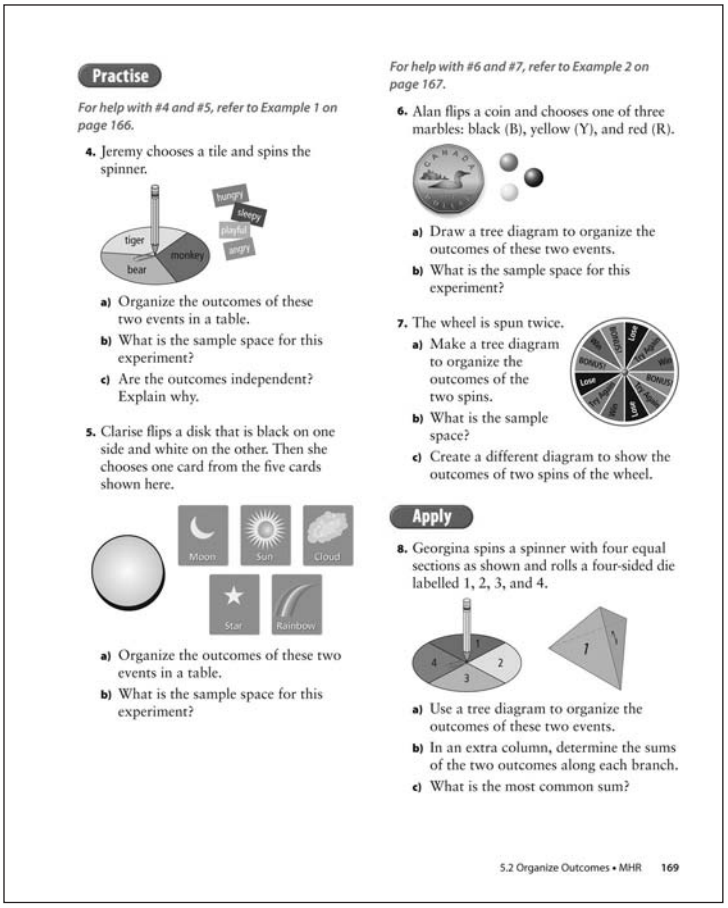
Key Ideas

The Key Ideas emphasize the concept of independent events and using organizers to organize outcomes. Many students (and adults) do not realize that if a fair die is rolled five times and the number 6 appears each time, the likelihood of the number 6 appearing on the next roll is still $\frac{1}{6}$ (no more and no less). Students could prepare their own list of Key Ideas and put it in their chapter Foldable.

Communicate the Ideas

In #1, students reinforce their understanding of independent events. In #2 and #3, students reinforce their understanding of using tree diagrams and tables to organize outcomes.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Rather than asking students to complete all three questions, you may wish to assign #2 and have them choose one other question. Have students work in pairs to answer #1. Students could work individually or in groups to answer #2 and #3, before summarizing the main points in a class discussion.	• Check answers to #1. Make sure that students understand the concept. • Tell students that the error in #3 (incorrectly ordering an outcome) is common. Encourage students to explain the strategy for listing outcomes: list the result of the first toss (T) and then list the result of the second toss (H). The outcome should be listed as T, H.



Practise

For help with #4 and #5, refer to Example 1 on page 166.

4. Jeremy chooses a tile and spins the spinner.

a) Organize the outcomes of these two events in a table.
b) What is the sample space for this experiment?
c) Are the outcomes independent? Explain why.

5. Clarise flips a disk that is black on one side and white on the other. Then she chooses one card from the five cards shown here.

a) Organize the outcomes of these two events in a table.
b) What is the sample space for this experiment?

For help with #6 and #7, refer to Example 2 on page 167.

6. Alan flips a coin and chooses one of three marbles: black (B), yellow (Y), and red (R).

a) Draw a tree diagram to organize the outcomes of these two events.
b) What is the sample space for this experiment?

7. The wheel is spun twice.

a) Make a tree diagram to organize the outcomes of the two spins.
b) What is the sample space?
c) Create a different diagram to show the outcomes of two spins of the wheel.

Apply

8. Georgina spins a spinner with four equal sections as shown and rolls a four-sided die labelled 1, 2, 3, and 4.

a) Use a tree diagram to organize the outcomes of these two events.
b) In an extra column, determine the sums of the two outcomes along each branch.
c) What is the most common sum?

5.2 Organize Outcomes • MHR 169

Practise

Allow students to use abbreviations for listing the sample space when appropriate.

Assessment *for* Learning	Supported Learning
Practise Have students do #4 and # 6. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #4 will need additional coaching with Example 1. Review this material with them, coach them as they correct #4, and then have them do #5 on their own. Check back with students several times to make sure that they understand the concepts. • Students who have problems with #6 will need additional coaching with Example 2. Review this material with them, coach them as they correct #6, and then have them do #7 on their own. Check back with students several times to make sure that they understand the concepts.

Apply and Extend

The Apply questions provide a variety of fairly straightforward contexts. For #8, the sums are not equally distributed. By direct counting, students can determine which sum occurs most frequently. For #12, students may need to be reminded that *product* refers to the result of multiplying two or more numbers.

The Extend problems require more analysis than those in the Apply section. In #13, there are eight equally likely branches for the third flip of the coin. You may wish to point out that the total number of possible outcomes (8), results from two options for each of three identical trials: $2^3 = 8$.

Answers

Communicate the Ideas

1. a) independent
b) not independent
c) independent

2. Answers will vary. Look for a tree diagram with two branches for the coin flip and two sets of six branches for the die.

3. Kevin is correct. The result of the first toss is listed first in each pair of outcomes in the table. The result of the first toss for this outcome is tails; the result of the second toss is heads. The outcome should be listed as T, H.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1 *or* 3, 2, 4 *or* 5, 6 *or* 7, 8, Math Link
Typical	1 *or* 3, 2, 4 *or* 5, 6 *or* 7, 8, 9 *or* 10, 11, 12, Math Link
Extension/Enrichment	1 *or* 3, 2, 13, 14

Supported Learning

Learning Style and Language

- Encourage students to share their understanding of the Key Ideas during group discussion.

Learning Style

- Post examples of tables and tree diagrams for students to use as guides. Label the different parts of the tables and tree diagrams. Encourage students to refer to the organizers to help them.
- Encourage students to use manipulatives, if necessary, to help them record possible outcomes.

Common Errors

- Tree diagrams can be arranged both vertically (down) and horizontally (right). Some students may get confused with one or both of the arrangements.

R_x Ensure that students practise both ways of organizing the tree diagram branches.

Answers

Math Link

1. a) Look for a tree diagram or a table.

		Die 2					
		1	2	3	4	5	6
Die 1	1	1, 1	1, 2	1, 3	1, 4	1, 5,	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

b) 7 **c)** 2 and 12

2. a) Look for a tree diagram or a table.

		Die 2			
		1	2	3	4
Die 1	1	1, 1	1, 2	1, 3	1, 4
	2	2, 1	2, 2	2, 3	2, 4
	3	3, 1	3, 2	3, 3	3, 4
	4	4, 1	4, 2	4, 3	4, 4

b) 5 **c)** 2 and 8

Supported Learning

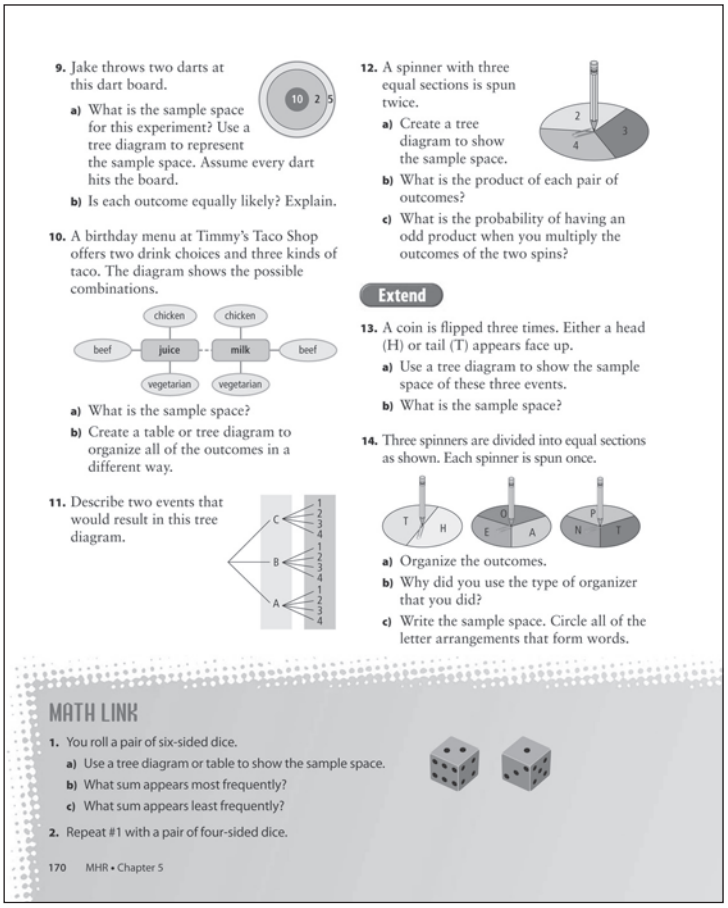
Learning Style and Memory

- Provide **BLM 5–5 Section 5.2 Extra Practice** to students who require extra practice.

Math Link

The Math Link provides students with an opportunity to explore the results of rolling two dice. You may wish to have students discuss their results as a class.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 170 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 193.	• Consider providing students with dice and have them use the dice to help find all the possible outcomes. • Students who are having difficulty getting started could use **BLM 5–6 Section 5.2 Math Link**, which provides scaffolding for this activity.



9. Jake throws two darts at this dart board.
   a) What is the sample space for this experiment? Use a tree diagram to represent the sample space. Assume every dart hits the board.
   b) Is each outcome equally likely? Explain.

10. A birthday menu at Timmy's Taco Shop offers two drink choices and three kinds of taco. The diagram shows the possible combinations.
   a) What is the sample space?
   b) Create a table or tree diagram to organize all of the outcomes in a different way.

11. Describe two events that would result in this tree diagram.

12. A spinner with three equal sections is spun twice.
   a) Create a tree diagram to show the sample space.
   b) What is the product of each pair of outcomes?
   c) What is the probability of having an odd product when you multiply the outcomes of the two spins?

Extend

13. A coin is flipped three times. Either a head (H) or tail (T) appears face up.
   a) Use a tree diagram to show the sample space of these three events.
   b) What is the sample space?

14. Three spinners are divided into equal sections as shown. Each spinner is spun once.
   a) Organize the outcomes.
   b) Why did you use the type of organizer that you did?
   c) Write the sample space. Circle all of the letter arrangements that form words.

MATH LINK

1. You roll a pair of six-sided dice.
   a) Use a tree diagram or table to show the sample space.
   b) What sum appears most frequently?
   c) What sum appears least frequently?
2. Repeat #1 with a pair of four-sided dice.

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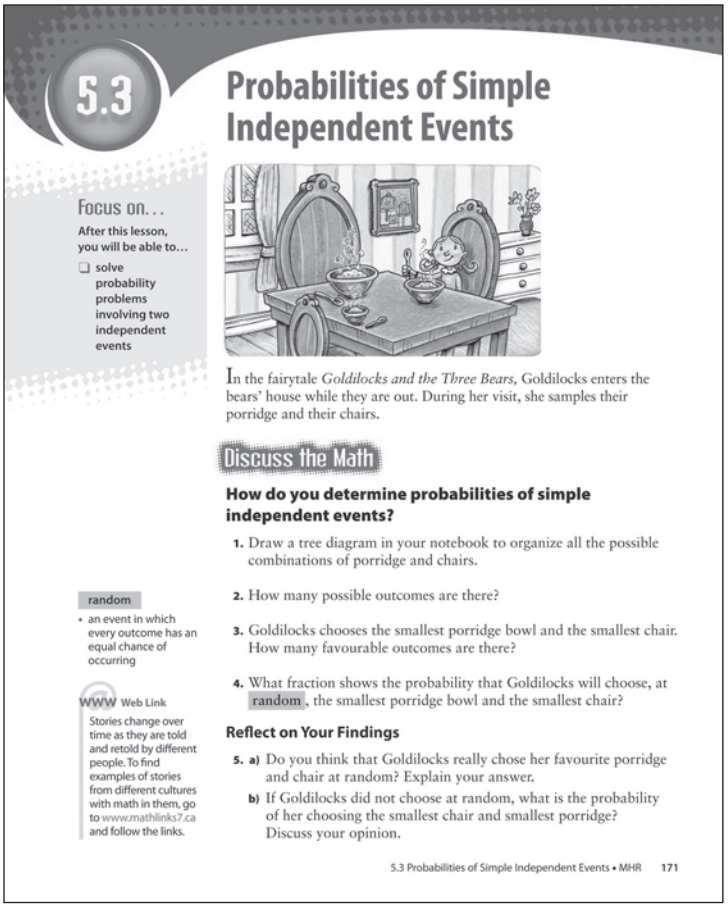
Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • What method do you prefer for organizing outcomes? Why? • What do you find interesting about this work on probability? • What is causing some confusion? What can you do to solve this problem?	• Encourage students to explain which organizer they prefer to use. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • You may wish to have students review the part related to Section 5.2 in **BLM 5–1 Chapter 5 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

5.3 Probabilities of Simple Independent Events



5.3

Probabilities of Simple Independent Events

Focus on...

After this lesson, you will be able to...

- solve probability problems involving two independent events

In the fairytale *Goldilocks and the Three Bears*, Goldilocks enters the bears' house while they are out. During her visit, she samples their porridge and their chairs.

Discuss the Math

How do you determine probabilities of simple independent events?

1. Draw a tree diagram in your notebook to organize all the possible combinations of porridge and chairs.
2. How many possible outcomes are there?
3. Goldilocks chooses the smallest porridge bowl and the smallest chair. How many favourable outcomes are there?
4. What fraction shows the probability that Goldilocks will choose, at random, the smallest porridge bowl and the smallest chair?

Reflect on Your Findings

5. a) Do you think that Goldilocks really chose her favourite porridge and chair at random? Explain your answer.
   b) If Goldilocks did not choose at random, what is the probability of her choosing the smallest chair and smallest porridge? Discuss your opinion.

random
- an event in which every outcome has an equal chance of occurring

WWW Web Link

Stories change over time as they are told and retold by different people. To find examples of stories from different cultures with math in them, go to www.mathlinks7.ca and follow the links.

5.3 Probabilities of Simple Independent Events • MHR 171

Suggested Timing
80–100 minutes

Materials
- ruler

Blackline Masters
BLM 5–1 Chapter 5 Self-Assessment
BLM 5–7 Section 5.3 Extra Practice

Mathematical Processes

- [x] Communication
- [x] Connections
- [ ] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization

WWW Web Link

For a site with stories from different cultures with math in them, go to **www.mathlinks7.ca** and follow the links.

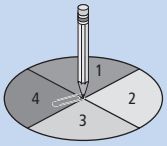
Specific Outcomes

SP6 Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.

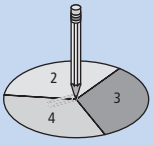
Warm-Up

Use the following spinners for questions #1 to #5.

Spinner A



Spinner B



1. What is the probability of spinning a letter on Spinner A?
2. What is the probability of spinning a number on Spinner A?
3. What is the probability of spinning 4 on Spinner A?
4. Show all of the possible outcomes for spinning Spinner A and then Spinner B.
5. What is the sample space for #4?

Mental Math

6. Find the percent mentally. Show your thinking.
   a) 95% of 42 **b)** 20% of 2048
7. Estimate the following. Show your thinking.
   a) 3.45×2.1 **b)** 125×22.4
8. Estimate a discount of $33\frac{1}{3}\%$ on a coat costing $65.99.
9. One bag of apples has a mass of 5 kg. Another is 1.6 times that size. Estimate the mass of the second bag. Show your thinking.
10. Estimate to place the decimal point. Show your thinking. $336.096 \div 19.45 = 1728$

Answers

Warm-Up

1. $P(\text{letter}) = 0\%$

2. $P(\text{number}) = 100\%$

3. $P(4) = 1{:}4$ or $\frac{1}{4}$ or 25%

4. Students can use a table, tree diagram, or other organizer.

	2	3	4
1	1, 2	1, 3	1, 4
2	2, 2	2, 3	2, 4
3	3, 2	3, 3	3, 4
4	4, 2	4, 3	4, 4

5. The sample space is (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4).

6. a) $10\% = 4.2$; $5\% = 2.1$; $95\% = 42 - 2.1 = 42 - 2 - 0.1 = 39.9$

b) $10\% = 204.8$; $20\% = (200 \times 2) + (4 \times 2) + (0.8 \times 2) = 400 + 8 + 1.6 = 409.6$

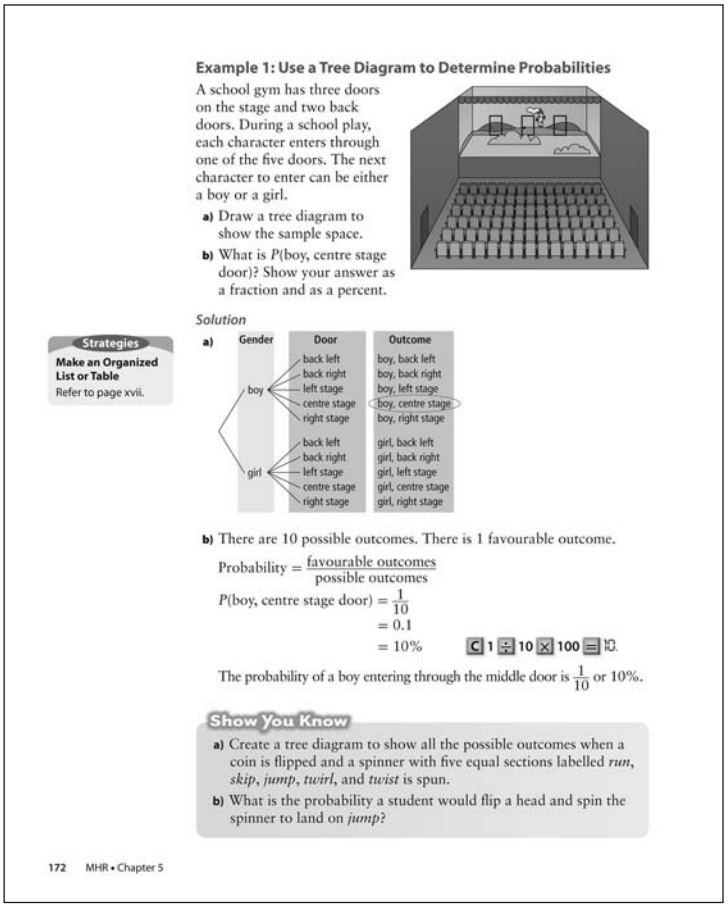
7. a) $3 \times 2 \approx 6$ or $3.5 \times 2 \approx 7$

b) $100 \times 20 \approx 2000$ or $120 \times 20 \approx 2400$

8. $\$66 \div 3 = \$22$

9. $5 \times 2 = 10$ kg or $5 \times 1.5 = 5 + 2.5 = 7.5$ kg

10. $100 \div 20 = 5 \times 3 = 15$ (There are five 20s in each 100.)
17.28



Example 1: Use a Tree Diagram to Determine Probabilities

A school gym has three doors on the stage and two back doors. During a school play, each character enters through one of the five doors. The next character to enter can be either a boy or a girl.

a) Draw a tree diagram to show the sample space.

b) What is P(boy, centre stage door)? Show your answer as a fraction and as a percent.

