Introduction to Fraction Operations

6

General Outcome

- Develop number sense.

Specific Outcomes

N1 Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0.

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
6.1	✓ determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9, 10
	✓ show why a number is not divisible by 0
	✓ use organizers to sort a set of numbers based on their divisiblity
	✓ determine the factors of a number using divisibility rules
	✓ write a fraction in lowest terms using common factors
6.2	✓ use models, diagrams, and addition statements to add fractions with like denominators
	✓ solve a problem involving addition of fractions and determine if the solution is reasonable
6.3	✓ use models, diagrams, and subtraction statements to subtract fractions with like denominators
	✓ solve a problem involving subtraction of fractions and determine if the solution is reasonable

Assessment *as* Learning	Supported Learning
Use the Before column of **BLM 6–1 Chapter 6 Self-Assessment** to provide students with the big picture for this chapter and to help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• As students complete each section of the chapter or complete the Chapter 6 Review, have them review the related parts of **BLM 6–1 Chapter 6 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Chapter 6 Planning Chart

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter Opener • 20–30 minutes		BLM 6–1 Chapter 6 Self-Assessment BLM 6–2 Introduction to Fraction Operations	• ruler • glue • grid paper • stapler
6.1 Divisibility • 120–150 minutes	**Essential:** 1a) *or* b), 2–5, 7, 9, 11, 13, 15, 19, 21, 23, Math Link **Typical:** 1a) *or* b), 2–5, 7, 9, 11, 13, 15, 17–21, 23, Math Link **Extension/Enrichment:** 1a) *or* b), 2–4, 22, 26–28	Master 2 Two Stars and One Wish Master 11 Hundred Chart BLM 6–1 Chapter 6 Self-Assessment BLM 6–3 Section 6.1 Extra Practice BLM 6–4 Section 6.1 Math Link	• grid paper • ruler • stapler • coloured pencils • calculator (optional) • counters or coins
6.2 Add Fractions With Like Denominators • 80–100 minutes	**Essential:** 1–3, 5, 7, 9, Math Link **Typical:** 1–3, 5, 7, 9, 11, 13–15, Math Link **Extension/Enrichment:** 1–4, 12, 15–18, Math Link	Master 13 Pattern Blocks Master 14 Fraction Circles Master 15 Fraction Strips BLM 6–1 Chapter 6 Self-Assessment BLM 6–5 Section 6.2 Extra Practice BLM 6–6 Section 6.2 Math Link BLM 6–10 24-Hour Fraction Circle	• pattern blocks • fraction strips • scissors • coloured pencils • ruler
6.3 Subtract Fractions With Like Denominators • 80–100 minutes	**Essential:** 1–4, 6, 8, 11, Math Link **Typical:** 1–4, 6, 8, 10, 11, Math Link **Extension/Enrichment:** 1–3, 10, 13–15	Master 13 Pattern Blocks Master 14 Fraction Circles Master 15 Fraction Strips BLM 6–1 Chapter 6 Self-Assessment BLM 6–7 Section 6.3 Extra Practice BLM 6–8 Section 6.3 Math Link	• pattern blocks • fraction strips • coloured pencils • ruler
Chapter 6 Review • 40–50 minutes	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Master 13 Pattern Blocks Master 14 Fraction Circles Master 15 Fraction Strips BLM 6–1 Chapter 6 Self-Assessment BLM 6–3 Section 6.1 Extra Practice BLM 6–5 Section 6.2 Extra Practice BLM 6–7 Section 6.3 Extra Practice	• coloured pencils • pattern blocks • fraction strips
Chapter 6 Practice Test • 40–50 minutes	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1–5, 7–13	Master 13 Pattern Blocks Master 14 Fraction Circles Master 15 Fraction Strips BLM 6–1 Chapter 6 Self-Assessment BLM 6–9 Chapter 6 Test	• coloured pencils • ruler • pattern blocks • fraction strips
Chapter 6 Wrap It Up! • 40–50 minutes		Master 1 Project Rubric Master 2 Two Stars and One Wish BLM 6–4 Section 6.1 Math Link BLM 6–6 Section 6.2 Math Link BLM 6–8 Section 6.3 Math Link BLM 6–10 24-Hour Fraction Circle BLM 6–11 Chapter 6 Wrap It Up!	• coloured pencils • ruler
Chapter 6 Math Games • 40–50 minutes		BLM 6–12 It's Divisible Spinner	• 3 six-sided dice per pair or group of students • paper clip per pair or group of students

Chapter 6 Planning Chart (continued)

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter 6 Challenge in Real Life • 60–75 minutes		**Green Mathematics** Master 1 Project Rubric **Math Mosaics** Master 1 Project Rubric Master 7 Isometric Dot Paper BLM 6–13 Chapter 6 *Mathlinks 7* Student Resource Answers BLM 6–14 Chapter 6 BLM Answers	**Green Mathematics** • poster paper • coloured pencils, paints, and/or markers • PowerPoint software (optional) • video equipment (optional) **Math Mosaics** • ruler • coloured pencils or markers • construction paper • fraction blocks (optional)

Chapter 6 Assessment Planner

Assessment Options	Type of Assessment	Assessment Tool
Chapter Opener	Assessment *as* Learning (TR pages i, 196)	BLM 6–1 Chapter 6 Self-Assessment Chapter 6 Foldable
6.1 Divisibility	Assessment *as* Learning (TR pages 201, 206, 208) Assessment *for* Learning (TR pages 202, 203, 205, 207, 209)	Master 2 Two Stars and One Wish Math Learning Log (TR page 208) BLM 6–1 Chapter 6 Self-Assessment
6.2 Add Fractions With Like Denominators	Assessment *as* Learning (TR pages 211, 214, 215) Assessment *for* Learning (TR pages 212, 213, 215, 216)	Math Learning Log (TR page 215) BLM 6–1 Chapter 6 Self-Assessment
6.3 Subtract Fractions With Like Denominators	Assessment *as* Learning (TR pages 218, 220, 221) Assessment *for* Learning (TR pages 218, 219, 220, 221)	Math Learning Log (TR page 221) BLM 6–1 Chapter 6 Self-Assessment
Chapter 6 Review	Assessment *for* Learning (TR page 222) Assessment *as* Learning (TR page 223)	Math Learning Log (TR page 223) BLM 6–1 Chapter 6 Self-Assessment
Chapter 6 Practice Test	Assessment *as* Learning (TR page 224) Assessment *of* Learning (TR page 225)	BLM 6–1 Chapter 6 Self-Assessment BLM 6–9 Chapter 6 Test
Chapter 6 Wrap It Up!	Assessment *of* Learning (TR page 224a)	Master 1 Project Rubric Master 2 Two Stars and One Wish
Chapter 6 Math Games	Assessment *for* Learning (TR page 226)	
Chapter 6 Challenge in Real Life	Assessment *for* Learning (TR pages 226a, 226b) Assessment *of* Learning (TR pages 226a, 226b)	Master 1 Project Rubric

You may wish to use one or more of the following materials to help you assess student readiness for Chapter 6.

Assessment *for* Learning	Supported Learning
Method 1: Have students write about one of these topics: • Choose two equivalent fractions. How are they the same? How are they different? • Why do we need to have the same-sized wholes when comparing fractions? • When we speak, how is my "halfway to school" different from your halfway? How are they the same? **Method 2:** Have students complete **BLM 6–2 Introduction to Fraction Operations** to check their conceptual understanding. Remind students that you are looking for the scope of their knowledge.	• Students who require reinforcement of prerequisite skills may wish to complete the Get Ready materials available in the *MathLinks 7 Workbook* and at the **www.mathlinks7.ca** book site.

Chapter Opener

Suggested Timing
20–30 minutes

Materials
- ruler
- glue
- grid paper
- stapler

Blackline Masters
BLM 6–1 Chapter 6 Self-Assessment

Key Words
divisible
common factor
lowest terms

What's the Math?

In this chapter, students will learn about the divisibility rules for the numbers 0, 2, 3, 4, 5, 6, 8, 9, and 10. They will be able to use their understanding of divisibility to write fractions in lowest terms. Students will also use hands-on materials to explore the concept of adding and subtracting fractions with like denominators. Allow students to explore and understand these concepts before moving on to the more formal algorithmic mathematical statements.

Activity Planning Notes

Begin Chapter 6 by explaining that the chapter is an introduction to arithmetic operations with fractions. This will involve determining if a number is divisible by another, how to write fractions in lowest terms, and how to add and subtract fractions that have the same denominator. Discuss with students where they have seen or used fractions in their lives.

Assessment *as* Learning	Supported Learning
Chapter 6 Foldable As students work on each section in Chapter 6, have them keep track of any problems they are having under the What I Need to Work On section in their chapter Foldable.	• As students complete each section, have them review the list of items they need to work on and then have them check off any that have been handled.

Math Link

The Math Link encourages students to begin thinking about how their day is divided up among various activities. Have students read this Math Link and the chapter problem wrap-up titled Wrap It Up! on page 225. In the Wrap It Up! students will have an opportunity to draw a fraction circle similar to Joseph's on page 196 of the student resource, to show how they spend their day.

FOLDABLES™
Study Tool

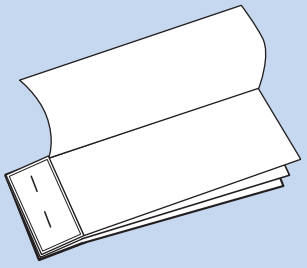
Have students make the Foldable in the student resource to keep track of the information in the chapter. You may wish to have students use these Foldables as a bookmark in their student resource or post them around the room.

You may prefer to have students keep track of Key Words using a design specifically for that purpose. Students can make the following Foldable and write vocabulary terms on the front of each tab. Have them use the space beneath the tab to write definitions and provide examples.

Step 1 Cut a sheet of grid paper horizontally in half. Fold the half sheet in two horizontally.

Step 2 Draw a line 2 cm from the left side of the folded paper. Cut the top part of the fold along this line.

Step 3 Make one of these folded sheets for each Key Word. Staple the tabs together to make a booklet.



Step 4 Write a Key Word on the front of each tab. Write definitions and give examples underneath the tabs.

Note: You can have students make the complete vocabulary Foldable at the beginning of the chapter, or have them add the number of tabs needed for each section as they do it. This will be a small Foldable as there are only three Key Words in the chapter. Depending on students needs, you may ask them to add other terms.

Supported Learning

Learning Style

- You may wish to create the Foldable ahead of time to use as a model with students.

ESL

- Some English language learners may need assistance in understanding the following terms in the chapter opener: *average* and *h* (short form for hour).

Meeting the Needs of All Learners

- To introduce this concept, give students some opportunity to explore, experience, and live the idea of fractions. Immerse them in a fraction environment, relating fractions to real-life experiences. Make students aware of what they already know.
- Discuss with students whether their behaviour patterns change from season to season. For example, when the nights are long, such as during the winter, people may do more indoor activity; whereas, they may do more outdoors during the longer summer daylight hours. Use fractions to discuss the time spent indoors and outdoors during the different seasons.

6.1 Divisibility

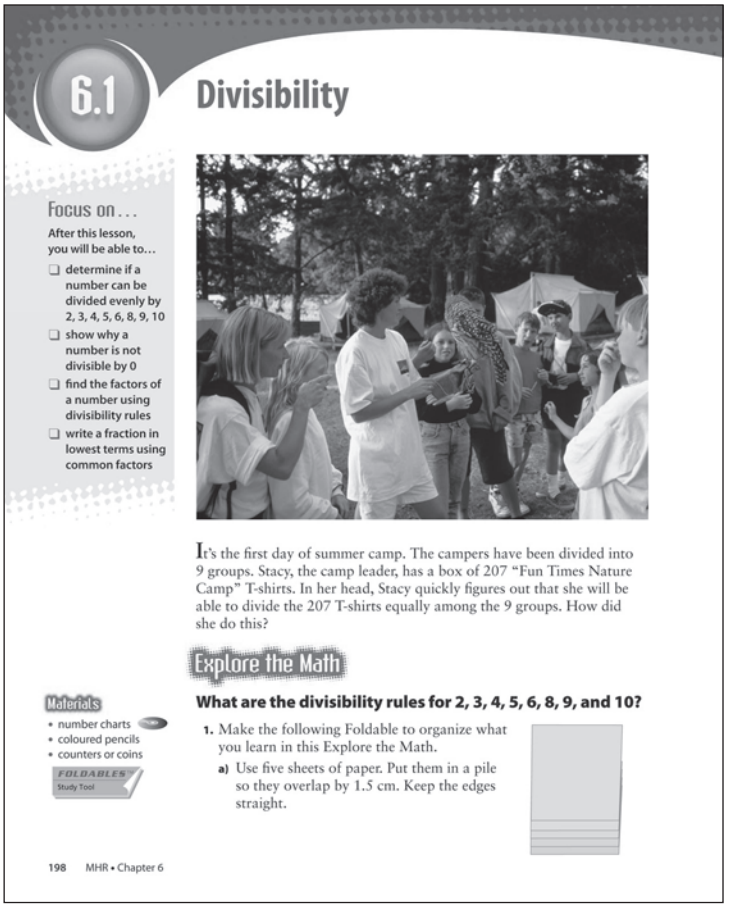
Suggested Timing
120–150 minutes

Materials
- grid paper
- ruler
- stapler
- coloured pencils
- calculator (optional)
- counters or coins

Blackline Masters
Master 2 Two Stars and One Wish
Master 11 Hundred Chart
BLM 6–1 Chapter 6 Self-Assessment
BLM 6–3 Section 6.1 Extra Practice
BLM 6–4 Section 6.1 Math Link

Mathematical Processes

- [x] Communication
- [ ] Connections
- [ ] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [x] Technology
- [x] Visualization



6.1 Divisibility

Focus on...
After this lesson, you will be able to...
- determine if a number can be divided evenly by 2, 3, 4, 5, 6, 8, 9, 10
- show why a number is not divisible by 0
- find the factors of a number using divisibility rules
- write a fraction in lowest terms using common factors

It's the first day of summer camp. The campers have been divided into 9 groups. Stacy, the camp leader, has a box of 207 "Fun Times Nature Camp" T-shirts. In her head, Stacy quickly figures out that she will be able to divide the 207 T-shirts equally among the 9 groups. How did she do this?

Explore the Math

Materials
- number charts
- coloured pencils
- counters or coins

FOLDABLES Study Tool

What are the divisibility rules for 2, 3, 4, 5, 6, 8, 9, and 10?

1. Make the following Foldable to organize what you learn in this Explore the Math.
   a) Use five sheets of paper. Put them in a pile so they overlap by 1.5 cm. Keep the edges straight.

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Specific Outcomes

N1 Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0.

Warm-Up

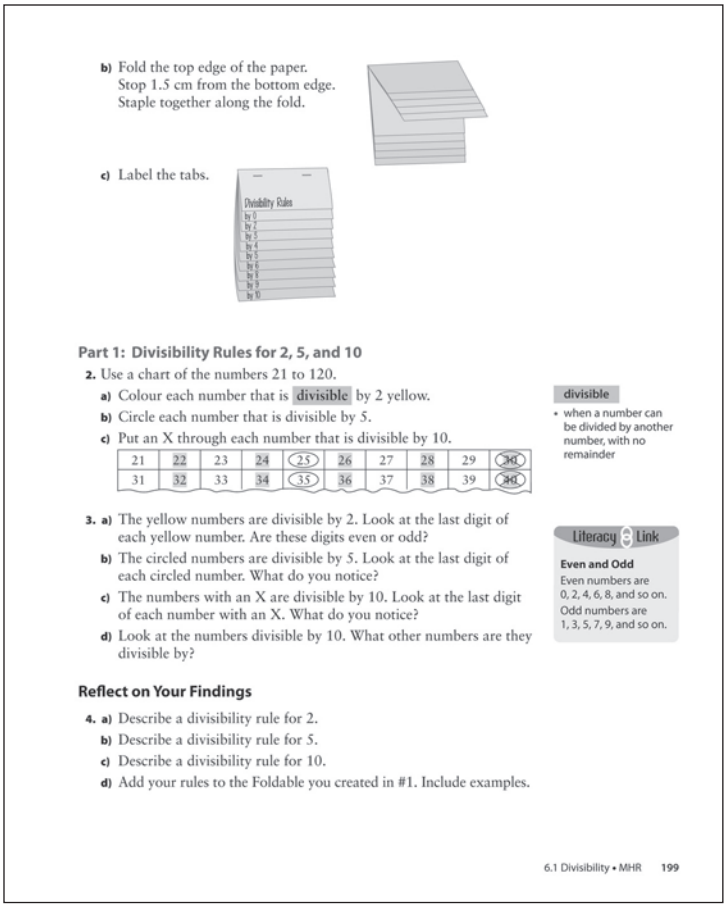
The table shows the experimental probability of having two girls, two boys, a boy and a girl, or a girl and a boy in a family with two children. Use the table to answer #1 to #4.

Experimental Outcome	Number of Results
GG	32
BB	15
BG	24
GB	29

1. How many times was this experiment conducted?
2. What is the experimental probability of a family having two girls?
3. What is the experimental probability of a family having one boy and one girl, in any order?
4. The theoretical probability of a family having two girls is 25%. Why is this different from your answer to #2?
5. Show using bar notation: 0.525252 ...

Mental Math

6. Use your money skills to give the decimal equivalent and percent for each of the following fractions.
   a) $\frac{1}{4}$ b) $\frac{1}{10}$
7. Mentally calculate \$80.40 ÷2. What fraction does ÷2 suggest?
8. What can you divide by to find $\frac{1}{4}$ of \$52?
   Calculate $\frac{1}{4}$ of \$52.



b) Fold the top edge of the paper. Stop 1.5 cm from the bottom edge. Staple together along the fold.

c) Label the tabs.

Divisibility Rules
by 0
by 1
by 2
by 3
by 4
by 5
by 6
by 7
by 8
by 9
by 10

Part 1: Divisibility Rules for 2, 5, and 10

2. Use a chart of the numbers 21 to 120.

a) Colour each number that is divisible by 2 yellow.

b) Circle each number that is divisible by 5.

c) Put an X through each number that is divisible by 10.

21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

divisible
- when a number can be divided by another number, with no remainder

3. a) The yellow numbers are divisible by 2. Look at the last digit of each yellow number. Are these digits even or odd?

b) The circled numbers are divisible by 5. Look at the last digit of each circled number. What do you notice?

c) The numbers with an X are divisible by 10. Look at the last digit of each number with an X. What do you notice?

d) Look at the numbers divisible by 10. What other numbers are they divisible by?

Literacy Link

Even and Odd

Even numbers are 0, 2, 4, 6, 8, and so on.

Odd numbers are 1, 3, 5, 7, 9, and so on.

Reflect on Your Findings

4. a) Describe a divisibility rule for 2.

b) Describe a divisibility rule for 5.

c) Describe a divisibility rule for 10.

d) Add your rules to the Foldable you created in #1. Include examples.

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N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Activity Planning Notes

Start the lesson by asking students about a time that they had to share an equal portion among several people. Find out how they decided what a fair portion was.

Explore the Math

Explain that students will be creating a Foldable to record their findings about divisibility rules. Have students turn to page 198 in their student resource and follow the directions for the Foldable.

Answers

Warm-Up

1. 100

2. $\frac{32}{100} = 0.32 = 32\%$

3. $\frac{24 + 29}{100} = 0.53 = 53\%$

4. Experimental probability shows a smaller sample. Only 100 families were included. Results could change with a larger number of families.

5. $0.\overline{52}$

6. **a)** $\frac{1}{4} = 0.25$ or 25% **b)** $\frac{1}{10} = 0.10$ or 10%

7. \$40.20; $\frac{1}{2}$ **8.** 4; \$13

Explore the Math

2.

21	22	23	24	(25)	26	27	28	29	~~(30)~~
31	32	33	34	(35)	36	37	38	39	~~(40)~~
41	42	43	44	(45)	46	47	48	49	~~(50)~~
51	52	53	54	(55)	56	57	58	59	~~(60)~~
61	62	63	64	(65)	66	67	68	69	~~(70)~~
71	72	73	74	(75)	76	77	78	79	~~(80)~~
81	82	83	84	(85)	86	87	88	89	~~(90)~~
91	92	93	94	(95)	96	97	98	99	~~(100)~~
101	102	103	104	(105)	106	107	108	109	~~(110)~~
111	112	11	114	(115)	116	117	118	119	~~(120)~~

3. **a)** even **b)** The last digit is 5 or 0.
c) The last digit is 0. **d)** 2 and 5

4. Answers may vary. For example:
a) A number is divisible by 2 if the last digit is even.
b) A number is divisible by 5 if the last digit is 5 or 0.
c) A number is divisible by 10 if the last digit is 0.
d) Answers will vary.

5.

