7 Add and Subtract Fractions

General Outcome

- Develop number sense.

Specific Outcomes

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
7.1	✓ find multiples of a number
	✓ use concrete materials to find a common denominator for a set of fractions
	✓ determine a common denominator using multiples
	✓ write equivalent fractions
7.2	✓ use concrete materials and diagrams to add and subtract fractions with unlike denominators
	✓ use a common denominator to add and subtract fractions with unlike denominators
	✓ solve problems involving the addition and/or subtraction of fractions
	✓ check that a solution to a fraction addition or subtraction statement is reasonable
7.3	✓ use concrete materials and diagrams to add mixed numbers with like or unlike denominators
	✓ use an addition statement to add mixed numbers with like or unlike denominators
	✓ solve problems involving the addition of mixed numbers
	✓ check that a solution to a mixed number addition statement is reasonable
7.4	✓ use concrete materials and diagrams to subtract mixed numbers with like or unlike denominators
	✓ use a subtraction statement to subtract mixed numbers with like or unlike denominators
	✓ solve problems involving the subtraction of mixed numbers
	✓ check that a solution to a mixed number subtraction statement is reasonable

Assessment *as* Learning	Supported Learning
Use the Before column of **BLM 7–1 Chapter 7 Self-Assessment** to provide students with the big picture for this chapter and to help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• As students complete each section of the chapter or complete the Chapter 7 Review, have them review the related parts of **BLM 7–1 Chapter 7 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Chapter 7 Planning Chart

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter Opener • 20–30 minutes		BLM 7–1 Chapter 7 Self-Assessment BLM 7–2 Add and Subtract Fractions	• stapler • notebook paper • ruler • scissors • coloured pencils
7.1 Common Denominators • 40–50 minutes	**Essential:** 1, 2, 4 *or* 5, 6 *or* 7, 8 *or* 9, 10, 12, Math Link **Typical:** 1–3, 4 *or* 5, 6 *or* 7, 8 *or* 9, 10–15, 18, Math Link **Extension/Enrichment:** 1–3, 16–21	Master 13 Pattern Blocks Master 15 Fraction Strips BLM 7–1 Chapter 7 Self-Assessment BLM 7–3 Section 7.1 Extra Practice BLM 7–4 Section 7.1 Math Link	• coloured pencils
7.2 Add and Subtract Fractions With Unlike Denominators • 80–100 minutes	**Essential:** 1–4, 6, 9, 11, 16, 17, Math Link **Typical:** 1–4, 6, 9, 11, 14, 16, 17, 18 *or* 19, 22, Math Link **Extension/Enrichment:** 1–3, 15, 18 *or* 19, 20–23	Master 2 Two Stars and One Wish Master 4 Vertical and Horizontal Number Lines Master 5 Tangram Master 13 Pattern Blocks Master 15 Fraction Strips BLM 7–1 Chapter 7 Self-Assessment BLM 7–5 Section 7.2 Extra Practice BLM 7–6 Section 7.2 Math Link	• pattern blocks • coloured pencils
7.3 Add Mixed Numbers • 80–100 minutes	**Essential:** 1–4, 6, 8, 10, 14, 15, 17, Math Link **Typical:** 1–4, 6, 8, 10, 12–15, 17, 19, Math Link **Extension/Enrichment:** 1–3, 17–20	Master 13 Pattern Blocks BLM 7–1 Chapter 7 Self-Assessment BLM 7–7 Section 7.3 Extra Practice BLM 7–8 Section 7.3 Math Link	• coloured pencils • pattern blocks
7.4 Subtract Mixed Numbers • 80–100 minutes	**Essential:** 1–3, 5, 7, 9, 11, 12, Math Link **Typical:** 1–3, 5, 7, 9, 11–13, 15–17, 21, 22, Math Link **Extension/Enrichment:** 1, 2, 13, 14, 16, 18, 20–23	Master 15 Fraction Strips BLM 7–1 Chapter 7 Self-Assessment BLM 7–9 Section 7.4 Extra Practice BLM 7–10 Section 7.4 Math Link	• coloured pencils • calculator (optional)
Chapter 7 Review • 40–50 minutes	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Master 9 0.5 Centimetre Grid Paper Master 13 Pattern Blocks Master 15 Fraction Strips BLM 7–1 Chapter 7 Self-Assessment BLM 7–3 Section 7.1 Extra Practice BLM 7–5 Section 7.2 Extra Practice BLM 7–7 Section 7.3 Extra Practice BLM 7–9 Section 7.4 Extra Practice	• coloured pencils
Chapter 7 Practice Test • 40–50 minutes	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1–8, 10, 11	Master 9 0.5 Centimetre Grid Paper Master 13 Pattern Blocks Master 15 Fraction Strips BLM 7–1 Chapter 7 Self-Assessment BLM 7–11 Chapter 7 Test	• coloured pencils • calculator (optional)
Chapter 7 Wrap It Up! • 60–75 minutes		Master 1 Project Rubric BLM 7–4 Section 7.1 Math Link BLM 7–6 Section 7.2 Math Link BLM 7–8 Section 7.3 Math Link BLM 7–10 Section 7.4 Math Link BLM 7–12 Chapter 7 Wrap It Up!	• materials to create a board game, such as scissors, coloured pencils, cardboard, playing pieces, and so on

Chapter 7 Planning Chart (continued)

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter 7 Math Games • 40–50 minutes		BLM 7–13 Fraction Race Spinners	• six-sided dice (red and blue) • paper clips • coloured pencils or coloured paper
Chapter 7 Challenge in Real Life • 60–75 minutes		Master 1 Project Rubric BLM 7–14 Chapter 7 *MathLinks 7* Student Resource Answers BLM 7–15 Chapter 7 BLM Answers	• sample magazines • 11 × 17 paper • ruler • coloured pencils or markers • scissors • glue or tape • drawing software (optional) • spreadsheet software (optional)

Chapter 7 Assessment Planner

Assessment Options	Type of Assessment	Assessment Tool
Chapter Opener	Assessment *as* Learning (TR pages i, 229)	BLM 7–1 Chapter 7 Self-Assessment Chapter 7 Foldable
7.1 Common Denominators	Assessment *as* Learning (TR pages 232, 233, 235) Assessment *for* Learning (TR pages 232, 234, 236)	Math Learning Log (TR page 235) BLM 7–1 Chapter 7 Self-Assessment
7.2 Add and Subtract Fractions With Unlike Denominators	Assessment *as* Learning (TR pages 238, 241, 244) Assessment *for* Learning (TR pages 240, 241, 244)	Master 2 Two Stars and One Wish Math Learning Log (TR page 244) BLM 7–1 Chapter 7 Self-Assessment
7.3 Add Mixed Numbers	Assessment *for* Learning (TR pages 247, 248, 250, 251) Assessment *as* Learning (TR pages 249, 251)	Math Learning Log (TR page 251) BLM 7–1 Chapter 7 Self-Assessment
7.4 Subtract Mixed Numbers	Assessment *for* Learning (TR pages 254, 256, 258, 259) Assessment *as* Learning (TR pages 257, 259)	Math Learning Log (TR page 259) BLM 7–1 Chapter 7 Self-Assessment
Chapter 7 Review	Assessment *for* Learning (TR page 260) Assessment *as* Learning (TR page 261)	Math Learning Log (TR page 261) BLM 7–1 Chapter 7 Self-Assessment
Chapter 7 Practice Test	Assessment *as* Learning (TR page 262) Assessment *of* Learning (TR page 263)	BLM 7–1 Chapter 7 Self-Assessment BLM 7–11 Chapter 7 Test
Chapter 7 Wrap It Up!	Assessment *of* Learning (TR page 262a)	Master 1 Project Rubric
Chapter 7 Math Games	Assessment *for* Learning (TR page 264)	
Chapter 7 Challenge in Real Life	Assessment *for* Learning (TR page 264a) Assessment *of* Learning (TR page 264a)	Master 1 Project Rubric

You may wish to use one or more of the following materials to help you assess student readiness for Chapter 7.

Assessment *for* Learning	Supported Learning
Method 1: Have students develop a journal entry to explain what they personally know about multiples, fractions, identifying and ordering unit fractions, and equivalent fractions. **Method 2:** Have students complete **BLM 7–2 Add and Subtract Fractions** to check their conceptual understanding. Remind students that you are looking for the scope of their knowledge.	• Students who require reinforcement of prerequisite skills may wish to complete the Get Ready materials available in the *MathLinks 7 Workbook* and at the **www.mathlinks7.ca** book site.

Chapter Opener

Suggested Timing
20–30 minutes

Materials
- stapler
- notebook paper
- ruler
- scissors
- coloured pencils

Blackline Masters
BLM 7–1 Chapter 7 Self-Assessment

Key Words
multiple
common denominator
improper fraction
mixed number

Supported Learning

Meeting the Needs of All Learners

- To introduce the addition and subtraction of fractions, you may wish to bring in food, such as pizza, and explore adding and subtracting fractions before eating the materials. For example, have two pizzas available and ask the class to discuss what fraction of pizza remains when different numbers of slices are removed. This activity will give students an opportunity to look at fractions in a way that might relate to their daily life.
- To catch student interest, you might want to broaden their knowledge of Egypt by reading and discussing Egyptian mythology, particularly about Horus. Have students look for some parallels between Egyptian mythology and other mythologies students are familiar with. Alternatively, you could discuss the ways in which different cultures explain their history and beliefs through myths.