Solution

Strategies
Make an Organized List or Table
Refer to page xvii.

a)

Gender	Door	Outcome
boy	back left	boy, back left
	back right	boy, back right
	left stage	boy, left stage
	centre stage	boy, centre stage
	right stage	boy, right stage
girl	back left	girl, back left
	back right	girl, back right
	left stage	girl, left stage
	centre stage	girl, centre stage
	right stage	girl, right stage

b) There are 10 possible outcomes. There is 1 favourable outcome.

$$\text{Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$P(\text{boy, centre stage door}) = \frac{1}{10} = 0.1 = 10\%$$

C 1 ÷ 10 × 100 = 10.

The probability of a boy entering through the middle door is $\frac{1}{10}$ or 10%.

Show You Know

a) Create a tree diagram to show all the possible outcomes when a coin is flipped and a spinner with five equal sections labelled *run*, *skip*, *jump*, *twirl*, and *twist* is spun.

b) What is the probability a student would flip a head and spin the spinner to land on *jump*?

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Supported Learning

Learning Style

- Provide students with many visual examples and hands-on learning activities to help them learn the concepts.

ESL and Language

- Have students work with a partner to complete the Discuss the Math.

Meeting the Needs of All Learners

- Some students may not be familiar with the story *Goldilocks and the Three Bears*. Consider reading a simple version of the story to the class. Alternatively, ask a volunteer to share the storyline with the class and explain the term *porridge*.

Activity Planning Notes

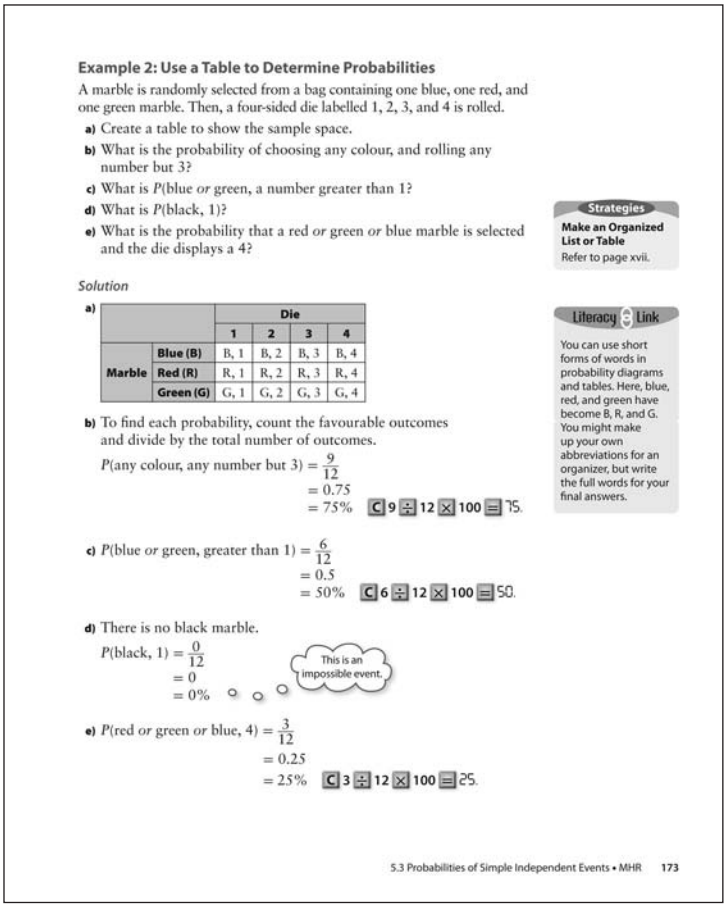
This section focuses on using organizers to determine probabilities. The concept of *random* is important to develop. Discuss what it means for Goldilocks to choose a bowl of porridge or chair randomly.

Discuss the Math

As a class, read the introduction, and then have students construct a tree diagram to organize all of the outcomes for choosing a bowl of porridge and a chair. As you circulate through the class, assist students who need help with completing the tree diagram. When everyone has had a chance to complete the diagram, ask a volunteer to draw his/her diagram on the board. Ask if anyone has organized their tree diagram in another way. Explore the alternate tree diagram where the order of the porridge and chairs is reversed. Point out that there are nine outcomes in either case.

You may wish to have students draw both a tree diagram and a table for #1 of Discuss the Math. Have them compare the two organizers and discuss the advantages of each.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss their ideas on how Goldilocks chose the chair and the porridge. Have students conclude their findings.	• Review the meaning of *random* and the plot of the Goldilocks story. They may remember that Goldilocks chose the large chair and bowl first, then the medium-sized ones, and then the small ones. Ask if Goldilocks made random choices.



Example 2: Use a Table to Determine Probabilities

A marble is randomly selected from a bag containing one blue, one red, and one green marble. Then, a four-sided die labelled 1, 2, 3, and 4 is rolled.

a) Create a table to show the sample space.

b) What is the probability of choosing any colour, and rolling any number but 3?

c) What is P(blue *or* green, a number greater than 1?

d) What is P(black, 1)?

e) What is the probability that a red *or* green *or* blue marble is selected and the die displays a 4?

Strategies
Make an Organized List or Table
Refer to page xvii.

Solution

a)

		Die			
		1	2	3	4
Marble	Blue (B)	B, 1	B, 2	B, 3	B, 4
	Red (R)	R, 1	R, 2	R, 3	R, 4
	Green (G)	G, 1	G, 2	G, 3	G, 4

Literacy Link
You can use short forms of words in probability diagrams and tables. Here, blue, red, and green have become B, R, and G. You might make up your own abbreviations for an organizer, but write the full words for your final answers.

b) To find each probability, count the favourable outcomes and divide by the total number of outcomes.

$P(\text{any colour, any number but 3}) = \frac{9}{12}$
$= 0.75$
$= 75\%$ C 9 ÷ 12 × 100 = 75.

c) $P(\text{blue } or \text{ green, greater than 1}) = \frac{6}{12}$
$= 0.5$
$= 50\%$ C 6 ÷ 12 × 100 = 50.

d) There is no black marble.

$P(\text{black, 1}) = \frac{0}{12}$
$= 0$
$= 0\%$

This is an impossible event.

e) $P(\text{red } or \text{ green } or \text{ blue, 4}) = \frac{3}{12}$
$= 0.25$
$= 25\%$ C 3 ÷ 12 × 100 = 25.

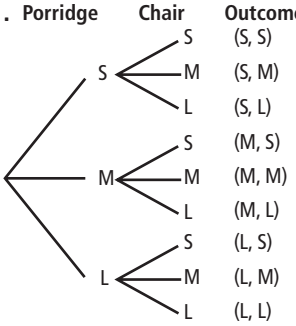
5.3 Probabilities of Simple Independent Events • MHR 173

Example 1 models using a tree diagram to determine probabilities. Explain that this example is an application of converting fractions to decimals and percents, which students learned in Chapter 4. You may need to review converting decimals to percents.

Answers

Discuss the Math

1.



2. 9

3. 1

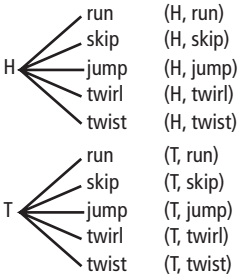
4. $\frac{1}{9}$ or 11.1%

5. a) Answers will vary.

b) Answers will vary. For example: If she did not choose at random, the probability could be 1 because she would have tried all the possibilities before she made her choice.

Show You Know: Example 1

a)



b) $\frac{1}{10}$ or 10%

Supported Learning

Motor

- Students may have difficulty drawing tree diagrams efficiently. You may wish to allow them to use virtual manipulatives.
- The buttons on most calculators are typically too small and close together for students with motor difficulties to use accurately. Students may benefit from using a calculator with oversized keys.

Common Errors

- When converting a fraction to a decimal, some students may incorrectly divide the denominator by the numerator.

R$_x$ Encourage students to apply number sense to the result. If the original fraction was proper, the resulting decimal should be less than 1.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1 on page 172.	• Note that students are asked to determine the probability. Ensure they express the probability as a fraction, a decimal, and a percent. • You may wish to provide additional questions to students who would benefit from them: **a)** Create a tree diagram to show all the possible outcomes when a coin is flipped and a spinner with four equal sections labelled *front, back, left*, and *right* is spun. (The tree diagram will follow the same format as the one in Example 1 part a), except that the Gender column will be titled Coin and show a head and tail choice. The Door column will be headed Spinner and show choices front, back, left, and right. The Outcome column will show the following: (H, front), (H, back), (H, left), (H, right), (T, front), (T, back), (T, left), (T, right).) **b)** What is the probability a student would flip a tail and spin the spinner to land on *left*? (Once students have the correct tree diagram, have them look at the sample space to check this probability. It would be 1:8 or 12.5%.) If students are having problems drawing a tree diagram, coach them through a), and then have them do b) on their own.

Answers

Show You Know: Example 2

a)

		Spinner				
		fly	swim	glide	walk	hop
Die	1	1, fly	1, swim	1, glide	1, walk	1, hop
	2	2, fly	2, swim	2, glide	2, walk	2, hop
	3	3, fly	3, swim	3, glide	3, walk	3, hop
	4	4, fly	4, swim	4, glide	4, walk	4, hop

b) $\frac{1}{20}$ or 5%

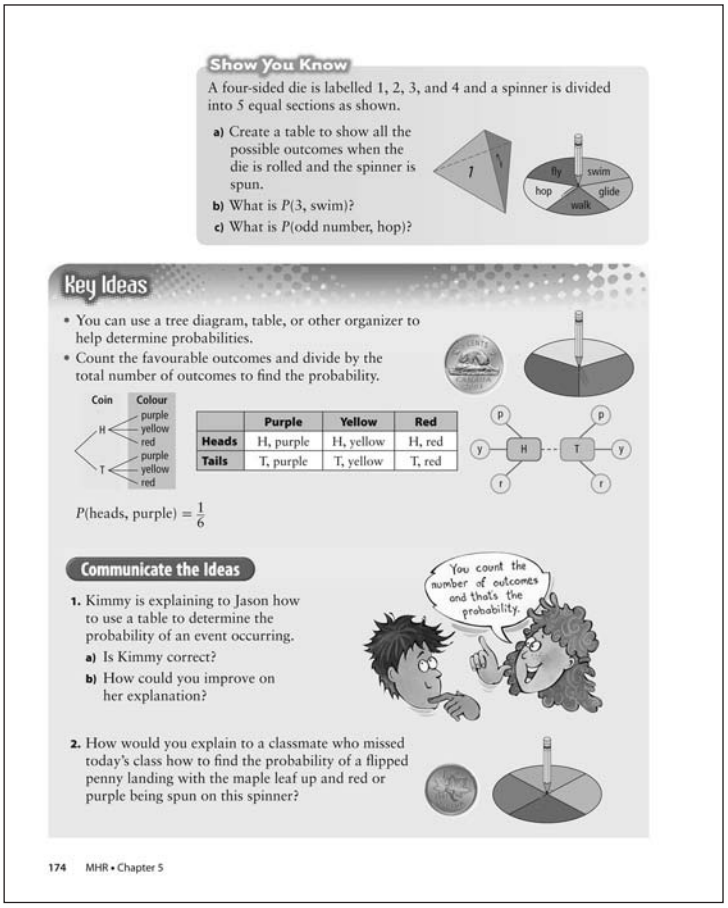
c) $\frac{2}{10}$ or 10%

Communicate the Ideas

1. a) No.

b) Answers may vary. For example: Circle the outcomes that you are looking for and then divide that number by the number of possible outcomes.

2. Answers will vary. Students should mention using an organizer such as a table or a tree diagram to help determine the probabilities.



Show You Know

A four-sided die is labelled 1, 2, 3, and 4 and a spinner is divided into 5 equal sections as shown.

a) Create a table to show all the possible outcomes when the die is rolled and the spinner is spun.

b) What is $P(3, \text{swim})$?

c) What is $P(\text{odd number, hop})$?

Key Ideas

- You can use a tree diagram, table, or other organizer to help determine probabilities.
- Count the favourable outcomes and divide by the total number of outcomes to find the probability.

	Purple	Yellow	Red
Heads	H, purple	H, yellow	H, red
Tails	T, purple	T, yellow	T, red

$P(\text{heads, purple}) = \frac{1}{6}$

Communicate the Ideas

1. Kimmy is explaining to Jason how to use a table to determine the probability of an event occurring.
   a) Is Kimmy correct?
   b) How could you improve on her explanation?

2. How would you explain to a classmate who missed today's class how to find the probability of a flipped penny landing with the maple leaf up and red or purple being spun on this spinner?

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Supported Learning

Learning Style, ESL, and Language

- Partner students for good one-on-one discussion. This facilitates comprehension of vocabulary.

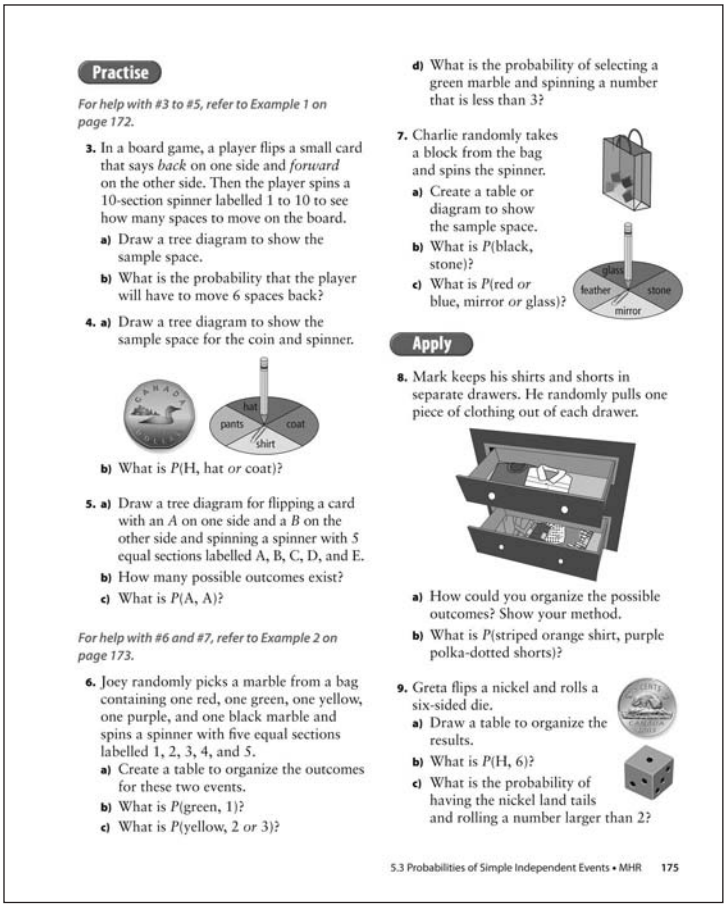
ESL and Language

- Be sure to explicitly teach using abbreviations in probability diagrams and tables. Some English language learners may not pick up on the short forms.

Example 2 models using a table to determine probabilities. Part b) asks students to determine the probability of choosing any colour marble and spinning any number but 3. Students need to understand that a successful spin would be 1, 2, or 4. As a class, read the Literacy Link on page 173 about using abbreviations. Discuss what constitutes a reasonable abbreviation for different words. For example, if there are blue and black marbles, then the abbreviations *B* and *Bl* are not sufficient to differentiate between the two colours.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• You may wish to provide additional questions for students who would benefit from them: A coin is flipped and a spinner divided into 3 equal sections labelled 1, 2, 3 is spun. **a)** Create a table to show all of the possible outcomes when the coin in flipped and the spinner is spun. **b)** What is $P(\text{H } or \text{ T}, 3)$? ($P(\text{H } or \text{ T}, 3) = 2{:}6 = 33\frac{1}{3}\%$.) **c)** What is $P(\text{H } or \text{ T, odd number})$? ($P(\text{H } or \text{ T, odd number}) = 4{:}6 = 66\frac{2}{3}\%$.) Coach students through drawing the table, assist them with b), and then have them do c) on their own.

	1	2	3
H	H, 1	H, 2	H, 3
T	T, 1	T, 2	T, 3



Practise

For help with #3 to #5, refer to Example 1 on page 172.

3. In a board game, a player flips a small card that says *back* on one side and *forward* on the other side. Then the player spins a 10-section spinner labelled 1 to 10 to see how many spaces to move on the board.
- **a)** Draw a tree diagram to show the sample space.
- **b)** What is the probability that the player will have to move 6 spaces back?

4. a) Draw a tree diagram to show the sample space for the coin and spinner.

b) What is P(H, hat *or* coat)?