1044	1045	1046	1047	(1048)	1049	1050	1051	1052	1053
1054	1055	(1056)	1057	1058	1059	1060	1061	1062	1063
(1064)	1065	1066	1067	1068	1069	1070	1071	(1072)	1073
1074	1075	1076	1077	1078	1079	(1080)	1081	1082	1083
1084	1085	1086	1087	(1088)	1089	1090	1091	1092	1093
1094	1095	(1096)	1097	1098	1099	1100	1101	1002	1003
(1104)	1105	1106	1107	1108	1109	1110	1111	(1112)	1113
1114	1115	1116	1117	1118	1119	(1120)	1121	1122	1123
1124	1125	1126	1127	(1128)	1129	1130	1131	1132	1133
1134	1135	(1136)	1137	1138	1139	1140	1141	1142	1143

Answers

Explore the Math

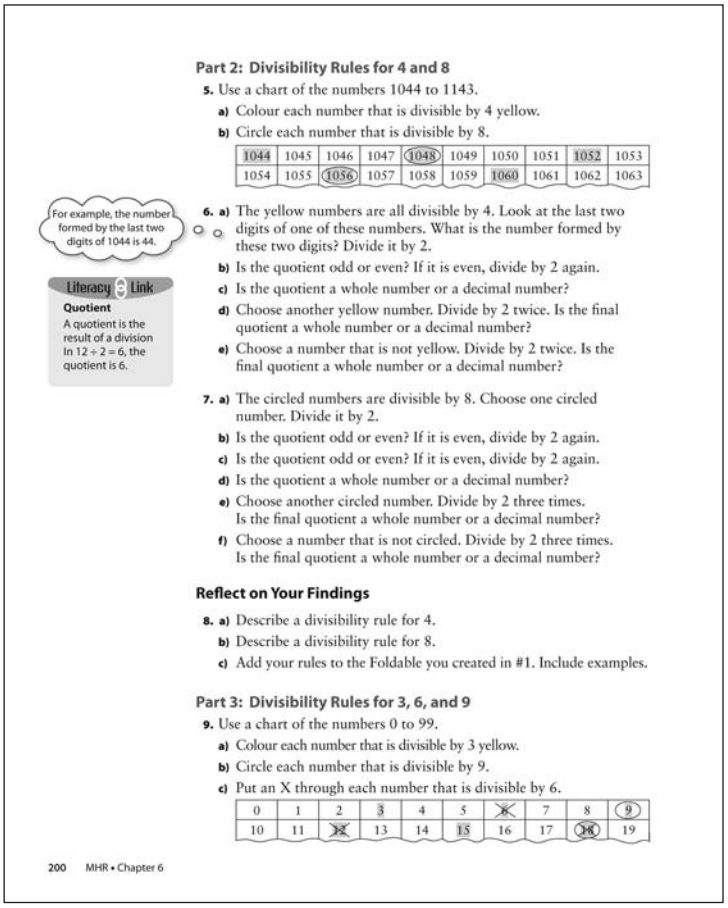
6. **a)** Answers may vary. **b)** even
c) whole number **d)** whole number
e) decimal number

7. **a)** Answers may very. **b)** even
c) even **d)** whole number
e) whole number **f)** decimal number

8. **a)** Answers may vary. For example: A number is divisible by 4 if the number formed by the last two digits is divisible by 2 at least twice.
b) Answers may vary. For example: A number is divisible by 8 if the number is divisible by 2 at least three times.
c) Answers will vary.

9.

0	1	2	3	4	5	~~6~~	7	8	(9)
10	11	~~12~~	13	14	15	16	17	~~18~~	19
20	21	22	23	~~24~~	25	26	(27)	28	29
~~30~~	31	32	33	34	35	~~36~~	37	38	39
40	41	~~42~~	43	44	(45)	46	47	~~48~~	49
50	51	52	53	~~54~~	55	56	57	58	59
~~60~~	61	62	(63)	64	65	~~66~~	67	68	69
70	71	(72)	73	74	75	76	77	~~78~~	79
80	(81)	82	83	~~84~~	85	86	87	88	89
(~~90~~)	91	92	93	94	95	~~96~~	97	98	(99)



Part 2: Divisibility Rules for 4 and 8

5. Use a chart of the numbers 1044 to 1143.
a) Colour each number that is divisible by 4 yellow.
b) Circle each number that is divisible by 8.

1044	1045	1046	1047	(1048)	1049	1050	1051	1052	1053
1054	1055	(1056)	1057	1058	1059	(1060)	1061	1062	1063

For example, the number formed by the last two digits of 1044 is 44.

Literacy Link
Quotient
A quotient is the result of a division In 12 ÷ 2 = 6, the quotient is 6.

6. **a)** The yellow numbers are all divisible by 4. Look at the last two digits of one of these numbers. What is the number formed by these two digits? Divide it by 2.
b) Is the quotient odd or even? If it is even, divide by 2 again.
c) Is the quotient a whole number or a decimal number?
d) Choose another yellow number. Divide by 2 twice. Is the final quotient a whole number or a decimal number?
e) Choose a number that is not yellow. Divide by 2 twice. Is the final quotient a whole number or a decimal number?

7. **a)** The circled numbers are divisible by 8. Choose one circled number. Divide it by 2.
b) Is the quotient odd or even? If it is even, divide by 2 again.
c) Is the quotient odd or even? If it is even, divide by 2 again.
d) Is the quotient a whole number or a decimal number?
e) Choose another circled number. Divide by 2 three times. Is the final quotient a whole number or a decimal number?
f) Choose a number that is not circled. Divide by 2 three times. Is the final quotient a whole number or a decimal number?

Reflect on Your Findings

8. **a)** Describe a divisibility rule for 4.
b) Describe a divisibility rule for 8.
c) Add your rules to the Foldable you created in #1. Include examples.

Part 3: Divisibility Rules for 3, 6, and 9

9. Use a chart of the numbers 0 to 99.
a) Colour each number that is divisible by 3 yellow.
b) Circle each number that is divisible by 9.
c) Put an X through each number that is divisible by 6.

0	1	2	3	4	5	~~6~~	7	8	(9)
10	11	~~12~~	13	14	15	16	17	~~18~~	19

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Supported Learning

Meeting the Needs of All Learners

- You may wish to substitute the day camp in the section opener with something more relevant to students' lives, such as a baseball, soccer, or hockey tournament, a fishing trip, or a science fair.

Learning Style and Gifted and Enrichment

- You may wish to read the book *The Doorbell Rang* by Pat Hutchings (HarperCollins Canada, 1989). In this story, a girl and a boy have to share cookies equally among their friends as they come into the house. The cookies always divide evenly, giving the factors of the number. As an extend activity, students may want to create a story of their own in the vein of *The Doorbell Rang*.

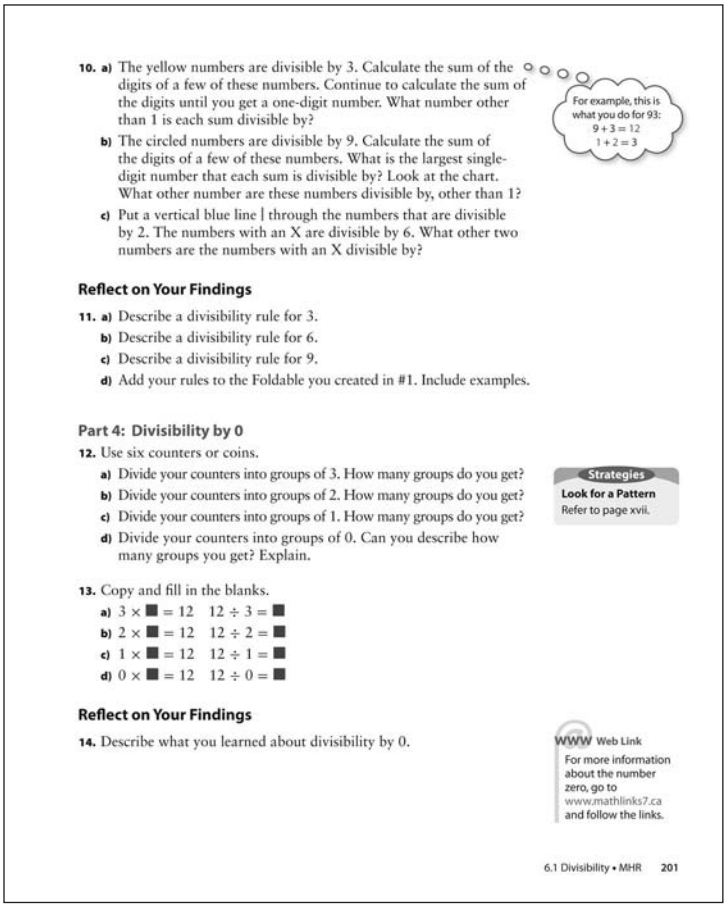
Part 1: Invite students to share their understanding of the terms *odd* and *even*. Explain that with an even amount of things, all things can be paired, with nothing left over. With an odd amount of things, there is always one thing that will be left over and cannot make a pair.

Part 2: Students may have difficulty dividing the larger numbers by 4 and 8. Allow students to use calculators for Part 2. It is important that students focus on divisibility, not on doing long division by hand.

Part 3: As students gain confidence in their work on divisibility, they will likely start recognizing the patterns for 3 and 9.

Part 4: Students may have difficulty with the concept of divisibility by 0. Have them work through several examples of sharing an amount with a number of people, working down to sharing with 0 people.

Reinforce understanding of divisibility by 0 by dividing 1000 by 1, then by 0.1, then by 0.01, then by 0.001, etc. Students will realize what is happening as you divide by numbers closer and closer to 0. You may wish to introduce the term *undefined* when you talk about dividing by 0. Explain to students that *undefined* is another way of saying that there is no possible answer.



10. a) The yellow numbers are divisible by 3. Calculate the sum of the digits of a few of these numbers. Continue to calculate the sum of the digits until you get a one-digit number. What number other than 1 is each sum divisible by?

For example, this is what you do for 93:
9 + 3 = 12
1 + 2 = 3

b) The circled numbers are divisible by 9. Calculate the sum of the digits of a few of these numbers. What is the largest single-digit number that each sum is divisible by? Look at the chart. What other number are these numbers divisible by, other than 1?

c) Put a vertical blue line | through the numbers that are divisible by 2. The numbers with an X are divisible by 6. What other two numbers are the numbers with an X divisible by?

Reflect on Your Findings

11. a) Describe a divisibility rule for 3.
b) Describe a divisibility rule for 6.
c) Describe a divisibility rule for 9.
d) Add your rules to the Foldable you created in #1. Include examples.

Part 4: Divisibility by 0

12. Use six counters or coins.
a) Divide your counters into groups of 3. How many groups do you get?
b) Divide your counters into groups of 2. How many groups do you get?
c) Divide your counters into groups of 1. How many groups do you get?
d) Divide your counters into groups of 0. Can you describe how many groups you get? Explain.

Strategies
Look for a Pattern
Refer to page xvii.

13. Copy and fill in the blanks.
a) 3 × ■ = 12 12 ÷ 3 = ■
b) 2 × ■ = 12 12 ÷ 2 = ■
c) 1 × ■ = 12 12 ÷ 1 = ■
d) 0 × ■ = 12 12 ÷ 0 = ■

Reflect on Your Findings

14. Describe what you learned about divisibility by 0.

WWW Web Link
For more information about the number zero, go to www.mathlinks7.ca and follow the links.

6.1 Divisibility • MHR 201

Answers

Explore the Math

10. **a)** 3 **b)** 9, 3 **c)** 2 and 3

11. **a)** Answers may vary. For example: A number is divisible by 3 if the sum of its digits is divisible by 3.
b) Answers may vary. For example: A number is divisible by 6 if the number is divisible by both 2 and 3.
c) Answers may vary. For example: A number is divisible by 9 if the sum of its digits is divisible by 9.
d) Answers will vary.

12. **a)** 2 **b)** 3 **c)** 6
d) No. It is not possible to divide six counters or coins into groups of 0.

13. **a)** 4, 4 **b)** 6, 6 **c)** 12, 12 **d)** No value, no value

14. Answers may vary. For example: There are no numbers that are divisible by 0.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss the divisibility rules for each part. During this process, they are generalizing what they have learned during the Explore the Math.	• Check that students have an understanding of the words *divisible, quotient, factor, common factor, lowest terms, even, odd,* and *pair* as they work through the lesson. • Along with completing the Reflect on Your Findings questions, have students note that any number divisible by 10 is also divisible by 2 and 5, because 2 and 5 are factors of 10. Also encourage students to recognize that any number divisible by 8 is divisible by 4, and any number divisible by 9 is divisible by 3. • Have students test their rules on larger numbers that are not on the chart. Allow them to check their work with a calculator. • To help students remember the divisibility rules, hang a poster in the classroom that lists these rules. You may wish to ask an artistic student to make this poster. • Suggest that students keep the Explore the Math Foldable in a handy place for easy reference.

Supported Learning

ESL

- You may need to do some direct teaching of the terms that are commonly used to indicate mathematical operations. Brainstorm terms that are synonymous with the various operations. For example, *multiply* can be referred to as *times*. Explicitly point out that the term *quotient* means the answer in a division question.

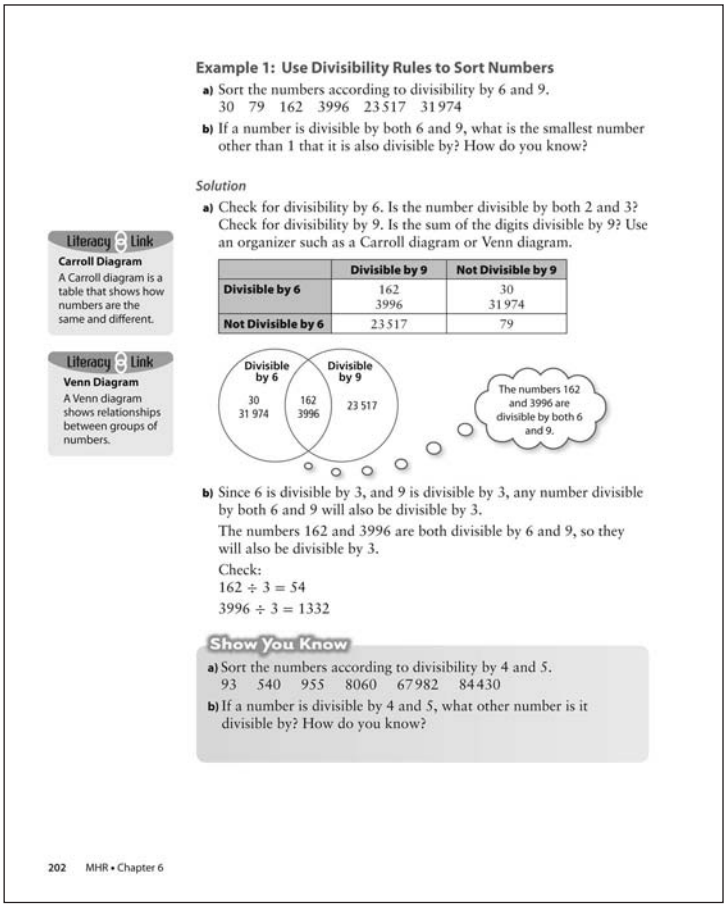
Answers

Show You Know: Example 1

a)

	Divisible by 4	Not Divisible by 4
Divisible by 5	540 8060	955 84 430
Not Divisible by 5		67 982

b) Answers may vary. For example: The number is divisible by 20 because it is divisible by 4 and 5.



Example 1: Use Divisibility Rules to Sort Numbers

a) Sort the numbers according to divisibility by 6 and 9.
30 79 162 3996 23 517 31 974

b) If a number is divisible by both 6 and 9, what is the smallest number other than 1 that it is also divisible by? How do you know?

Solution

a) Check for divisibility by 6. Is the number divisible by both 2 and 3? Check for divisibility by 9. Is the sum of the digits divisible by 9? Use an organizer such as a Carroll diagram or Venn diagram.

Literacy Link
Carroll Diagram
A Carroll diagram is a table that shows how numbers are the same and different.

	Divisible by 9	Not Divisible by 9
Divisible by 6	162 3996	30 31 974
Not Divisible by 6	23 517	79

Literacy Link
Venn Diagram
A Venn diagram shows relationships between groups of numbers.

b) Since 6 is divisible by 3, and 9 is divisible by 3, any number divisible by both 6 and 9 will also be divisible by 3.
The numbers 162 and 3996 are both divisible by 6 and 9, so they will also be divisible by 3.
Check:
$162 \div 3 = 54$
$3996 \div 3 = 1332$

Show You Know

a) Sort the numbers according to divisibility by 4 and 5.
93 540 955 8060 67 982 84 430

b) If a number is divisible by 4 and 5, what other number is it divisible by? How do you know?

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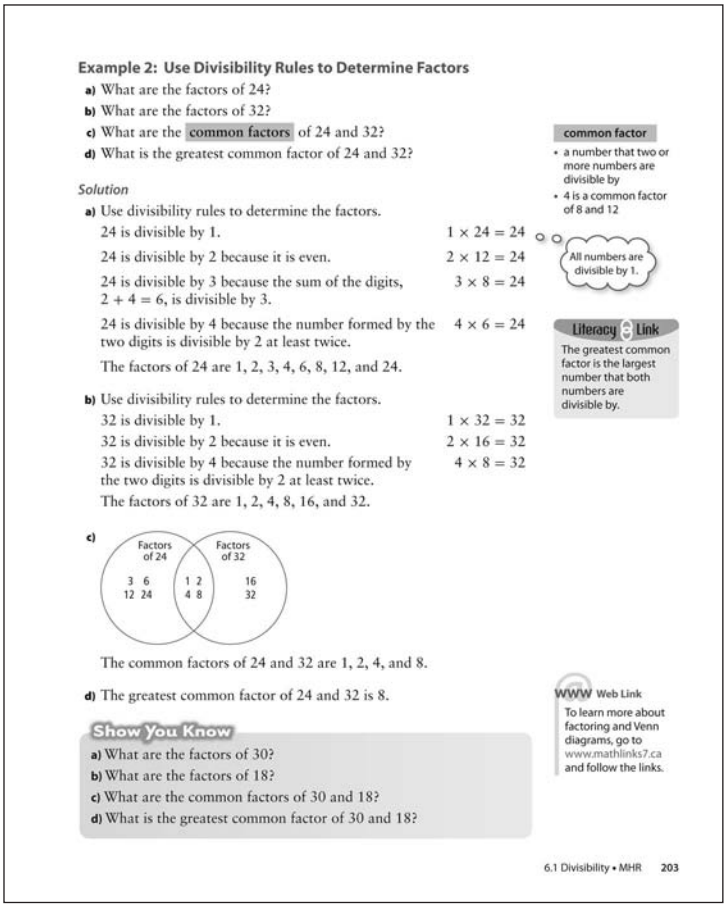
Web Link

For more information about Venn diagrams and Carroll diagrams, go to **www.mathlinks7.ca** and follow the links.

Have students work through Example 1 by using their Foldable to help in determining divisibility. Guide students in how to read and construct Carroll and Venn diagrams, for example, by asking them what the numbers in the middle of the Venn diagram represent. You may also wish to discuss other types of organizers in addition to the Carroll and Venn diagrams, explaining how they help to organize information.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• Have students use an organizer for the Show You Know exercise. • Provide kinesthetic and concrete learners with cut-out numbers and an enlarged Venn diagram so that they can physically place each number in its appropriate place on the diagram. • Give students who will benefit from it additional practice in sorting numbers according to their divisibility by 6 and 9: 78, 132, 342, 711, 9513, 52 272. Let students use a calculator. (Look for the following answers: – Divisible by 6 only: 78, 132 – Divisible by 9 only: 711, 9513 – Divisible by 6 and 9: 342, 52 272) Coach students through 78 and 342, and then have them try the other numbers on their own. It is important that students fully explore this skill before moving on to Example 2.



Example 2: Use Divisibility Rules to Determine Factors

a) What are the factors of 24?
b) What are the factors of 32?
c) What are the common factors of 24 and 32?
d) What is the greatest common factor of 24 and 32?

common factor
- a number that two or more numbers are divisible by
- 4 is a common factor of 8 and 12

Solution

a) Use divisibility rules to determine the factors.

24 is divisible by 1. $1 \times 24 = 24$

24 is divisible by 2 because it is even. $2 \times 12 = 24$

24 is divisible by 3 because the sum of the digits, $2 + 4 = 6$, is divisible by 3. $3 \times 8 = 24$

24 is divisible by 4 because the number formed by the two digits is divisible by 2 at least twice. $4 \times 6 = 24$

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

All numbers are divisible by 1.

Literacy Link
The greatest common factor is the largest number that both numbers are divisible by.

b) Use divisibility rules to determine the factors.

32 is divisible by 1. $1 \times 32 = 32$

32 is divisible by 2 because it is even. $2 \times 16 = 32$

32 is divisible by 4 because the number formed by the two digits is divisible by 2 at least twice. $4 \times 8 = 32$

The factors of 32 are 1, 2, 4, 8, 16, and 32.

c)

The common factors of 24 and 32 are 1, 2, 4, and 8.

d) The greatest common factor of 24 and 32 is 8.

WWW Web Link
To learn more about factoring and Venn diagrams, go to www.mathlinks7.ca and follow the links.

Show You Know

a) What are the factors of 30?
b) What are the factors of 18?
c) What are the common factors of 30 and 18?
d) What is the greatest common factor of 30 and 18?

Answers

Show You Know: Example 2

a) 1, 2, 3, 5, 6, 10, 15, 30

b) 1, 2, 3, 6, 9, 18

c) 1, 2, 3 6

d) 6

For Example 2, have students develop their own list of factors for 24 and 32 before reviewing the method here. You may wish to discuss the various methods that students use.

In part a) of Example 2, you may need to explain why students do not need to test 24 for divisibility by 24, 12, 8, and 6. Show them that the multiplication indicates that these numbers are factors of 24, which means that 24 is divisible by them.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• Students may need a posted list of the steps involved in determining the greatest common factor. These include finding all the factors of each number, identifying which ones are common, and then listing the greatest common factor. • You may wish to give students who will benefit from the practice two additional numbers for which they must determine the greatest common factor: 10 and 20. (Look for the following: Factors of 10: 1, 2, 5, 10 Factors of 20: 1, 2, 4, 5, 10, 20 Common factors: 1, 2, 5, 10 Greatest common factor: 10) • Students should not need help in determining the factors and drawing a Venn diagram since they practised these steps in both Examples 1 and 2; however, you may need to coach them through determining common factors and the greatest common factor.

Supported Learning

Learning Style

- For Example 2 part b), direct students' attention to the Venn diagram, which shows in a visual way how common factors can be identified.

WWW Web Link

Encourage students to go to **www.mathlinks7.ca** and follow the links to learn more about factoring and Venn diagrams. This site may be particularly beneficial to visual learners.