WWW Web Link

Students can explore the Egyptian myth of the Eye of Horus by going to **www.mathlinks7.ca** and following the links.

What's the Math?

In this chapter, students use paper folding, diagrams, and multiples to find a common denominator. Then, they use pattern blocks, fraction strips, diagrams, and a common denominator to add and subtract fractions with unlike denominators. Students add mixed numbers with like and unlike denominators using pattern blocks and diagrams as well as mathematical statements. In the final section of the chapter, students subtract mixed numbers with like and unlike denominators using fraction strips, diagrams, and mathematical statements. Throughout the chapter, students solve real-world problems involving fractions and are exposed to ways in which other cultures have used fractions.

Activity Planning Notes

Have students complete a web showing what they know about fractions operations, including what they learned in Chapter 6. Allow students to use words, diagrams, and numbers to record their ideas. Collect this piece of paper to make you aware of what students may already know or have misconceptions about. You might revisit this activity at the end of the chapter to allow students to add new concepts that they have learned.

Math Link

The Math Link on page 228 is intended to get students thinking about ways in which fractions have been used in different cultures. It provides students with an opportunity to apply specific beliefs of Egyptian culture to fraction operations. Students begin thinking about common denominators and adding fractions as they predict whether the parts of the Eye of Horus combine to make a sum of 1. (Note that students will have the chance to add these fractions on page 244.) The theme of fractions through history and around the world will be carried throughout the chapter and will culminate in the chapter problem wrap-up titled Wrap It Up! on page 263.

FOLDABLES™
Study Tool

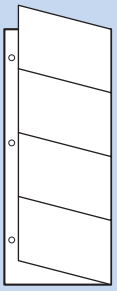
Have students make the Foldable in the student resource to keep track of the information in the chapter.

You may prefer to have students keep track of Key Words using a design specifically for that purpose. Students can make the following Foldable and write vocabulary terms on the front of each tab. Have them use the space beneath the tab to write definitions and provide examples.

Step 1 Fold a sheet of notebook paper in half lengthwise with the crease to the right.

Step 2 Measure the height of the page and draw lines to divide the height into four equal parts. Cut every quarter as far as the fold, creating tabs as you go. This will create four tabs.

Step 3 Label each tab with a math term. Write definitions and give examples underneath the tabs.



Assessment *as* Learning	Supported Learning
Chapter 7 Foldable As students work on each section in Chapter 7, have them keep track of any problems they are having under the What I Need to Work On tab in their chapter Foldable.	• As students complete each section, have them review the list of items they need to work on, and then have them check off any that have been handled.

Supported Learning

ESL and Language

- Students may benefit from having new vocabulary written on study cards and displayed in the classroom on a math word wall.

ESL and Memory

- Students to create their own vocabulary/picture dictionary in their notebook for the Key Words. Matching a picture and/or symbol with each Key Word and its definition may help students consolidate their understanding of the vocabulary.

7.1 Common Denominators

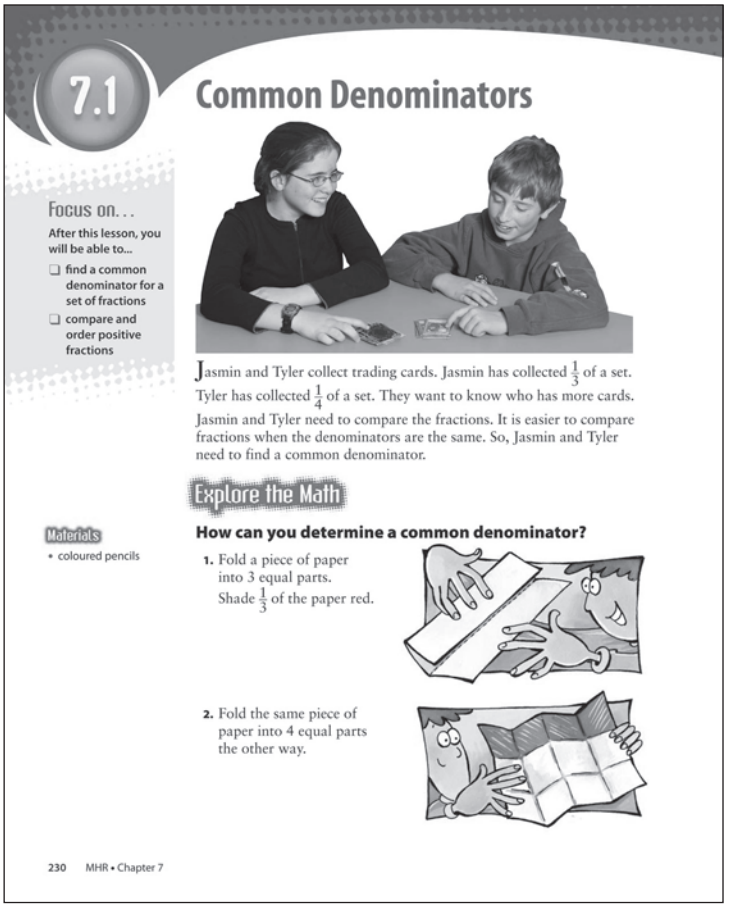
Suggested Timing
40–50 minutes

Materials
- coloured pencils

Blackline Masters
Master 13 Pattern Blocks
Master 15 Fraction Strips
BLM 7–1 Chapter 7 Self-Assessment
BLM 7–3 Section 7.1 Extra Practice
BLM 7–4 Section 7.1 Math Link

Mathematical Processes

- [x] Communication
- [x] Connections
- [ ] Mental Mathematics and Estimation
- [ ] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



7.1 Common Denominators

Focus on...
After this lesson, you will be able to...
- find a common denominator for a set of fractions
- compare and order positive fractions

Jasmin and Tyler collect trading cards. Jasmin has collected $\frac{1}{3}$ of a set. Tyler has collected $\frac{1}{4}$ of a set. They want to know who has more cards. Jasmin and Tyler need to compare the fractions. It is easier to compare fractions when the denominators are the same. So, Jasmin and Tyler need to find a common denominator.

Explore the Math

Materials
- coloured pencils

How can you determine a common denominator?

1. Fold a piece of paper into 3 equal parts. Shade $\frac{1}{3}$ of the paper red.

2. Fold the same piece of paper into 4 equal parts the other way.

230 MHR • Chapter 7

Specific Outcomes

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

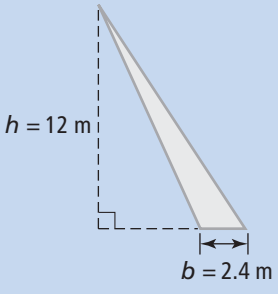
Warm-Up

For #1 to #3, subtract and then write your answer in lowest terms.

1. $\frac{5}{12} - \frac{1}{12}$ **2.** $\frac{7}{8} - \frac{3}{8}$ **3.** $1 - \frac{3}{10}$

4. Change each fraction to a repeating decimal. Use bar notation.
a) $\frac{2}{3}$ **b)** $\frac{2}{9}$

5. Estimate and calculate the area of the triangle shown.



Mental Math

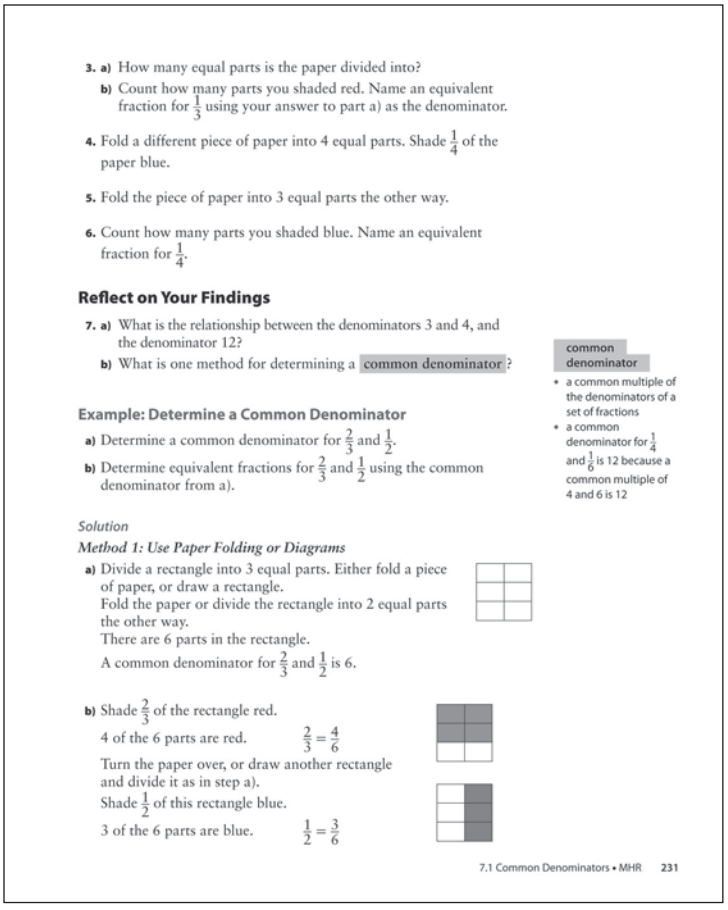
6. Which of these numbers are divisible by 4? How do you know?
48, 96, 196, 248, 353, 1056

7. Which of these numbers are divisible by 9? How do you know?
81, 108, 181, 208, 281, 1063, 9054

8. List the first 10 multiples of 8.

9. List the first 10 multiples of 6.

10. Give two common multiples for 8 and 6.



3. a) How many equal parts is the paper divided into?
b) Count how many parts you shaded red. Name an equivalent fraction for $\frac{1}{3}$ using your answer to part a) as the denominator.