5. a) Draw a tree diagram for flipping a card with an A on one side and a B on the other side and spinning a spinner with 5 equal sections labelled A, B, C, D, and E.
- **b)** How many possible outcomes exist?
- **c)** What is P(A, A)?

For help with #6 and #7, refer to Example 2 on page 173.

6. Joey randomly picks a marble from a bag containing one red, one green, one yellow, one purple, and one black marble and spins a spinner with five equal sections labelled 1, 2, 3, 4, and 5.
- **a)** Create a table to organize the outcomes for these two events.
- **b)** What is P(green, 1)?
- **c)** What is P(yellow, 2 *or* 3)?
- **d)** What is the probability of selecting a green marble and spinning a number that is less than 3?

7. Charlie randomly takes a block from the bag and spins the spinner.
- **a)** Create a table or diagram to show the sample space.
- **b)** What is P(black, stone)?
- **c)** What is P(red *or* blue, mirror *or* glass)?

Apply

8. Mark keeps his shirts and shorts in separate drawers. He randomly pulls one piece of clothing out of each drawer.

- **a)** How could you organize the possible outcomes? Show your method.
- **b)** What is P(striped orange shirt, purple polka-dotted shorts)?

9. Greta flips a nickel and rolls a six-sided die.
- **a)** Draw a table to organize the results.
- **b)** What is P(H, 6)?
- **c)** What is the probability of having the nickel land tails and rolling a number larger than 2?

Key Ideas

The Key Ideas emphasize the strategy of using tree diagrams and tables and directly counting outcomes to solve probability problems. This section of the chapter provides another opportunity to convert fractions, decimals, and percents. Students could prepare their own list of Key Ideas and put it in their chapter Foldable.

Communicate the Ideas

The Communicate the Ideas questions on page 174 allow students to review solving probability problems.

Practise

These questions give students additional practice in drawing and interpreting tree diagrams and tables to determine probability. In #4b), ensure students understand that (Head, hat) and (Head, coat) are both favourable outcomes.

Supported Learning

Learning Style and Language

- Allow students to present their answers to Communicate the Ideas orally, in writing, or using a combination of both.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students do both questions. Once they have finished, have them share their analysis in a class discussion.	• Have students work in groups to answer the questions. • As you circulate, listen for students' explanations and assess whether they have a basic understanding of how to solve probability problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8, 9
Typical	1–4, 6, 8–11, 12 *or* 13
Extension/Enrichment	1, 2, 13–15

Assessment *for* Learning	Supported Learning
Practise Have students do #3, #4, and #6. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #3 and #4 will need additional coaching with Example 1. Work with them to correct #3 and #4, and then have them complete #5 on their own. Check back with students several times to make sure that they understand the concepts. • Students who have problems with #6 will need additional coaching with Example 2. Work with them to correct #6, and then have them complete #7 on their own. Check back with students several times to make sure that they understand the concepts.

Supported Learning

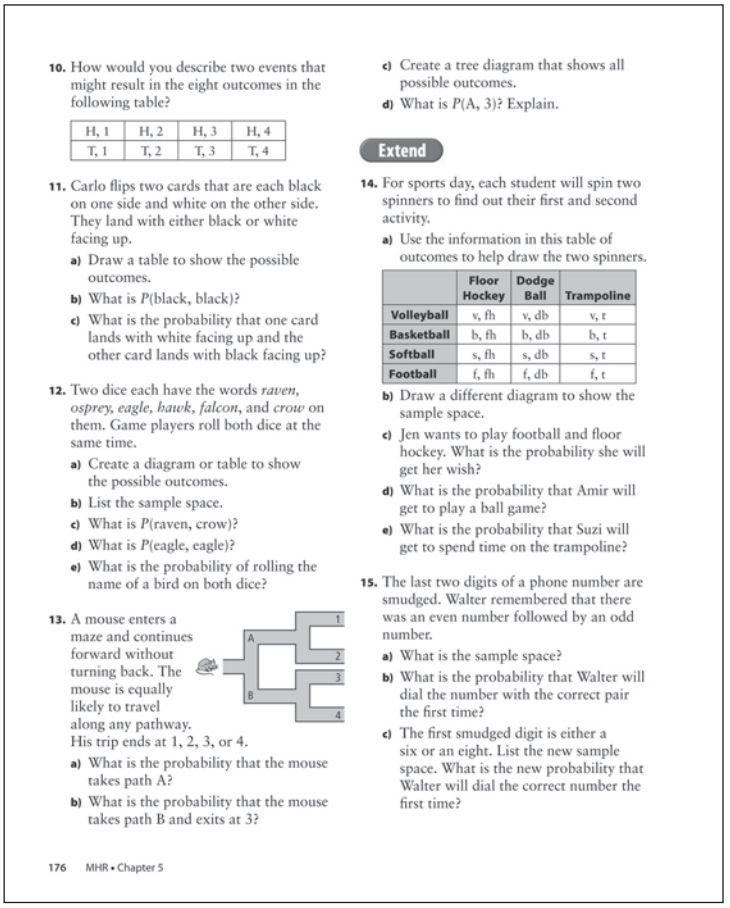
Learning Style and Memory

- Provide **BLM 5–7 Section 5.3 Extra Practice** to students who require more practice.

Common Errors

- Students may be confused about the meaning of the word *or* in probability.

R$_x$ Ensure students understand that the use of *or* in probability is inclusive. For example, when a spinner with the letters A, B, and C is spun and you are asked to find the probability of spinning an A or B, then both of these letters are considered favourable outcomes.



10. How would you describe two events that might result in the eight outcomes in the following table?

H, 1	H, 2	H, 3	H, 4
T, 1	T, 2	T, 3	T, 4

11. Carlo flips two cards that are each black on one side and white on the other side. They land with either black or white facing up.

a) Draw a table to show the possible outcomes.

b) What is P(black, black)?

c) What is the probability that one card lands with white facing up and the other card lands with black facing up?

12. Two dice each have the words *raven, osprey, eagle, hawk, falcon,* and *crow* on them. Game players roll both dice at the same time.

a) Create a diagram or table to show the possible outcomes.

b) List the sample space.

c) What is P(raven, crow)?

d) What is P(eagle, eagle)?

e) What is the probability of rolling the name of a bird on both dice?

13. A mouse enters a maze and continues forward without turning back. The mouse is equally likely to travel along any pathway. His trip ends at 1, 2, 3, or 4.

a) What is the probability that the mouse takes path A?

b) What is the probability that the mouse takes path B and exits at 3?

c) Create a tree diagram that shows all possible outcomes.

d) What is P(A, 3)? Explain.

Extend

14. For sports day, each student will spin two spinners to find out their first and second activity.

a) Use the information in this table of outcomes to help draw the two spinners.

	Floor Hockey	Dodge Ball	Trampoline
Volleyball	v, fh	v, db	v, t
Basketball	b, fh	b, db	b, t
Softball	s, fh	s, db	s, t
Football	f, fh	f, db	f, t

b) Draw a different diagram to show the sample space.

c) Jen wants to play football and floor hockey. What is the probability she will get her wish?

d) What is the probability that Amir will get to play a ball game?

e) What is the probability that Suzi will get to spend time on the trampoline?

15. The last two digits of a phone number are smudged. Walter remembered that there was an even number followed by an odd number.

a) What is the sample space?

b) What is the probability that Walter will dial the number with the correct pair the first time?

c) The first smudged digit is either a six or an eight. List the new sample space. What is the new probability that Walter will dial the correct number the first time?

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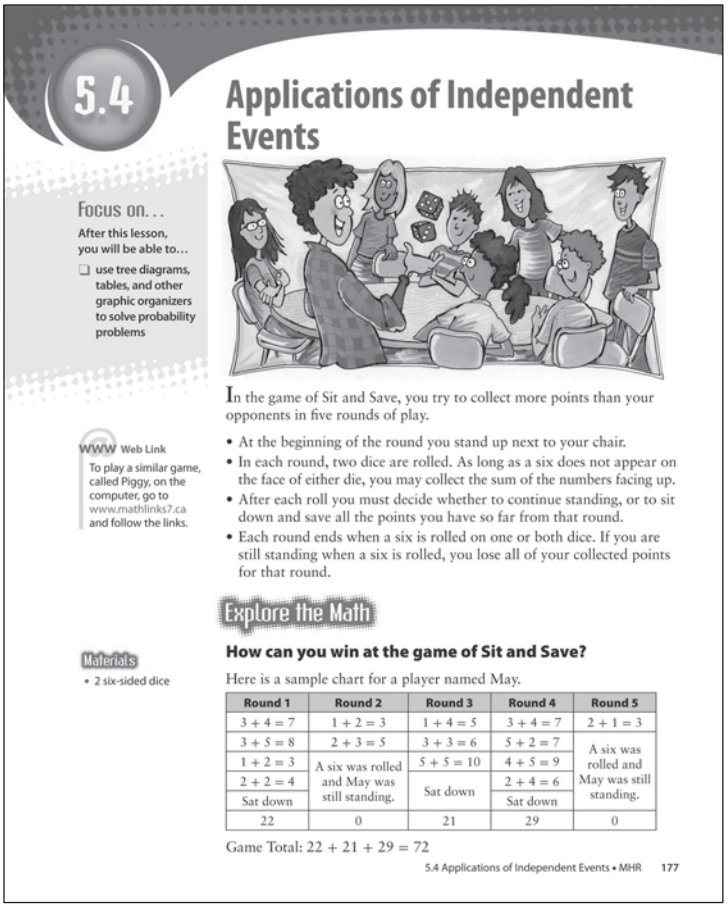
Apply and Extend

Students able to complete #8 to #13 demonstrate a good understanding of solving probability problems using the material in this section.

For #14 and #15, students need to use the information provided to determine different probabilities and determine the sample space.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • Why are tree diagrams a useful way to help determine probabilities?	• Encourage students to recognize the advantages of using tree diagrams to organize the outcomes in probability problems. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • You may wish to have students review the part related to Section 5.3 in **BLM 5–1 Chapter 5 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

5.4 Applications of Independent Events



5.4

Applications of Independent Events

Focus on...

After this lesson, you will be able to...

- use tree diagrams, tables, and other graphic organizers to solve probability problems

Web Link

To play a similar game, called Piggy, on the computer, go to www.mathlinks7.ca and follow the links.

In the game of Sit and Save, you try to collect more points than your opponents in five rounds of play.

- At the beginning of the round you stand up next to your chair.
- In each round, two dice are rolled. As long as a six does not appear on the face of either die, you may collect the sum of the numbers facing up.
- After each roll you must decide whether to continue standing, or to sit down and save all the points you have so far from that round.
- Each round ends when a six is rolled on one or both dice. If you are still standing when a six is rolled, you lose all of your collected points for that round.

Explore the Math

Materials
- 2 six-sided dice

How can you win at the game of Sit and Save?

Here is a sample chart for a player named May.

Round 1	Round 2	Round 3	Round 4	Round 5
3 + 4 = 7	1 + 2 = 3	1 + 4 = 5	3 + 4 = 7	2 + 1 = 3
3 + 5 = 8	2 + 3 = 5	3 + 3 = 6	5 + 2 = 7	A six was rolled and May was still standing.
1 + 2 = 3	A six was rolled and May was still standing.	5 + 5 = 10	4 + 5 = 9	
2 + 2 = 4		Sat down	2 + 4 = 6	
Sat down			Sat down	
22	0	21	29	0

Game Total: 22 + 21 + 29 = 72

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Suggested Timing
80–100 minutes

Materials
- 2 dice
- ruler
- integer chips or coins

Blackline Masters
BLM 5–1 Chapter 5 Self-Assessment
BLM 5–8 Section 5.4 Extra Practice
BLM 5–9 Crunch Time Game Board
BLM 5–10 Section 5.4 Math Link

Mathematical Processes

- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization

Web Link

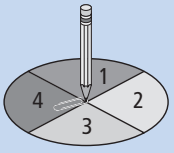
For a game called Piggy, go to **www.mathlinks7.ca** and follow the links.

Specific Outcomes

SP6 Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.

Warm-Up

Use a coin and the spinner to answer #1 to #5.



1. You flip a coin and spin the spinner. Draw a tree diagram showing the possible outcomes.
2. How many possible outcomes exist for #1?
3. List an impossible outcome for #1.
4. What is P(H, a number)?
5. What is P(T, a letter)?

Mental Math

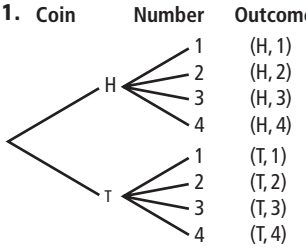
Show your thinking for each of #6 to #10.

6. Estimate 55% of 109.
7. Calculate 75% of $64.
8. Estimate the area of the following triangle: $b = 6.9$ m, $h = 15.2$ m
9. Estimate the change from $20 for a purchase of $15.83.
10. Estimate the number of items worth $0.49 you could buy for $6.

Answers

Warm-Up

1.



2. 8 **3.** Answers may vary. For example: H, 5.

4. P(H, a number) = 4:8 *or* $\frac{4}{8}$ *or* $\frac{1}{2}$ or 50%

5. P(T, a letter) = 0%

6. 50% ≈ 55; 10% ≈ 5.5; 5% ≈ 2. 55% ≈ 55 + 2 ≈ 57

7. 25% = \$64 ÷ 4 = \$16. 75% = 3 × \$16 = \$48

8. $7 \times 15 \div 2 = (7 \times 10 + 7 \times 5) \div 2 = (70 + 35) \div 2 = 105 \div 2 = 52.5\ m^2$

9. \$20 − \$16 = \$4 A little low

10. Use a pattern: \$1 = 2 halves; \$2 = 4 halves; \$3 = 6 halves; \$4 = 8 halves; \$5 = 10 halves; \$6 = 12 halves

Explore the Math

1. a) 10 **b)** 29

2. Outcomes of the games will vary.

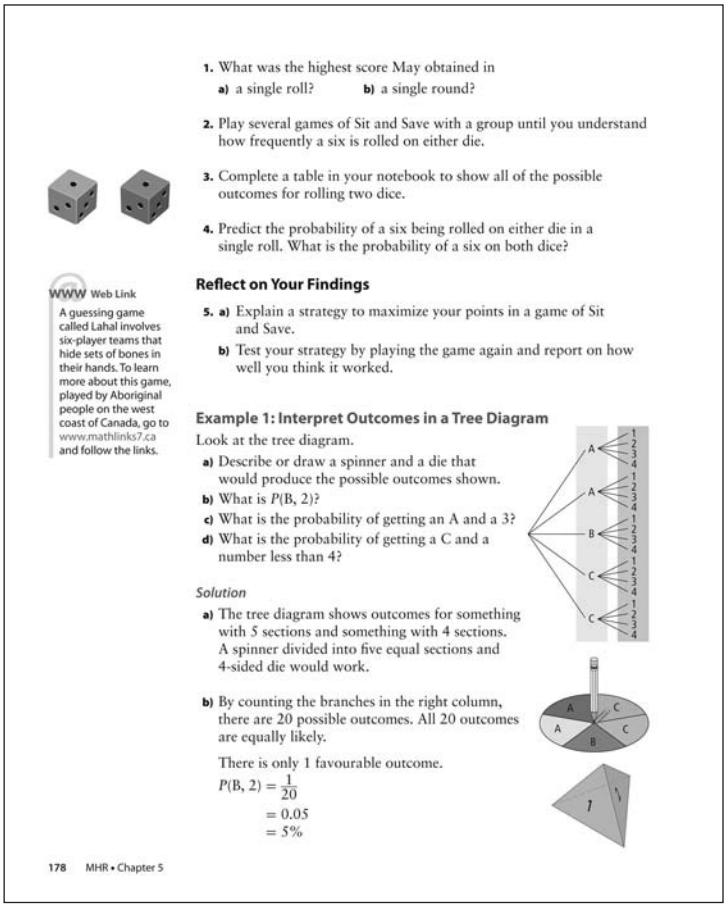
3.

		Die 2					
		1	2	3	4	5	6
Die 1	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

4. $\frac{1}{6}$; $\frac{1}{36}$

5. a), b) Strategies will vary.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss and test their strategies.	• Ask students who do well to share their strategy with the class. Discuss each strategy and how well it works. • Have students test various strategies. Encourage them to keep to one strategy for each game, rather than jumping from strategy to strategy.



1. What was the highest score May obtained in
   a) a single roll? b) a single round?
2. Play several games of Sit and Save with a group until you understand how frequently a six is rolled on either die.
3. Complete a table in your notebook to show all of the possible outcomes for rolling two dice.
4. Predict the probability of a six being rolled on either die in a single roll. What is the probability of a six on both dice?

WWW Web Link
A guessing game called Lahal involves six-player teams that hide sets of bones in their hands. To learn more about this game, played by Aboriginal people on the west coast of Canada, go to www.mathlinks7.ca and follow the links.

Reflect on Your Findings

5. a) Explain a strategy to maximize your points in a game of Sit and Save.
   b) Test your strategy by playing the game again and report on how well you think it worked.

Example 1: Interpret Outcomes in a Tree Diagram

Look at the tree diagram.

a) Describe or draw a spinner and a die that would produce the possible outcomes shown.
b) What is P(B, 2)?
c) What is the probability of getting an A and a 3?
d) What is the probability of getting a C and a number less than 4?

Solution

a) The tree diagram shows outcomes for something with 5 sections and something with 4 sections. A spinner divided into five equal sections and 4-sided die would work.

b) By counting the branches in the right column, there are 20 possible outcomes. All 20 outcomes are equally likely.

There is only 1 favourable outcome.

$P(B, 2) = \frac{1}{20}$
$= 0.05$
$= 5\%$

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Activity Planning Notes

Carefully explain the rules of the game Sit and Save to students. Students will learn the game quickly. It is important that they play the game to get a feel for the likelihood of a six being rolled.