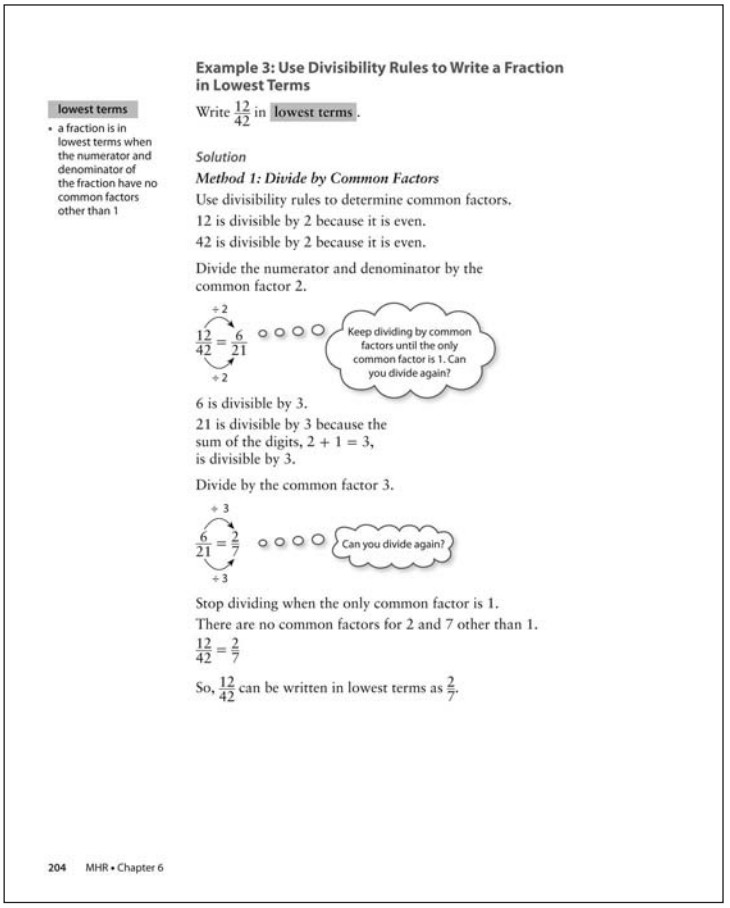
Supported Learning

ESL, Language, and Memory

- Some students may need help with the meanings of *numerator* and *denominator*. Show students that the numerator is at the top of the fraction and the denominator is at the bottom.

Meeting the Needs of All Learners

- Pace your lesson delivery, use concrete materials, and allow students to discuss the concepts among themselves.



Example 3: Use Divisibility Rules to Write a Fraction in Lowest Terms

lowest terms
- a fraction is in lowest terms when the numerator and denominator of the fraction have no common factors other than 1

Write $\frac{12}{42}$ in lowest terms.

Solution

Method 1: Divide by Common Factors

Use divisibility rules to determine common factors.

12 is divisible by 2 because it is even.

42 is divisible by 2 because it is even.

Divide the numerator and denominator by the common factor 2.

$\frac{12}{42} = \frac{6}{21}$ (÷2 above, ÷2 below)

Keep dividing by common factors until the only common factor is 1. Can you divide again?

6 is divisible by 3.

21 is divisible by 3 because the sum of the digits, 2 + 1 = 3, is divisible by 3.

Divide by the common factor 3.

$\frac{6}{21} = \frac{2}{7}$ (÷3 above, ÷3 below)

Can you divide again?

Stop dividing when the only common factor is 1.

There are no common factors for 2 and 7 other than 1.

$\frac{12}{42} = \frac{2}{7}$

So, $\frac{12}{42}$ can be written in lowest terms as $\frac{2}{7}$.

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For Example 3, introduce the concept of lowest terms. Some students may be more familiar with other words used to describe this process, such as *simplify*. Make sure students understand that simplifying a fraction and writing a fraction in lowest terms involve the same steps.

Encourage students to explore both methods that are presented in Example 3. Once students have explored each method, they may then decide which one they prefer to use. Have students use their divisibility Foldables for Example 3, Method 2, to help in finding a greatest common factor.

Note that in section 6.2 students will explore other methods for expressing fractions in lowest terms, such as using models and diagrams.

Method 2: Divide by the Greatest Common Factor

Use divisibility rules to determine the factors of 12:

12 is divisible by 1.	$1 \times 12 = 12$
12 is divisible by 2 because it is even.	$2 \times 6 = 12$
12 is divisible by 3 because the sum of the digits, $1 + 2 = 3$, is divisible by 3.	$3 \times 4 = 12$

The factors of 12 are 1, 2, 3, 4, 6, and 12.

You do not need to divide by 4, 6, and 12 because the multiplication shows that they are factors of 12.

Use divisibility rules to determine the factors of 42:

42 is divisible by 1.	$1 \times 42 = 42$
42 is divisible by 2 because it is even.	$2 \times 21 = 42$
42 is divisible by 3 because the sum of the digits, $4 + 2 = 6$, is divisible by 3.	$3 \times 14 = 42$
42 is divisible by 6 because it is divisible by 2 and 3.	$6 \times 7 = 42$

The factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42.

The common factors of 12 and 42 are 1, 2, 3, and ⑥.

The greatest common factor is 6.

Write $\frac{12}{42}$ in lowest terms by dividing the numerator and denominator by 6.

$$\frac{12}{42} \overset{\div 6}{=} \frac{2}{7}$$

So, $\frac{12}{42}$ can be written in lowest terms as $\frac{2}{7}$.

Show You Know

Write each fraction in lowest terms.

a) $\frac{20}{24}$ b) $\frac{12}{18}$

Answers

Show You Know: Example 3

a) $\frac{5}{6}$ b) $\frac{2}{3}$

Assessment *for* Learning	Supported Learning
Example 3 Have students do the Show You Know related to Example 3.	• You may wish to have students use both methods as they work on this Show You Know. Once they have finished, ask them which method they prefer, and why. Students may notice that one method is easier for some questions and the other method for other questions. • Give students who will benefit from the practice additional fractions to write in lowest terms: **a)** $\frac{4}{6}$ (Allow students to practise first with numbers that have fewer factors. 4 and 6 have a common factor of 2. $\frac{4}{6} = \frac{2}{3}$) **b)** $\frac{20}{32}$ (Move on to numbers with more factors. If they do not use the greatest common factor of 20 and 32, students may need multiple steps to put this in lowest terms. $\frac{20}{32} = \frac{10}{16} = \frac{5}{8}$. Point out that this method is okay too. There are many different ways to get the correct answer.) Coach students through a), and then have them try b) on their own.

Answers

Communicate the Ideas

1. a) Answers may vary. For example: 2 and 3 are factors of 6.

b) 2 and 5. Answers may vary. For example: 2 and 5 are factors of 10.

2. a) Answers may vary. For example: List all of the factors for 36 and all of the factors for 20. Then find the greatest factor that they have in common.

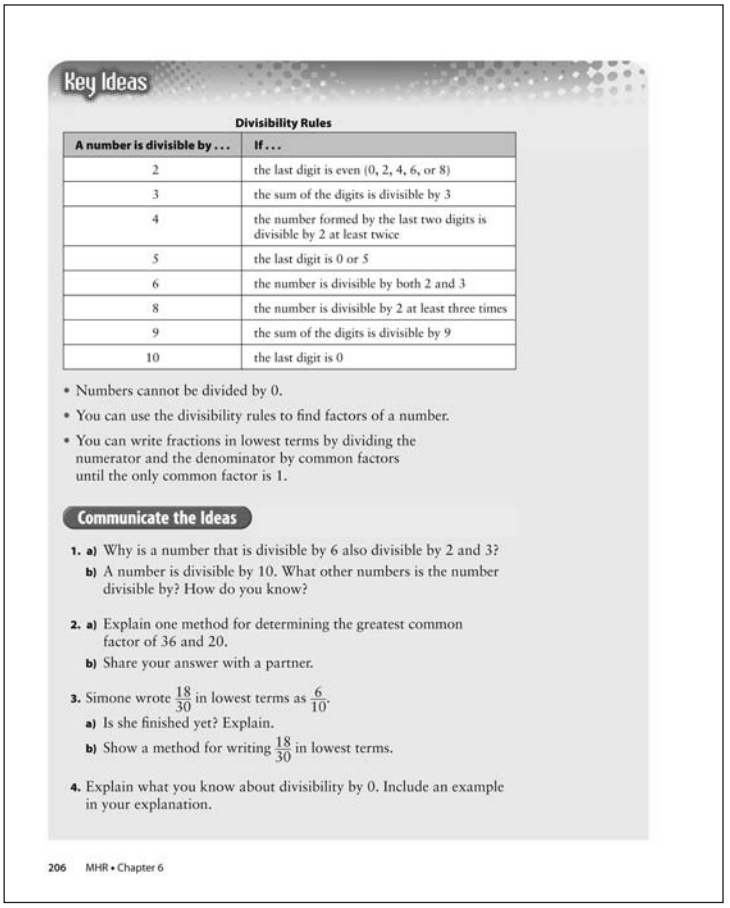
b) Answers will vary.

3. a) No. 6 and 10 are also divisible by 2.

b) Answers may vary. For example:

$$\frac{18}{30} \overset{\div 6}{=} \frac{3}{5}$$

4. Answers may vary. For example: It is not possible to divide a number by 0. If you try to divide 8 toys among 0 children, there is no possible way to do it.



Key Ideas

Divisibility Rules

A number is divisible by . . .	If . . .
2	the last digit is even (0, 2, 4, 6, or 8)
3	the sum of the digits is divisible by 3
4	the number formed by the last two digits is divisible by 2 at least twice
5	the last digit is 0 or 5
6	the number is divisible by both 2 and 3
8	the number is divisible by 2 at least three times
9	the sum of the digits is divisible by 9
10	the last digit is 0

- Numbers cannot be divided by 0.
- You can use the divisibility rules to find factors of a number.
- You can write fractions in lowest terms by dividing the numerator and the denominator by common factors until the only common factor is 1.

Communicate the Ideas

1. a) Why is a number that is divisible by 6 also divisible by 2 and 3?

b) A number is divisible by 10. What other numbers is the number divisible by? How do you know?

2. a) Explain one method for determining the greatest common factor of 36 and 20.

b) Share your answer with a partner.

3. Simone wrote $\frac{18}{30}$ in lowest terms as $\frac{6}{10}$.

a) Is she finished yet? Explain.

b) Show a method for writing $\frac{18}{30}$ in lowest terms.

4. Explain what you know about divisibility by 0. Include an example in your explanation.

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Supported Learning

Learning Style, ESL, and Language

- Students should be encouraged to share their understanding of the Key Ideas during group discussion.
- Allow students to choose to complete the Communicate the Ideas questions either in writing or orally or using a combination of both.

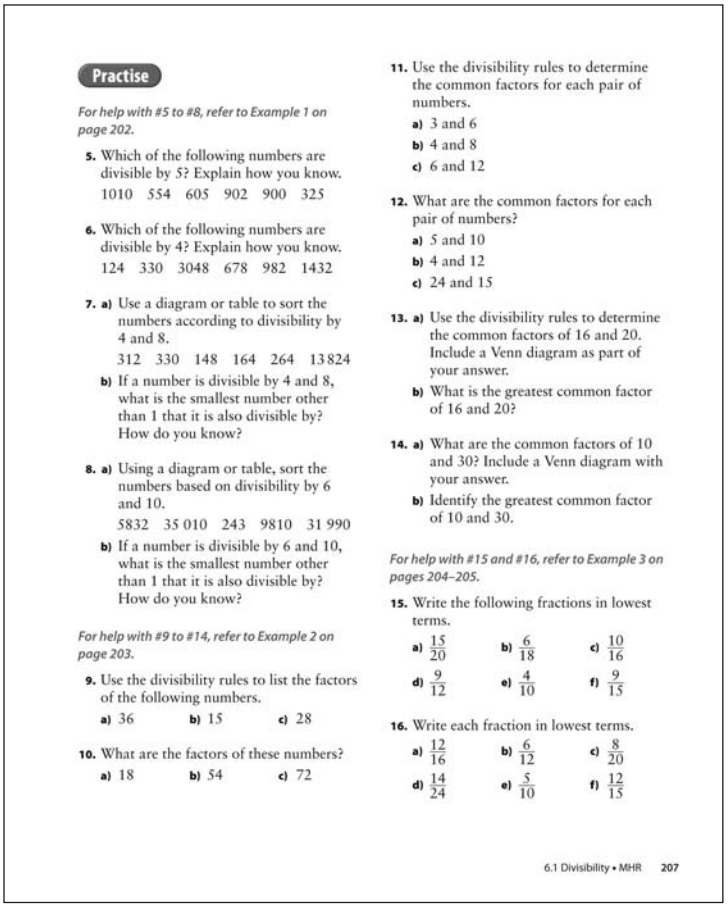
Key Ideas

Have students compare the Foldables they created in the Explore the Math to the Key Ideas. Instruct them to correct any errors that they may have made about divisibility. Have them explain how understanding divisibility helps in determining factors. Also discuss when a fraction is in lowest terms.

Communicate the Ideas

The Communicate the Ideas allows students to show their understanding of using divisibility rules, determining greatest common factors, and writing a fraction in lowest terms. It also provides students with the opportunity to express their understanding of why division by 0 is not possible.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students complete #1a) *or* b), #2, and #3. Have a class discussion about #4, and then have students write their answer independently.	• For #1, pay attention to students' explanations to determine whether they have a basic understanding of the divisibility rules. • Use **Master 2 Two Stars and One Wish** to have students critique other students' writing pieces. This master allows them to write two things they like about a piece and one thing they would improve. • Work with the class to develop criteria for judging a classmate's answer. For example, criteria for #3 might include – described how to determine common factors – described how to determine the greatest common factor – used mathematical words – was logical and easy to follow



Practise

For help with #5 to #8, refer to Example 1 on page 202.

5. Which of the following numbers are divisible by 5? Explain how you know.
1010 554 605 902 900 325

6. Which of the following numbers are divisible by 4? Explain how you know.
124 330 3048 678 982 1432

7. **a)** Use a diagram or table to sort the numbers according to divisibility by 4 and 8.
312 330 148 164 264 13 824
b) If a number is divisible by 4 and 8, what is the smallest number other than 1 that it is also divisible by? How do you know?

8. **a)** Using a diagram or table, sort the numbers based on divisibility by 6 and 10.
5832 35 010 243 9810 31 990
b) If a number is divisible by 6 and 10, what is the smallest number other than 1 that it is also divisible by? How do you know?

For help with #9 to #14, refer to Example 2 on page 203.

9. Use the divisibility rules to list the factors of the following numbers.
a) 36 **b)** 15 **c)** 28

10. What are the factors of these numbers?
a) 18 **b)** 54 **c)** 72

11. Use the divisibility rules to determine the common factors for each pair of numbers.
a) 3 and 6
b) 4 and 8
c) 6 and 12

12. What are the common factors for each pair of numbers?
a) 5 and 10
b) 4 and 12
c) 24 and 15

13. **a)** Use the divisibility rules to determine the common factors of 16 and 20. Include a Venn diagram as part of your answer.
b) What is the greatest common factor of 16 and 20?

14. **a)** What are the common factors of 10 and 30? Include a Venn diagram with your answer.
b) Identify the greatest common factor of 10 and 30.

For help with #15 and #16, refer to Example 3 on pages 204–205.

15. Write the following fractions in lowest terms.
a) $\frac{15}{20}$ **b)** $\frac{6}{18}$ **c)** $\frac{10}{16}$
d) $\frac{9}{12}$ **e)** $\frac{4}{10}$ **f)** $\frac{9}{15}$

16. Write each fraction in lowest terms.
a) $\frac{12}{16}$ **b)** $\frac{6}{12}$ **c)** $\frac{8}{20}$
d) $\frac{14}{24}$ **e)** $\frac{5}{10}$ **f)** $\frac{12}{15}$

6.1 Divisibility • MHR 207

Supported Learning

Meeting the Needs of All Learners

- More Practise activities of the following type may be needed: "Which numbers are divisible by 5? Explain how you know."

Common Errors

- Students have difficulty remembering all of the divisibility rules.

R_x Allowing students to explore the divisibility of numbers using a calculator will help them become more familiar with the rules.

- Students have difficulty writing fractions in lowest terms.

R_x Have students go back over Example 3. Also, have them explore concrete materials as a way to determine lowest terms. (See section 6.2, Example 1.)

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1a) *or* b), 2–5, 7, 9, 11, 13, 15, 19, 21, 23, Math Link
Typical	1a) *or* b), 2–5, 7, 9, 11, 13, 15, 17–21, 23, Math Link
Extension/Enrichment	1a) *or* b), 2–4, 22, 26–28

Practise

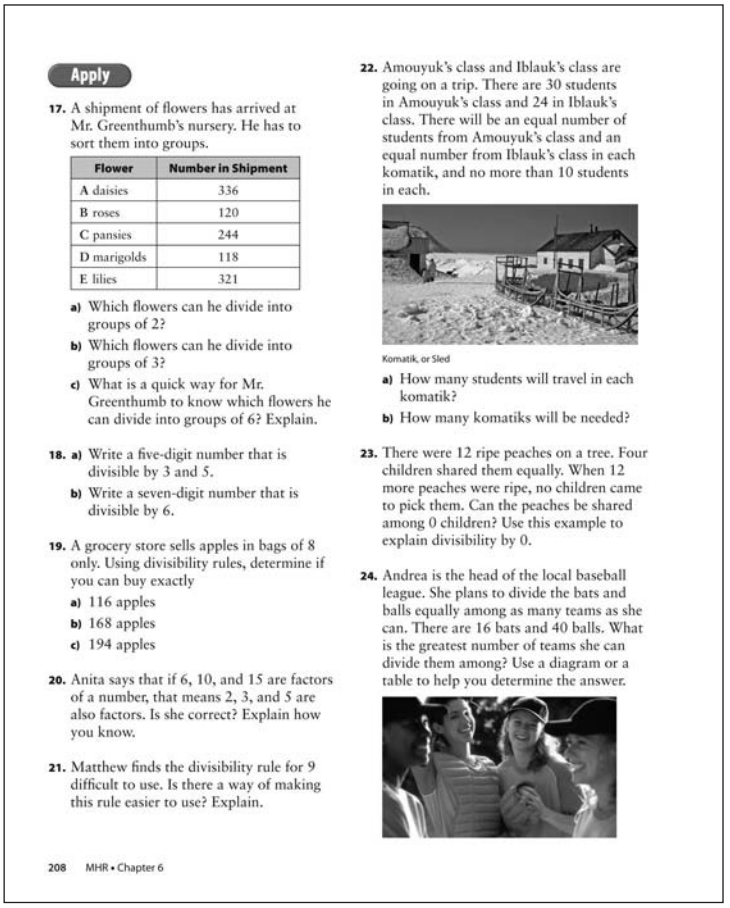
Have students work in pairs or groups of up to four to discuss the questions and agree on answers. Make sure that students become comfortable with a method that makes sense to them for finding common factors and writing fractions in lowest terms.

Assessment *for* Learning	Supported Learning
Practise Have students do #5, #7, #9, #11, #13, and #15. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #5 and #7 will need additional coaching with Example 1. Have these students explain how they are using divisibility rules. Clarify any misunderstandings. Coach students with 124 in #6 and 35 010 in #8a), and assist them with #8b) if necessary. Then have them complete the rest of #6 and #8 on their own. • Students who have problems with #9, #11, and #13 will need additional coaching with Example 2. Have these students explain how they are determining factors, common factors and greatest common factor. Clarify any misunderstandings. Coach students through #10a), #12a), and #14a). Then have them complete the rest of each question on their own. • Students who have problems with #15 will need additional coaching with Example 3. Have these students explain how they are writing fractions in lowest terms. Clarify any misunderstandings. Coach students through #16a). Then have them complete the rest of the question on their own. • Check back with students several times to make sure that they understand the concepts. • Have students do the Math Games activity on page 226, which provides a fun way to practise the divisibility rules.

Supported Learning

Learning Style and Memory

- You may wish to provide **BLM 6–3 Section 6.1 Extra Practice** to students who require more practice.



Apply

17. A shipment of flowers has arrived at Mr. Greenthumb's nursery. He has to sort them into groups.

Flower	Number in Shipment
A daisies	336
B roses	120
C pansies	244
D marigolds	118
E lilies	321

a) Which flowers can he divide into groups of 2?
b) Which flowers can he divide into groups of 3?
c) What is a quick way for Mr. Greenthumb to know which flowers he can divide into groups of 6? Explain.

18. a) Write a five-digit number that is divisible by 3 and 5.
b) Write a seven-digit number that is divisible by 6.

19. A grocery store sells apples in bags of 8 only. Using divisibility rules, determine if you can buy exactly
a) 116 apples
b) 168 apples
c) 194 apples

20. Anita says that if 6, 10, and 15 are factors of a number, that means 2, 3, and 5 are also factors. Is she correct? Explain how you know.

21. Matthew finds the divisibility rule for 9 difficult to use. Is there a way of making this rule easier to use? Explain.

22. Amouyuk's class and Iblauk's class are going on a trip. There are 30 students in Amouyuk's class and 24 in Iblauk's class. There will be an equal number of students from Amouyuk's class and an equal number from Iblauk's class in each komatik, and no more than 10 students in each.

Komatik, or Sled

a) How many students will travel in each komatik?
b) How many komatiks will be needed?

23. There were 12 ripe peaches on a tree. Four children shared them equally. When 12 more peaches were ripe, no children came to pick them. Can the peaches be shared among 0 children? Use this example to explain divisibility by 0.

24. Andrea is the head of the local baseball league. She plans to divide the bats and balls equally among as many teams as she can. There are 16 bats and 40 balls. What is the greatest number of teams she can divide them among? Use a diagram or a table to help you determine the answer.