4. Fold a different piece of paper into 4 equal parts. Shade $\frac{1}{4}$ of the paper blue.

5. Fold the piece of paper into 3 equal parts the other way.

6. Count how many parts you shaded blue. Name an equivalent fraction for $\frac{1}{4}$.

Reflect on Your Findings

7. a) What is the relationship between the denominators 3 and 4, and the denominator 12?
b) What is one method for determining a common denominator?

common denominator
- a common multiple of the denominators of a set of fractions
- a common denominator for $\frac{1}{4}$ and $\frac{1}{6}$ is 12 because a common multiple of 4 and 6 is 12

Example: Determine a Common Denominator

a) Determine a common denominator for $\frac{2}{3}$ and $\frac{1}{2}$.
b) Determine equivalent fractions for $\frac{2}{3}$ and $\frac{1}{2}$ using the common denominator from a).

Solution

Method 1: Use Paper Folding or Diagrams

a) Divide a rectangle into 3 equal parts. Either fold a piece of paper, or draw a rectangle.
Fold the paper or divide the rectangle into 2 equal parts the other way.
There are 6 parts in the rectangle.
A common denominator for $\frac{2}{3}$ and $\frac{1}{2}$ is 6.

b) Shade $\frac{2}{3}$ of the rectangle red.
4 of the 6 parts are red. $\frac{2}{3} = \frac{4}{6}$
Turn the paper over, or draw another rectangle and divide it as in step a).
Shade $\frac{1}{2}$ of this rectangle blue.
3 of the 6 parts are blue. $\frac{1}{2} = \frac{3}{6}$

7.1 Common Denominators • MHR 231

Answers

Warm-Up

1. $\frac{4}{12} = \frac{1}{3}$ **2.** $\frac{4}{8} = \frac{1}{2}$ **3.** $\frac{7}{10}$

4. a) $0.\overline{6}$ **b)** $0.\overline{2}$

5. Estimate: $12 \times 2 \div 2 = 12$ m^2
Calculate: $12 \times 2.4 \div 2 = 14.4$ m^2

6. 48, 96, 196, 248, 1056. The number formed by the last two digits is divisible by 2 at least twice.

7. 81, 108, 9054. The sum of the digits is divisible by 9.

8. 8, 16, 24, 32, 40, 48, 56, 64, 72, 80

9. 6, 12, 18, 24, 30, 36, 42, 48, 54, 60

10. 24, 48

Explore the Math

3. a) 12 **b)** 4, $\frac{4}{12}$

6. 3, $\frac{3}{12}$

7. a) $3 \times 4 = 12$
b) Answers may vary. For example: Multiply the denominators of the fractions.

Activity Planning Notes

Have students discuss and describe ways of comparing thirds and fourths before you introduce the topic in this section opener. You may wish to ask students, "When comparing thirds and fourths, how do you know which one is greater? How do you know you are right?"

Have students read the section opener. Explain to students that one way to compare fractions is to find a common denominator and then write and compare equivalent fractions using this denominator. Make sure students understand that they are discussing the fraction of a whole set of trading cards and that the value of the fraction is not changing.

Explore the Math

Have students work through the Explore the Math individually or with a partner. In this activity, make sure students recognize that they can represent the same paper in two ways using different denominators, and that the values of these fractions are equivalent, i.e., $\frac{3}{3} = \frac{4}{4} = \frac{12}{12}$.

Supported Learning

ESL

- Work through the Explore the Math holding up the paper after each step to provide visual support.
- Words that English language learners may need assistance with include *trading cards*, *set*, and *shade*.

Motor

- Students with motor challenges may find it difficult to do the folding and shading activity for finding a common denominator. Consider allowing them the alternative of using virtual manipulatives.

Answers

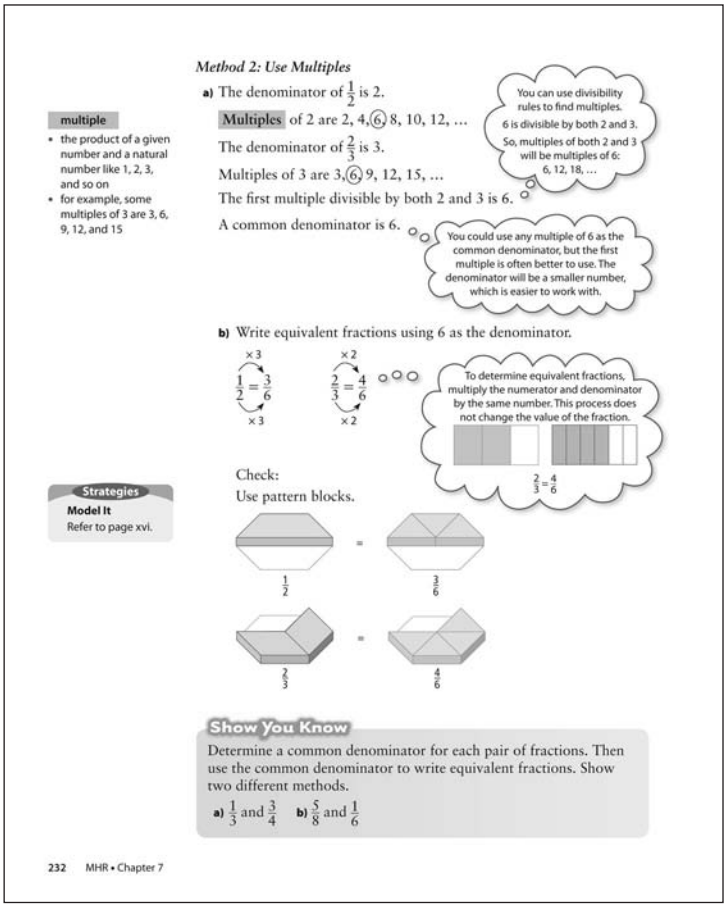
Show You Know: Example

Answers may vary. For example:

a) common denominator: 12. $\frac{1}{3} = \frac{4}{12}$; $\frac{3}{4} = \frac{9}{12}$

b) common denominator: 24. $\frac{5}{8} = \frac{15}{24}$; $\frac{1}{6} = \frac{4}{24}$

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math activity, or read students' responses to #7a) and b). Encourage students to generalize conclusions about their findings during the activity.	• When they see the paper folding, make sure students realize that the 12 parts show the common denominator for 3 and 4. Then get students to predict how many parts will be in a paper if they fold it in two parts one way and five another. Have them do the folding to check their prediction. • Ask students to identify any pattern between the number of folds each way and the number of boxes in the folded paper.



Method 2: Use Multiples

a) The denominator of $\frac{1}{2}$ is 2.

Multiples of 2 are 2, 4, 6, 8, 10, 12, ...

The denominator of $\frac{2}{3}$ is 3.

Multiples of 3 are 3, 6, 9, 12, 15, ...

The first multiple divisible by both 2 and 3 is 6.

A common denominator is 6.

multiple
- the product of a given number and a natural number like 1, 2, 3, and so on
- for example, some multiples of 3 are 3, 6, 9, 12, and 15

You can use divisibility rules to find multiples. 6 is divisible by both 2 and 3. So, multiples of both 2 and 3 will be multiples of 6: 6, 12, 18, ...

You could use any multiple of 6 as the common denominator, but the first multiple is often better to use. The denominator will be a smaller number, which is easier to work with.

b) Write equivalent fractions using 6 as the denominator.

$\frac{1}{2} = \frac{3}{6}$ (×3 above and ×3 below)

$\frac{2}{3} = \frac{4}{6}$ (×2 above and ×2 below)

To determine equivalent fractions, multiply the numerator and denominator by the same number. This process does not change the value of the fraction.

$\frac{2}{3} = \frac{4}{6}$

Check:

Use pattern blocks.

Strategies
Model It
Refer to page xvi.

Show You Know

Determine a common denominator for each pair of fractions. Then use the common denominator to write equivalent fractions. Show two different methods.

a) $\frac{1}{3}$ and $\frac{3}{4}$ **b)** $\frac{5}{8}$ and $\frac{1}{6}$

Supported Learning

Learning Style

- Allow students to find common denominators using their method of choice (when possible).
- Have students use **Master 13 Pattern Blocks** to check their answers for Method 1 part b) in the Example.
- Have students check their answers for the Show You Know using **Master 15 Fraction Strips** or other manipulatives.