Explore the Math

Review the sample chart for May with students. Ask students to predict the likelihood of a six being rolled. Explain how to keep score for the game in their notebook. Play the game Sit and Save a few times with the entire class. Again, ask students to predict the probability of rolling a six. As a class, create a chart indicating all of the possible outcomes for rolling two dice. Using the chart and direct counting, have students determine the probability of a six being rolled. Compare this value with students' earlier prediction.

WWW Web Link

For a guessing game called Lahal played by Aboriginal people on the west coast of Canada, go to **www.mathlinks7.ca** and follow the links.

c) There are 2 favourable outcomes.

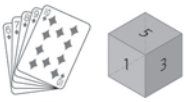
$P(\text{A, 3}) = \frac{2}{20}$
$= 0.1$
$= 10\%$

d) There are 3 numbers that are less than 4: 1, 2, and 3.
For each of these numbers, there are 2 possible regions labelled C.
By counting, there are 6 favourable outcomes.

$P(\text{C, less than 4}) = \frac{6}{20}$
$= 0.3$
$= 30\%$

Example 2: Interpret Outcomes in a Table

A card is chosen at random and a die labelled 1 to 6 is rolled.



a) Organize the outcomes in a table.
b) What is the probability of getting only one 6?
c) What is the probability of getting at least one 6?
d) What is the probability of the two numbers having a sum of 10?
e) What is the probability of the two numbers having a sum of 10 or more?

Solution

a)

		Six-Sided Die					
		1	2	3	4	5	6
Number Cards	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6
	7	7, 1	7, 2	7, 3	7, 4	7, 5	7, 6
	8	8, 1	8, 2	8, 3	8, 4	8, 5	8, 6
	9	9, 1	9, 2	9, 3	9, 4	9, 5	9, 6
	10	10, 1	10, 2	10, 3	10, 4	10, 5	10, 6

b) There are 9 favourable outcomes: (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (7, 6), (8, 6), (9, 6), (10, 6).

$P(\text{one 6}) = \frac{9}{30}$
$= 0.3$
$= 30\%$ C 9 ÷ 30 × 100 = 30.

(6, 6) is not included because it has two 6s.

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Example 1 illustrates how to interpret outcomes in a tree diagram. Ensure students realize that some outcomes appear more than once, and that each favourable outcome must be counted.

Example 2 illustrates how to interpret outcomes in a table.

Supported Learning

Learning Style

- Students who prefer to learn visually will appreciate the opportunity to play a game and test their strategy. Encourage them to develop their skills in this area.

ESL

- Explain terms such as *opponents*, *round*, *roll*, *highest*, *frequently*, and *maximize* to English language learners who may have difficulty with these terms. Have students add any new terms to their dictionary.

Common Errors

- Students may be confused about the increasing complexity of outcomes in many probability experiments.

R_x Remind students to record each outcome for each face of a die or section of a spinner, even if it is the same outcome (e.g., a spinner with A, B, C, and C).

TECH LINK

You may wish to have students play Lahal, which is described in the Web Link on page 178, and organize their findings using a tree diagram or a table.

Answers

Communicate the Ideas

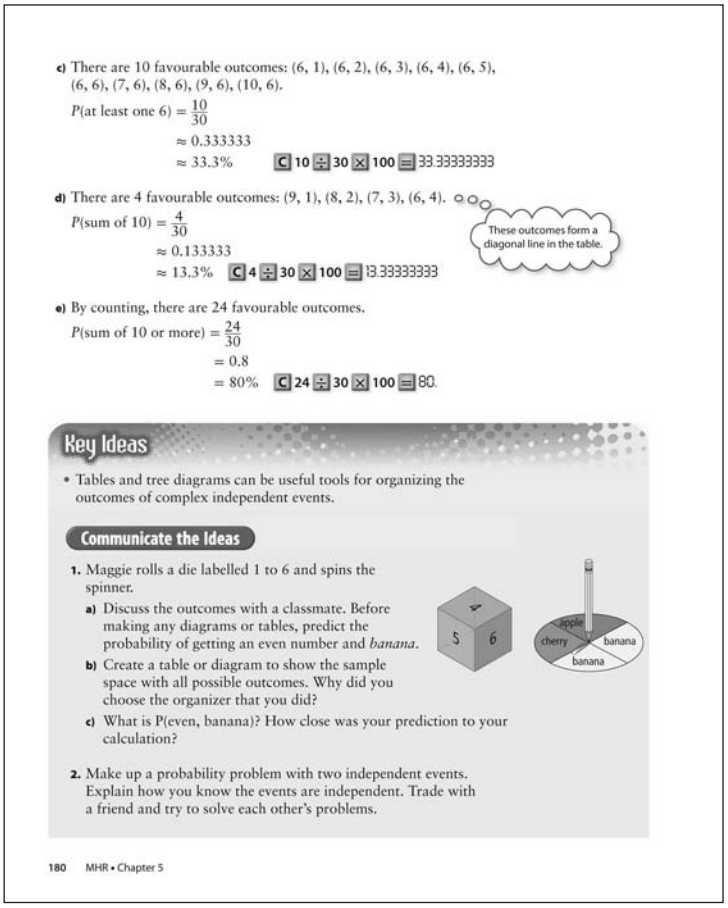
1. a) Predictions will vary. For example: 25%

b) Answers may vary.

		Spinner			
		apple	banana	banana	cherry
Die 1	1	1, apple	1, banana	1, banana	1, cherry
	2	2, apple	2, banana	2, banana	2, cherry
	3	3, apple	3, banana	3, banana	3, cherry
	4	4, apple	4, banana	4, banana	4, cherry
	5	5, apple	5, banana	5, banana	5, cherry
	6	6, apple	6, banana	6, banana	6, cherry

c) $\frac{6}{24}$ or 25%

2. Answers will vary.



c) There are 10 favourable outcomes: (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), (7, 6), (8, 6), (9, 6), (10, 6).

$P(\text{at least one } 6) = \frac{10}{30}$

≈ 0.333333

$\approx 33.3\%$ C 10 ÷ 30 × 100 = 33.33333333

d) There are 4 favourable outcomes: (9, 1), (8, 2), (7, 3), (6, 4).

$P(\text{sum of } 10) = \frac{4}{30}$

≈ 0.133333

$\approx 13.3\%$ C 4 ÷ 30 × 100 = 13.33333333

These outcomes form a diagonal line in the table.

e) By counting, there are 24 favourable outcomes.

$P(\text{sum of 10 or more}) = \frac{24}{30}$

$= 0.8$

$= 80\%$ C 24 ÷ 30 × 100 = 80.

Key Ideas

- Tables and tree diagrams can be useful tools for organizing the outcomes of complex independent events.

Communicate the Ideas

1. Maggie rolls a die labelled 1 to 6 and spins the spinner.

a) Discuss the outcomes with a classmate. Before making any diagrams or tables, predict the probability of getting an even number and *banana*.

b) Create a table or diagram to show the sample space with all possible outcomes. Why did you choose the organizer that you did?

c) What is P(even, banana)? How close was your prediction to your calculation?

2. Make up a probability problem with two independent events. Explain how you know the events are independent. Trade with a friend and try to solve each other's problems.

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Key Ideas

The Key Ideas emphasize using tables and tree diagrams to organize the outcomes of complex independent events. Stress that as the number of possible outcomes increases, students need to be careful to count favourable and possible outcomes. Generally, tables are more appropriate than tree diagrams when there are more than 20 possible outcomes. Students could prepare their own list of Key Ideas and put it in their chapter Foldable.

Supported Learning

ESL

- For #2, pair English language learners with students who have a good understanding of the terminology.

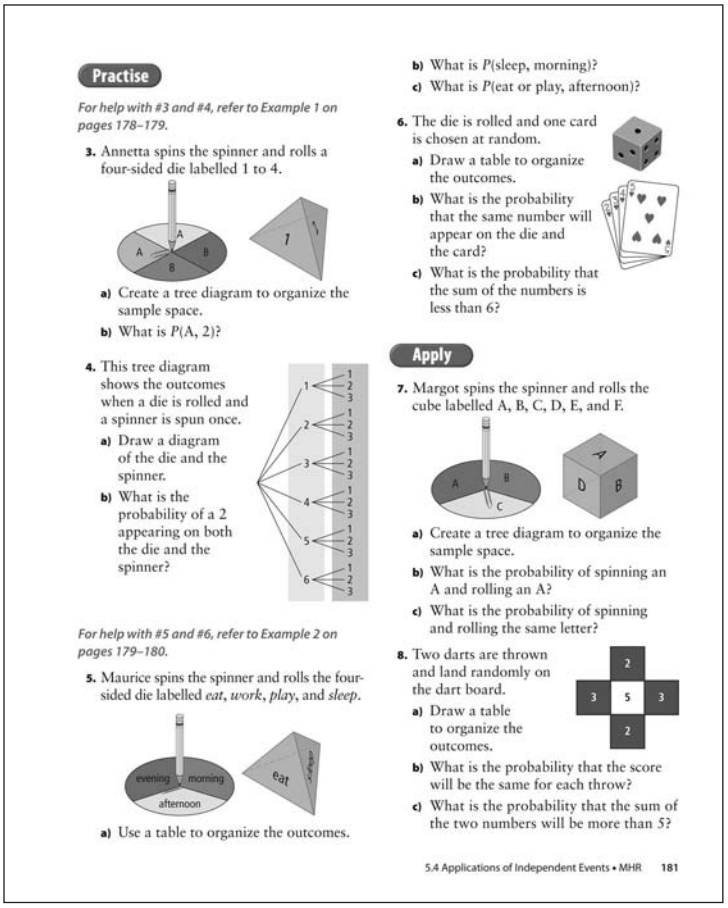
Meeting the Needs of All Learners

- Post examples of tables and tree diagrams in the classroom. Label the different parts of the tables and tree diagrams. Encourage students to refer to the charts for help.

Communicate the Ideas

These questions allow students to explain their understanding of calculating the probability of two independent events.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students do both questions working individually or in groups. Consider having students work with a partner for #2 and then exchange problems with another group. Encourage them to share their answers and listen to each other's explanations. Once they have finished, discuss the questions as a class.	• For #2, pair students who have a similar understanding of probability. Students should be able to justify their choice of tool for determining the sample space. • For #2, consider providing students with a die and spinner to help them make up a problem.



Practise

For help with #3 and #4, refer to Example 1 on pages 178–179.

3. Annetta spins the spinner and rolls a four-sided die labelled 1 to 4.

a) Create a tree diagram to organize the sample space.

b) What is $P(A, 2)$?

4. This tree diagram shows the outcomes when a die is rolled and a spinner is spun once.

a) Draw a diagram of the die and the spinner.

b) What is the probability of a 2 appearing on both the die and the spinner?

For help with #5 and #6, refer to Example 2 on pages 179–180.

5. Maurice spins the spinner and rolls the four-sided die labelled *eat*, *work*, *play*, and *sleep*.

a) Use a table to organize the outcomes.

b) What is P(sleep, morning)?

c) What is P(eat or play, afternoon)?

6. The die is rolled and one card is chosen at random.

a) Draw a table to organize the outcomes.

b) What is the probability that the same number will appear on the die and the card?

c) What is the probability that the sum of the numbers is less than 6?

Apply

7. Margot spins the spinner and rolls the cube labelled A, B, C, D, E, and F.

a) Create a tree diagram to organize the sample space.

b) What is the probability of spinning an A and rolling an A?

c) What is the probability of spinning and rolling the same letter?

8. Two darts are thrown and land randomly on the dart board.

a) Draw a table to organize the outcomes.

b) What is the probability that the score will be the same for each throw?

c) What is the probability that the sum of the two numbers will be more than 5?

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Supported Learning

Learning Style and Memory

- Provide **BLM 5–8 Section 5.4 Extra Practice** to students who require extra practice with the skills in this section.

ESL

- Explain terms such as *prime*, *composite,* and *initials* to English language learners who may have difficulty with these terms.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 7, Math Link
Typical	1–3, 5 *two of* 7, 8, 9, Math Link
Extension/Enrichment	1, 2, 10, 11, Math Link

Practise

For #6c), students need to determine the sums of the numbers. Students may wish to record each sum inside each cell of the table in order to count the number of sums that are less than 6. Remind students that "less than 6" does not include 6.

Assessment *for* Learning	Supported Learning
Practise Have students do #3 and #5. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #3 will need additional coaching with Example 1. Work with them to correct #3, and then have them do #4. Check back with students several times to make sure that they understand the concepts. • Students who have problems with #5 will need additional coaching with Example 2. Work with them to correct #5, and then have them do #6. Check back with students several times to make sure that they understand the concepts.

Apply and Extend

The Apply questions provide a variety of fairly straightforward contexts for solving probability problems. For #8, stress that each thrown dart lands somewhere on the board.

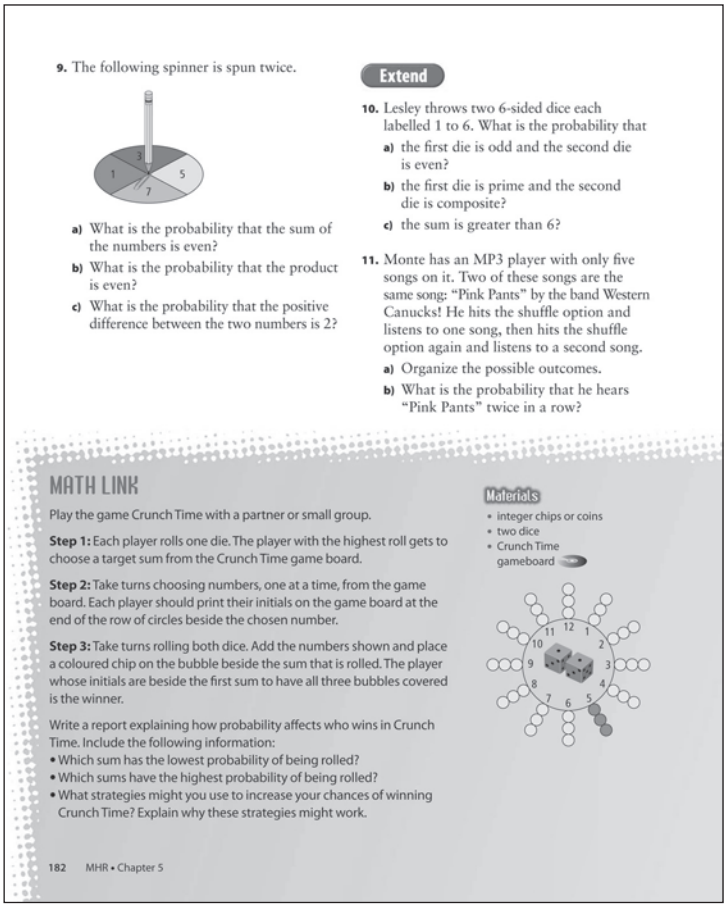
The Extend problems require significantly more analysis than those in the Apply section. For #10, remind students that 1 is not a prime number.

Supported Learning

Learning Style, ESL, Language, and Motor

- Consider allowing students to use a computer to write the report for the Math Link.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • Did Crunch Time work out the way you expected? Explain why or why not.	• In Section 5.5, students conduct their own probability experiments. Have them begin to discuss that probability does not always work. For example, a player on #3 may have won Crunch Time, even when it is more probable for a player on #7 to win. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • You may wish to have students review the part related to Section 5.4 in **BLM 5–1 Chapter 5 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.



9. The following spinner is spun twice.

a) What is the probability that the sum of the numbers is even?
b) What is the probability that the product is even?
c) What is the probability that the positive difference between the two numbers is 2?

Extend

10. Lesley throws two 6-sided dice each labelled 1 to 6. What is the probability that
a) the first die is odd and the second die is even?
b) the first die is prime and the second die is composite?
c) the sum is greater than 6?

11. Monte has an MP3 player with only five songs on it. Two of these songs are the same song: "Pink Pants" by the band Western Canucks! He hits the shuffle option and listens to one song, then hits the shuffle option again and listens to a second song.
a) Organize the possible outcomes.
b) What is the probability that he hears "Pink Pants" twice in a row?

MATH LINK

Play the game Crunch Time with a partner or small group.

Step 1: Each player rolls one die. The player with the highest roll gets to choose a target sum from the Crunch Time game board.

Step 2: Take turns choosing numbers, one at a time, from the game board. Each player should print their initials on the game board at the end of the row of circles beside the chosen number.

Step 3: Take turns rolling both dice. Add the numbers shown and place a coloured chip on the bubble beside the sum that is rolled. The player whose initials are beside the first sum to have all three bubbles covered is the winner.

Write a report explaining how probability affects who wins in Crunch Time. Include the following information:
• Which sum has the lowest probability of being rolled?
• Which sums have the highest probability of being rolled?
• What strategies might you use to increase your chances of winning Crunch Time? Explain why these strategies might work.

Materials
• integer chips or coins
• two dice
• Crunch Time gameboard

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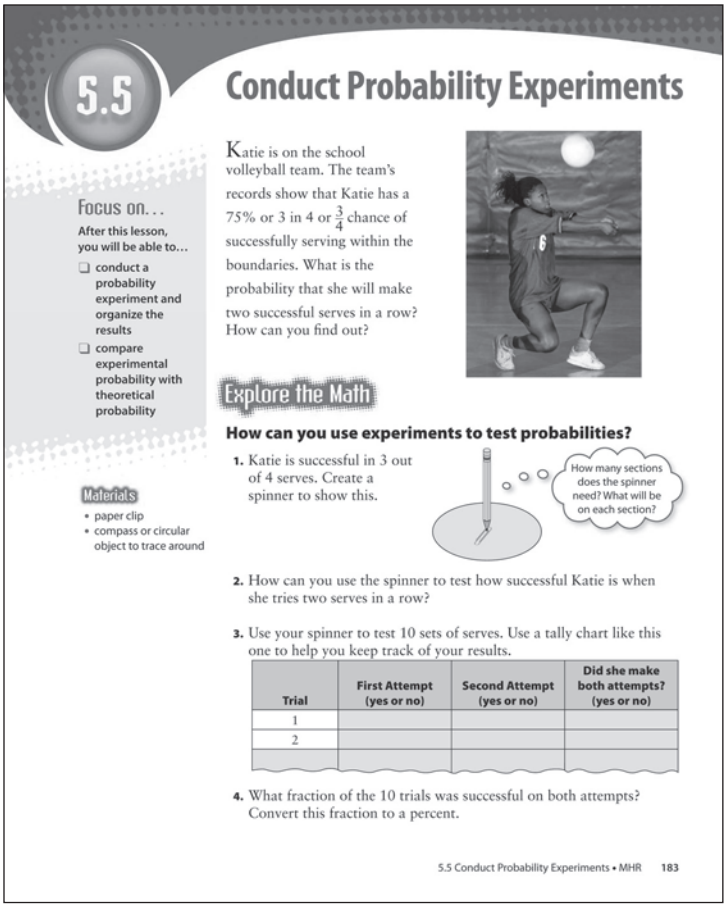
Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 182 is intended to help students work toward the chapter problem titled Wrap It Up! on page 193.	• As students create the table to show the sample space and determine the respective sums, stress the importance of counting all of the repeated sums. For example, the most commonly occurring sum will be 7, which can be generated by the following six arrangements: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1). • Students who are having difficulty getting started could use **BLM 5–10 Section 5.4 Math Link**, which provides scaffolding for the activity.