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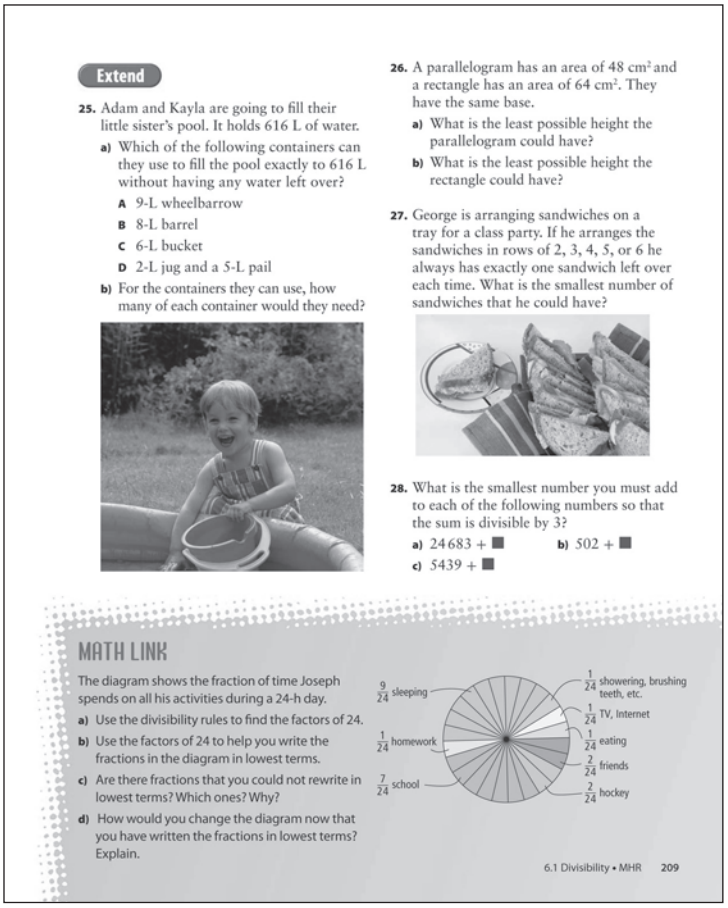
Apply and Extend

You may wish to discuss #20 as a class as this will help reinforce students' understanding of divisibility and factors. For #21, have students share their methods for using divisibility rules and any tricks they have. This type of discussion is helpful for other students who may not yet have solidified their understanding.

You may choose to have students work on #26 in groups. When students come up with an answer, ask, "How do you know you are correct?"

Assessment *as* Learning	Supported Learning
Math Learning Log Have students reflect on two or three items they have improved on and how they think they have improved. Have students also pick one area in which they feel they did not learn as much as they should or could have. Have them reflect on how they plan to improve in this area of learning.	• Have students check the What I Need to Work On section of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Work with students to develop a plan for dealing with the areas in which they are having difficulty. • Depending on students' learning style, have them provide oral or written answers. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • Have students review the part related to section 6.1 in **BLM 6–1 Chapter 6 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.



Extend

25. Adam and Kayla are going to fill their little sister's pool. It holds 616 L of water.

a) Which of the following containers can they use to fill the pool exactly to 616 L without having any water left over?

A 9-L wheelbarrow

B 8-L barrel

C 6-L bucket

D 2-L jug and a 5-L pail

b) For the containers they can use, how many of each container would they need?

26. A parallelogram has an area of 48 cm² and a rectangle has an area of 64 cm². They have the same base.

a) What is the least possible height the parallelogram could have?

b) What is the least possible height the rectangle could have?

27. George is arranging sandwiches on a tray for a class party. If he arranges the sandwiches in rows of 2, 3, 4, 5, or 6 he always has exactly one sandwich left over each time. What is the smallest number of sandwiches that he could have?

28. What is the smallest number you must add to each of the following numbers so that the sum is divisible by 3?

a) 24 683 + ■ **b)** 502 + ■ **c)** 5439 + ■

MATH LINK

The diagram shows the fraction of time Joseph spends on all his activities during a 24-h day.

a) Use the divisibility rules to find the factors of 24.

b) Use the factors of 24 to help you write the fractions in the diagram in lowest terms.

c) Are there fractions that you could not rewrite in lowest terms? Which ones? Why?

d) How would you change the diagram now that you have written the fractions in lowest terms? Explain.

$\frac{9}{24}$ sleeping; $\frac{1}{24}$ homework; $\frac{7}{24}$ school; $\frac{1}{24}$ showering, brushing teeth, etc.; $\frac{1}{24}$ TV, Internet; $\frac{1}{24}$ eating; $\frac{2}{24}$ friends; $\frac{2}{24}$ hockey

Answers

Math Link

a) 1, 2, 3, 4, 6, 8, 12, 24

b) sleeping = $\frac{3}{8}$; homework = $\frac{1}{24}$; school = $\frac{7}{24}$; showering, brushing teeth, etc. = $\frac{1}{24}$; TV, Internet = $\frac{1}{24}$; eating = $\frac{1}{24}$; friends = $\frac{1}{12}$; hockey = $\frac{1}{12}$

c) Yes. $\frac{1}{24}$ and $\frac{7}{24}$ are already in lowest terms.

d) The diagram would not change. The fractions that are written in lowest terms are equivalent in value to the original fractions.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 209 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 225.	• You may wish to have students do this Math Link in order to further their understanding of using divisibility to determine factors and of writing fractions in lowest terms. • Students who are having difficulty getting started could use **BLM 6–4 Section 6.1 Math Link**, which provides scaffolding for this activity. • Having students write their answer to d) in their chapter Foldable might help them to clarify their understanding of fractions in lowest terms. • Observe students as they work on the Math Link. Make sure that they use divisibility correctly to write the fractions in lowest terms. Also check that they understand that the part of a whole a fraction represents does not change when the fraction is written in lowest terms.

MATH LINK

As a class, discuss what each fraction in the fraction circle means in terms of time; for example, $\frac{2}{24}$ is 2 h out of 24 h. This activity will ensure that the fraction circle is meaningful to students and will also prepare them for the Math Links in the following sections of the chapter.

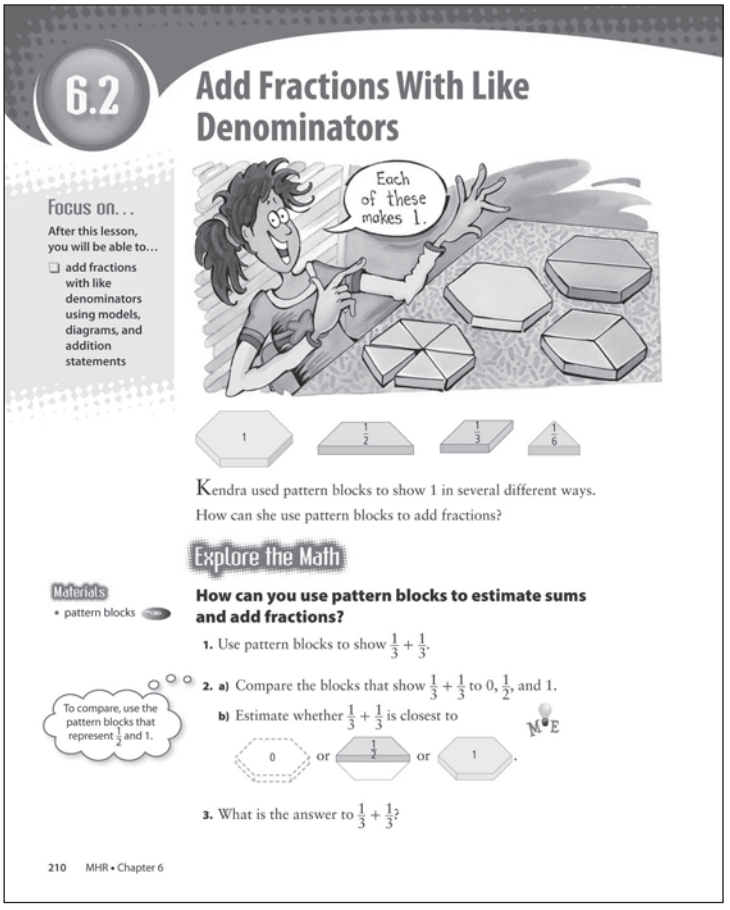
6.2 Add Fractions With Like Denominators

Suggested Timing
80–100 minutes

Materials
- pattern blocks
- fraction strips
- scissors
- coloured pencils
- ruler

Blackline Masters
Master 13 Pattern Blocks
Master 14 Fraction Circles
Master 15 Fraction Strips
BLM 6–1 Chapter 6 Self-Assessment
BLM 6–5 Section 6.2 Extra Practice
BLM 6–6 Section 6.2 Math Link
BLM 6–10 24-Hour Fraction Circle

Mathematical Processes
- ☑ Communication
- ☑ Connections
- ☑ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☑ Technology
- ☑ Visualization


6.2 Add Fractions With Like Denominators

Focus on...
After this lesson, you will be able to...
- add fractions with like denominators using models, diagrams, and addition statements

Each of these makes 1.

Kendra used pattern blocks to show 1 in several different ways.
How can she use pattern blocks to add fractions?

Explore the Math

Materials
- pattern blocks

How can you use pattern blocks to estimate sums and add fractions?

1. Use pattern blocks to show $\frac{1}{3} + \frac{1}{3}$.

To compare, use the pattern blocks that represent $\frac{1}{2}$ and 1.

2. a) Compare the blocks that show $\frac{1}{3} + \frac{1}{3}$ to 0, $\frac{1}{2}$, and 1.
b) Estimate whether $\frac{1}{3} + \frac{1}{3}$ is closest to 0 or $\frac{1}{2}$ or 1.

3. What is the answer to $\frac{1}{3} + \frac{1}{3}$?

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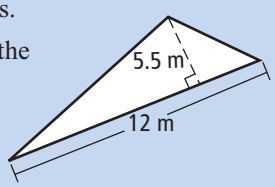
Specific Outcomes

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Warm-Up

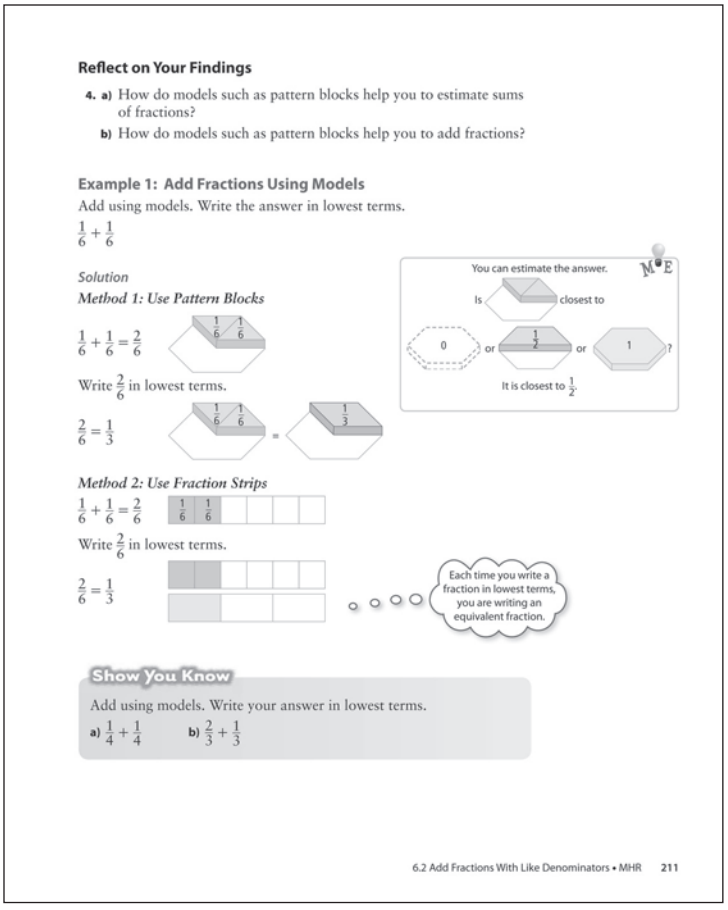
1. How can you tell if a number is divisible by 5?
2. How can you tell if a number is divisible by 2?
3. Use the rules from #1 and #2 to sort the following numbers: 12, 15, 20, 25, 30, 32, 40. Use a Venn diagram to show your results.
4. Estimate and then calculate the area of the triangle shown.



5. Show using bar notation: 6.6666666 ...

Mental Math

6. What is 75% of 120? Show your thinking.
7. What is 40% of 150? Show your thinking.
8. Show the following fractions as decimals. Round to hundredths.
   a) $\frac{1}{8}$ b) $\frac{2}{5}$
9. Show the following decimals as percents.
   a) 0.53 b) 0.765
10. Estimate 19 out of 92 as a percent. Show your thinking.



Reflect on Your Findings

4. a) How do models such as pattern blocks help you to estimate sums of fractions?

b) How do models such as pattern blocks help you to add fractions?

Example 1: Add Fractions Using Models

Add using models. Write the answer in lowest terms.

$\frac{1}{6} + \frac{1}{6}$

Solution

Method 1: Use Pattern Blocks

$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

Write $\frac{2}{6}$ in lowest terms.

$\frac{2}{6} = \frac{1}{3}$

You can estimate the answer.

Is closest to 0 or $\frac{1}{2}$ or 1?

It is closest to $\frac{1}{2}$.

Method 2: Use Fraction Strips

$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

Write $\frac{2}{6}$ in lowest terms.

$\frac{2}{6} = \frac{1}{3}$

Each time you write a fraction in lowest terms, you are writing an equivalent fraction.

Show You Know

Add using models. Write your answer in lowest terms.

a) $\frac{1}{4} + \frac{1}{4}$ **b)** $\frac{2}{3} + \frac{1}{3}$

6.2 Add Fractions With Like Denominators • MHR 211

Activity Planning Notes

Have students use pattern blocks to discover how to make one whole in as many ways as they can. Tell them that four ways are shown in the student resource and encourage them to discover how many other ways are possible.

Explore the Math

Estimation of fractions is difficult for many students. Establish benchmark numbers that students should use throughout the chapter for comparing, such as 0, $\frac{1}{2}$, and 1. This type of estimating will help reinforce their number sense and assist them in deciding if an answer is reasonable or not. Discuss how making estimates before adding fractions is helpful for checking answers after adding.

You may wish to hand out to students **Master 13 Pattern Blocks** if actual pattern blocks are not available.

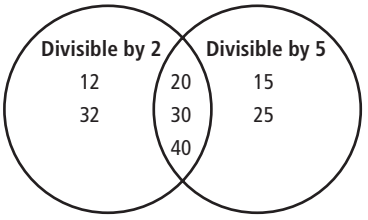
Answers

Warm-Up

1. It ends in 5 or 0.

2. It is an even number, which means it ends in 0, 2, 4, 6, or 8.

3.



4. Estimate: $12 \times 5 \div 2 = 60 \div 2 = 30$ m^2
Calculate: $12 \times 5.5 \div 2 = 66 \div 2 = 33$ m^2

5. $6.\overline{6}$

6. 25% of 120 $= 120 \div 4 = 30$
75% of 120 $= 3 \times 30 = 90$

7. 10% of 150 $= 15$
40% of 150 $= 4 \times 15 = 60$

8. **a)** $0.125 = 0.13$ **b)** 0.4

9. **a)** 53% **b)** 76.5%

10. 50% of 92 $= 46$
25% of 92 $= 23$ Too high
10% of 92 $= 9.2$
20% $= 18.4$ Too low
The answer is between 20% and 25%.

Explore the Math

1. two blue rhombuses

2. a), b) $\frac{1}{2}$

3. $\frac{2}{3}$

4. a) Answers may vary. For example: Models such as pattern blocks allow you to see a range of possible solutions.

b) Answers may vary. For example: Models such as pattern blocks allow you to see the answers visually.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss how models such as pattern blocks help in estimating and in adding fractions. During this process, they are generalizing what they have learned during the Explore the Math.	• To remind students what the different pattern blocks represent, you may wish to label paper copies of pattern blocks and hang them in the classroom for students to refer to.

Answers

Show You Know: Example 1

a) $\frac{1}{2}$ **b)** 1

Show You Know: Example 2

a) $\frac{3}{5}$ **b)** 1

Show You Know: Example 3

a) $\frac{1}{2}$ **b)** $\frac{2}{3}$

Supported Learning

ESL

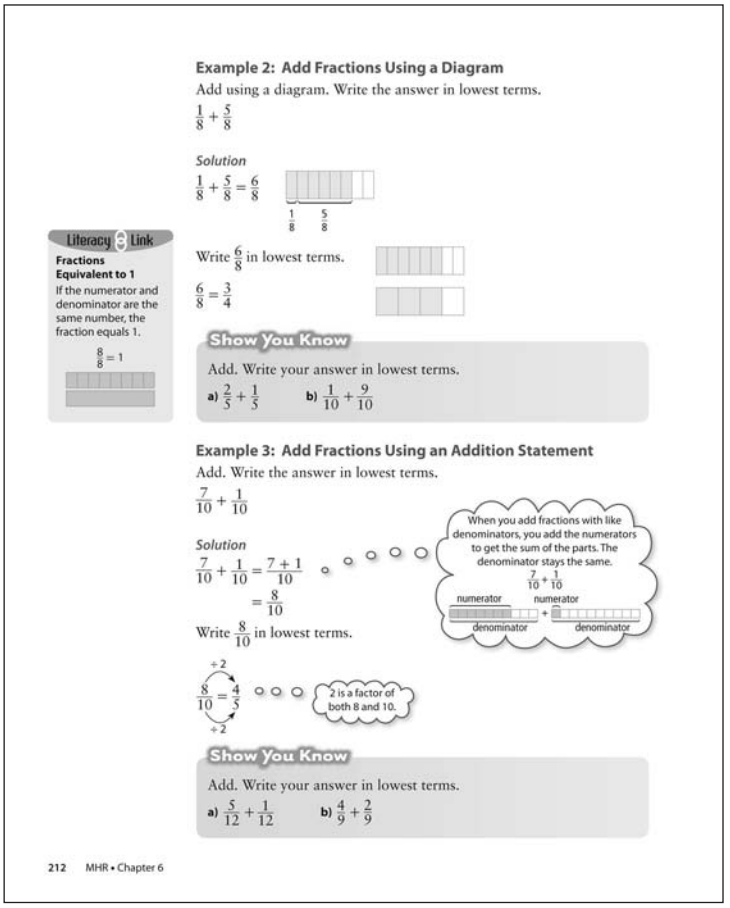
- Some English language learners may need to be taught that "like" denominators are denominators that are the same.
- In Example 3, students may need assistance with the term *addition statement*.

Motor

- Students may find it difficult to manipulate the pattern blocks and/or fraction strips. In order for them to be successful, ensure that they are given adequate time to practise.
- You may wish to allow students to use virtual manipulatives instead of the pattern blocks.

Meeting the Needs of All Learners

- To ease students into the concept of adding fractions, have them use any concrete and/or visual materials they work most comfortably with, such as charts and pictures.



Example 2: Add Fractions Using a Diagram

Add using a diagram. Write the answer in lowest terms.

$\frac{1}{8} + \frac{5}{8}$

Solution

$\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$

Write $\frac{6}{8}$ in lowest terms.

$\frac{6}{8} = \frac{3}{4}$

Literacy Link

Fractions Equivalent to 1

If the numerator and denominator are the same number, the fraction equals 1.

$\frac{8}{8} = 1$

Show You Know

Add. Write your answer in lowest terms.

a) $\frac{2}{5} + \frac{1}{5}$ **b)** $\frac{1}{10} + \frac{9}{10}$

Example 3: Add Fractions Using an Addition Statement

Add. Write the answer in lowest terms.

$\frac{7}{10} + \frac{1}{10}$

Solution

$\frac{7}{10} + \frac{1}{10} = \frac{7+1}{10}$

$= \frac{8}{10}$

Write $\frac{8}{10}$ in lowest terms.

When you add fractions with like denominators, you add the numerators to get the sum of the parts. The denominator stays the same.

$\frac{7}{10} + \frac{1}{10}$

numerator numerator

denominator denominator

$\frac{8}{10} = \frac{4}{5}$ (÷2)

2 is a factor of both 8 and 10.

Show You Know

Add. Write your answer in lowest terms.

a) $\frac{5}{12} + \frac{1}{12}$ **b)** $\frac{4}{9} + \frac{2}{9}$

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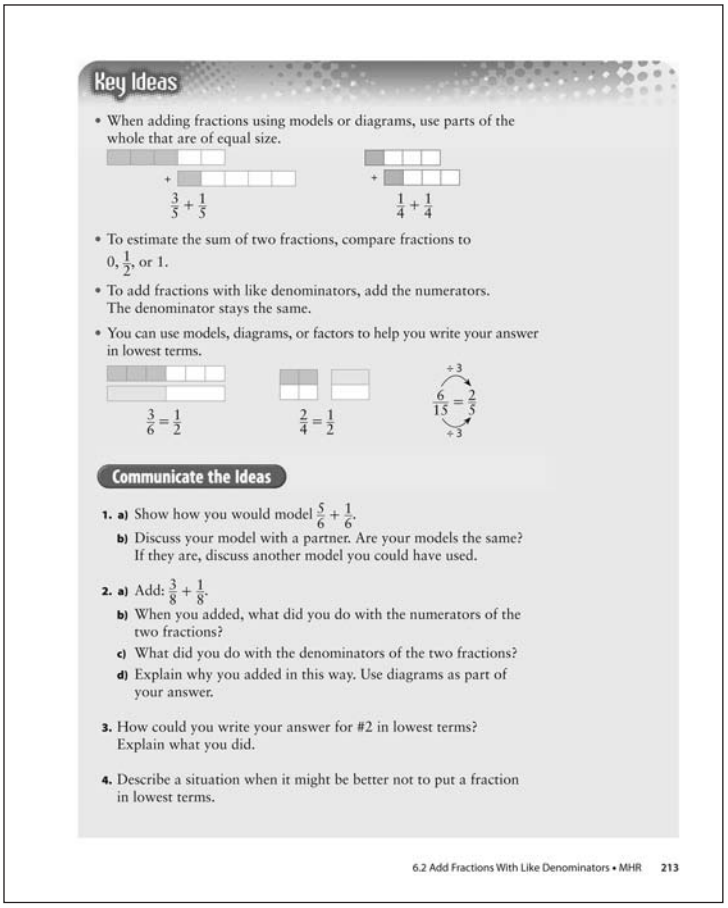
Example 1 provides two methods for adding fractions: pattern blocks and fraction strips. Guide students to see the relationship between the two: they are both models used to help us understand how to add fractions. Have students complete the Show You Know using both pattern blocks and fraction strips. You may wish to provide students with **Master 13 Pattern Blocks** and **Master 15 Fraction Strips**.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know on page 211 related to Example 1.	• Encourage students to include estimation in their work. • You may wish to provide additional questions for students who will benefit from them: **a)** $\frac{1}{6} + \frac{5}{6}$ (Have students practise using pattern blocks. They should note that the number of sixths covers the pattern block base. $\frac{6}{6}$) **b)** $\frac{1}{12} + \frac{1}{12}$ (Have students practise using fraction strips. Remind students to put their answer in lowest terms. Ask them what both 2 and 12 are divisible by. $\frac{2}{12} = \frac{1}{6}$) Coach students through a), and then have them try b) on their own.