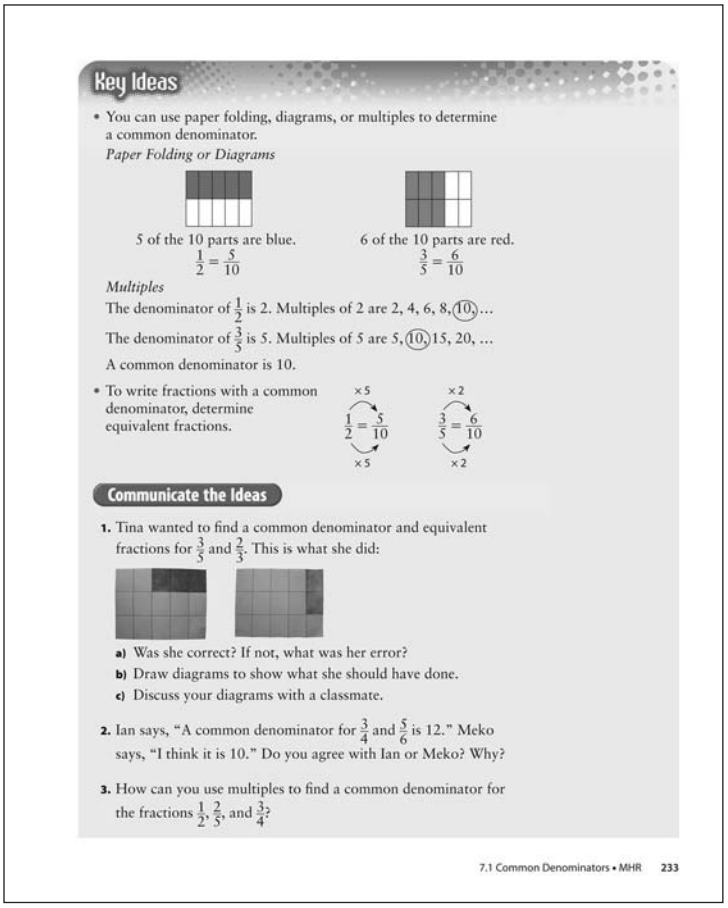
ESL, Language, and Memory

- Some students may need assistance with the term *equivalent fraction*. Ensure they understand that *equivalent* means the same as *equal*. Show what equivalent fractions are using visual examples involving fraction strips, pattern blocks, and number lines.

The Example shows two methods of finding a common denominator. Method 1 demonstrates how to use paper folding and diagrams, and Method 2 shows how to use multiples. Students then determine equivalent fractions using a common denominator.

Students may observe that multiplying the denominators also gives them a common multiple. Explain to students that the product of two numbers is a common multiple, but it may not be the lowest common multiple. Encourage students to find common denominators using a variety of methods. Reinforce to students that when finding an equivalent fraction they must not change the value of the fraction. For example, if the denominator becomes greater, the numerator must become greater by the same operation.

Assessment *for* Learning	Supported Learning
Example Have students complete the Show You Know related to the Example.	• When students complete the Show You Know, check to see which methods of finding common denominators they chose. Alternatively, have students determine which method might be most effective before they attempt each question. Ask students to explain why they chose each method. • Students may benefit from having practice determining equivalent fractions for unit fractions: **a)** $\frac{1}{2}$ and $\frac{1}{3}$ (common denominator: 6. $\frac{1}{2} = \frac{3}{6}$; $\frac{1}{3} = \frac{2}{6}$) **b)** $\frac{1}{5}$ and $\frac{1}{4}$ (common denominator: 20. $\frac{1}{5} = \frac{4}{20}$; $\frac{1}{4} = \frac{5}{20}$) Coach students through part a). Have them do part b) on their own.



Key Ideas

- You can use paper folding, diagrams, or multiples to determine a common denominator.

Paper Folding or Diagrams

5 of the 10 parts are blue.

$\frac{1}{2} = \frac{5}{10}$

6 of the 10 parts are red.

$\frac{3}{5} = \frac{6}{10}$

Multiples

The denominator of $\frac{1}{2}$ is 2. Multiples of 2 are 2, 4, 6, 8, (10), ...

The denominator of $\frac{3}{5}$ is 5. Multiples of 5 are 5, (10), 15, 20, ...

A common denominator is 10.

- To write fractions with a common denominator, determine equivalent fractions.

$\frac{1}{2} = \frac{5}{10}$ (×5 above, ×5 below) $\qquad \frac{3}{5} = \frac{6}{10}$ (×2 above, ×2 below)

Communicate the Ideas

1. Tina wanted to find a common denominator and equivalent fractions for $\frac{3}{5}$ and $\frac{2}{3}$. This is what she did:

a) Was she correct? If not, what was her error?
b) Draw diagrams to show what she should have done.
c) Discuss your diagrams with a classmate.

2. Ian says, "A common denominator for $\frac{3}{4}$ and $\frac{5}{6}$ is 12." Meko says, "I think it is 10." Do you agree with Ian or Meko? Why?

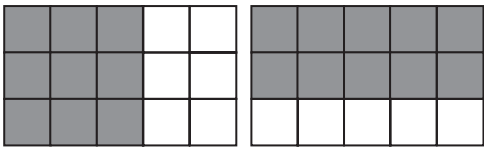
3. How can you use multiples to find a common denominator for the fractions $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{4}$?

Answers

Communicate the Ideas

1. a) Answers will vary. For example: No. Tina should have shaded 9 out of 15 parts red, and 10 out of 15 parts blue.

b)



$\frac{3}{5}$ $\qquad$ $\frac{2}{3}$

c) Answers will vary.

2. Ian is correct. Answers may vary. For example: 10 is not a multiple of 6 or 4.

3. Answers may vary. For example: Some multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20. Some multiples of 5 are 5, 10, 15, and 20. Some multiples of 4 are 4, 8, 12, 16, and 20. The first multiple divisible by 2, 5, and 4 is 20. A common denominator is 20.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 2, 4 *or* 5, 6 *or* 7, 8 *or* 9, 10, 12, Math Link
Typical	1–3, 4 *or* 5, 6 *or* 7, 8 *or* 9, 10–15, 18, Math Link
Extension/Enrichment	1–3, 16–21

Key Ideas

This section explores finding common denominators. Students learn several methods: paper folding, diagrams, and multiples. Once students determine a common denominator, they find equivalent fractions. Have students describe in their chapter Foldable which method of finding a common denominator they prefer and why they prefer that method.

WWW Web Link

To explore comparing fractions and determining equivalent fractions further, students can go to **www.mathlinks7.ca** and follow the links.

Communicate the Ideas

Discuss #1 with the class. Make sure students understand that Tina neglected to convert the fractions to equivalent fractions. Remind students of the importance of maintaining the value of the original fraction.

When students complete #3, ask them to explain how they found a common multiple for three fractions.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students do #1 and #2. Students who have a good grasp of the concepts may be assigned all three questions.	• In #1, if students state that Tina was correct, ask them to explain their thinking. Have students write the fractions shown in the diagram in #1, and ask them to determine whether they are equivalent to the fractions given in the problem. • In #3, some students may benefit from demonstrating all the methods they know and identifying which one is easiest for them to use.

Supported Learning

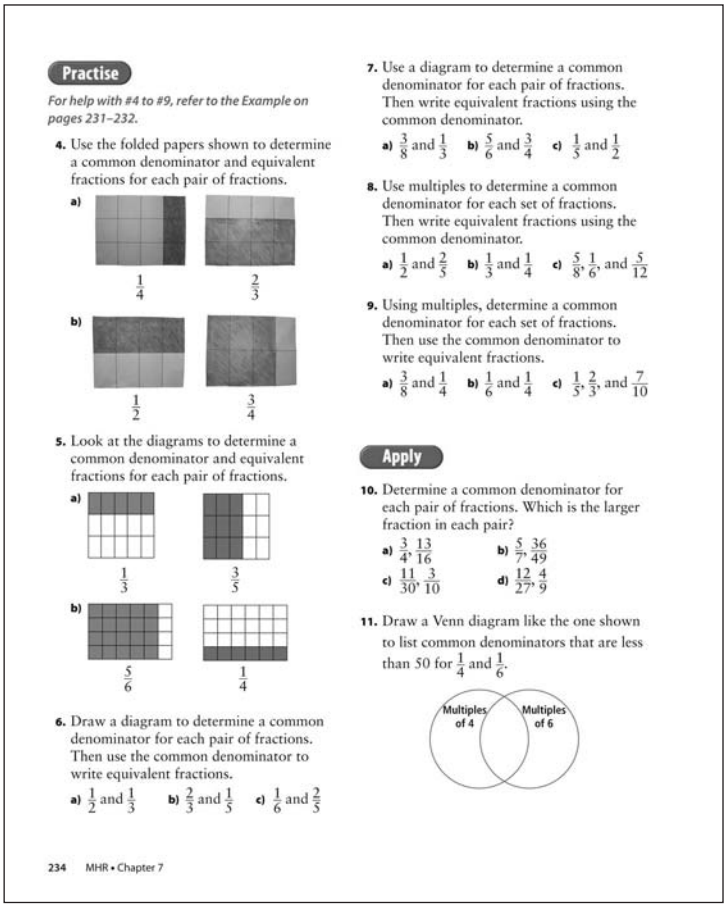
Learning Style

- In #10, if students do not recognize that one denominator is a multiple of the other denominator, you may wish to have students explore the denominators using paper folding and/or diagrams.

Common Errors

- Some students may confuse expressing a fraction in lowest terms with finding a common multiple.

R_x Make sure students can differentiate the meaning of a multiple and a factor. Remind students to read questions carefully and to make sure they understand what they are being asked.



Practise

For help with #4 to #9, refer to the Example on pages 231–232.