Math Link

Have students play Crunch Time to see how probability works out in a game situation. Provide integer chips or coins, pairs of dice, and copies of **BLM 5–9 Crunch Time Game Board** to play the game. When photocopying the blackline master, enlarge it to 200% and use 11 by 17 paper. The result should accommodate the chips or coins.

Once they have played several rounds, have students analyse the probability of each sum, and then play the game again. Have students write a report that explains who wins in Crunch Time.

5.5 Conduct Probability Experiments



5.5 Conduct Probability Experiments

Katie is on the school volleyball team. The team's records show that Katie has a 75% or 3 in 4 or $\frac{3}{4}$ chance of successfully serving within the boundaries. What is the probability that she will make two successful serves in a row? How can you find out?

Focus on...

After this lesson, you will be able to...

- conduct a probability experiment and organize the results
- compare experimental probability with theoretical probability

Materials

- paper clip
- compass or circular object to trace around

Explore the Math

How can you use experiments to test probabilities?

1. Katie is successful in 3 out of 4 serves. Create a spinner to show this.

How many sections does the spinner need? What will be on each section?

2. How can you use the spinner to test how successful Katie is when she tries two serves in a row?

3. Use your spinner to test 10 sets of serves. Use a tally chart like this one to help you keep track of your results.

Trial	First Attempt (yes or no)	Second Attempt (yes or no)	Did she make both attempts? (yes or no)
1			
2			

4. What fraction of the 10 trials was successful on both attempts? Convert this fraction to a percent.

5.5 Conduct Probability Experiments • MHR 183

Suggested Timing
80–100 minutes

Materials
- paper clip
- pencil
- compass or circular object to trace around
- 2 coloured pencils
- ruler

Blackline Masters
BLM 5–1 Chapter 5 Self-Assessment
BLM 5–11 Random Number Generator
BLM 5–12 Section 5.5 Extra Practice

Mathematical Processes

- ☐ Communication
- ☑ Connections
- ☐ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☑ Technology
- ☑ Visualization

Specific Outcomes

SP6 Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.

Warm-Up

Use the table below to answer #1 to #5.

	Breakfast	Lunch	Snack	Supper
Apple	apple, breakfast	apple, lunch	apple, snack	apple, supper
Pear	pear, breakfast	pear, lunch	pear, snack	pear, supper
Fig	fig, breakfast	fig, lunch	fig, snack	fig, supper

1. The table shows outcomes from a probability experiment. Draw or describe what might have been used to get these results.
2. What is the probability of getting a fig?
3. What is the probability of getting a fruit?
4. What is P(pear, snack)?
5. What is P(carrot, snack)?

Mental Math

Show your thinking for each of #6 to #10.

6. Calculate 65% of 126.
7. Calculate 30% of $176.
8. Estimate the area of the following parallelogram: $b = 163.2$ m, $h = 24.6$ m
9. Estimate the change from $30 for a purchase of $27.12.
10. Estimate how many items worth $0.74 you could buy for $15.

Answers

Warm-Up

1. Answers will vary. For example: One spinner has 3 equal sections: apple, pear, fig. A second spinner has 4 equal sections: breakfast, lunch, snack, supper. Answers will vary. Instead of a spinner, students may suggest using two bags.

2. $P(\text{fig}) = 4{:}12$ or $\frac{1}{3}$ or $33.\overline{3}\%$ **3.** $P(\text{fruit}) = 100\%$

4. $P(\text{pear, snack}) = 1{:}12$ or $\frac{1}{12}$ or $0.08\overline{3}\%$

5. $P(\text{carrot, snack}) = 0\%$

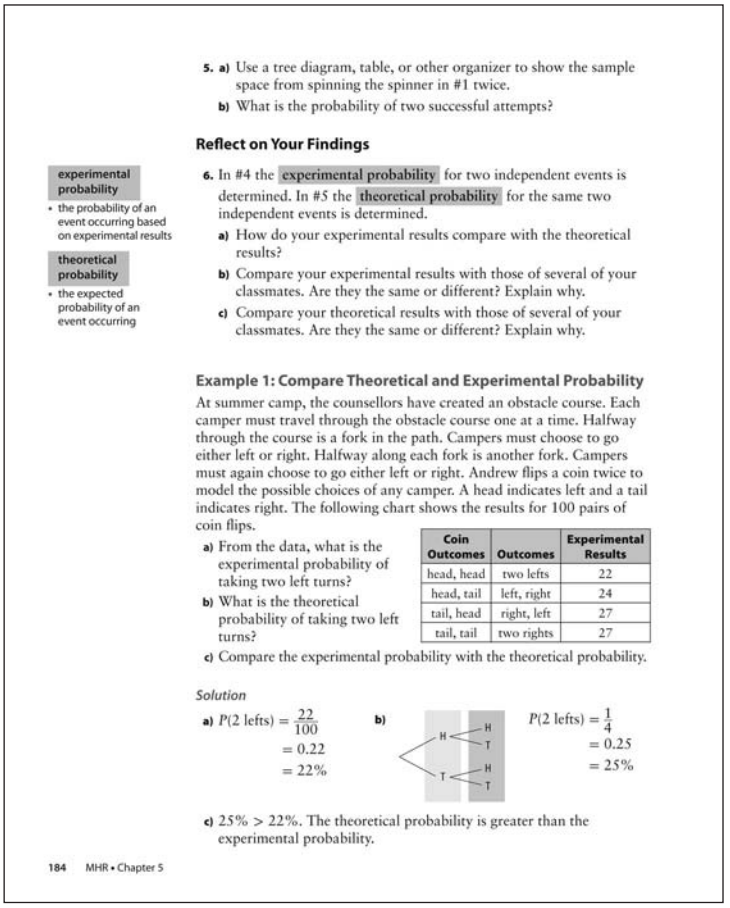
6. 50% of 126 = 63; 10% of 126 = 12.6; 5% of 126 = 6.3. 65% = 63 + 12.6 + 6.3 = 81.9

7. 10% of \$176 = \$17.60. 30% = \$17.60 × 3 = (\$17 × 3) + (\$0.60 × 3) = \$51 + \$1.80 = \$52.80

8. $160 \times 20 \approx 3200\ \text{m}^2$

9. \$30 − \$27 = \$3 A little high

10. 2 × \$0.75 = \$1.50. 10 × \$1.50 = \$15. You could buy about 20 items.



5. a) Use a tree diagram, table, or other organizer to show the sample space from spinning the spinner in #1 twice.
b) What is the probability of two successful attempts?

Reflect on Your Findings

experimental probability
- the probability of an event occurring based on experimental results

theoretical probability
- the expected probability of an event occurring

6. In #4 the experimental probability for two independent events is determined. In #5 the theoretical probability for the same two independent events is determined.
a) How do your experimental results compare with the theoretical results?
b) Compare your experimental results with those of several of your classmates. Are they the same or different? Explain why.
c) Compare your theoretical results with those of several of your classmates. Are they the same or different? Explain why.

Example 1: Compare Theoretical and Experimental Probability

At summer camp, the counsellors have created an obstacle course. Each camper must travel through the obstacle course one at a time. Halfway through the course is a fork in the path. Campers must choose to go either left or right. Halfway along each fork is another fork. Campers must again choose to go either left or right. Andrew flips a coin twice to model the possible choices of any camper. A head indicates left and a tail indicates right. The following chart shows the results for 100 pairs of coin flips.

Coin Outcomes	Outcomes	Experimental Results
head, head	two lefts	22
head, tail	left, right	24
tail, head	right, left	27
tail, tail	two rights	27

a) From the data, what is the experimental probability of taking two left turns?
b) What is the theoretical probability of taking two left turns?
c) Compare the experimental probability with the theoretical probability.

Solution

a) $P(2\text{ lefts}) = \frac{22}{100} = 0.22 = 22\%$

b) $P(2\text{ lefts}) = \frac{1}{4} = 0.25 = 25\%$

c) 25% > 22%. The theoretical probability is greater than the experimental probability.

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Supported Learning

ESL

- Some students may be unfamiliar with volleyball. If so, ask a volunteer to explain how to play the game and what a serve is.

Motor

- Students may find it difficult to create the spinner independently. Consider allowing them to use virtual manipulatives.
- Make sure that students use a ruler to draw the tally chart. Alternatively, consider allowing them to use a computer to create the chart.

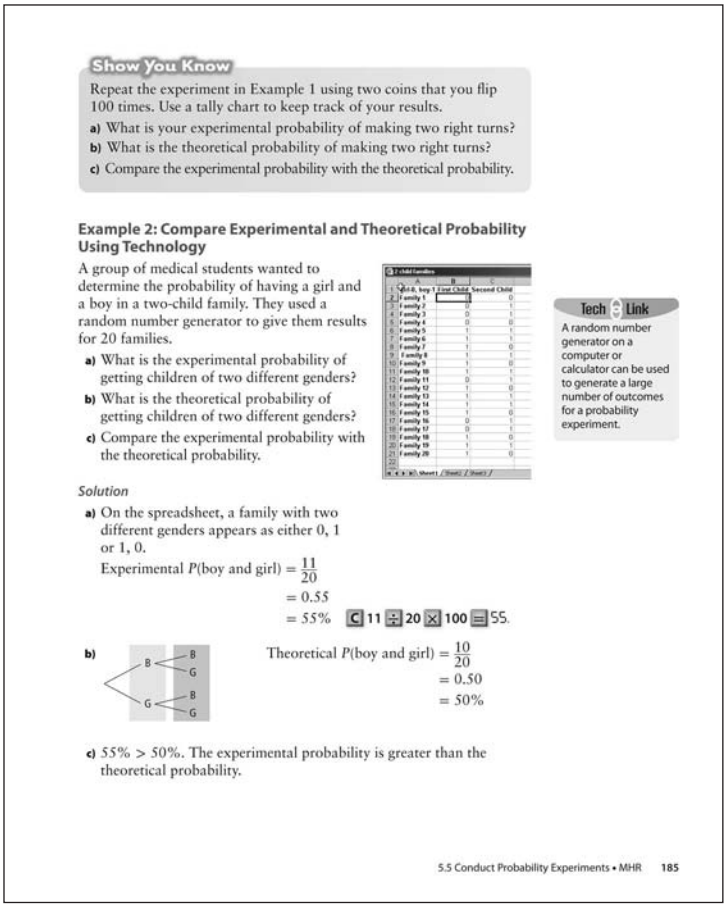
Activity Planning Notes

Read the introduction with the students. Make sure that students know what a *serve* is in volleyball. Ask students to predict what the probability is for Katie to successfully serve the ball twice in a row.

Explore the Math

Discuss how to create a spinner with four equal sections to simulate Katie's success with serving a volleyball. On an overhead, show the class a spinner with four equal regions labelled Y, Y, Y, N. Explain that a successful serve occurs if a Y is spun. Have students work in pairs to design and construct the spinner using a round object to trace around, two coloured pencils, a paper clip, and a pencil. Prompt students to determine the central angle for each section ($360° \div 4 = 90°$). Have the students copy the tally chart on page 183 into their notebook and then conduct the probability experiment 10 times and record the results. Then have students answer #4 and #5.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss experimental and theoretical probability, and whether or not the two probabilities are the same.	• Ensure that students understand the difference between experimental probability and theoretical probability. • Record class results. Encourage students to notice that experimental probability is different from theoretical probability. Discuss how the two might approach each other if you continued to conduct the experiment enough times. Some students may have difficulty with this concept and may need to see it played out many times.



Show You Know

Repeat the experiment in Example 1 using two coins that you flip 100 times. Use a tally chart to keep track of your results.

a) What is your experimental probability of making two right turns?

b) What is the theoretical probability of making two right turns?

c) Compare the experimental probability with the theoretical probability.

Example 2: Compare Experimental and Theoretical Probability Using Technology

A group of medical students wanted to determine the probability of having a girl and a boy in a two-child family. They used a random number generator to give them results for 20 families.

a) What is the experimental probability of getting children of two different genders?

b) What is the theoretical probability of getting children of two different genders?

c) Compare the experimental probability with the theoretical probability.

Tech Link

A random number generator on a computer or calculator can be used to generate a large number of outcomes for a probability experiment.

Solution

a) On the spreadsheet, a family with two different genders appears as either 0, 1 or 1, 0.

Experimental $P(\text{boy and girl}) = \frac{11}{20}$

$= 0.55$

$= 55\%$ C 11 ÷ 20 × 100 = 55.

b)

Theoretical $P(\text{boy and girl}) = \frac{10}{20}$

$= 0.50$

$= 50\%$

c) 55% > 50%. The experimental probability is greater than the theoretical probability.

5.5 Conduct Probability Experiments • MHR 185

Answers

Explore the Math

1. Spinners may vary. For example: A spinner divided into four equal sections labelled Y, Y, Y, N.

2. Spin twice and record the outcomes.

3., **4.** Answers will vary.

5. a) Look for an organizer such as a table or a tree diagram.

		Spin 2			
		Y	Y	Y	N
Spin 1	Y	Y, Y	Y, Y	Y, Y	Y, N
	Y	Y, Y	Y, Y	Y, Y	Y, N
	Y	Y, Y	Y, Y	Y, Y	Y, N
	N	N, Y	N, Y	N, Y	N, N

b) $\frac{9}{16}$ or 56.25%

6. a)–c) Answers will vary.

Show You Know: Example 1

a) Answers will vary. For example: 27%

b) $\frac{1}{4}$ or 25% **c)** Answers will vary.

Common Errors

- Students may be confused between theoretical and experimental probabilities.

R_x Emphasize that all of their work in the chapter prior to this section has involved theoretical probabilities. When students construct tree diagrams and tables to solve probability problems, they are calculating theoretical probabilities. Students will be familiar with the concept of doing experiments in science. In probability experiments, students generate experimental data and then compare the data with the expected probability, which is theoretical probability.

Example 1 explores the concepts of experimental probability and theoretical probability. Work through the solution with students to ensure they understand the concepts.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• Ensure students know how to accurately complete a tally chart. • You may wish to provide additional questions to students who would benefit from them: Use the spinner you developed in Explore the Math. This time, spin the spinner 100 times. Use a tally chart to keep track of your results. **a)** What is your experimental probability of two successful attempts? (You may wish to have the group of students work together to see how many times they have to spin the spinner to get the experimental probability close to $\frac{9}{16}$ or 56.25%.) **b)** What is the theoretical probability of two successful attempts? ($\frac{9}{16}$ or 56.25%) **c)** Compare the experimental probability with the theoretical probability. (Again, emphasize that experimental probability usually approaches theoretical probability when the experiment is repeated enough times. Some groups will get results that approach theoretical probability right away; others won't.)

Example 2 explores the results of a random number generator. As a class, read and discuss the Tech Link about random number generators. Ensure that they understand that the numbers 0 and 1 in the computer screen shot refer to the gender of a child.

Supported Learning

ESL and Language

- Some students may have difficulty understanding the scenario in Example 1. Consider asking a student to sketch the obstacle course on the board. Clarify the meaning of the word *fork* in the context of the obstacle course.
- Pair students to read Examples 1 and 2.

Meeting the Needs of All Learners

- Make sure that students understand the terms. Modelling examples will help ensure they understand the concepts. In addition, consider having students work in groups, use computers and manipulatives, or work out problems on the board to help them conduct probability experiments.

Answers

Show You Know: Example 2

a) Answers will vary. For example: 20%

b) $\frac{1}{4}$ or 25%

c) Answers will vary.

Communicate the Ideas

1. Answers may vary. For example: The chart probably provides information about experimental probability. The experimental outcomes have been collected in a tally chart and can be counted.
2. Answers may vary. For example: Experimental probability is the probability of an event determined from experimental outcomes. Theoretical probability is the probability of an event determined from a list of all possible outcomes.
3. Answers may vary. For example: Yes. As you increase the number of trials in an experiment, the experimental probability will approach the theoretical probability.

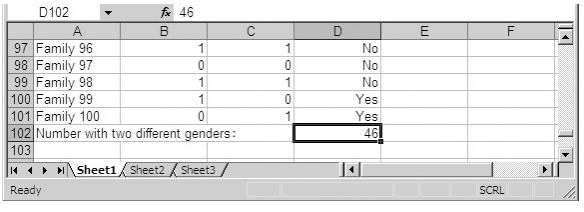
Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• Have students use **BLM 5–11 Random Generator** to help answer the questions. Students may try this several times and compare their results.