For Example 2, remind students that the whole can be of any shape, as long as the parts within the whole are the same size and the wholes are the same size.



Key Ideas

- When adding fractions using models or diagrams, use parts of the whole that are of equal size.

$\frac{3}{5} + \frac{1}{5}$ $\qquad$ $\frac{1}{4} + \frac{1}{4}$

- To estimate the sum of two fractions, compare fractions to 0, $\frac{1}{2}$, or 1.
- To add fractions with like denominators, add the numerators. The denominator stays the same.
- You can use models, diagrams, or factors to help you write your answer in lowest terms.

$\frac{3}{6} = \frac{1}{2}$ $\qquad$ $\frac{2}{4} = \frac{1}{2}$ $\qquad$ $\frac{6}{15} = \frac{2}{5}$ (×3 / ×3)

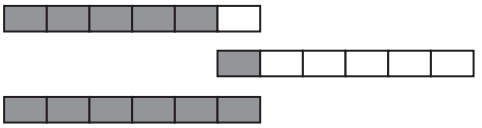
Communicate the Ideas

1. a) Show how you would model $\frac{5}{6} + \frac{1}{6}$.
   b) Discuss your model with a partner. Are your models the same? If they are, discuss another model you could have used.
2. a) Add: $\frac{3}{8} + \frac{1}{8}$.
   b) When you added, what did you do with the numerators of the two fractions?
   c) What did you do with the denominators of the two fractions?
   d) Explain why you added in this way. Use diagrams as part of your answer.
3. How could you write your answer for #2 in lowest terms? Explain what you did.
4. Describe a situation when it might be better not to put a fraction in lowest terms.

Answers

Communicate the Ideas

1. a) Answers may vary. For example:

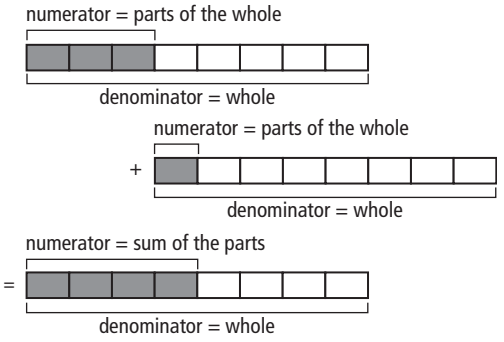


b) Answers will vary.

2. a) $\frac{4}{8}$ **b)** I added the numerators.

c) I kept the denominators the same.

d) Answers may vary. For example: The denominator represents the whole. The numerator represents parts of the whole. When you add fractions with like denominators, the whole does not change. You need to add the numerators to get the sum of the parts.



Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know on page 212 related to Example 2.	• Have students complete the Show You Know using a diagram. • You may wish to provide students with more practice: **a)** $\frac{1}{5} + \frac{3}{5}$ ($\frac{4}{5}$. Ask students how they know the answer is in lowest terms.) **b)** $\frac{1}{10} + \frac{7}{10}$ ($\frac{8}{10} = \frac{4}{5}$. Ask students what number they need to divide into both 8 and 10 to write the answer in lowest terms.) Coach students through a), and then have them try b) on their own.

Example 3 shows students how to add fractions using an addition statement.

Assessment *for* Learning	Supported Learning
Example 3 Have students do the Show You Know on page 212 related to Example 3.	• Note which students recognize that both answers must be rewritten in lowest terms and which do not. • You may wish to provide students with extra practice in adding fractions: **a)** $\frac{1}{7} + \frac{3}{7}$ (Have students first complete a question with an answer that is already in lowest terms. $\frac{4}{7}$) **b)** $\frac{1}{10} + \frac{3}{10}$ (Students will need to rewrite the answer in lowest terms. $\frac{4}{10} = \frac{2}{5}$ Coach students through a), and then have them try b) on their own.

Supported Learning

Learning Style

- Encourage kinesthetic and concrete learners to continue to use pattern blocks and/or fraction strips to help them as they add in Example 3.
- You may wish to allow kinesthetic and concrete learners to use **Master 15 Fraction Strips** and **Master 14 Fraction Circles** instead of drawing diagrams.

Common Errors

- Students have difficulty understanding that you add the numerators but the denominator stays the same.

R$_x$ Have them spend more time adding using manipulatives, ensuring students pay close attention to how the denominator (or whole) does not change.

Answers

Communicate the Ideas

3. $\frac{4}{8} \overset{\div 4}{=} \frac{1}{2}$

Answers may vary. For example: Divide the numerator and the denominator of the fraction by the greatest common factor of 4.

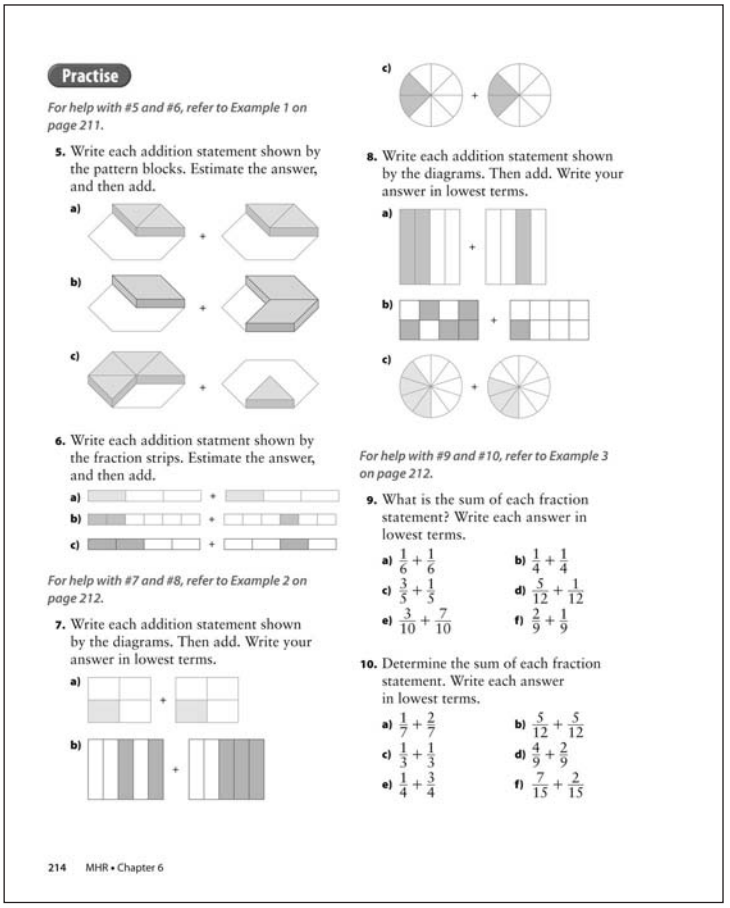
4. Answers will vary. For example: It would be better to know that you have $\frac{3}{12}$ of a carton of eggs than to know that you have $\frac{1}{4}$ of a carton, because there are 12 eggs in a carton and you immediately know that you have 3 eggs.

Supported Learning

ESL

- Some English language learners may have difficulty with the wording in Communicate the Ideas #4. Clarify what this question is asking. If students are very new to the English language, eliminate this question or have them answer it in their own language.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students complete #1 to #3. You may wish to use #4 as an extension question.	• Have volunteers explain the process they used in #1 and why. • Make available pattern blocks and fraction strips to help students as they answer #1 and #2. • Allow students with motor difficulties to use **Master 13 Pattern Blocks**, **Master 15 Fraction Strips**, and/or **Master 14 Fraction Circles** to answer #1 and #2, instead of drawing the diagrams themselves. • Have students share their response to #3 and check that their classmate has the answer in lowest terms.



Practise

For help with #5 and #6, refer to Example 1 on page 211.

5. Write each addition statement shown by the pattern blocks. Estimate the answer, and then add.

a)

b)

c)

6. Write each addition statment shown by the fraction strips. Estimate the answer, and then add.

a)

b)

c)

For help with #7 and #8, refer to Example 2 on page 212.

7. Write each addition statement shown by the diagrams. Then add. Write your answer in lowest terms.

a)

b)

c)

8. Write each addition statement shown by the diagrams. Then add. Write your answer in lowest terms.

a)

b)

c)

For help with #9 and #10, refer to Example 3 on page 212.

9. What is the sum of each fraction statement? Write each answer in lowest terms.

a) $\frac{1}{6} + \frac{1}{6}$ b) $\frac{1}{4} + \frac{1}{4}$

c) $\frac{3}{5} + \frac{1}{5}$ d) $\frac{5}{12} + \frac{1}{12}$

e) $\frac{3}{10} + \frac{7}{10}$ f) $\frac{2}{9} + \frac{1}{9}$

10. Determine the sum of each fraction statement. Write each answer in lowest terms.

a) $\frac{1}{7} + \frac{2}{7}$ b) $\frac{5}{12} + \frac{5}{12}$

c) $\frac{1}{3} + \frac{1}{3}$ d) $\frac{4}{9} + \frac{2}{9}$

e) $\frac{1}{4} + \frac{3}{4}$ f) $\frac{7}{15} + \frac{2}{15}$

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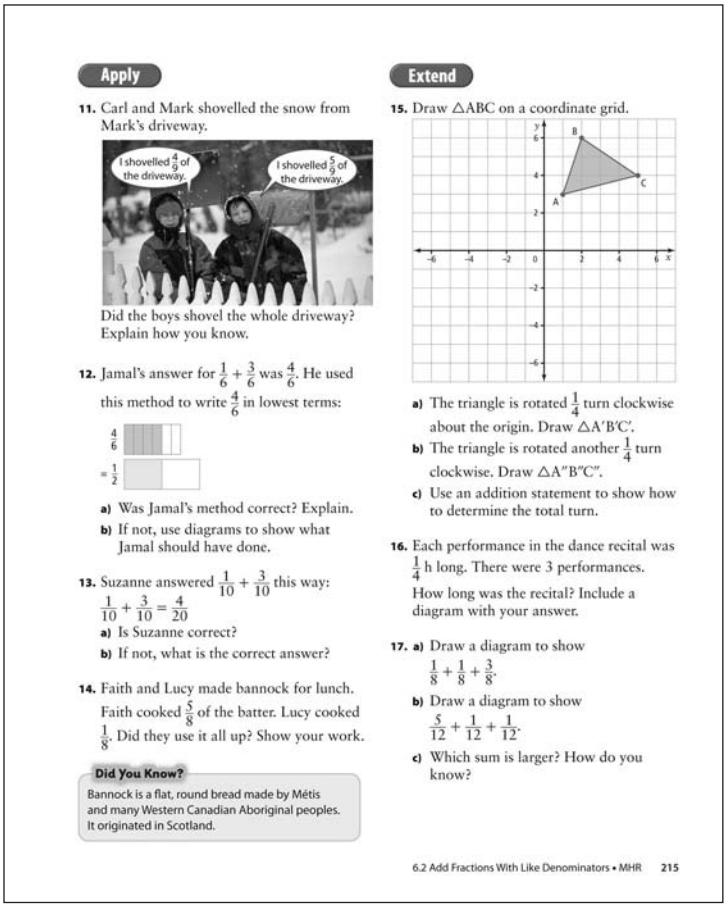
Communicate the Ideas

The Communicate the Ideas allows students to show their understanding of modelling fractions. It also demonstrates their understanding of how to write an answer in lowest terms and when it may be advantageous not to rewrite an answer in lowest terms.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 7, 9, Math Link
Typical	1–3, 5, 7, 9, 11, 13–15, Math Link
Extension/Enrichment	1–4, 12, 15–18, Math Link

Practise

Instruct students to estimate prior to adding, and then have students state whether or not their answers are reasonable.



Apply

11. Carl and Mark shovelled the snow from Mark's driveway.

Did the boys shovel the whole driveway? Explain how you know.

12. Jamal's answer for $\frac{1}{6} + \frac{3}{6}$ was $\frac{4}{6}$. He used this method to write $\frac{4}{6}$ in lowest terms:

$\frac{4}{6}$

$= \frac{1}{2}$

a) Was Jamal's method correct? Explain.

b) If not, use diagrams to show what Jamal should have done.

13. Suzanne answered $\frac{1}{10} + \frac{3}{10}$ this way:

$\frac{1}{10} + \frac{3}{10} = \frac{4}{20}$

a) Is Suzanne correct?

b) If not, what is the correct answer?

14. Faith and Lucy made bannock for lunch. Faith cooked $\frac{5}{8}$ of the batter. Lucy cooked $\frac{1}{8}$. Did they use it all up? Show your work.

Did You Know?

Bannock is a flat, round bread made by Métis and many Western Canadian Aboriginal peoples. It originated in Scotland.

Extend

15. Draw $\triangle ABC$ on a coordinate grid.

a) The triangle is rotated $\frac{1}{4}$ turn clockwise about the origin. Draw $\triangle A'B'C'$.

b) The triangle is rotated another $\frac{1}{4}$ turn clockwise. Draw $\triangle A''B''C''$.

c) Use an addition statement to show how to determine the total turn.

16. Each performance in the dance recital was $\frac{1}{4}$ h long. There were 3 performances. How long was the recital? Include a diagram with your answer.

17. a) Draw a diagram to show $\frac{1}{8} + \frac{1}{8} + \frac{3}{8}$.

b) Draw a diagram to show $\frac{5}{12} + \frac{1}{12} + \frac{1}{12}$.

c) Which sum is larger? How do you know?

6.2 Add Fractions With Like Denominators • MHR 215

Apply and Extend

The answer to #18 is larger than one. Explain to students that when the numerator is larger than the denominator the fraction shows more than one whole.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students reflect on two or three items they have improved on and how they think they have improved. Have students also pick one area in which they feel they did not learn as much as they should or could have. Have them reflect on how they plan to improve in this area of learning.	• Have students check the What I Need to Work On section of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Work with students to develop a plan for dealing with the areas in which they are having difficulty. • Depending on students' learning style, have them provide oral or written answers. • Have students review the part related to section 6.2 in **BLM 6–1 Chapter 1 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Common Errors

- Students may draw diagrams of unequal wholes.

R_x Remind students to use wholes that are the same size. To aid in their understanding, have them cut out two circles of different sizes and then cut each in half. Tell students that they are going to show $\frac{1}{2} + \frac{1}{2} = 1$ and tell them to put the large half circle together with the small half circle. Ask them if they can add fractions in this way and then have them explain why not.

Supported Learning

Learning Style

- Allow students to use pattern blocks, fraction strips, diagrams, and other materials of their choice to answer the questions in the Practise, Apply, and Extend.

Learning Style and Memory

- You may wish to provide **BLM 6–5 Section 6.2 Extra Practice** to students who require more practice.

Assessment *for* Learning	Supported Learning
Practise Have students do #5, #7, and #9. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #5 will need to be coached through #6a). Then have them complete the rest of #6 on their own. • Students who have problems with #7 will need to be coached through #8b). Then have them complete the rest of #8 on their own. • Students who have problems with #9 will need to be coached through #10b). Then have them complete the rest of #10 on their own.

Answers

Math Link

a) Answers may vary. **b)** Answers may vary.

c) The fractions should add up to $\frac{24}{24} = 1$ because one day has 24 h.

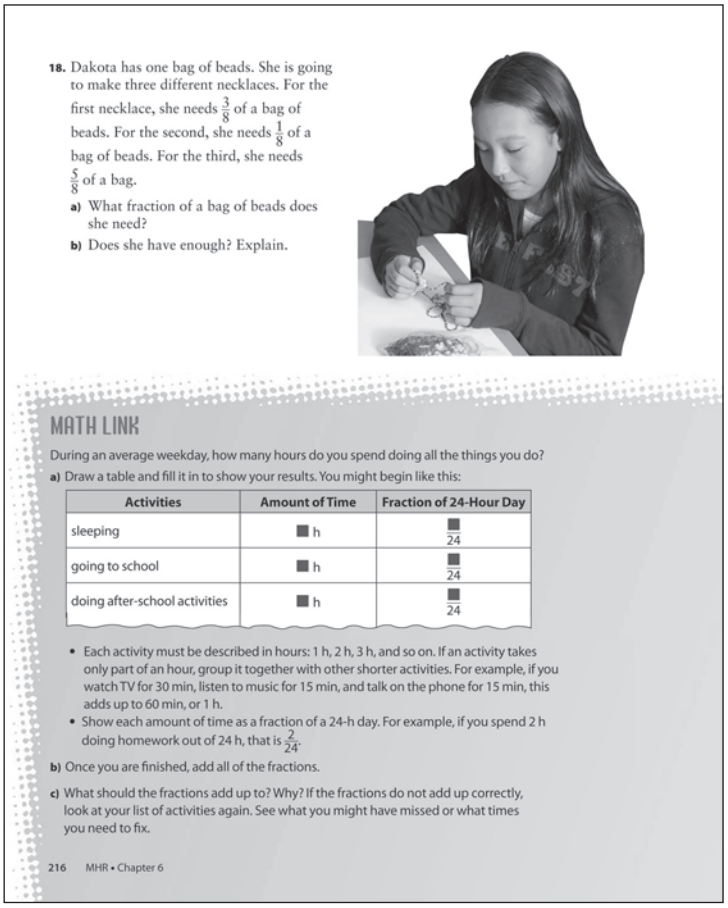
Supported Learning

Learning Style

- Some students may find it easier to divide their day into fractions using a fraction circle. You may wish to provide students with **BLM 6–10 24-Hour Fraction Circle**, which all students will use for the Wrap It Up! on page 225.

Gifted and Enrichment

- Encourage students to explore how to show activities that take less than an hour. For example, talking on the phone for 15 min is $\frac{1}{4}$ of an hour, or $\frac{1}{96}$ of the day.



18. Dakota has one bag of beads. She is going to make three different necklaces. For the first necklace, she needs $\frac{3}{8}$ of a bag of beads. For the second, she needs $\frac{1}{8}$ of a bag of beads. For the third, she needs $\frac{5}{8}$ of a bag.

a) What fraction of a bag of beads does she need?

b) Does she have enough? Explain.

MATH LINK

During an average weekday, how many hours do you spend doing all the things you do?

a) Draw a table and fill it in to show your results. You might begin like this:

Activities	Amount of Time	Fraction of 24-Hour Day
sleeping	■ h	$\frac{■}{24}$
going to school	■ h	$\frac{■}{24}$
doing after-school activities	■ h	$\frac{■}{24}$

- Each activity must be described in hours: 1 h, 2 h, 3 h, and so on. If an activity takes only part of an hour, group it together with other shorter activities. For example, if you watch TV for 30 min, listen to music for 15 min, and talk on the phone for 15 min, this adds up to 60 min, or 1 h.
- Show each amount of time as a fraction of a 24-h day. For example, if you spend 2 h doing homework out of 24 h, that is $\frac{2}{24}$.

b) Once you are finished, add all of the fractions.

c) What should the fractions add up to? Why? If the fractions do not add up correctly, look at your list of activities again. See what you might have missed or what times you need to fix.

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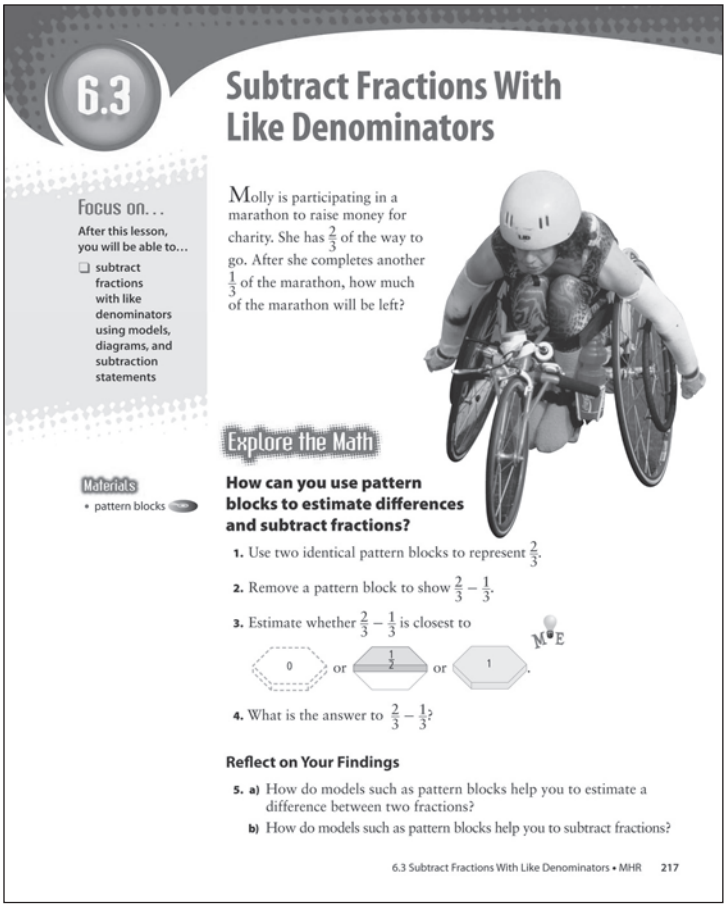
Math Link

It is necessary for students to do this Math Link in order to complete the Math Link on page 221 and the Wrap It Up! on page 225. Students will use a table to record all the things that they do in a 24-h period. Give students examples of activities to include: going to school, doing homework, sleeping, time with friends, eating, personal grooming, after-school activities, sports, watching television, reading, listening to music, talking on the phone, or emailing and chatting on the computer. Students then express these activities as fractions of a day. For example, 2 h doing chores is represented as $\frac{2}{24}$. Students must then make sure their fractions add up to one whole day. Once students have completed their tables, you may wish to post their work in the classroom.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 216 provides students with a fractions activity that relates to their own lives. It is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 225.	• You may want to provide students with the table from the Math Link. Give them a copy of **BLM 6–6 Section 6.2 Math Link**. • Assist students who have trouble grouping their activities into hour units. Suggest that they do not need to be overly precise about timing; the main focus of the activity is fractions, not exactly how they spend each minute of their day. • Observe students as they work on the Math Link, making sure that they are able to show hours in the fraction form and add the fractions so that they equal 1. • Have students discuss with a classmate why the fractions must add up to 1. Circulate, checking for student understanding. • Encourage students to check each other's calculations and discuss any answers that are discrepant. • Students who are having difficulty getting started could use **BLM 6–6 Section 6.2 Math Link**, which provides scaffolding for this activity.