4. Use the folded papers shown to determine a common denominator and equivalent fractions for each pair of fractions.

a) $\frac{1}{4}$ $\frac{2}{3}$

b) $\frac{1}{2}$ $\frac{3}{4}$

5. Look at the diagrams to determine a common denominator and equivalent fractions for each pair of fractions.

a) $\frac{1}{3}$ $\frac{3}{5}$

b) $\frac{5}{6}$ $\frac{1}{4}$

6. Draw a diagram to determine a common denominator for each pair of fractions. Then use the common denominator to write equivalent fractions.

a) $\frac{1}{2}$ and $\frac{1}{3}$ b) $\frac{2}{3}$ and $\frac{1}{5}$ c) $\frac{1}{6}$ and $\frac{2}{5}$

7. Use a diagram to determine a common denominator for each pair of fractions. Then write equivalent fractions using the common denominator.

a) $\frac{3}{8}$ and $\frac{1}{3}$ b) $\frac{5}{6}$ and $\frac{3}{4}$ c) $\frac{1}{5}$ and $\frac{1}{2}$

8. Use multiples to determine a common denominator for each set of fractions. Then write equivalent fractions using the common denominator.

a) $\frac{1}{2}$ and $\frac{2}{5}$ b) $\frac{1}{3}$ and $\frac{1}{4}$ c) $\frac{5}{8}$, $\frac{1}{6}$, and $\frac{5}{12}$

9. Using multiples, determine a common denominator for each set of fractions. Then use the common denominator to write equivalent fractions.

a) $\frac{3}{8}$ and $\frac{1}{4}$ b) $\frac{1}{6}$ and $\frac{1}{4}$ c) $\frac{1}{5}$, $\frac{2}{3}$, and $\frac{7}{10}$

Apply

10. Determine a common denominator for each pair of fractions. Which is the larger fraction in each pair?

a) $\frac{3}{4}$, $\frac{13}{16}$ b) $\frac{5}{7}$, $\frac{36}{49}$

c) $\frac{11}{30}$, $\frac{3}{10}$ d) $\frac{12}{27}$, $\frac{4}{9}$

11. Draw a Venn diagram like the one shown to list common denominators that are less than 50 for $\frac{1}{4}$ and $\frac{1}{6}$.

Multiples of 4 | Multiples of 6

234 MHR • Chapter 7

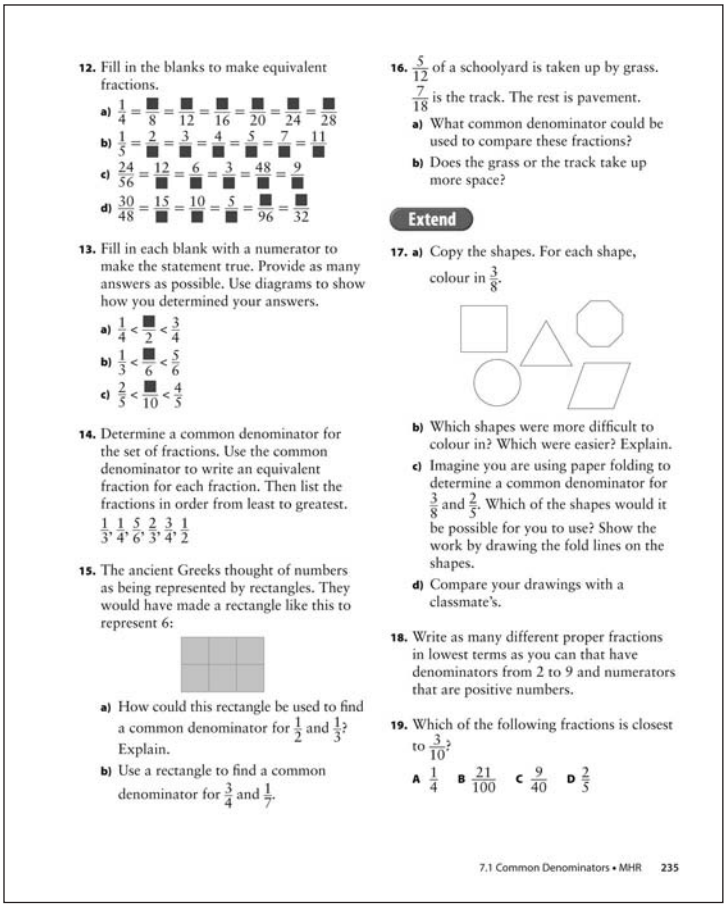
Practise

Have students work in groups to discuss methods of finding common denominators.

Parts c) of #8 and #9 ask students to find a common denominator for three fractions. Explain to students that finding the lowest common denominator will make the arithmetic easier when writing equivalent fractions. In some questions, one denominator is a multiple of another denominator. Encourage students to look for this situation before determining other common multiples for the denominators.

In #11, students may need to be reminded about how to read a Venn diagram. Make sure students list the common multiples of 4 and 6 in the middle of the diagram where the circles overlap.

Assessment *for* Learning	Supported Learning
Practise and Apply Have students do #4, #6, and #8. Students who have no difficulty with these questions can do #10 and then go on to the rest of the Apply questions.	• Students who have problems with #4, #6, and #8 will need additional support. Have these students explain their thinking on these questions. Clarify any misconceptions. Refer them to the Example in the student resource. Ensure that students are comfortable finding equivalent fractions. Remind them that an operation performed on the denominator must be performed on the numerator. Coach students through #5a), #7a), and #9a) using manipulatives. Then have students complete the remaining parts of each question on their own.



12. Fill in the blanks to make equivalent fractions.

a) $\frac{1}{4} = \frac{\blacksquare}{8} = \frac{\blacksquare}{12} = \frac{\blacksquare}{16} = \frac{\blacksquare}{20} = \frac{\blacksquare}{24} = \frac{\blacksquare}{28}$

b) $\frac{1}{5} = \frac{2}{\blacksquare} = \frac{3}{\blacksquare} = \frac{4}{\blacksquare} = \frac{5}{\blacksquare} = \frac{7}{\blacksquare} = \frac{11}{\blacksquare}$

c) $\frac{24}{56} = \frac{12}{\blacksquare} = \frac{6}{\blacksquare} = \frac{3}{\blacksquare} = \frac{48}{\blacksquare} = \frac{9}{\blacksquare}$

d) $\frac{30}{48} = \frac{15}{\blacksquare} = \frac{10}{\blacksquare} = \frac{5}{\blacksquare} = \frac{\blacksquare}{96} = \frac{\blacksquare}{32}$

13. Fill in each blank with a numerator to make the statement true. Provide as many answers as possible. Use diagrams to show how you determined your answers.

a) $\frac{1}{4} < \frac{\blacksquare}{2} < \frac{3}{4}$

b) $\frac{1}{3} < \frac{\blacksquare}{6} < \frac{5}{6}$

c) $\frac{2}{5} < \frac{\blacksquare}{10} < \frac{4}{5}$

14. Determine a common denominator for the set of fractions. Use the common denominator to write an equivalent fraction for each fraction. Then list the fractions in order from least to greatest.

$\frac{1}{3}, \frac{1}{4}, \frac{5}{6}, \frac{2}{3}, \frac{3}{4}, \frac{1}{2}$

15. The ancient Greeks thought of numbers as being represented by rectangles. They would have made a rectangle like this to represent 6:

a) How could this rectangle be used to find a common denominator for $\frac{1}{2}$ and $\frac{1}{3}$? Explain.

b) Use a rectangle to find a common denominator for $\frac{3}{4}$ and $\frac{1}{7}$.

16. $\frac{5}{12}$ of a schoolyard is taken up by grass. $\frac{7}{18}$ is the track. The rest is pavement.

a) What common denominator could be used to compare these fractions?

b) Does the grass or the track take up more space?

Extend

17. a) Copy the shapes. For each shape, colour in $\frac{3}{8}$.

b) Which shapes were more difficult to colour in? Which were easier? Explain.

c) Imagine you are using paper folding to determine a common denominator for $\frac{3}{8}$ and $\frac{2}{5}$. Which of the shapes would it be possible for you to use? Show the work by drawing the fold lines on the shapes.

d) Compare your drawings with a classmate's.

18. Write as many different proper fractions in lowest terms as you can that have denominators from 2 to 9 and numerators that are positive numbers.

19. Which of the following fractions is closest to $\frac{3}{10}$?

A $\frac{1}{4}$ B $\frac{21}{100}$ C $\frac{9}{40}$ D $\frac{2}{5}$

7.1 Common Denominators • MHR 235

Supported Learning

Learning Style and Memory

- You may wish to provide **BLM 7–3 Section 7.1 Extra Practice** to students who require additional practice.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • In what ways are finding common denominators and expressing fractions in lowest terms similar? In what ways are they different?	• Have students describe at least two methods of finding a common denominator. Tell students to use diagrams when appropriate in their explanations. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • You may wish to have students fill in the During column for section 7.1 of **BLM 7–1 Chapter 7 Self-Assessment**. Have students indicate how they might improve any items that they have marked either red or yellow.

Apply and Extend

In #12, have students discuss or describe any patterns they notice when writing the equivalent fractions.

Students are provided with a real-world context in #16. Suggest that they use diagrams to help explain their answer.

Encourage students who are having difficulty with #17c) to cut pieces of paper into each shape and actually do the folding.

In #19, students compare five fractions. Encourage students to consider their strategy prior to attempting this question. They will likely need to find a common denominator for all five fractions and then write the equivalent fractions.

In #20, students might use the fact that oil and water don't "mix" to help them determine which beaker would give a reading of the amount of liquid inside.