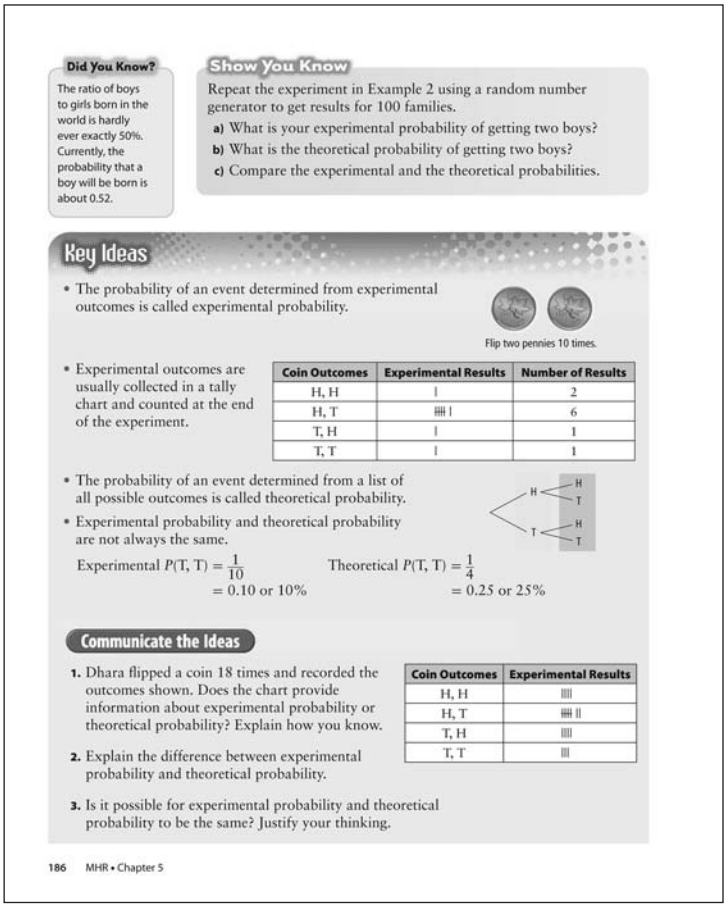
Supported Learning

Learning Style and Gifted and Enrichment

- Students who are familiar with spreadsheet programs may wish to use the following information to get their spreadsheets to tally the results from the random number generator.
  - Type "Number with two different genders:" in cell A102.
  - Type "=countif(d2:d101,"Yes")" in cell D102. This formula will count the number of families that have two children of different genders. Note that the count may vary from the count shown, depending on what the random generator does for that experiment. Every time students enter new data, the spreadsheet recalculates all cells.

  Resample for another 100 families by holding down the CTRL key, and pressing "R". Every time students do this, the spreadsheet recalculates all of the random numbers, and adjusts the total accordingly.




Did You Know?

The ratio of boys to girls born in the world is hardly ever exactly 50%. Currently, the probability that a boy will be born is about 0.52.

Show You Know

Repeat the experiment in Example 2 using a random number generator to get results for 100 families.

a) What is your experimental probability of getting two boys?

b) What is the theoretical probability of getting two boys?

c) Compare the experimental and the theoretical probabilities.

Key Ideas

- The probability of an event determined from experimental outcomes is called experimental probability.
- Experimental outcomes are usually collected in a tally chart and counted at the end of the experiment.
- The probability of an event determined from a list of all possible outcomes is called theoretical probability.
- Experimental probability and theoretical probability are not always the same.

Flip two pennies 10 times.

Coin Outcomes	Experimental Results	Number of Results
H, H	I	2
H, T	𝍸 I	6
T, H	I	1
T, T	I	1

Experimental $P(T, T) = \frac{1}{10}$ $= 0.10$ or 10%

Theoretical $P(T, T) = \frac{1}{4}$ $= 0.25$ or 25%

Communicate the Ideas

1. Dhara flipped a coin 18 times and recorded the outcomes shown. Does the chart provide information about experimental probability or theoretical probability? Explain how you know.
2. Explain the difference between experimental probability and theoretical probability.
3. Is it possible for experimental probability and theoretical probability to be the same? Justify your thinking.

Coin Outcomes	Experimental Results
H, H	IIII
H, T	𝍸 II
T, H	IIII
T, T	III

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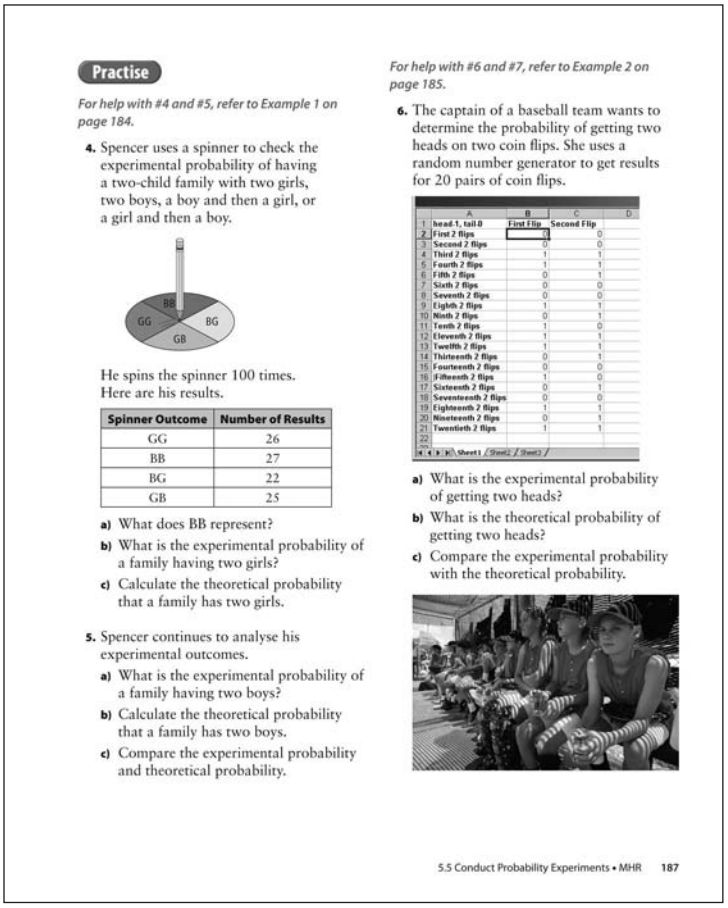
Key Ideas

The Key Ideas emphasize the differences between experimental probability and theoretical probability. Remind students that these ratios are rarely identical in a probability experiment. Students could prepare their own list of Key Ideas and put it in their chapter Foldable.

Communicate the Ideas

These questions allow students to reinforce their understanding of theoretical and experimental probability. You may wish to summarize the Key Ideas in a class discussion before having students attempt these questions.

Assessment *as* Learning	Supported Learning
Communicate the Ideas You may wish to have students answer these questions individually or as a class.	• Students who have a clear understanding of the differences between theoretical and experimental probability will be able to justify their thinking in #3. Those with a basic understanding may have difficulty explaining when the two could be the same. Encourage students to consider what would happen if the experiment were conducted additional times.



Practise

For help with #4 and #5, refer to Example 1 on page 184.

4. Spencer uses a spinner to check the experimental probability of having a two-child family with two girls, two boys, a boy and then a girl, or a girl and then a boy.

He spins the spinner 100 times. Here are his results.

Spinner Outcome	Number of Results
GG	26
BB	27
BG	22
GB	25

a) What does BB represent?
b) What is the experimental probability of a family having two girls?
c) Calculate the theoretical probability that a family has two girls.

5. Spencer continues to analyse his experimental outcomes.
a) What is the experimental probability of a family having two boys?
b) Calculate the theoretical probability that a family has two boys.
c) Compare the experimental probability and theoretical probability.

For help with #6 and #7, refer to Example 2 on page 185.

6. The captain of a baseball team wants to determine the probability of getting two heads on two coin flips. She uses a random number generator to get results for 20 pairs of coin flips.

	A	B	C	D
1	head-1, tail-0	First Flip	Second Flip	
2	First 2 flips	0	0	
3	Second 2 flips	0	0	
4	Third 2 flips	1	1	
5	Fourth 2 flips	1	1	
6	Fifth 2 flips	0	1	
7	Sixth 2 flips	0	0	
8	Seventh 2 flips	0	0	
9	Eighth 2 flips	1	1	
10	Ninth 2 flips	0	1	
11	Tenth 2 flips	1	0	
12	Eleventh 2 flips	1	1	
13	Twelfth 2 flips	1	1	
14	Thirteenth 2 flips	0	1	
15	Fourteenth 2 flips	0	0	
16	Fifteenth 2 flips	1	0	
17	Sixteenth 2 flips	0	1	
18	Seventeenth 2 flips	0	0	
19	Eighteenth 2 flips	1	1	
20	Nineteenth 2 flips	0	1	
21	Twentieth 2 flips	1	1	
22				

a) What is the experimental probability of getting two heads?
b) What is the theoretical probability of getting two heads?
c) Compare the experimental probability with the theoretical probability.

5.5 Conduct Probability Experiments • MHR 187

Supported Learning

Learning Style, ESL, and Language

- Consider allowing students to answer the Communicate the Ideas questions orally.

Learning Style

- Use cooperative groups to explore, study, and discuss the concepts. Make sure to clearly identify concepts and terminology. Have students use guided practice until they can recognize patterns.

Meeting the Needs of All Learners

- Review the Key Words. Have students locate the words in the student resource and explain their meaning to the class or a classmate.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8
Typical	1–4, 6, 8–11
Extension/Enrichment	1–3, 9, 11–13

Common Errors

- Students may have difficulty with the text-heavy nature of the questions in the Practise and Apply sections.

R_x Help students extract the information they need to answer each question. Alternatively, have students work in pairs.

Practise

Assessment *for* Learning	Supported Learning
Practise Have students do #4 and #6. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #4 will need additional coaching with Example 1. Work with them to correct #4 and then have them do #5. Check back with students several times to make sure that they understand the concepts. • Students who have problems with #6 will need additional coaching with Example 2. Make sure that they understand how to read the spreadsheet. Work with them to correct #6, and then have them do #7. If computers are available, you may wish to have students use **BLM 5–11 Random Number Generator** to get new results for this question and redo the question using the new results. Check back several times to make sure that they understand the concepts.

7. Grandpa has baked muffins to share. He puts a bran raisin muffin and a blueberry muffin in one bag, and then a bran raisin muffin and a blueberry muffin in a second bag. You get to pick one muffin from each bag. Use a random number generator to check the probability of picking two different muffins.

	A	B	C	D
1	blueberry 1, raisin 0	First Pick	Second Pick	
2	First 2 picks	0	1	
3	Second 2 picks	1	1	
4	Third 2 picks	1	0	
5	Fourth 2 picks	1	1	
6	Fifth 2 picks	0	1	
7	Sixth 2 picks	0	1	
8	Seventh 2 picks	0	1	
9	Eighth 2 picks	1	0	
10	Ninth 2 picks	0	1	
11	Tenth 2 picks	1	1	
12	Eleventh 2 picks	1	0	
13	Twelfth 2 picks	0	0	
14	Thirteenth 2 picks	0	0	
15	Fourteenth 2 picks	0	0	
16	Fifteenth 2 picks	0	1	
17	Sixteenth 2 picks	0	0	
18	Seventeenth 2 picks	0	1	
19	Eighteenth 2 picks	0	0	
20	Nineteenth 2 picks	0	1	
21	Twentieth 2 picks	1	0	
22				

Sheet1 / Sheet2 / Sheet3

a) What is the experimental probability of picking a muffin of each type?

b) What is the theoretical probability of picking a muffin of each type?

c) Compare the experimental probability with the theoretical probability.

Apply

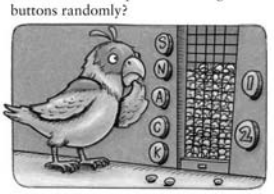
8. a) Build a spinner and conduct the probability experiment described in #4. Organize your results in a tally chart.

b) According to your results, what is the experimental probability of having a family with two children of different genders?

c) What is the theoretical probability of having this type of family?

d) Compare the experimental probability and theoretical probability.

9. Scientists are working with a parrot. The parrot knows it has to push one button on the left and then one button on the right to get into the food bin. To open the door, the parrot has to push C, 2. What are the chances of the parrot choosing these buttons randomly?



Scientists collect the experimental results for the first 50 tries in a tally chart.

	1	2
S	I	II
N	𝍸	𝍸
A	𝍸 𝍸 I	III
C	𝍸 I	𝍸 II
K	𝍸 II	𝍸

a) The correct combination is C, 2. How many times did the parrot open the food bin?

b) What is the experimental probability of selecting C, 2?

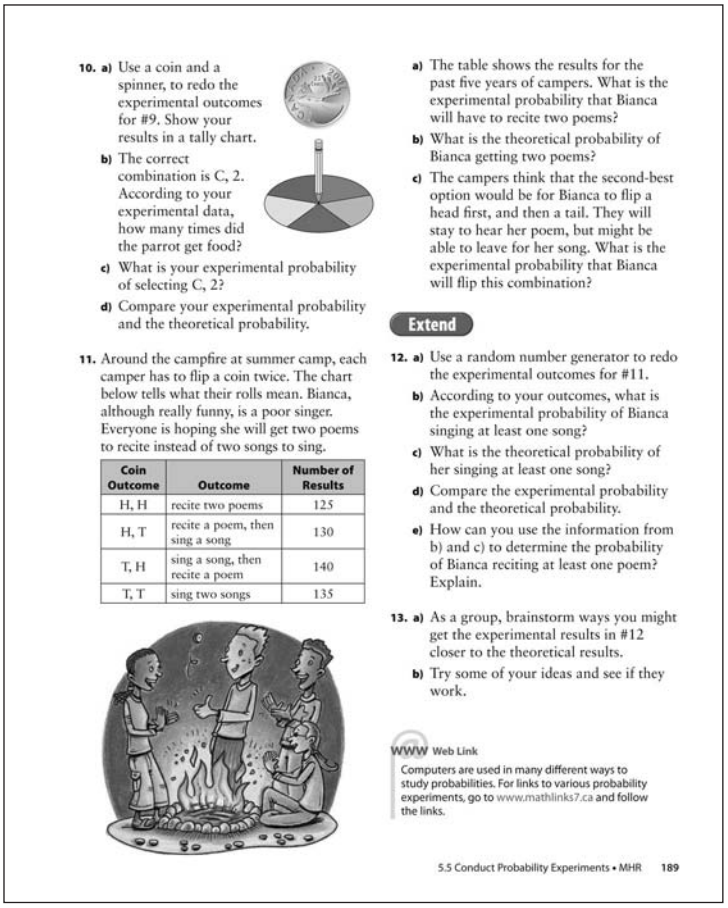
c) Show the sample space for this probability event. What is the theoretical probability of selecting C, 2?

d) Compare the experimental and theoretical probability. What might this suggest?

Apply and Extend

The Apply questions provide a variety of fairly straightforward contexts for solving problems involving experimental and theoretical probabilities. Note that #8 and #10 require students to conduct probability experiments.

Students must use a random number generator to conduct an experiment in #12. The most important answer for #13 involves increasing the number of trials. As the number of trials increases, the experimental data should more closely approach the theoretical outcomes.



10. a) Use a coin and a spinner, to redo the experimental outcomes for #9. Show your results in a tally chart.
b) The correct combination is C, 2. According to your experimental data, how many times did the parrot get food?
c) What is your experimental probability of selecting C, 2?
d) Compare your experimental probability and the theoretical probability.

11. Around the campfire at summer camp, each camper has to flip a coin twice. The chart below tells what their rolls mean. Bianca, although really funny, is a poor singer. Everyone is hoping she will get two poems to recite instead of two songs to sing.

Coin Outcome	Outcome	Number of Results
H, H	recite two poems	125
H, T	recite a poem, then sing a song	130
T, H	sing a song, then recite a poem	140
T, T	sing two songs	135

a) The table shows the results for the past five years of campers. What is the experimental probability that Bianca will have to recite two poems?
b) What is the theoretical probability of Bianca getting two poems?
c) The campers think that the second-best option would be for Bianca to flip a head first, and then a tail. They will stay to hear her poem, but might be able to leave for her song. What is the experimental probability that Bianca will flip this combination?

Extend

12. a) Use a random number generator to redo the experimental outcomes for #11.
b) According to your outcomes, what is the experimental probability of Bianca singing at least one song?
c) What is the theoretical probability of her singing at least one song?
d) Compare the experimental probability and the theoretical probability.
e) How can you use the information from b) and c) to determine the probability of Bianca reciting at least one poem? Explain.

13. a) As a group, brainstorm ways you might get the experimental results in #12 closer to the theoretical results.
b) Try some of your ideas and see if they work.

WWW Web Link
Computers are used in many different ways to study probabilities. For links to various probability experiments, go to www.mathlinks7.ca and follow the links.

5.5 Conduct Probability Experiments • MHR 189

Supported Learning

Learning Style and Memory

- Provide **BLM 5–12 Section 5.5 Extra Practice** to students who need extra practice.

Gifted and Enrichment

- Encourage students to visit **www.mathlinks7.ca** and follow the links to try various probability experiments.

WWW Web Link

For sites that allow students to calculate experimental probabilities and compare them to theoretical probabilities, go to **www.mathlinks7.ca** and follow the links.