6.3 Subtract Fractions With Like Denominators



6.3

Subtract Fractions With Like Denominators

Focus on...

After this lesson, you will be able to...

- subtract fractions with like denominators using models, diagrams, and subtraction statements

Molly is participating in a marathon to raise money for charity. She has $\frac{2}{3}$ of the way to go. After she completes another $\frac{1}{3}$ of the marathon, how much of the marathon will be left?

Explore the Math

Materials
- pattern blocks

How can you use pattern blocks to estimate differences and subtract fractions?

1. Use two identical pattern blocks to represent $\frac{2}{3}$.
2. Remove a pattern block to show $\frac{2}{3} - \frac{1}{3}$.
3. Estimate whether $\frac{2}{3} - \frac{1}{3}$ is closest to 0 or $\frac{1}{2}$ or 1.
4. What is the answer to $\frac{2}{3} - \frac{1}{3}$?

Reflect on Your Findings

5. a) How do models such as pattern blocks help you to estimate a difference between two fractions?
   b) How do models such as pattern blocks help you to subtract fractions?

6.3 Subtract Fractions With Like Denominators • MHR 217

Suggested Timing
80–100 minutes

Materials
- pattern blocks
- fraction strips
- coloured pencils
- ruler

Blackline Masters
Master 13 Pattern Blocks
Master 14 Fraction Circles
Master 15 Fraction Strips
BLM 6–1 Chapter 6 Self-Assessment
BLM 6–7 Section 6.3 Extra Practice
BLM 6–8 Section 6.3 Math Link

Mathematical Processes

- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization

Specific Outcomes

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Warm-Up

For #1 and #2, use diagrams to help you add.

1. $\frac{2}{6} + \frac{3}{6}$ **2.** $\frac{2}{9} + \frac{3}{9}$

For #3 and #4, write your answer in lowest terms.

3. $\frac{5}{12} + \frac{5}{12}$ **4.** $\frac{1}{4} + \frac{3}{4}$

5. Compare 59%, 0.62, and $\frac{61}{100}$. Write them in order from least to greatest.

Mental Math

6. Estimate 142 out of 180 as a percent.

7. Which is larger: 50% of 202 or 25% of 400?

8. Estimate the area of the parallelogram: $b = 8.2$ m, $h = 3.9$ m

9. Estimate the area of the triangle: $b = 4.1$ m, $h = 6.9$ m

10. Use estimation to place the decimal point in 3851 cm: 282 cm + 24.6 cm + 78.5 cm = 3851 cm

Answers

Warm-Up

1. $\frac{5}{6}$ **2.** $\frac{5}{9}$ **3.** $\frac{10}{12} = \frac{5}{6}$ **4.** $\frac{4}{4} = 1$

5. $59\% = 0.59$, $\frac{61}{100} = 0.61$, 0.62

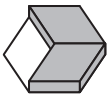
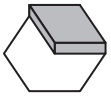
6. 50% of $180 = 90$
25% of $180 = 45$
10% of $180 = 18$
5% of $180 = 9$
$80\% = 90 + 45 + 9 = 144$ Too high
$75\% = 45 \times 3 = 135$ Too low
The answer is between 75% and 80%.

7. 50% of $202 = 101$
25% of $400 = 100$
50% of 202 is larger.

8. $8 \times 4 = 32\ m^2$

9. $4 \times 7 \div 2 = 14\ m^2$ **10.** 385.1 cm

Explore the Math

1.  **2.**  **3.** $\frac{1}{2}$ **4.** $\frac{1}{3}$

5. a) Answers may vary. For example: Models such as pattern blocks allow you to see a range of possible solutions.

b) Answers may vary. For example: Models such as pattern blocks allow you to see the answers visually.

Show You Know: Example 1

a) $\frac{1}{2}$ **b)** 0

Supported Learning

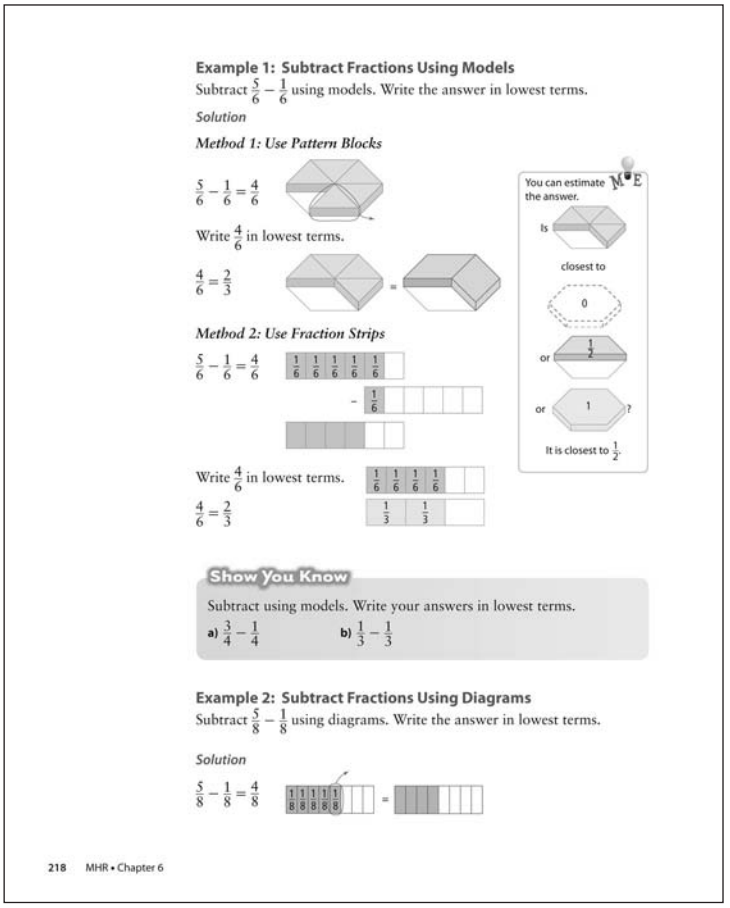
Learning Style

- Use plenty of visual examples and hands-on learning to help students learn the concepts.
- When using diagrams to demonstrate subtraction, begin by having students physically cut out the quantity to be subtracted from the diagram to reinforce the concept of subtraction.

ESL

- Students may need help understanding the word *charity*.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss how models help in estimating and in subtracting fractions.	• Discuss that is it useful to estimate before subtracting so that you can then compare your answer to the estimate to check that the answer is reasonable.



Example 1: Subtract Fractions Using Models

Subtract $\frac{5}{6} - \frac{1}{6}$ using models. Write the answer in lowest terms.

Solution

Method 1: Use Pattern Blocks

$\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$

Write $\frac{4}{6}$ in lowest terms.

$\frac{4}{6} = \frac{2}{3}$

You can estimate the answer.

is closest to 0, $\frac{1}{2}$, or 1?

It is closest to $\frac{1}{2}$.

Method 2: Use Fraction Strips

$\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$

Write $\frac{4}{6}$ in lowest terms.

$\frac{4}{6} = \frac{2}{3}$

Show You Know

Subtract using models. Write your answers in lowest terms.

a) $\frac{3}{4} - \frac{1}{4}$ **b)** $\frac{1}{3} - \frac{1}{3}$

Example 2: Subtract Fractions Using Diagrams

Subtract $\frac{5}{8} - \frac{1}{8}$ using diagrams. Write the answer in lowest terms.

Solution

$\frac{5}{8} - \frac{1}{8} = \frac{4}{8}$

218 MHR • Chapter 6

Activity Planning Notes

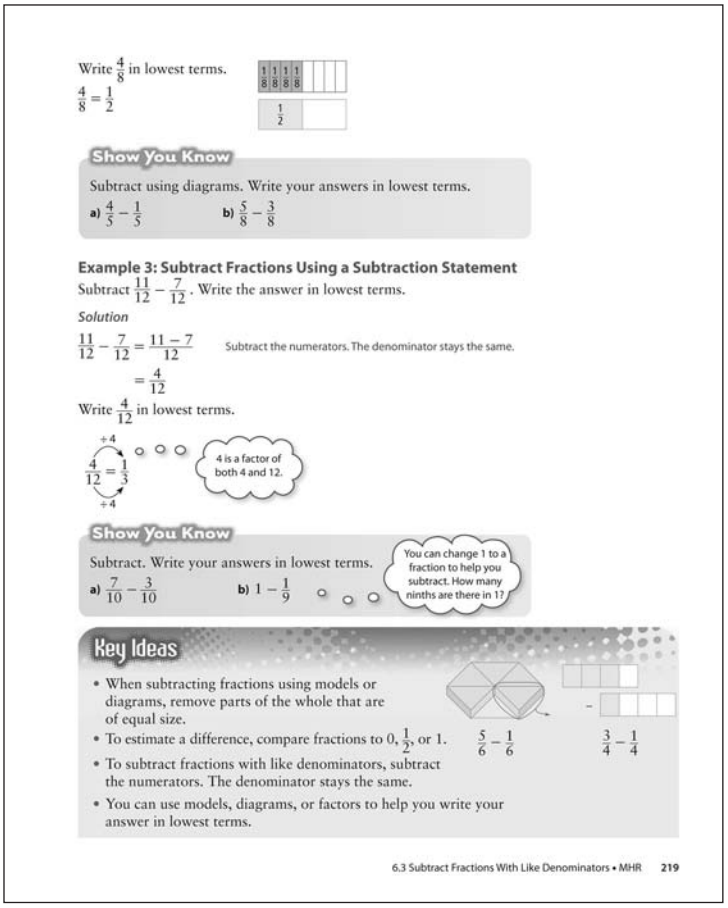
Have students brainstorm examples of a fraction of something being subtracted, for example, a slice taken from a 12-slice pizza that only has 7 slices left: $\frac{7}{12} - \frac{1}{12}$.

Explore the Math

Remind students that modelling the subtraction of fractions requires a part of the fraction to be taken away.

For Example 1, have students estimate an answer before determining the actual answer.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• Have students use both methods to discover which is more useful for them in understanding subtraction. • You may wish to provide additional subtraction questions for students who will benefit from them: **a)** $\frac{2}{6} - \frac{1}{6}$ (Students practise using pattern blocks. $\frac{1}{6}$) **b)** $\frac{7}{8} - \frac{1}{8}$ (Students practise using fraction strips. $\frac{6}{8} = \frac{3}{4}$)



Write $\frac{4}{8}$ in lowest terms.

$\frac{4}{8} = \frac{1}{2}$

Show You Know

Subtract using diagrams. Write your answers in lowest terms.

a) $\frac{4}{5} - \frac{1}{5}$ b) $\frac{5}{8} - \frac{3}{8}$

Example 3: Subtract Fractions Using a Subtraction Statement

Subtract $\frac{11}{12} - \frac{7}{12}$. Write the answer in lowest terms.

Solution

$\frac{11}{12} - \frac{7}{12} = \frac{11-7}{12}$ Subtract the numerators. The denominator stays the same.

$= \frac{4}{12}$

Write $\frac{4}{12}$ in lowest terms.

$\frac{4}{12} = \frac{1}{3}$ (+4, ÷4)

4 is a factor of both 4 and 12.

Show You Know

Subtract. Write your answers in lowest terms.

a) $\frac{7}{10} - \frac{3}{10}$ b) $1 - \frac{1}{9}$

You can change 1 to a fraction to help you subtract. How many ninths are there in 1?

Key Ideas

- When subtracting fractions using models or diagrams, remove parts of the whole that are of equal size.
- To estimate a difference, compare fractions to 0, $\frac{1}{2}$, or 1. $\frac{5}{6} - \frac{1}{6}$ $\frac{3}{4} - \frac{1}{4}$
- To subtract fractions with like denominators, subtract the numerators. The denominator stays the same.
- You can use models, diagrams, or factors to help you write your answer in lowest terms.

6.3 Subtract Fractions With Like Denominators • MHR 219

For Example 2, remind students that to represent the subtraction of fractions, all diagrams need to be the same size.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• You may wish to provide extra subtraction questions to students who will benefit from them: **a)** $\frac{4}{7} - \frac{1}{7}$ (Students first do a question that does not require them to rewrite the fraction in lowest terms. $\frac{3}{7}$) **b)** $\frac{8}{9} - \frac{2}{9}$ (Students must rewrite the answer in lowest terms. $\frac{6}{9} = \frac{2}{3}$)

Move on to Example 3 once all students understand the concept of subtracting fractions with models and diagrams. Say to students, "In the first line of the solution, why does the numerator show $11 - 7$ over the denominator of 12?" Remind students that when subtracting fractions with like denominators the numerators are subtracted but the denominators stay the same.

Answers

Show You Know: Example 2

a) $\frac{3}{5}$ **b)** $\frac{1}{4}$

Show You Know: Example 3

a) $\frac{2}{5}$ **b)** $\frac{8}{9}$

Supported Learning

Learning Style

- Allow kinesthetic and concrete learners to use pattern blocks and fraction strips to model the questions in Example 3 and in the Practise.
- Some students may prefer to complete the questions using **Master 15 Fractions Strips**, **Master 13 Pattern Blocks**, and/or **Master 14 Fraction Circles**.

ESL

- For Example 3, students may need assistance understanding the term *subtraction statement*.

Web Link

To give students more practice with fractions, including subtracting fractions, go to **www.mathlinks7.ca** and follow the links.

Assessment *for* Learning	Supported Learning
Example 3 Have students do the Show You Know related to Example 3.	• Have students work with a partner to complete and discuss the math. • Check students' work to make sure that the steps are being followed correctly and that all work is shown. • You may wish to provide extra subtraction questions to students who will benefit from them: **a)** $\frac{9}{11} - \frac{3}{11}$ (Students first do a question that does not require them to rewrite the fraction in lowest terms. $\frac{6}{11}$) **b)** $\frac{4}{9} - \frac{1}{9}$ (Students must rewrite the answer in lowest terms. $\frac{3}{9} = \frac{1}{3}$) Coach students through a), and then have them try b) on their own.

Answers

Communicate the Ideas

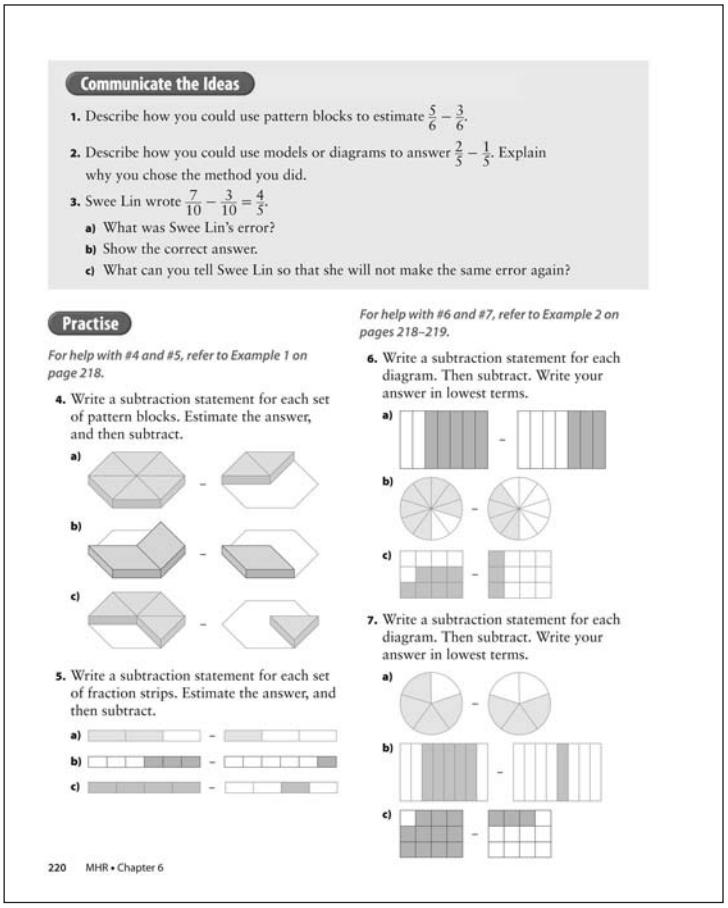
1. Answers may vary. For example: Compare two green triangles to the blue rhombus.

2. $\frac{1}{5}$. Methods may vary.

3. a) Answers may vary. For example: She should have kept the denominator as 10.

b) $\frac{4}{10} = \frac{2}{5}$

c) Answers may vary. For example: When subtracting fractions with like denominators the denominator remains the same, and when rewriting a fraction in lowest terms both the numerator and denominator must be divided by the same number.



Communicate the Ideas

1. Describe how you could use pattern blocks to estimate $\frac{5}{6} - \frac{3}{6}$.
2. Describe how you could use models or diagrams to answer $\frac{2}{5} - \frac{1}{5}$. Explain why you chose the method you did.
3. Swee Lin wrote $\frac{7}{10} - \frac{3}{10} = \frac{4}{5}$.
   a) What was Swee Lin's error?
   b) Show the correct answer.
   c) What can you tell Swee Lin so that she will not make the same error again?

Practise

For help with #4 and #5, refer to Example 1 on page 218.

4. Write a subtraction statement for each set of pattern blocks. Estimate the answer, and then subtract.
   a)
   b)
   c)
5. Write a subtraction statement for each set of fraction strips. Estimate the answer, and then subtract.
   a)
   b)
   c)

For help with #6 and #7, refer to Example 2 on pages 218–219.

6. Write a subtraction statement for each diagram. Then subtract. Write your answer in lowest terms.
   a)
   b)
   c)
7. Write a subtraction statement for each diagram. Then subtract. Write your answer in lowest terms.
   a)
   b)
   c)

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Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students complete #1 and #2, and then discuss #3 as a class.	• Encourage students to consider their own learning style when answering #2.

Assessment *for* Learning	Supported Learning
Practise Have students do #4, #6, and #8. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #4 will need to be coached through #5a). Then have them complete the rest of #5 on their own. • Students who have problems with #6 will need to be coached through #7b). Then have them complete the rest of #7 on their own. • Students who have problems with #8 will need to be coached through #9a). Then have them complete the rest of #9 on their own.

Key Ideas

Have students write the steps for subtracting in their journal, using their own words and their own example.

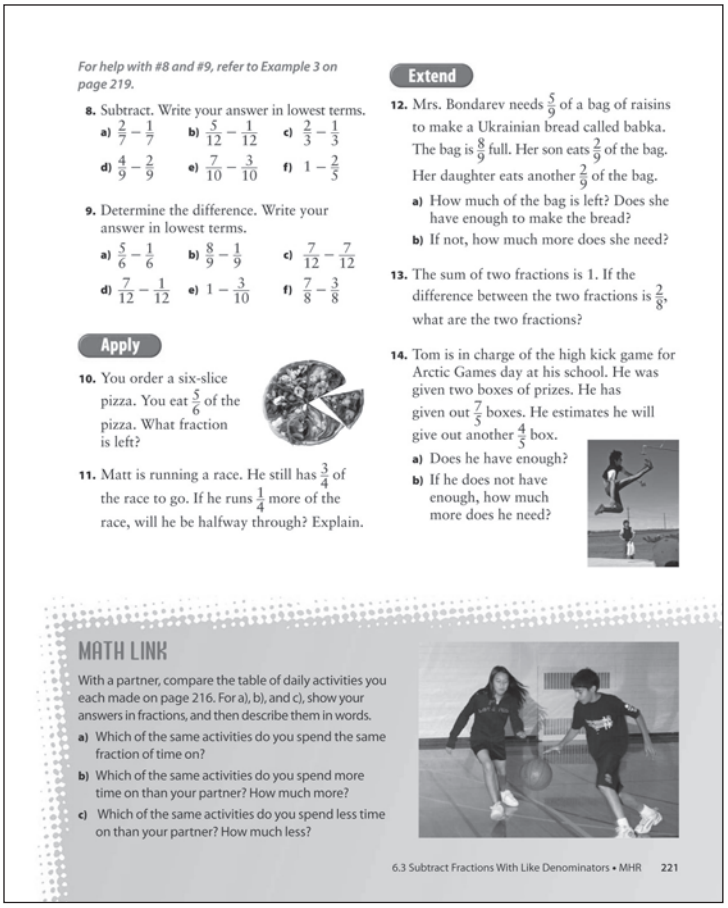
Communicate the Ideas

The Communicate the Ideas is intended to allow students to express their understanding of estimating fractions, subtracting fractions with like denominators, and writing answers in lowest terms.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8, 11, Math Link
Typical	1–4, 6, 8, 10, 11, Math Link
Extension/Enrichment	1–3, 10, 13–15

Practise

Students should have a choice of which manipulatives they would like to use to answer the questions in this section. Check to make sure that students show all work for each question that they complete. Good habits now will make it easier for them to work with algebra questions later on.