Answers

Math Link

a) 64

b) $\frac{1}{2} = \frac{32}{64}, \frac{1}{4} = \frac{16}{64}, \frac{1}{8} = \frac{8}{64}, \frac{1}{16} = \frac{4}{64}, \frac{1}{32} = \frac{2}{64}, \frac{1}{64}$

Supported Learning

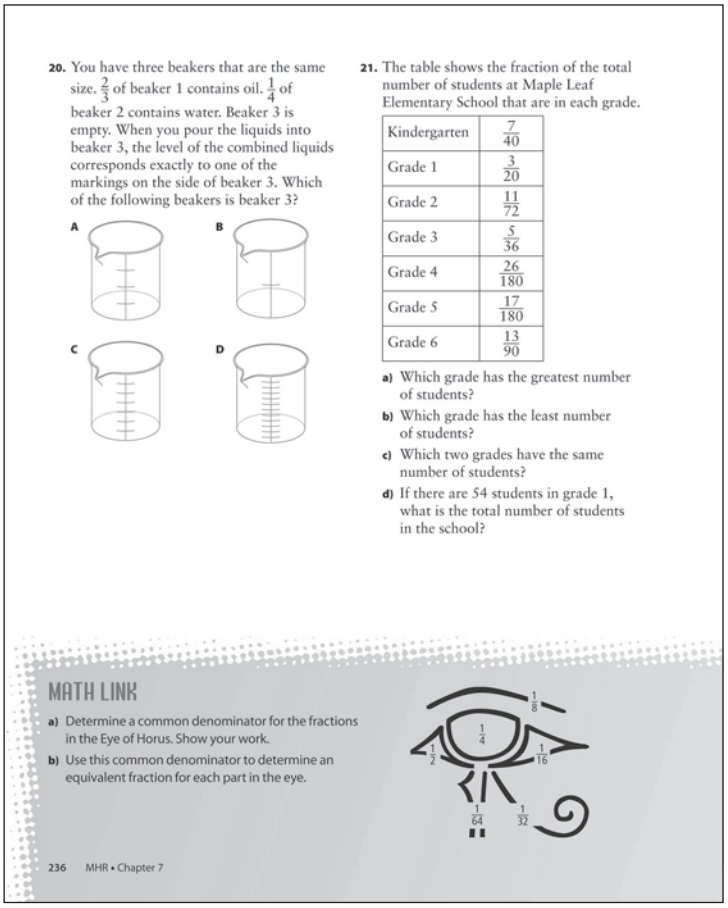
Learning Style, ESL, Language, and Memory

- Have students complete the Math Link to help them prepare for the Wrap It Up! on page 263.

Common Errors

- Some students may find it difficult to determine a common denominator for the Eye of Horus.

R_x Assist students in seeing that each of the denominators is a multiple of 64. Refer them back to Method 2 of the Example, which shows how to use multiples to find a common denominator.



20. You have three beakers that are the same size. $\frac{2}{5}$ of beaker 1 contains oil. $\frac{1}{4}$ of beaker 2 contains water. Beaker 3 is empty. When you pour the liquids into beaker 3, the level of the combined liquids corresponds exactly to one of the markings on the side of beaker 3. Which of the following beakers is beaker 3?

A B C D

21. The table shows the fraction of the total number of students at Maple Leaf Elementary School that are in each grade.

Kindergarten	$\frac{7}{40}$
Grade 1	$\frac{3}{20}$
Grade 2	$\frac{11}{72}$
Grade 3	$\frac{5}{36}$
Grade 4	$\frac{26}{180}$
Grade 5	$\frac{17}{180}$
Grade 6	$\frac{13}{90}$

a) Which grade has the greatest number of students?

b) Which grade has the least number of students?

c) Which two grades have the same number of students?

d) If there are 54 students in grade 1, what is the total number of students in the school?

MATH LINK

a) Determine a common denominator for the fractions in the Eye of Horus. Show your work.

b) Use this common denominator to determine an equivalent fraction for each part in the eye.

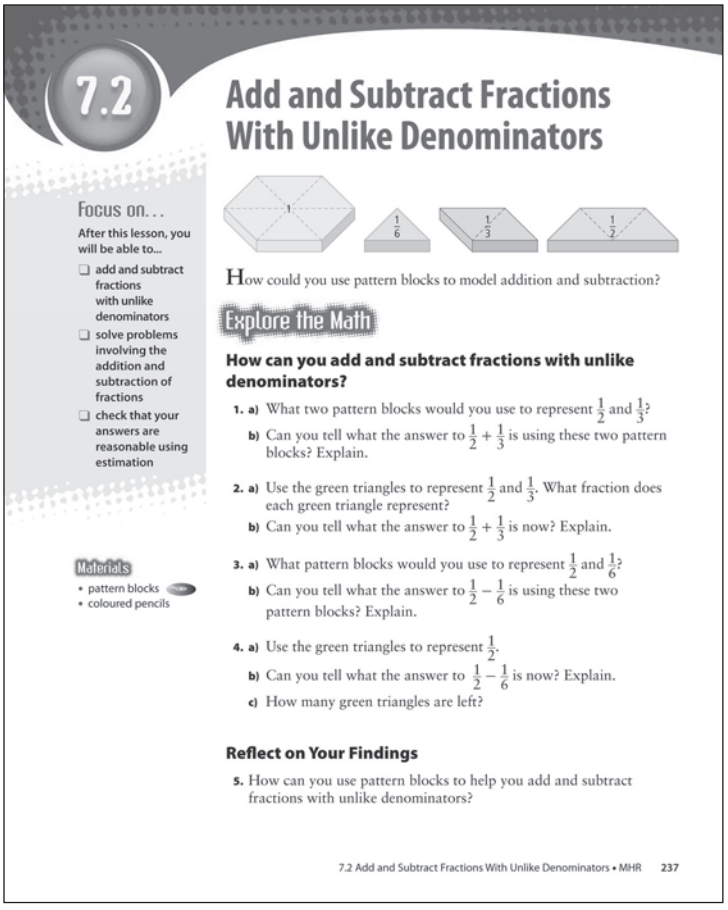
236 MHR • Chapter 7

Math Link

This Math Link is part of the chapter problem introduced in the Math Link box on page 228. It provides students with an opportunity to explain and demonstrate their understanding of finding common denominators and equivalent fractions as it applies to the Eye of Horus.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 236 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 263.	• For part b), you may wish to have students show their work by labelling the parts of the eye with the equivalent fractions; they can draw a copy of the eye themselves or use **BLM 7-4 Section 7.1 Math Link**, which includes a visual of the Eye of Horus. • After students determine equivalent fractions, you may wish to have them write the fractions in order from least to greatest. • Refer students who are having difficulty getting started to **BLM 7–4 Section 7.1 Math Link**, which provides scaffolding for this activity.

7.2 Add and Subtract Fractions With Unlike Denominators



7.2

Add and Subtract Fractions With Unlike Denominators

Focus on...

After this lesson, you will be able to...

- add and subtract fractions with unlike denominators
- solve problems involving the addition and subtraction of fractions
- check that your answers are reasonable using estimation

Materials

- pattern blocks
- coloured pencils

How could you use pattern blocks to model addition and subtraction?

Explore the Math

How can you add and subtract fractions with unlike denominators?

1. a) What two pattern blocks would you use to represent $\frac{1}{2}$ and $\frac{1}{3}$?
   b) Can you tell what the answer to $\frac{1}{2} + \frac{1}{3}$ is using these two pattern blocks? Explain.
2. a) Use the green triangles to represent $\frac{1}{2}$ and $\frac{1}{3}$. What fraction does each green triangle represent?
   b) Can you tell what the answer to $\frac{1}{2} + \frac{1}{3}$ is now? Explain.
3. a) What pattern blocks would you use to represent $\frac{1}{2}$ and $\frac{1}{6}$?
   b) Can you tell what the answer to $\frac{1}{2} - \frac{1}{6}$ is using these two pattern blocks? Explain.
4. a) Use the green triangles to represent $\frac{1}{2}$.
   b) Can you tell what the answer to $\frac{1}{2} - \frac{1}{6}$ is now? Explain.
   c) How many green triangles are left?

Reflect on Your Findings

5. How can you use pattern blocks to help you add and subtract fractions with unlike denominators?

7.2 Add and Subtract Fractions With Unlike Denominators • MHR 237

Suggested Timing
80–100 minutes

Materials
- pattern blocks
- coloured pencils

Blackline Masters
Master 2 Two Stars and One Wish
Master 4 Vertical and Horizontal Number Lines
Master 5 Tangram
Master 13 Pattern Blocks
Master 15 Fraction Strips
BLM 7–1 Chapter 7 Self-Assessment
BLM 7–5 Section 7.2 Extra Practice
BLM 7–6 Section 7.2 Math Link

Mathematical Processes

- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization

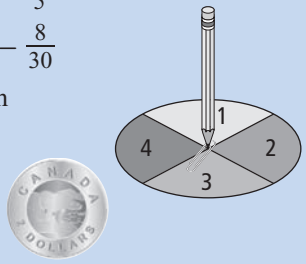
Specific Outcomes

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Warm-Up

For #1 and #2, use a diagram to determine a common denominator. Then, write equivalent fractions using the common denominator.

1. $\frac{5}{8}$ and $\frac{1}{2}$
2. $\frac{2}{3}$ and $\frac{3}{5}$
3. $\frac{3}{8} + \frac{2}{8}$
4. $\frac{27}{30} - \frac{8}{30}$
5. Use a table or tree diagram to show all possible outcomes from flipping the coin and spinning the spinner.



Mental Math

6. Which numbers are divisible by 3? How do you know?
   56, 69, 103, 123, 156, 169, 568, 891
7. Which numbers are divisible by 8? How do you know?
   124, 130, 160, 200, 206, 568, 1000
8. Use multiples to find two common multiples for 3 and 6.
9. Change the following decimals to percents.
   a) 0.575 **b)** $0.\overline{3}$
10. Convert each percent to a decimal.
    a) 23.3% **b)** $11.\overline{1}\%$

Answers

Warm-Up

1. Common denominator: 8. $\frac{5}{8}, \frac{4}{8}$

2. Common denominator: 15. $\frac{10}{15}, \frac{9}{15}$

3. $\frac{5}{8}$ **4.** $\frac{19}{30}$

5.