WWW Web Link

For a site with links to probability experiments, go to **www.mathlinks7.ca** and follow the links.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • What is the difference between experimental probability and theoretical probability? Explain.	• Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • You may wish to have students review the part related to Section 5.5 in **BLM 5–1 Chapter 5 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

5 Chapter Review

Suggested Timing
40–50 minutes

Materials
- ruler
- loading-strip model
- calculator

Blackline Masters
BLM 5–1 Chapter 5 Self-Assessment
BLM 5–3 Section 5.1 Extra Practice
BLM 5–5 Section 5.2 Extra Practice
BLM 5–7 Section 5.3 Extra Practice
BLM 5–8 Section 5.4 Extra Practice
BLM 5–12 Section 5.5 Extra Practice

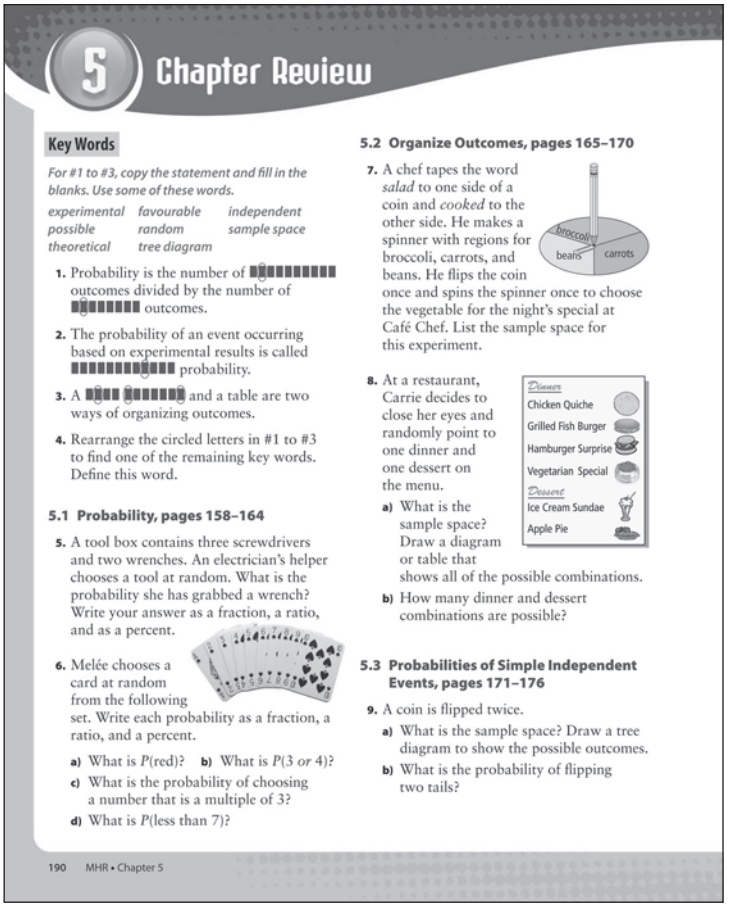
Supported Learning

Learning Style and Memory

- Students who require more practice on a particular topic may refer to **BLM 5–3 Section 5.1 Extra Practice, BLM 5–5 Section 5.2 Extra Practice, BLM 5–7 Section 5.3 Extra Practice, BLM 5–8 Section 5.4 Extra Practice**, and **BLM 5–12 Section 5.5 Extra Practice**.

Learning Style

- Allow students to complete the Chapter 5 Review using any combination of oral and written answers, including diagrams.



5 Chapter Review

Key Words

For #1 to #3, copy the statement and fill in the blanks. Use some of these words.

experimental, favourable, independent, possible, random, sample space, theoretical, tree diagram

1. Probability is the number of [redacted] outcomes divided by the number of [redacted] outcomes.
2. The probability of an event occurring based on experimental results is called [redacted] probability.
3. A [redacted] and a table are two ways of organizing outcomes.
4. Rearrange the circled letters in #1 to #3 to find one of the remaining key words. Define this word.

5.1 Probability, pages 158–164

5. A tool box contains three screwdrivers and two wrenches. An electrician's helper chooses a tool at random. What is the probability she has grabbed a wrench? Write your answer as a fraction, a ratio, and as a percent.
6. Melée chooses a card at random from the following set. Write each probability as a fraction, a ratio, and a percent.
   a) What is P(red)? b) What is P(3 *or* 4)?
   c) What is the probability of choosing a number that is a multiple of 3?
   d) What is P(less than 7)?

5.2 Organize Outcomes, pages 165–170

7. A chef tapes the word *salad* to one side of a coin and *cooked* to the other side. He makes a spinner with regions for broccoli, carrots, and beans. He flips the coin once and spins the spinner once to choose the vegetable for the night's special at Café Chef. List the sample space for this experiment.

8. At a restaurant, Carrie decides to close her eyes and randomly point to one dinner and one dessert on the menu.
   a) What is the sample space? Draw a diagram or table that shows all of the possible combinations.
   b) How many dinner and dessert combinations are possible?

Dinner
Chicken Quiche
Grilled Fish Burger
Hamburger Surprise
Vegetarian Special
Dessert
Ice Cream Sundae
Apple Pie

5.3 Probabilities of Simple Independent Events, pages 171–176

9. A coin is flipped twice.
   a) What is the sample space? Draw a tree diagram to show the possible outcomes.
   b) What is the probability of flipping two tails?

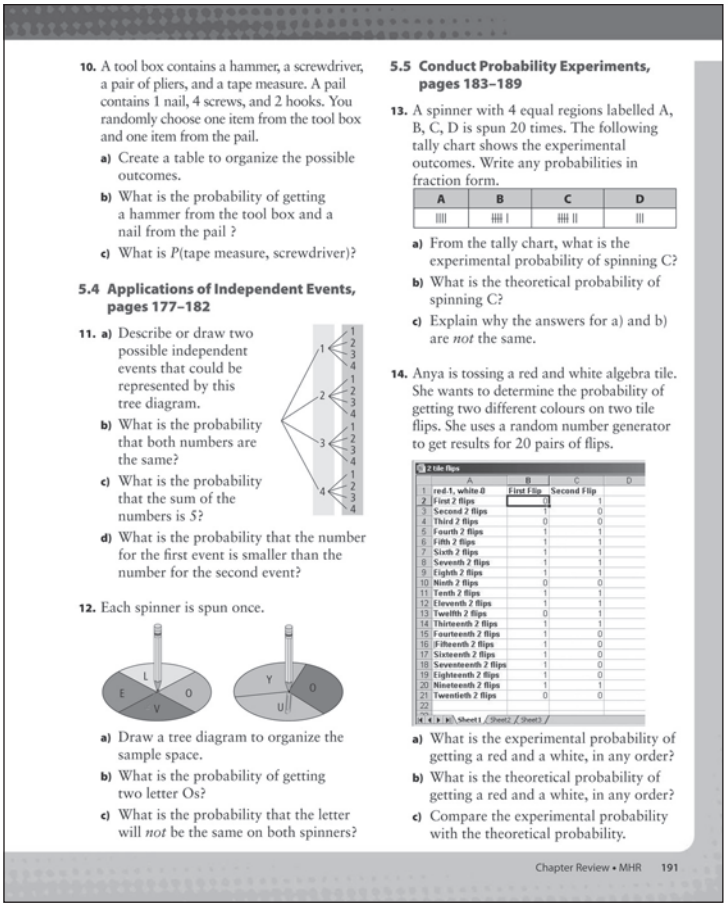
190 MHR • Chapter 5

Activity Planning Notes

Have students work independently or in pairs to complete the review questions. If students encounter difficulties, they could discuss strategies with other students. Encourage them to refer to the information in their chapter Foldable and then to the specific section in the student resource and/or their notebooks. Once they have found a suitable strategy, students should include it in the appropriate section of their Foldable.

Alternatively, you may wish to assign questions to reinforce skills and concepts in preparation for the Chapter 5 Test.

Assessment *for* Learning	Supported Learning
Chapter 5 Review The chapter review provides an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On tab of their chapter Foldable. Have students do at least one question related to each item in that tab. • Have students revisit any section that they are having difficulty with prior to working on the Chapter 5 Practice Test.



10. A tool box contains a hammer, a screwdriver, a pair of pliers, and a tape measure. A pail contains 1 nail, 4 screws, and 2 hooks. You randomly choose one item from the tool box and one item from the pail.
a) Create a table to organize the possible outcomes.
b) What is the probability of getting a hammer from the tool box and a nail from the pail ?
c) What is P(tape measure, screwdriver)?

5.4 Applications of Independent Events, pages 177–182

11. a) Describe or draw two possible independent events that could be represented by this tree diagram.
b) What is the probability that both numbers are the same?
c) What is the probability that the sum of the numbers is 5?
d) What is the probability that the number for the first event is smaller than the number for the second event?

12. Each spinner is spun once.

a) Draw a tree diagram to organize the sample space.
b) What is the probability of getting two letter Os?
c) What is the probability that the letter will *not* be the same on both spinners?

5.5 Conduct Probability Experiments, pages 183–189

13. A spinner with 4 equal regions labelled A, B, C, D is spun 20 times. The following tally chart shows the experimental outcomes. Write any probabilities in fraction form.

A	B	C	D
IIII	IIII I	IIII II	III

a) From the tally chart, what is the experimental probability of spinning C?
b) What is the theoretical probability of spinning C?
c) Explain why the answers for a) and b) are *not* the same.

14. Anya is tossing a red and white algebra tile. She wants to determine the probability of getting two different colours on two tile flips. She uses a random number generator to get results for 20 pairs of flips.

a) What is the experimental probability of getting a red and a white, in any order?
b) What is the theoretical probability of getting a red and a white, in any order?
c) Compare the experimental probability with the theoretical probability.

Chapter Review • MHR 191

Supported Learning

ESL, Language, and Memory

- Allow students to practise the vocabulary terms using flash cards. Have students work together to quiz each other on the Key Words for the chapter.

Gifted and Enrichment

- Students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge for gifted students, go to **www.mathlinks7.ca** and follow the links.

Meeting the Needs of All Learners

- You may wish to incorporate games from other cultures in this section. Encourage students to share simple games of chance that they know.

Assessment *as* Learning	Supported Learning
Math Learning Log Once students have completed the chapter review, have students reflect on their progress and complete a journal entry for each statement: – I am comfortable with the following parts of the chapter ... – I am having difficulty with ... – Here's how I plan to address the areas I am still having difficulty with ...	• Have students use the What I Need to Work On tab of their chapter Foldable and answer these questions from the contents of that section. • You may wish to have students refer to **BLM 5–1 Chapter 5 Self-Assessment** when they report on what they are comfortable with, what they continue to have difficulty with, and what they plan to do about it.

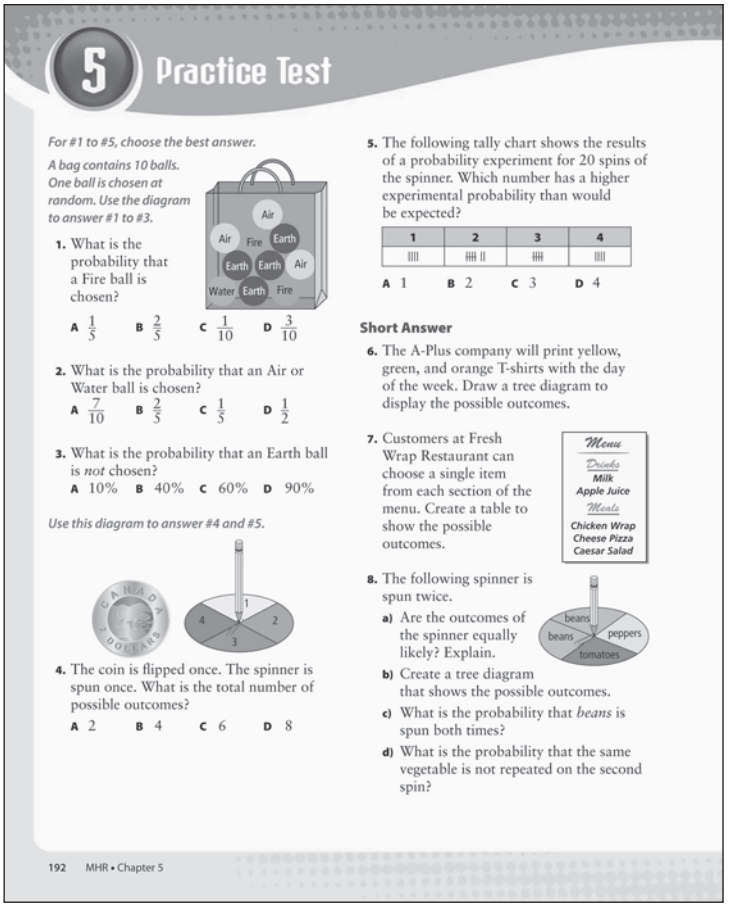
5 Practice Test

Suggested Timing
40–50 minutes

Materials
- ruler

Blackline Masters
BLM 5–1 Chapter 5 Self-Assessment
BLM 5–13 Chapter 5 Test

Assessment *as* Learning	Supported Learning
Chapter 5 Self-Assessment Have students review their earlier responses on **BLM 5–1 Chapter 5 Self-Assessment**.	• Have students use their responses on the Chapter 5 Practice Test and work they completed earlier in the chapter to complete the After column of this self-assessment. • Before students do the Chapter 5 Test, coach them in the areas in which they are having problems.



5 Practice Test

For #1 to #5, choose the best answer.

A bag contains 10 balls. One ball is chosen at random. Use the diagram to answer #1 to #3.

1. What is the probability that a Fire ball is chosen?
A $\frac{1}{5}$ B $\frac{2}{5}$ C $\frac{1}{10}$ D $\frac{3}{10}$

2. What is the probability that an Air or Water ball is chosen?
A $\frac{7}{10}$ B $\frac{2}{5}$ C $\frac{1}{5}$ D $\frac{1}{2}$

3. What is the probability that an Earth ball is *not* chosen?
A 10% B 40% C 60% D 90%

Use this diagram to answer #4 and #5.

4. The coin is flipped once. The spinner is spun once. What is the total number of possible outcomes?
A 2 B 4 C 6 D 8

5. The following tally chart shows the results of a probability experiment for 20 spins of the spinner. Which number has a higher experimental probability than would be expected?

1	2	3	4
||||	|||| ||	||||	||||

A 1 B 2 C 3 D 4

Short Answer

6. The A-Plus company will print yellow, green, and orange T-shirts with the day of the week. Draw a tree diagram to display the possible outcomes.

7. Customers at Fresh Wrap Restaurant can choose a single item from each section of the menu. Create a table to show the possible outcomes.

Menu
Drinks
Milk
Apple Juice
Meals
Chicken Wrap
Cheese Pizza
Caesar Salad

8. The following spinner is spun twice.
a) Are the outcomes of the spinner equally likely? Explain.
b) Create a tree diagram that shows the possible outcomes.
c) What is the probability that *beans* is spun both times?
d) What is the probability that the same vegetable is not repeated on the second spin?

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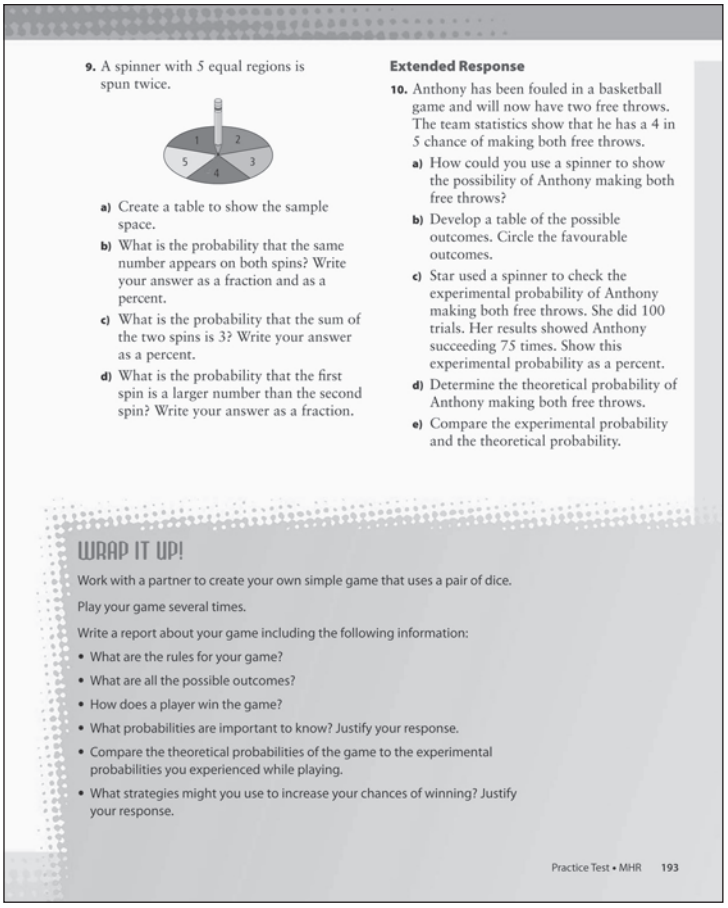
Study Guide

Question(s)	Section(s)	Refer to	I can …
1, 2, 3	5.1	Example 2	✓ give answers as probabilities from 0% to 100%
4	5.2	Example 1	✓ determine the outcomes of two independent events
5	5.5	Example 1	✓ compare experimental probability with theoretical probability
6, 7	5.4	Examples 1, 2	✓ use tree diagrams, tables, and other graphic organizers to solve probability problems
8, 9	5.4 5.1 5.2 5.3	Examples 1, 2 Example 2 Examples 1, 2 Examples 1, 2	✓ use tree diagrams, tables, and other graphic organizers to solve probability problems ✓ find the probability of an event in several different ways ✓ give answers as probabilities from 0% to 100% ✓ organize the outcomes of two independent events using tables and tree diagrams ✓ solve probability problems involving two independent events
10	5.4 5.5	Examples 1, 2 Example 1 Example 1	✓ use tree diagrams, tables, and other graphic organizers to solve probability problems ✓ conduct a probability experiment and organize the results ✓ compare experimental probability with theoretical probability



9. A spinner with 5 equal regions is spun twice.

a) Create a table to show the sample space.
b) What is the probability that the same number appears on both spins? Write your answer as a fraction and as a percent.
c) What is the probability that the sum of the two spins is 3? Write your answer as a percent.
d) What is the probability that the first spin is a larger number than the second spin? Write your answer as a fraction.

Extended Response

10. Anthony has been fouled in a basketball game and will now have two free throws. The team statistics show that he has a 4 in 5 chance of making both free throws.
a) How could you use a spinner to show the possibility of Anthony making both free throws?
b) Develop a table of the possible outcomes. Circle the favourable outcomes.
c) Star used a spinner to check the experimental probability of Anthony making both free throws. She did 100 trials. Her results showed Anthony succeeding 75 times. Show this experimental probability as a percent.
d) Determine the theoretical probability of Anthony making both free throws.
e) Compare the experimental probability and the theoretical probability.

WRAP IT UP!

Work with a partner to create your own simple game that uses a pair of dice.

Play your game several times.

Write a report about your game including the following information:

- What are the rules for your game?
- What are all the possible outcomes?
- How does a player win the game?
- What probabilities are important to know? Justify your response.
- Compare the theoretical probabilities of the game to the experimental probabilities you experienced while playing.
- What strategies might you use to increase your chances of winning? Justify your response.