For help with #8 and #9, refer to Example 3 on page 219.

8. Subtract. Write your answer in lowest terms.

a) $\frac{2}{7} - \frac{1}{7}$ b) $\frac{5}{12} - \frac{1}{12}$ c) $\frac{2}{3} - \frac{1}{3}$

d) $\frac{4}{9} - \frac{2}{9}$ e) $\frac{7}{10} - \frac{3}{10}$ f) $1 - \frac{2}{5}$

9. Determine the difference. Write your answer in lowest terms.

a) $\frac{5}{6} - \frac{1}{6}$ b) $\frac{8}{9} - \frac{1}{9}$ c) $\frac{7}{12} - \frac{7}{12}$

d) $\frac{7}{12} - \frac{1}{12}$ e) $1 - \frac{3}{10}$ f) $\frac{7}{8} - \frac{3}{8}$

Apply

10. You order a six-slice pizza. You eat $\frac{5}{6}$ of the pizza. What fraction is left?

11. Matt is running a race. He still has $\frac{3}{4}$ of the race to go. If he runs $\frac{1}{4}$ more of the race, will he be halfway through? Explain.

Extend

12. Mrs. Bondarev needs $\frac{5}{9}$ of a bag of raisins to make a Ukrainian bread called babka. The bag is $\frac{8}{9}$ full. Her son eats $\frac{2}{9}$ of the bag. Her daughter eats another $\frac{2}{9}$ of the bag.

a) How much of the bag is left? Does she have enough to make the bread?

b) If not, how much more does she need?

13. The sum of two fractions is 1. If the difference between the two fractions is $\frac{2}{8}$, what are the two fractions?

14. Tom is in charge of the high kick game for Arctic Games day at his school. He was given two boxes of prizes. He has given out $\frac{7}{5}$ boxes. He estimates he will give out another $\frac{4}{5}$ box.

a) Does he have enough?

b) If he does not have enough, how much more does he need?

MATH LINK

With a partner, compare the table of daily activities you each made on page 216. For a), b), and c), show your answers in fractions, and then describe them in words.

a) Which of the same activities do you spend the same fraction of time on?

b) Which of the same activities do you spend more time on than your partner? How much more?

c) Which of the same activities do you spend less time on than your partner? How much less?

6.3 Subtract Fractions With Like Denominators • MHR 221

Apply and Extend

Have students come up with questions similar to the ones in the Apply and share them with classmates.

For #13, you may wish to have students work in groups. Have each group explain the strategies that they used to solve the problem.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 221 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 225.	• Observe students as they work on the Math Link, making sure that they are able to compare fractions and that they understand they must subtract fractions in order to determine how much more or less time they spend on activities than their partner. • Students who are having difficulty getting started could use **BLM 6–8 Section 6.3 Math Link**, which provides scaffolding for this activity.

Supported Learning

Learning Style and Memory

- You may wish to provide **BLM 6–7 Section 6.3 Extra Practice** to students who require more practice.

ESL and Language

- Have students work on the Math Link with partners who have a good understanding of the terminology.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students reflect on two or three items that they have improved on and how they think they have improved.	• Work with students to develop a plan for dealing with the areas in which they are having difficulty. • Depending on students' learning style, have them provide oral or written answers. • Have students review the part related to section 6.3 in **BLM 6–1 Chapter 6 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

MATH LINK

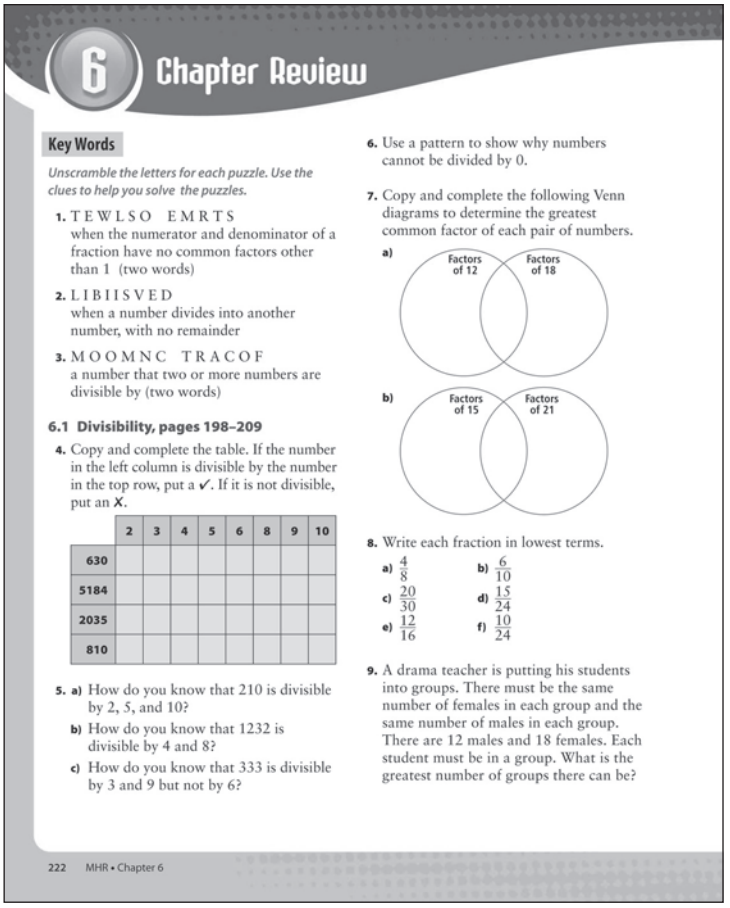
In this activity, students analyse how they spend their time over a 24-h period, using the table that they created in the section 6.2 Math Link on page 216. Through comparisons and fraction subtraction, students draw conclusions about their daily activities.

6 Chapter Review

Suggested Timing
40–50 minutes

Materials
- coloured pencils
- pattern blocks
- fraction strips

Blackline Masters
Master 13 Pattern Blocks
Master 14 Fraction Circles
Master 15 Fraction Strips
BLM 6–1 Chapter 6 Self-Assessment
BLM 6–3 Section 6.1 Extra Practice
BLM 6–5 Section 6.2 Extra Practice
BLM 6–7 Section 6.3 Extra Practice



6 Chapter Review

Key Words

Unscramble the letters for each puzzle. Use the clues to help you solve the puzzles.

1. TEWLSO EMRTS
when the numerator and denominator of a fraction have no common factors other than 1 (two words)

2. LIBIISVED
when a number divides into another number, with no remainder

3. MOOMNC TRACOF
a number that two or more numbers are divisible by (two words)

6.1 Divisibility, pages 198–209

4. Copy and complete the table. If the number in the left column is divisible by the number in the top row, put a ✓. If it is not divisible, put an X.

	2	3	4	5	6	8	9	10
630								
5184								
2035								
810								

5. a) How do you know that 210 is divisible by 2, 5, and 10?
b) How do you know that 1232 is divisible by 4 and 8?
c) How do you know that 333 is divisible by 3 and 9 but not by 6?

6. Use a pattern to show why numbers cannot be divided by 0.

7. Copy and complete the following Venn diagrams to determine the greatest common factor of each pair of numbers.
a) Factors of 12 / Factors of 18
b) Factors of 15 / Factors of 21

8. Write each fraction in lowest terms.
a) $\frac{4}{8}$ b) $\frac{6}{10}$ c) $\frac{20}{30}$ d) $\frac{15}{24}$ e) $\frac{12}{16}$ f) $\frac{10}{24}$

9. A drama teacher is putting his students into groups. There must be the same number of females in each group and the same number of males in each group. There are 12 males and 18 females. Each student must be in a group. What is the greatest number of groups there can be?

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Supported Learning

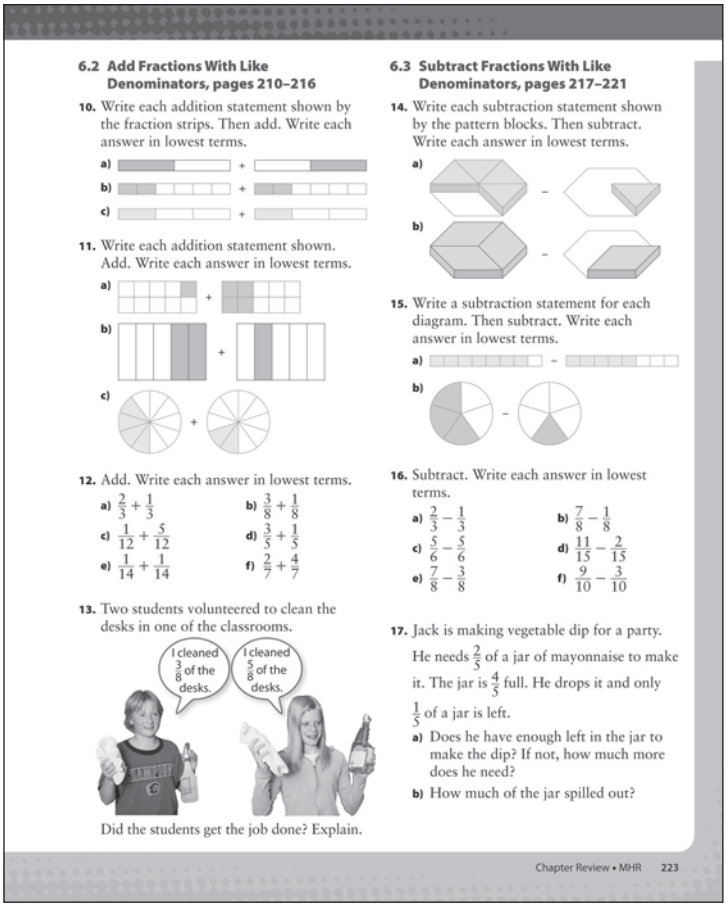
Learning Style and Memory
- Students who require more practice on a particular topic may refer to **BLM 6–3 Section 6.1 Extra Practice**, **BLM 6–5 Section 6.2 Extra Practice**, and **BLM 6–7 Section 6.3 Extra Practice**.

Activity Planning Notes

For #1 to #3, allow students to review the Key Words in pairs. One student could read the definitions and the other student could decipher the scrambled words.

Have students place the numbers 4 to 17 in two columns in their notebooks. Have them look at the question related to the number in their student resource. Students then use the colours that they used on **BLM 6–1 Chapter 6 Self-Assessment** to circle the questions they fully understand, need help with, or do not yet understand at all.

Assessment *for* Learning	Supported Learning
Chapter 6 Review The Chapter 6 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Tell students to check the contents of the What I Need to Work On section of their chapter Foldable. Have students do at least one question related to any concept, skill, or process that has been giving them trouble. • Prior to working on the chapter test, have students revisit any section that they are having difficulty with.



6.2 Add Fractions With Like Denominators, pages 210–216

10. Write each addition statement shown by the fraction strips. Then add. Write each answer in lowest terms.

a)
b)
c)

11. Write each addition statement shown. Add. Write each answer in lowest terms.

a)
b)
c)

12. Add. Write each answer in lowest terms.

a) $\frac{2}{3} + \frac{1}{3}$
b) $\frac{3}{8} + \frac{1}{8}$
c) $\frac{1}{12} + \frac{5}{12}$
d) $\frac{3}{5} + \frac{1}{5}$
e) $\frac{1}{14} + \frac{1}{14}$
f) $\frac{2}{7} + \frac{4}{7}$

13. Two students volunteered to clean the desks in one of the classrooms.

Did the students get the job done? Explain.

6.3 Subtract Fractions With Like Denominators, pages 217–221

14. Write each subtraction statement shown by the pattern blocks. Then subtract. Write each answer in lowest terms.

a)
b)

15. Write a subtraction statement for each diagram. Then subtract. Write each answer in lowest terms.

a)
b)

16. Subtract. Write each answer in lowest terms.

a) $\frac{2}{3} - \frac{1}{3}$
b) $\frac{7}{8} - \frac{1}{8}$
c) $\frac{5}{6} - \frac{5}{6}$
d) $\frac{11}{15} - \frac{2}{15}$
e) $\frac{7}{8} - \frac{3}{8}$
f) $\frac{9}{10} - \frac{3}{10}$

17. Jack is making vegetable dip for a party. He needs $\frac{2}{5}$ of a jar of mayonnaise to make it. The jar is $\frac{4}{5}$ full. He drops it and only $\frac{1}{5}$ of a jar is left.

a) Does he have enough left in the jar to make the dip? If not, how much more does he need?
b) How much of the jar spilled out?

Supported Learning

Learning Style

- Allow students to complete the chapter review using any combination of oral descriptions, diagrams, and written answers.
- Provide students who need them with pattern blocks, fraction strips, **Master 13 Pattern Blocks**, **Master 14 Fraction Circles**, and/or **Master 15 Fraction Strips** to use as they work on the questions in the review.

ESL, Language, and Memory

- Allow students to practise the vocabulary terms using flash cards. Have students work together to quiz each other.

Motor

- Students who have motor challenges may find copying the table in #4 to be a difficult task. Allow them to create the table using a computer.

Gifted and Enrichment

- Some students may already be familiar with the skills handled in this review. To provide additional questions, go to **www.mathlinks7.ca** and follow the links.

Assessment *as* Learning	Supported Learning
Math Learning Log Once students have completed the Chapter 6 Review, have them reflect on their progress and answer the following questions: • How well did I do? • What could I have done differently? • Is there anything I still need help with? How will I get this help?	• Have students use the What I Need to Work On section of their chapter Foldable to provide information about what they continue to have problems with and what problems they had that have now been resolved. • You may wish to have students refer to **BLM 6–1 Chapter 6 Self-Assessment** when they report on how well they understand the chapter. • There are many different ways to write fractions in lowest terms and to add and subtract fractions with like denominators. Encourage students to use the methods that they feel most comfortable with, but to try other methods. They might find that some methods work better for some types of questions and others for other types. Suggest to students that they record in their chapter Foldable the method that they found most useful for each type of question and that they refer back to these notes as necessary.

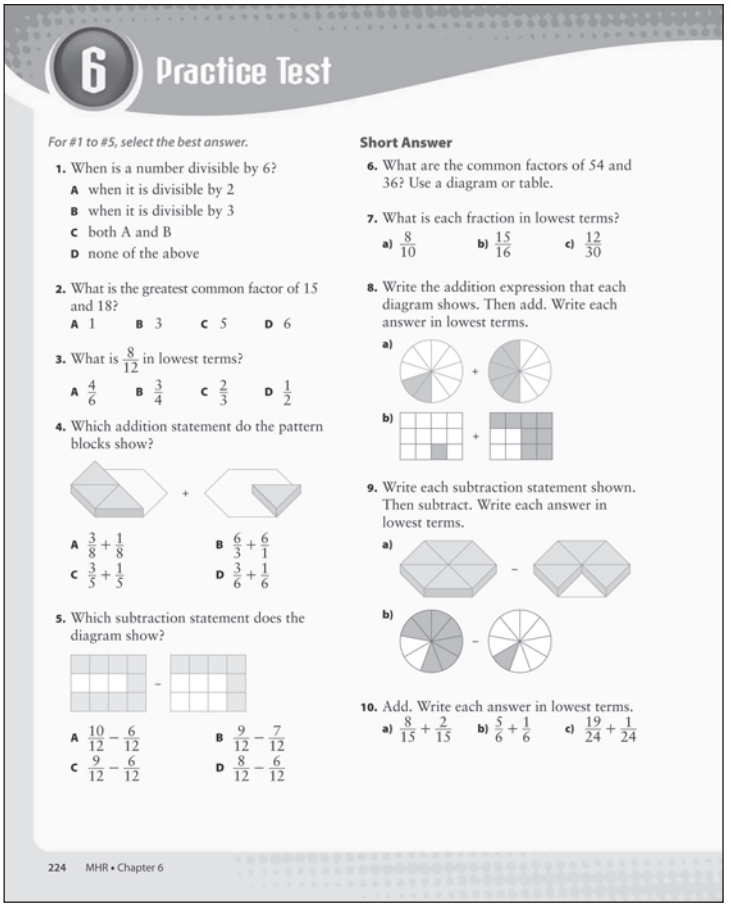
6 Practice Test

Suggested Timing
40–50 minutes

Materials
- coloured pencils
- ruler
- pattern blocks
- fraction strips

Blackline Masters
Master 13 Pattern Blocks
Master 14 Fraction Circles
Master 15 Fraction Strips
BLM 6–1 Chapter 6 Self-Assessment
BLM 6–9 Chapter 6 Test

Assessment *as* Learning	Supported Learning
Chapter 6 Self-Assessment Have students review their earlier responses on **BLM 6–1 Chapter 6 Self-Assessment**.	• Have students use their responses on the Chapter 6 Practice Test and work that they completed earlier in the chapter to complete the After column of this self-assessment. Before the chapter test, coach them in the areas in which they are having problems.



6 Practice Test

For #1 to #5, select the best answer.

1. When is a number divisible by 6?
   A when it is divisible by 2
   B when it is divisible by 3
   C both A and B
   D none of the above

2. What is the greatest common factor of 15 and 18?
   A 1 B 3 C 5 D 6

3. What is $\frac{8}{12}$ in lowest terms?
   A $\frac{4}{6}$ B $\frac{3}{4}$ C $\frac{2}{3}$ D $\frac{1}{2}$

4. Which addition statement do the pattern blocks show?
   A $\frac{3}{8}+\frac{1}{8}$ B $\frac{6}{3}+\frac{6}{1}$
   C $\frac{3}{5}+\frac{1}{5}$ D $\frac{3}{6}+\frac{1}{6}$

5. Which subtraction statement does the diagram show?
   A $\frac{10}{12}-\frac{6}{12}$ B $\frac{9}{12}-\frac{7}{12}$
   C $\frac{9}{12}-\frac{6}{12}$ D $\frac{8}{12}-\frac{6}{12}$

Short Answer

6. What are the common factors of 54 and 36? Use a diagram or table.

7. What is each fraction in lowest terms?
   a) $\frac{8}{10}$ b) $\frac{15}{16}$ c) $\frac{12}{30}$

8. Write the addition expression that each diagram shows. Then add. Write each answer in lowest terms.
   a)
   b)

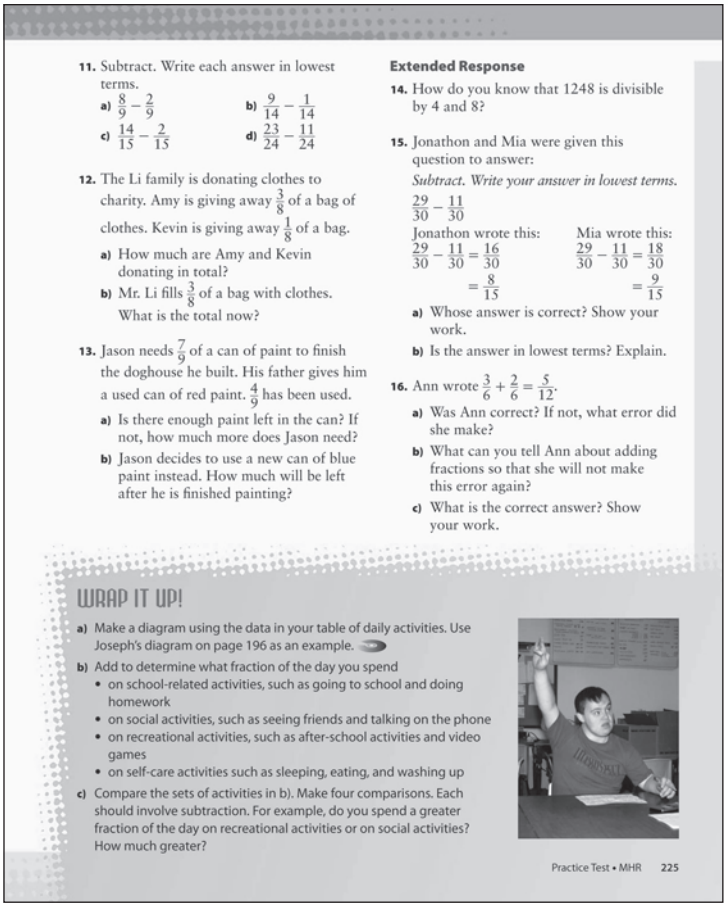
9. Write each subtraction statement shown. Then subtract. Write each answer in lowest terms.
   a)
   b)

10. Add. Write each answer in lowest terms.
    a) $\frac{8}{15}+\frac{2}{15}$ b) $\frac{5}{6}+\frac{1}{6}$ c) $\frac{19}{24}+\frac{1}{24}$

224 MHR • Chapter 6

Study Guide

Question(s)	Section(s)	Refer to	I can …
1, 14	6.1	Explore the Math Example 1	✓ determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9, 10
2, 6	6.1	Example 2	✓ determine factors of a number using the divisibility rules ✓ use an organizer to sort a set of numbers by divisibility
3, 7, 15	6.1	Example 3	✓ write a fraction in lowest terms by identifying the common factor between the numerator and denominator
4, 8	6.2	Explore the Math Examples 1, 2	✓ add fractions using models and diagrams ✓ write the answer in lowest terms
5, 9	6.3	Explore the Math Examples 1, 2	✓ subtract fractions using models and diagrams ✓ write the answer in lowest terms
10, 12, 16	6.2	Example 3	✓ add fractions using an addition statement ✓ solve a problem involving addition of fractions ✓ write the answer in lowest terms
11, 13, 15	6.3	Example 3	✓ subtract fractions using a subtraction statement ✓ solve a problem involving subtraction of fractions ✓ write the answer in lowest terms



11. Subtract. Write each answer in lowest terms.
a) $\frac{8}{9} - \frac{2}{9}$ b) $\frac{9}{14} - \frac{1}{14}$
c) $\frac{14}{15} - \frac{2}{15}$ d) $\frac{23}{24} - \frac{11}{24}$

12. The Li family is donating clothes to charity. Amy is giving away $\frac{3}{8}$ of a bag of clothes. Kevin is giving away $\frac{1}{8}$ of a bag.
a) How much are Amy and Kevin donating in total?
b) Mr. Li fills $\frac{3}{8}$ of a bag with clothes. What is the total now?