	1	2	3	4
H	H, 1	H, 2	H, 3	H, 4
T	T, 1	T, 2	T, 3	T, 4

6. 69, 123, 156, 891. The sum of the digits is divisible by 3.

7. 160, 200, 568, 1000. The number is divisible by 2 at least three times.

8. 3: 3, 6, 9, (12), 15, (18); 6: 6, (12), (18); 12, 18

9. a) 57.5% **b)** $33.\overline{3}\%$ **10. a)** 0.233 **b)** $0.\overline{1}$

Explore the Math

1. a) red trapezoid and blue rhombus

b) No. Answers may vary. For example: The parts of the whole are not equal in size.

2. a) $\frac{1}{6}$

b) Yes, $\frac{5}{6}$. Answers may vary. For example: The parts of the whole are now equal in size. $\frac{1}{2}$ is represented by three green triangles. $\frac{1}{3}$ is represented by two green triangles. To add, you count the green triangles.

3. a) red trapezoid and green triangle

b) No. Answers may vary. For example: The parts of the whole are not equal in size.

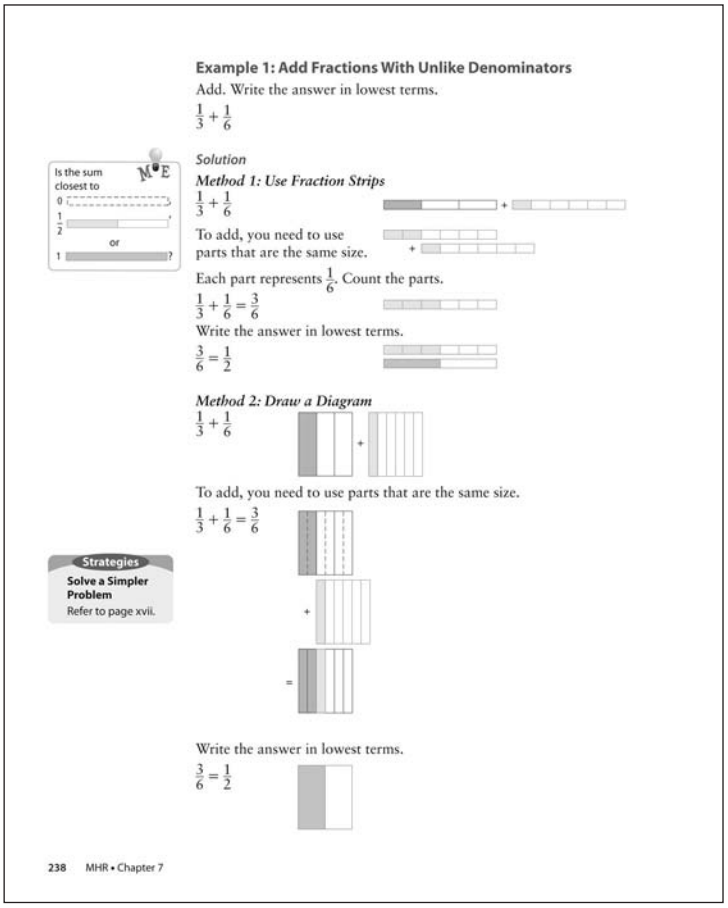
4. a)

b) Yes, $\frac{2}{6}$. Answers may vary. For example: The parts of the whole are now equal in size. $\frac{1}{2}$ is represented by three green triangles. To subtract $\frac{1}{6}$, you remove one green triangle.

c) 2

5. Answers may vary. For example: You can use pattern blocks that are equal in size to represent the fractions.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math activity, or read their responses to #5 on page 237.	• You may wish to create a class list of the relationships among the shapes and one whole. • Encourage students to consider times in their own lives when they add or subtract fractions.



Example 1: Add Fractions With Unlike Denominators

Add. Write the answer in lowest terms.

$\frac{1}{3} + \frac{1}{6}$

Solution

Method 1: Use Fraction Strips

$\frac{1}{3} + \frac{1}{6}$

To add, you need to use parts that are the same size.

Each part represents $\frac{1}{6}$. Count the parts.

$\frac{1}{3} + \frac{1}{6} = \frac{3}{6}$

Write the answer in lowest terms.

$\frac{3}{6} = \frac{1}{2}$

Method 2: Draw a Diagram

$\frac{1}{3} + \frac{1}{6}$

To add, you need to use parts that are the same size.

$\frac{1}{3} + \frac{1}{6} = \frac{3}{6}$

Write the answer in lowest terms.

$\frac{3}{6} = \frac{1}{2}$

Is the sum closest to 0, $\frac{1}{2}$, or 1?

Strategies
Solve a Simpler Problem
Refer to page xvii.

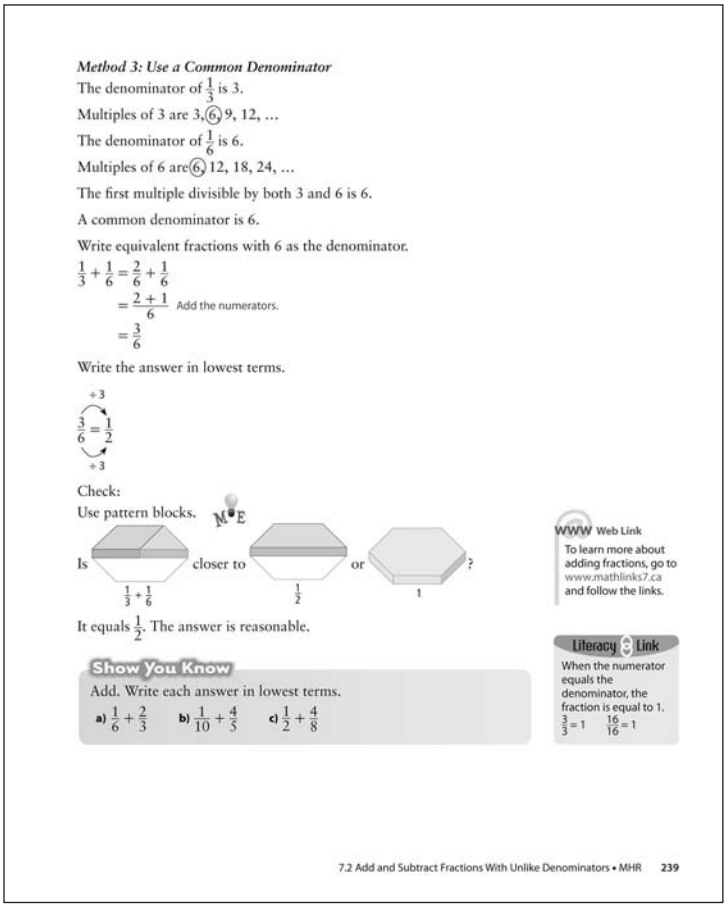
238 MHR • Chapter 7

Activity Planning Notes

As a class, discuss the section opener. Review the names of the shapes with students and encourage students to notice the combination of shapes that fit together to equal another shape. To promote a discussion, ask, "How can you make one whole? How many triangles, rhombuses, and trapezoids make one whole? What parts make $\frac{1}{2}$?"

Explore the Math

Students may not need to use pattern blocks of the same size to determine the answers to the addition and subtraction statements. In step 1b), students may see that adding $\frac{1}{2}$ and $\frac{1}{3}$ will result in $\frac{5}{6}$, as only the green $\frac{1}{6}$ triangle will fit into the empty space. In step 3b), students may see that when $\frac{1}{6}$ is laid on $\frac{1}{2}$ it leaves room for a blue rhombus or $\frac{1}{3}$. The purpose of the exercise is for students to realize that adding and subtracting fractions with unlike denominators may be easier if all the fractions have a common denominator. Point out to students that common denominators are particularly helpful when the questions become more challenging. If pattern blocks are unavailable, provide students with **Master 13 Pattern Blocks**.



Method 3: Use a Common Denominator

The denominator of $\frac{1}{3}$ is 3.

Multiples of 3 are 3, 6, 9, 12, ...

The denominator of $\frac{1}{6}$ is 6.

Multiples of 6 are 6, 12, 18, 24, ...

The first multiple divisible by both 3 and 6 is 6.

A common denominator is 6.

Write equivalent fractions with 6 as the denominator.

$\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6}$

$= \frac{2+1}{6}$ Add the numerators.

$= \frac{3}{6}$

Write the answer in lowest terms.

$\frac{3}{6} = \frac{1}{2}$ (÷3 above and ÷3 below)

Check:

Use pattern blocks.

Is $\frac{1}{3} + \frac{1}{6}$ closer to $\frac{1}{2}$ or 1?

It equals $\frac{1}{2}$. The answer is reasonable.

Show You Know

Add. Write each answer in lowest terms.

a) $\frac{1}{6} + \frac{2}{3}$ b) $\frac{1}{10} + \frac{4}{5}$ c) $\frac{1}{2} + \frac{4}{8}$

Web Link

To learn more about adding fractions, go to www.mathlinks7.ca and follow the links.

Literacy Link

When the numerator equals the denominator, the fraction is equal to 1. $\frac{3}{3} = 1$ $\frac{16}{16} = 1$

7.2 Add and Subtract Fractions With Unlike Denominators • MHR 239

In Example 1, students add fractions with unlike denominators. Three methods of adding fractions are presented: using fraction strips, drawing diagrams, and using a common denominator. Ask students which method they think will be easiest to use when adding fractions with unlike denominators. Remind students of the importance of estimating an answer before working through the problem.