Practice Test • MHR 193

Activity Planning Notes

The practice test can be assigned as an in-class or take-home assignment. These are the minimum questions that will meet the related curriculum outcomes: #1, #3, #4, #8, and #9.

Answers to the Chapter 5 Practice Test are provided on **BLM 5–16 Chapter 5 *MathLinks 7* Student Resource Answers**.

Assessment *of* Learning	Supported Learning
Chapter 5 Test After students complete the Chapter 5 Practice Test, you may wish to use **BLM 5–13 Chapter 5 Test** as a summative assessment.	• Consider using the Math Games on page 194 or the Challenge in Real Life on page 195 to assess the knowledge and skills of students who have difficulty with tests.

Supported Learning

Learning Style and Memory

- Consider using the report that students complete for the Wrap It Up! to assess the knowledge and skills of students who respond well to project work and struggle with formal tests.

ESL, Language, and Memory

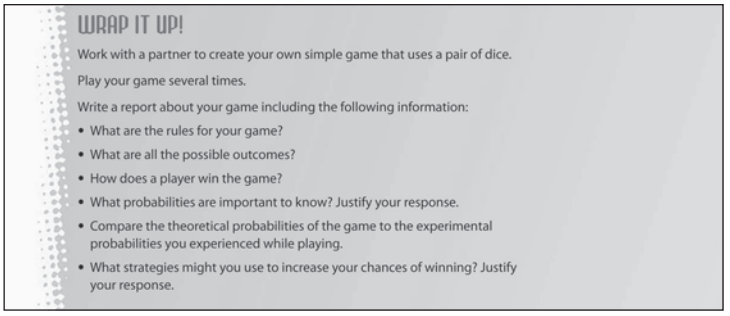
- Consider allowing students to use their chapter Foldable and/or a calculator during the practice test.

Wrap It Up!

Suggested Timing
40–50 minutes

Materials
- counters or coins
- 2 dice
- poster board or regular paper

Blackline Masters
Master 1 Project Rubric
BLM 5–4 Section 5.1 Math Link
BLM 5–6 Section 5.2 Math Link
BLM 5–10 Section 5.4 Math Link
BLM 5–14 Chapter 5 Wrap It Up!



WRAP IT UP!

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- How does a player win the game?
- What probabilities are important to know? Justify your response.
- Compare the theoretical probabilities of the game to the experimental probabilities you experienced while playing.
- What strategies might you use to increase your chances of winning? Justify your response.

Specific Outcomes

SP5 Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.

SP6 Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.

Supported Learning

Learning Style and Language
- Students who struggle with writing could present their report orally.

Motor
- Consider allowing students to use a computer to write their report.

Activity Planning Notes

This chapter problem can be scaled up or down to meet the needs of your students. For example, you could have students create the game without an analysis. Introduce the problem and clarify the assessment criteria. Make the activity as real as possible by providing students with dice, construction materials, and other materials for the game.

Assessment *of* Learning	Supported Learning
Wrap It Up! After students construct the game board, encourage them to play the game several times. Students should be able to justify the strategies that might increase their chances of winning. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this Wrap It Up! Page 193a provides notes on how to use the rubric for this Wrap It Up!	• Encourage students to create a game with simple rules. • If students have not completed the Math Links earlier in the chapter, you may wish to provide them with **BLM 5–4 Section 5.1 Math Link, BLM 5–6 Section 5.2 Math Link**, and **BLM 5–10 Section 5.4 Math Link**. • Some students may benefit from using **BLM 5–14 Chapter 5 Wrap It Up!**, which provides scaffolding for the chapter problem wrap-up.

The chart below shows **Master 1 Project Rubric** for tasks such as that in the Wrap It Up! and provides notes that specify how to identify the level of specific answers for the project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 (Standard of Excellence)	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete solution, with a minor mathematical, drawing, calculation, or communication error, which does not affect the final conclusion
4 (Above Acceptable)	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding. ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete solution, with one mathematical justification or outcome missing *or* • provides a complete solution, with no comparison between the theoretical and experimental probabilities *or* • provides a complete solution, with weak strategies for increasing the chances of winning
3 (Meets Acceptable)	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides a complete solution for the first four bullets, but justification is weak or absent, or a comparison may be attempted but is vague or flawed *or* • provides a report, with answers only, with no justification
2 (Below Acceptable)	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/ connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• defines a comparison between theoretical and experimental probability, but fails to make any links using evidence from the game *or* • provides a complete response to the first and third bullets, but none of the possible outcomes are mathematically justified or have many errors
1 (Beginning)	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• takes a correct step to address the problem

Math Games

Suggested Timing
40–50 minutes

Materials
- paper bag
- red and blue counters of the same shape and equal sizes

Assessment *for* Learning	Supported Learning
Is It a Match? Have students play the game with a partner.	• After students have played several rounds of Match or No Match, brainstorm how to make the game fair. Answers may vary from placing a different number of counters in the bag to awarding a different score for a win. • Explain that a tree diagram helps to determine the sample space and the probability of outcomes within the sample space. • After students have played several rounds of the game, consider collecting the experimental probability for that number of games, and then compare the difference between experimental and theoretical probability.

Common Errors

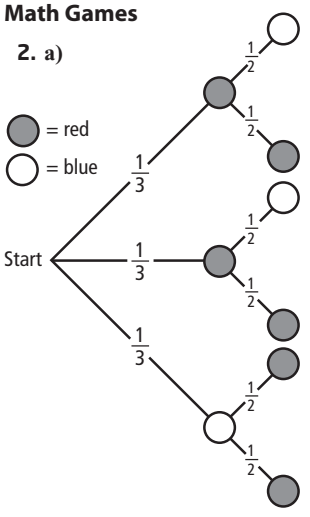
- Students may find that the tree diagrams are awkward to draw as the number of counters in the bag increases.

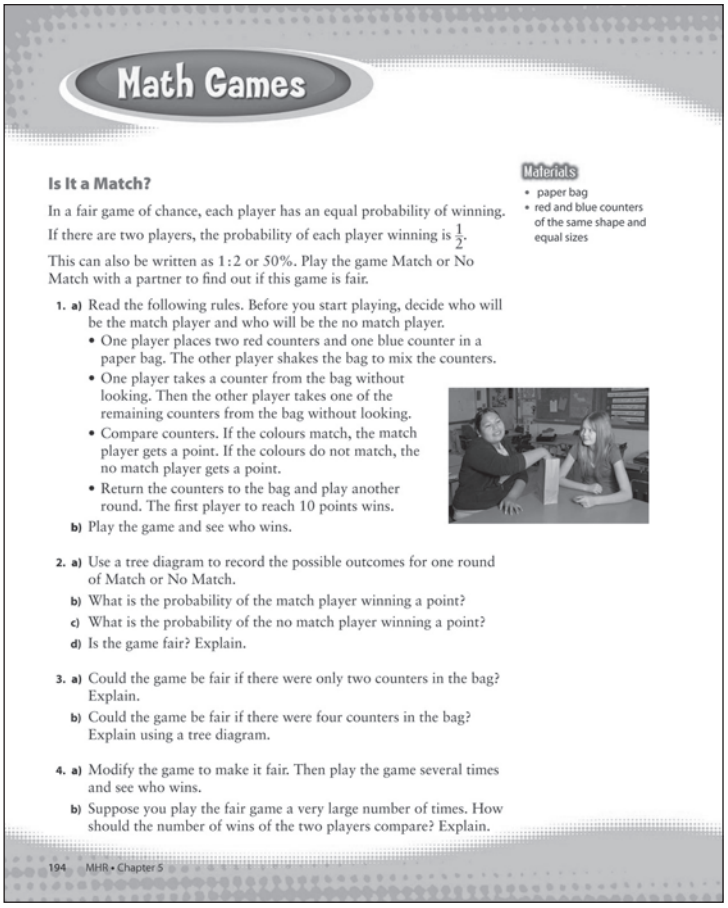
R_x Make sure students leave adequate space for the tree diagrams.

Answers

Math Games

2. a)





Math Games

Is It a Match?

In a fair game of chance, each player has an equal probability of winning. If there are two players, the probability of each player winning is $\frac{1}{2}$. This can also be written as 1:2 or 50%. Play the game Match or No Match with a partner to find out if this game is fair.

Materials
- paper bag
- red and blue counters of the same shape and equal sizes

1. a) Read the following rules. Before you start playing, decide who will be the match player and who will be the no match player.
   - One player places two red counters and one blue counter in a paper bag. The other player shakes the bag to mix the counters.
   - One player takes a counter from the bag without looking. Then the other player takes one of the remaining counters from the bag without looking.
   - Compare counters. If the colours match, the match player gets a point. If the colours do not match, the no match player gets a point.
   - Return the counters to the bag and play another round. The first player to reach 10 points wins.

   b) Play the game and see who wins.

2. a) Use a tree diagram to record the possible outcomes for one round of Match or No Match.
   b) What is the probability of the match player winning a point?
   c) What is the probability of the no match player winning a point?
   d) Is the game fair? Explain.

3. a) Could the game be fair if there were only two counters in the bag? Explain.
   b) Could the game be fair if there were four counters in the bag? Explain using a tree diagram.

4. a) Modify the game to make it fair. Then play the game several times and see who wins.
   b) Suppose you play the fair game a very large number of times. How should the number of wins of the two players compare? Explain.

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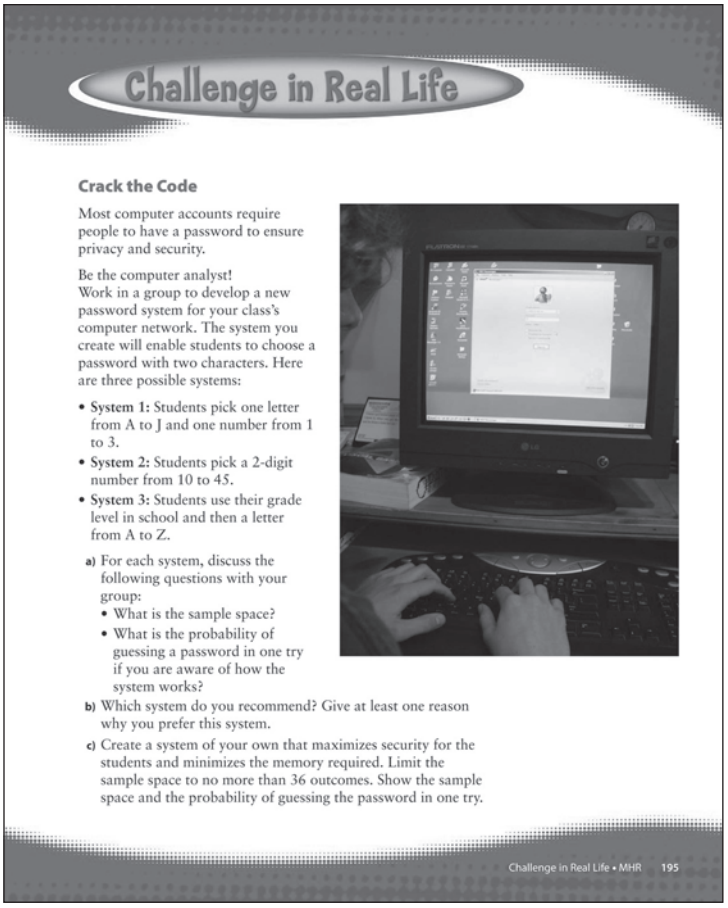
Specific Outcomes

SP6 Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.

Activity Planning Notes

Have students play several rounds before they begin their analysis. See the Answers for the six outcomes. The probability of each outcome is $\frac{1}{6}$. The probability that the outcomes will match is $\frac{2}{6}$ or $\frac{1}{3}$. The probability that the outcomes will not match is $\frac{4}{6}$ or $\frac{2}{3}$. This game is not fair since each player does not have an equal chance of winning. There are 12 outcomes in the sample space for the fair game: (red, blue), (red, red), (red, red), (red, blue), (red, red), (red, red), (red, blue), (red, red), (red, red), (blue, red), (blue, red), (blue, red). The probability of each outcome is $\frac{1}{12}$. The probability that the outcomes will match is $\frac{6}{12}$ or $\frac{1}{2}$. The probability that the outcomes will not match is $\frac{6}{12}$ or $\frac{1}{2}$. This game is fair since each player has an equal chance of winning.

Challenge in Real Life



Challenge in Real Life

Crack the Code

Most computer accounts require people to have a password to ensure privacy and security.

Be the computer analyst!
Work in a group to develop a new password system for your class's computer network. The system you create will enable students to choose a password with two characters. Here are three possible systems:

- **System 1:** Students pick one letter from A to J and one number from 1 to 3.
- **System 2:** Students pick a 2-digit number from 10 to 45.
- **System 3:** Students use their grade level in school and then a letter from A to Z.

a) For each system, discuss the following questions with your group:
- What is the sample space?
- What is the probability of guessing a password in one try if you are aware of how the system works?

b) Which system do you recommend? Give at least one reason why you prefer this system.

c) Create a system of your own that maximizes security for the students and minimizes the memory required. Limit the sample space to no more than 36 outcomes. Show the sample space and the probability of guessing the password in one try.

Challenge in Real Life • MHR 195

Specific Outcomes

SP4 Express probabilities as ratios, fractions and percents.
SP5 Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.

Activity Planning Notes

You may wish to use the following steps to introduce and complete this challenge:

1. Read through Crack the Code and discuss where passwords are used (e.g., computer networks, online banking, online shopping) and why it is important to keep them private. Make sure that students understand the meaning of the terms *sample space*, *probability*, and *outcome*. Note that students may have an idea for improving the password system that increases the probability of guessing the password. At this level, however, do not have students work with a sample space that has more than 36 outcomes.

Suggested Timing
60–75 minutes

Blackline Masters
Master 1 Project Rubric
BLM 5–15 System 1 Spinners

Mathematical Processes

- [x] Communication
- [x] Connections
- [ ] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [ ] Visualization

Answers

Challenge in Real Life

a) System 1

	1	2	3
A	(A, 1)	(A, 2)	(A, 3)
B	(B, 1)	(B, 2)	(B, 3)
C	(C, 1)	(C, 2)	(C, 3)
D	(D, 1)	(D, 2)	(D, 3)
E	(E, 1)	(E, 2)	(E, 3)
F	(F, 1)	(F, 2)	(F, 3)
G	(G, 1)	(G, 2)	(G, 3)
H	(H, 1)	(H, 2)	(H, 3)
I	(I, 1)	(I, 2)	(I, 3)
J	(J, 1)	(J, 2)	(J, 3)

Sample space = 30
Probability of guessing the password in one try = 1:30 or $\frac{1}{30}$ or $\approx 3.3\%$
Ways of improving the password system might include adding a symbol or changing the password.

System 2

Numbers 10 – 45 = 36 numbers
Sample space = 36
Probability of guessing the password in one try = 1:36 or $\frac{1}{36}$ or $\approx 2.8\%$
Ways of improving the password system might include adding letters or symbols, or changing the password weekly.

System 3

Sample space for grade 7 = 26
Probability of guessing the password in one try = 1:26 or $\frac{1}{26}$ or $\approx 3.8\%$
Ways of improving the password system might include not using the grade as part of the password since someone who knows a student's grade level would have an advantage, and adding double letters such as AA to JJ.

Answers

b) Answers will vary. Expect students to provide a reason. For example, they may prefer System 1 because it uses a letter combined with a number; System 2 because it has a greater sample space; or System 3 because they like to include the grade level in the password.

c) Answers will vary. Look for the sample space and the probability of guessing the password in one try.

Supported Learning

Learning Style, ESL, and Language

- Encourage concrete and kinesthetic learners to use spinners to help them identify the sample space for each of the three systems. For System 1, students might use **BLM 5–15 System 1 Spinners**. Make sure that the sections of all spinners are the same size. Place the spinner at A on the first spinner, and ask what numbers could come up on the second spinner. Record the pairs: (A, 1), (A, 2), and (A, 3). For System 2, students may pick the numbers from 10 to 45 out of a hat and record the sample space. System 3 could be made concrete in a similar way. Note that this is not a theoretical experiment, but a way of showing outcomes concretely. This method may also be helpful for English language learners and students with poor language skills.

Gifted and Enrichment

- Encourage students to consider other ways of putting together a password with a sample space with no more than 36 outcomes.

2. Consider reviewing how to use a tree diagram or other organizer by working through part a) for System 1 as a class. Consider using an overhead of **BLM 5–15 System 1 Spinners** to help you.
3. Have students work as a group to complete part a) for Systems 2 and 3, and then have them decide which system they prefer and why.
4. Have students work individually or in groups to develop their own password system and show its sample space and effectiveness.
5. Clarify that the task is to
   - show the sample space and probability of each combination for Systems 1, 2, and 3
   - assess three systems, recommend one system, and explain the reason why
   - create another password system and show the sample space and the probability of guessing the password in one try
6. Review **Master 1 Project Rubric** with students so that they will know what is expected.

This challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment *for* Learning	Supported Learning
Crack the Code Discuss the challenge with the class. Have students work in small groups to develop their own password system.	• Review with students how to determine the sample space. • As a class or a small group, discuss which system is preferable. • For a second challenge, complete with teaching notes and student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

Assessment *of* Learning	Supported Learning
Crack the Code Discuss the challenge with the class. Have students work in small groups to develop their own password system.	• Use **Master 1 Project Rubric** to assist you in assessing student work. Page 195a provides notes on how to use this rubric for this challenge. • To view student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as the Challenge in Real Life and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete solution, with weak communication *or* • provides a complete solution, with a sample space less than 36 done correctly
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding. ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a correct and complete solution for parts a), b), and a significant start to part c) *or* • provides a complete solution, with communication missing for one of part a), b), or c)
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides a complete solution for the sample space and probability of all systems *or* • provides a complete solution for the sample space of Systems 2 and 3, with a complete explanation of preference *or* • provides a complete solution for part c)
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a complete solution for two of three sample spaces *or* • provides a complete solution for one sample space, with one recommendation
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• makes a correct start to one part of the problem