13. Jason needs $\frac{7}{9}$ of a can of paint to finish the doghouse he built. His father gives him a used can of red paint. $\frac{4}{9}$ has been used.
a) Is there enough paint left in the can? If not, how much more does Jason need?
b) Jason decides to use a new can of blue paint instead. How much will be left after he is finished painting?

Extended Response

14. How do you know that 1248 is divisible by 4 and 8?

15. Jonathon and Mia were given this question to answer:
Subtract. Write your answer in lowest terms.
$\frac{29}{30} - \frac{11}{30}$
Jonathon wrote this: $\frac{29}{30} - \frac{11}{30} = \frac{16}{30} = \frac{8}{15}$
Mia wrote this: $\frac{29}{30} - \frac{11}{30} = \frac{18}{30} = \frac{9}{15}$
a) Whose answer is correct? Show your work.
b) Is the answer in lowest terms? Explain.

16. Ann wrote $\frac{3}{6} + \frac{2}{6} = \frac{5}{12}$.
a) Was Ann correct? If not, what error did she make?
b) What can you tell Ann about adding fractions so that she will not make this error again?
c) What is the correct answer? Show your work.

WRAP IT UP!

a) Make a diagram using the data in your table of daily activities. Use Joseph's diagram on page 196 as an example.
b) Add to determine what fraction of the day you spend
- on school-related activities, such as going to school and doing homework
- on social activities, such as seeing friends and talking on the phone
- on recreational activities, such as after-school activities and video games
- on self-care activities such as sleeping, eating, and washing up

c) Compare the sets of activities in b). Make four comparisons. Each should involve subtraction. For example, do you spend a greater fraction of the day on recreational activities or on social activities? How much greater?

Practice Test • MHR 225

Supported Learning

Learning Style

- Allow students to use pattern blocks, fractions strips, **Master 13 Pattern Blocks**, **Master 14 Fraction Circles**, and/or **Master 15 Fraction Strips** to complete the practice test.

ESL

- Note that #13 requires a good understanding of the English language. Avoid this question for new English language learners.

Activity Planning Notes

Have students write numbers 1 to 16 in their notebook. They will then circle each question in the colours that they used on **BLM 6–1 Chapter 6 Self-Assessment** to identify which questions they understand fully, need help with, or do not yet understand at all. Have them first complete the work that they know they can do, then complete the work that they know something about, and finally, do their best on the work that they are still struggling with.

This practice test can be assigned as an in-class or take-home assignment. These are the minimum questions that will meet the related curriculum outcomes: #1–#5, #7–#13.

Answers to the Chapter 6 Practice Test are provided on **BLM 6-13 Chapter 6 *MathLinks 7* Student Resource Answers**.

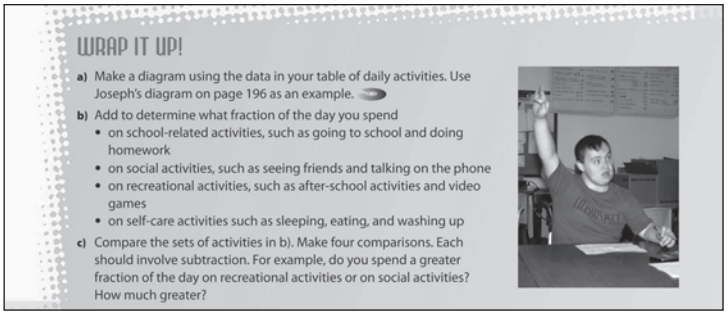
Assessment *of* Learning	Supported Learning
Chapter 6 Test After students complete the practice test, you may wish to use **BLM 6–9 Chapter 6 Test** as a summative assessment.	• You might wish to allow students to use their chapter Foldable. • Consider using the Math Games on page 226 or the Challenge in Real Life on page 227 to assess the knowledge and skills of students who have difficulty with tests.

Wrap It Up!

Suggested Timing
40–50 minutes

Materials
- coloured pencils
- ruler

Blackline Masters
Master 1 Project Rubric
Master 2 Two Stars and One Wish
BLM 6–4 Section 6.1 Math Link
BLM 6–6 Section 6.2 Math Link
BLM 6–8 Section 6.3 Math Link
BLM 6–10 24-Hour Fraction Circle
BLM 6–11 Chapter 6 Wrap It Up!


WRAP IT UP!

a) Make a diagram using the data in your table of daily activities. Use Joseph's diagram on page 196 as an example.

b) Add to determine what fraction of the day you spend
- on school-related activities, such as going to school and doing homework
- on social activities, such as seeing friends and talking on the phone
- on recreational activities, such as after-school activities and video games
- on self-care activities such as sleeping, eating, and washing up

c) Compare the sets of activities in b). Make four comparisons. Each should involve subtraction. For example, do you spend a greater fraction of the day on recreational activities or on social activities? How much greater?

Specific Outcomes

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Activity Planning Notes

You may find it necessary to adapt or add to the categories of activities that are suggested in the student resource, depending on how students spend their day. For example, you may wish to include a "chores" category.

Brainstorm with students what makes a good diagram. Set criteria with students before having them start their fraction circles; refer to the rubric on page 225a for ideas. Once they are finished, have students assess each other's work according to the criteria you have agreed on. Provide students with **Master 2 Two Stars and One Wish** for recording these assessments. Consider allowing students who have performed poorly to resubmit a completed project.

Display the diagrams around the room or have students place them in their portfolios. You may want to have a class discussion about the similarities and differences among the diagrams.

Assessment *of* Learning	Supported Learning
Wrap It Up! This chapter problem wrap-up is intended to allow students to apply and use the work from the previous Math Links to draw conclusions and make comparisons about their own daily activities. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this Wrap It Up! Page 225a provides notes on how to use this rubric for this Wrap It Up!	• You may wish to provide to students **BLM 6–10 24-Hour Fraction Circle** to assist them with their diagrams. • Have students review the work that they have completed in the Math Links in sections 6.1, 6.2, and 6.3 before they begin. • If students have not completed the Math Links earlier in the chapter, you may wish to provide them with **BLM 6–4 Section 6.1 Math Link**, **BLM 6–6 Section 6.2 Math Link**, and **BLM 6–8 Section 6.3 Math Link**. • Some students might benefit from using **BLM 6–11 Chapter 6 Wrap It Up!**, which provides scaffolding for the chapter problem wrap-up. • Observe how accurately students draw their diagrams and add, compare, and subtract fractions.

The chart below shows the **Master 1 Project Rubric** for tasks such as the Wrap It Up! and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution *or* • provides a complete solution with a calculation error that does not affect the overall solution and conclusion
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete solution with an addition error affecting the overall conclusion *or* • provides a complete solution with an error or omission in the comparison *or* • provides a complete solution with answers not written in lowest form
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• includes a diagram that accurately represents the data from the table, and related areas are correctly added • includes comparisons that are incorrect, or comparisons are omitted *or* • includes a diagram that accurately represents the data but errors in addition occur • includes comparisons that are correct for incorrect addition
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• includes a diagram that represents the data • provides no further work *or* • provides a diagram and some addition, but they are not complete
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides a correct start to the diagram

Math Games

Suggested Timing
40–50 minutes

Materials
- 3 six-sided dice per pair or group of students
- paper clip per pair or group of students

Blackline Masters
BLM 6–12 It's Divisible Spinner

Answers

Math Games

1. Answers will vary.
2. Ten will always give a score of 0 points. A number is divisible by 10 if the last digit is 0. You cannot roll 0 with the dice.

Supported Learning

Gifted and Enrichment

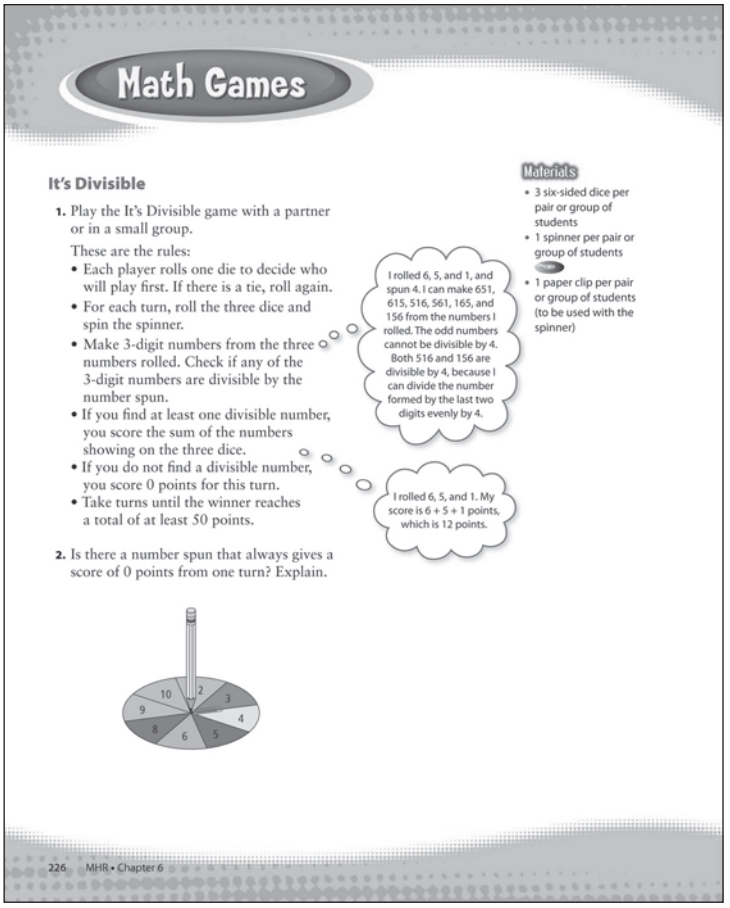
- For scoring, instead of finding the sum, students might find the product of the numbers showing on the three dice. That means if a student finds two divisible numbers, that student's score would be the product multiplied by 2.

Common Errors

- Students may forget the divisibility rules.

R_x Make sure students look for patterns in the dividends and divisors.

Assessment *for* Learning	Supported Learning
It's Divisible Have students play this game with a partner of similar math ability.	• Encourage students to record the three-digit number and its divisor. • After students have played the game one or more times, brainstorm winning strategies, such as knowledge of the divisibility rules. • Have students play with one partner, consider how they might increase their chances of winning, and then play with another partner of similar math ability.


Math Games

It's Divisible

1. Play the It's Divisible game with a partner or in a small group.

These are the rules:
- Each player rolls one die to decide who will play first. If there is a tie, roll again.
- For each turn, roll the three dice and spin the spinner.
- Make 3-digit numbers from the three numbers rolled. Check if any of the 3-digit numbers are divisible by the number spun.
- If you find at least one divisible number, you score the sum of the numbers showing on the three dice.
- If you do not find a divisible number, you score 0 points for this turn.
- Take turns until the winner reaches a total of at least 50 points.

I rolled 6, 5, and 1, and spun 4. I can make 651, 615, 516, 561, 165, and 156 from the numbers I rolled. The odd numbers cannot be divisible by 4. Both 516 and 156 are divisible by 4, because I can divide the number formed by the last two digits evenly by 4.

I rolled 6, 5, and 1. My score is 6 + 5 + 1 points, which is 12 points.

Materials
- 3 six-sided dice per pair or group of students
- 1 spinner per pair or group of students
- 1 paper clip per pair or group of students (to be used with the spinner)

2. Is there a number spun that always gives a score of 0 points from one turn? Explain.

226 MHR • Chapter 6

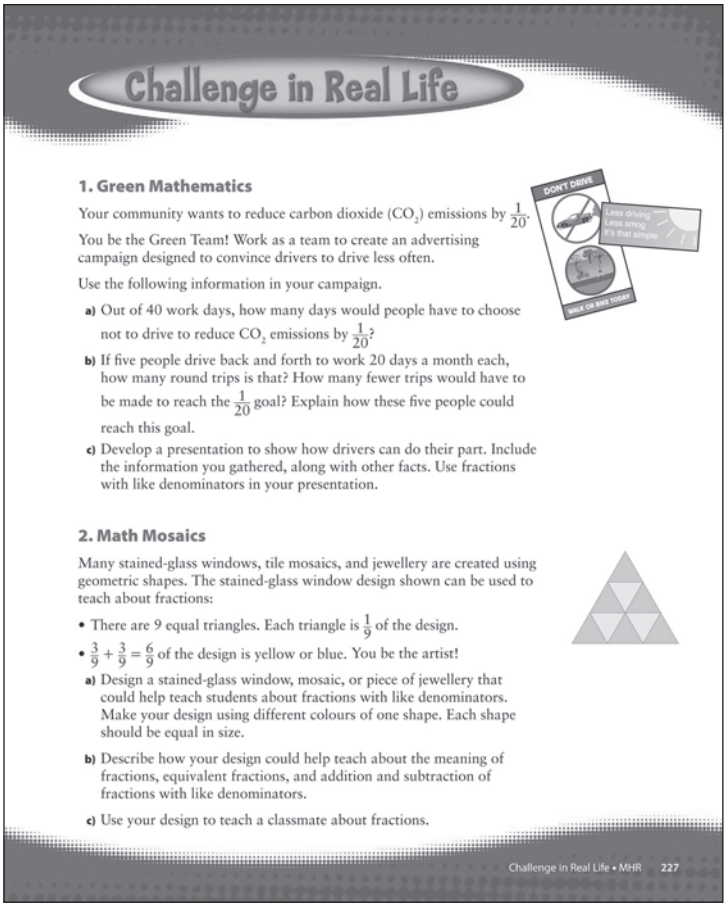
Specific Outcomes

N1 Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0.

Activity Planning Notes

Read through the game with students. Have them use **BLM 6–12 It's Divisible Spinner** and a paper clip to make the spinner. The scoring may be complicated for some students so work through several examples before having them play. As an alternative, players might explain their reasoning aloud for the other students to either confirm or challenge. If the player is incorrect in his or her reasoning, another student challenges the player by explaining the error in reasoning and thus scoring some or all of the points available. If the challenger is incorrect, he or she either loses points or loses a turn. A calculator might be used to settle the challenge. Discuss as a class what would be considered an acceptable explanation.

Challenge in Real Life



Challenge in Real Life

1. Green Mathematics

Your community wants to reduce carbon dioxide (CO_2) emissions by $\frac{1}{20}$.

You be the Green Team! Work as a team to create an advertising campaign designed to convince drivers to drive less often.

Use the following information in your campaign.

a) Out of 40 work days, how many days would people have to choose not to drive to reduce CO_2 emissions by $\frac{1}{20}$?

b) If five people drive back and forth to work 20 days a month each, how many round trips is that? How many fewer trips would have to be made to reach the $\frac{1}{20}$ goal? Explain how these five people could reach this goal.

c) Develop a presentation to show how drivers can do their part. Include the information you gathered, along with other facts. Use fractions with like denominators in your presentation.

2. Math Mosaics

Many stained-glass windows, tile mosaics, and jewellery are created using geometric shapes. The stained-glass window design shown can be used to teach about fractions:

- There are 9 equal triangles. Each triangle is $\frac{1}{9}$ of the design.
- $\frac{3}{9} + \frac{3}{9} = \frac{6}{9}$ of the design is yellow or blue. You be the artist!

a) Design a stained-glass window, mosaic, or piece of jewellery that could help teach students about fractions with like denominators. Make your design using different colours of one shape. Each shape should be equal in size.

b) Describe how your design could help teach about the meaning of fractions, equivalent fractions, and addition and subtraction of fractions with like denominators.

c) Use your design to teach a classmate about fractions.

Challenge in Real Life • MHR 227

Suggested Timing
60–75 minutes

Materials

Green Mathematics
- poster paper
- coloured pencils, paints, and/or markers
- PowerPoint software (optional)
- video equipment (optional)

Math Mosaics
- ruler
- coloured pencils or markers
- construction paper
- fraction blocks (optional)

Blackline Masters

Green Mathematics
Master 1 Project Rubric

Math Mosaics
Master 1 Project Rubric
Master 7 Isometric Dot Paper

Mathematical Processes

- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [x] Technology
- [ ] Visualization

Specific Outcomes

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Activity Planning Notes

Green Mathematics

You may wish to use the following steps to introduce and complete this challenge:

1. Read through Green Mathematics as a class. To introduce this activity, you might wish to talk about government programs aimed at improving air quality. Initiatives such as the one discussed in the challenge support these government programs. Discuss how using public or alternative transportation can reduce the amount of exhaust emissions.
2. Before students complete part c), discuss as a class what could be done to encourage people to participate in a program such as this.

Answers

Green Mathematics

a) $\frac{1}{20} = \frac{2}{40}$. People would have to choose not to drive on 2 of 40 days.

b) $5 \times 20 = 100$ round trips

$\frac{1}{20} = \frac{5}{100}$

It would be necessary to make five fewer round trips. This would be one round trip each. People could meet this target by carpooling or by working at home one day each month.

c) Answers will vary. Make sure that students use fractions in their ads. For example: To reduce smog by $\frac{1}{20}$, carpool one day a month.

Supported Learning

Meeting the Needs of All Learners

- Have students research what levels of emissions come from the vehicles their family members use, such as cars, vans, motorbikes, ATVs, and snowmobiles, and then develop plans for reducing the use of these vehicles. For example, discuss the pros and cons of using a bicycle rather than a van, motorbike, or ATV, and running a dog team versus a snowmobile.

Gifted and Enrichment

- Have students research the dangers of pollution (air quality, respiratory problems) and ways of reducing pollution (lowering the thermostat at night, turning off lights) and use this information in their presentation.

3. Clarify that the task is to
- calculate what would have to be done to produce a $\frac{1}{20}$ emission reduction
- explain how five people could meet this goal
- develop a public relations method to encourage people to participate

4. Review the **Master 1 Project Rubric** with students so that they will know what is expected.

This challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment *for* Learning	Supported Learning
Green Mathematics Discuss the challenge with the class. Have students work individually to answer parts a) and b). As a class, discuss ideas for promoting better air quality. Then have students complete part c) individually.	• Students might wish to review government programs aimed at promoting better air quality and consider how they might use similar ideas to advertise the type of program mentioned here. • Allow students to present their campaign in a variety of forms, such as written form, oral presentation, video, or PowerPoint presentation (limit PowerPoint presentations to six slides). • You may wish to allow students to develop skits to advertise driving reduction.

Assessment *of* Learning	Supported Learning
Green Mathematics Discuss the challenge with the class. Have students work individually to answer parts a) and b). As a class, discuss ideas for promoting better air quality. Then have students complete part c) individually.	• **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this challenge. Page 227a provides notes on how to use this rubric for this challenge. • To view student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as the Challenge in Real Life and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete solution with weak or unclear communication in part c) *or* • provides a solution that does not clearly link the mathematical values from parts a) and b) to the presentation *or* • provides a complete solution to part c) based on an incorrect calculation in either part a) or b)
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides a complete and correct solution to parts a) and b) with clear justification in part b)
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a correct mathematical response to parts a) and b) *or* • provides a complete and correct part b)
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides a correct response to part a)

Supported Learning

Meeting the Needs of All Learners

- Students may wish to use shapes to make cultural designs, such as part of a basket or blanket design.

Gifted and Enrichment

- Encourage students to note how many different-sized triangles (or other shapes) they can find in their design and the fraction of the design's total area each shape makes.
- Have them convert each fraction to a decimal and note any patterns.
- Have students make a second design and compare how the two designs break into different fractions. Encourage them to note both similarities and differences.

Math Mosaics

You may wish to use the following steps to introduce and complete this challenge:

1. Have the class look at the sample stained-glass window and discuss how it might be used to teach the meaning of fractions (3 yellow triangles out of 9 triangles represent $\frac{3}{9}$ of the design), equivalent fractions ($\frac{3}{9} = \frac{1}{3}$), addition of fractions with like denominators ($\frac{3}{9} + \frac{6}{9} = \frac{9}{9} = 1$), and subtraction of fractions with like denominators ($\frac{9}{9} - \frac{6}{9} = \frac{3}{9} = \frac{1}{3}$).
2. Challenge students to make their own design that could be used in a similar way. Students can use shapes of their choice, but should repeat the same shape throughout the visual. You may wish to provide **Master 7 Isometric Dot Paper**.
3. Tell students that as they develop their design they should think about how it might help other students to learn about fractions.
4. Clarify that the task is to
   - draw a design that shows fractions
   - use the design to teach or demonstrate the meaning of fractions, equivalent fractions, and addition and subtraction of fractions with like denominators
   - try the idea on a classmate
5. Review the **Master 1 Project Rubric** with students so that they will know what is expected.

This challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment *for* Learning	Supported Learning
Math Mosaics Discuss the stained-glass window example with the class. Have students work individually to answer parts a) and b). Have them complete part c) with a partner.	• Kinesthetic learners and those who need concrete representations might wish to trace and colour several copies of one of the fraction block shapes on construction paper, cut them out, and then manipulate these until they get a pleasing design. Make sure that they use at least two different colours of construction paper. • Allow students to present their description in part b) in written form or orally.

Assessment *of* Learning	Supported Learning
Math Mosaics Discuss the stained-glass window example with the class. Have students work individually to answer parts a) and b). Have them complete part c) with a partner.	• **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this challenge. Page 227b provides notes on how to use this rubric for this challenge. • To view student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as the Challenge in Real Life and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct response
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete response but one of the following explanations is weak or omitted: – the meaning of fractions – equivalent fractions – addition and subtraction of fractions with like denominators
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• includes a design that accurately represents equal fractions (with like denominators) of a whole, and addition, subtraction, and some equivalent fractions are represented, but explanations and justifications are omitted
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• includes a design that accurately represents equal fractions (with like denominators) of a whole, and both addition and subtraction are accurately represented *or* • includes a design that accurately represents equal fractions (with like denominators) of a whole, and equivalent fractions are accurately represented
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• draws the design but makes no mathematical link