In Method 3, explain to students that they can use *any* common denominator to add fractions; the number just needs to be a multiple of each denominator. Emphasize to students that the value of the fraction must not change, and have them check their equivalent fractions.

You may wish to have students write or explain in their own words the information in the Literacy Link on page 239. Encourage them to use a diagram in their explanation.

Answers

Show You Know: Example 1

a) $\frac{5}{6}$ b) $\frac{9}{10}$ c) $\frac{8}{8} = 1$

Supported Learning

Learning Style

- Once students have explored using all three methods of adding fractions with unlike denominators, allow them to complete the questions using the method of their choice (when possible).

Motor

- Students may find it difficult to manipulate the pattern blocks and/or fraction strips. This is a good opportunity for them to work on their motor skills, but ensure that they are given adequate time to practise.
- You may wish to have students use virtual manipulatives instead of concrete materials.

Meeting the Needs of All Learners

- Introduce the concepts in this lesson by providing many examples of fractions with unlike denominators. You might show fractions strips and ask students to come up with an equivalent fraction with a different denominator. Ask them how they found this fraction.
- Allow students extra time to analyse the diagrams, and make sure they understand the vocabulary. Use several types of manipulatives.

Common Errors

- Students add both the numerators and the denominators.

R_x Encourage students to state the common denominator and illustrate it. Discuss how the denominator represents the whole and that the parts are added, not the wholes. Review with them the thought bubble on page 212.

- Students may forget to express an answer in lowest terms.

R_x Remind students to read and understand all that a question is asking before answering it.

Web Link

For another way of approaching fractions, have students go to **www.mathlinks7.ca** and follow the links.

Supported Learning

Learning Style

- For the Show You Know for Examples 1 and 2, you may wish to have students work through the questions using fractions strips. They can then draw a diagram of the fractions strips in their notebook. Provide students with **Master 15 Fractions Strips**.
- Encourage students to draw diagrams and to use manipulatives to assist them in their understanding of subtraction of fractions with unlike denominators.

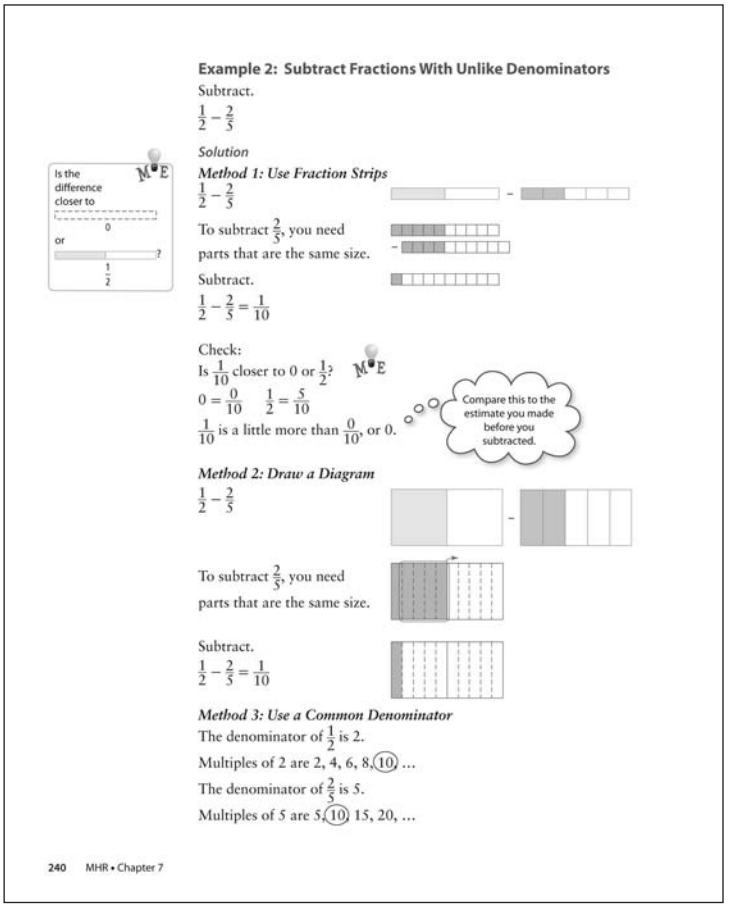
Common Errors

- Students do not find a common denominator before adding or subtracting.

R_x Refer students back to the Explore the Math where they explored that fractions can only be added or subtracted when the parts of the whole are the same size.

- Students do not find a correct common denominator.

R_x You may wish to have them review the Example in section 7.1 where they used paper folding and diagrams to find a common denominator.



Example 2: Subtract Fractions With Unlike Denominators

Subtract.

$\frac{1}{2} - \frac{2}{5}$

Solution

Method 1: Use Fraction Strips

$\frac{1}{2} - \frac{2}{5}$

To subtract $\frac{2}{5}$, you need parts that are the same size.

Subtract.

$\frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

Is the difference closer to 0 or $\frac{1}{2}$?

Check:

Is $\frac{1}{10}$ closer to 0 or $\frac{1}{2}$?

$0 = \frac{0}{10}$ $\frac{1}{2} = \frac{5}{10}$

$\frac{1}{10}$ is a little more than $\frac{0}{10}$, or 0.

Compare this to the estimate you made before you subtracted.

Method 2: Draw a Diagram

$\frac{1}{2} - \frac{2}{5}$

To subtract $\frac{2}{5}$, you need parts that are the same size.

Subtract.

$\frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

Method 3: Use a Common Denominator

The denominator of $\frac{1}{2}$ is 2.

Multiples of 2 are 2, 4, 6, 8, (10), ...

The denominator of $\frac{2}{5}$ is 5.

Multiples of 5 are 5, (10), 15, 20, ...

240 MHR • Chapter 7

Assessment *for* Learning	Supported Learning
Example 1 Have students complete the Show You Know on page 239 related to Example 1.	• Have students talk through their thinking in a group. • Make sure students check their answers against their estimates and/or check their answers using another method of adding fractions with unlike denominators, such as pattern blocks. • Students may benefit from doing the following additional questions, which use the unit fractions students worked with in section 7.1. Allow students to choose the method they use. **a)** $\frac{1}{2} + \frac{1}{3}$ $(\frac{3}{6} + \frac{2}{6} = \frac{5}{6})$ **b)** $\frac{1}{5} + \frac{1}{4}$ $(\frac{4}{20} + \frac{5}{20} = \frac{9}{20})$ Coach students through part a). Have them do part b) on their own.

In Example 2, students subtract fractions with unlike denominators using three methods. In Method 1, students use fraction strips. Draw students' attention to the Mental Math and Estimation box on the left. Emphasize the importance of estimating and how it can be used to check an answer. In Method 2, students draw a diagram to show subtraction, and in Method 3 they find a common denominator, write equivalent fractions, and subtract the numerators.

Clarify with students that subtracting fractions with unlike denominators is similar to adding fractions with unlike denominators. Remind students that they have worked with each of these methods in Example 1. Encourage students to identify similarities between adding and subtracting fractions with unlike denominators.

The first multiple divisible by both 2 and 5 is 10.

A common denominator is 10.

Write equivalent fractions with 10 as the denominator.

$\frac{1}{2} - \frac{2}{5} = \frac{5}{10} - \frac{4}{10}$

$= \frac{5-4}{10}$ Subtract the numerators.

$= \frac{1}{10}$

Show You Know

Subtract. Write each answer in lowest terms.

a) $\frac{3}{4} - \frac{1}{2}$ b) $\frac{2}{3} - \frac{1}{4}$ c) $\frac{3}{4} - \frac{1}{8}$

Key Ideas

- When adding and subtracting fractions using models or diagrams, show each fraction using parts of the whole that are of equal size.

Pattern Blocks *Diagram*

$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$ $\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6}$

- To add or subtract fractions with unlike denominators, use a common denominator.
- You can estimate when adding or subtracting fractions by comparing fractions to 0, $\frac{1}{2}$, or 1.

Communicate the Ideas

1. How are $\frac{1}{3}$ and $\frac{2}{6}$ alike? How are they different?

2. a) How would you use diagrams to calculate $\frac{1}{4} + \frac{1}{2}$?

   b) Compare your answer with a classmate's.

3. Why is it difficult to calculate $\frac{1}{2} - \frac{1}{8}$ without changing $\frac{1}{2}$ to $\frac{4}{8}$?

Assessment *for* Learning	Supported Learning
Example 2 Have students complete the Show You Know related to Example 2.	• Have students work in groups and use manipulatives. • Ask students which of the three methods they find easiest to use and why. • Students may benefit from doing the following additional questions, which use the unit fractions students worked with in section 7.1. Allow students to choose the method they use. **a)** $\frac{1}{2} - \frac{1}{3}$ ($\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$) **b)** $\frac{1}{4} - \frac{1}{5}$ ($\frac{5}{20} - \frac{4}{20} = \frac{1}{20}$) Coach students through part a). Have them do part b) on their own.

Key Ideas

Have students model an addition and subtraction statement in their journals using fractions with unlike denominators and outline the steps they would use to solve them. Ask students to explain how they could estimate an answer and why estimation is important.

Communicate the Ideas

In #2a), encourage students to describe in words how they would use diagrams and then to draw the diagrams.

Answers

Show You Know: Example 2

a) $\frac{1}{4}$ **b)** $\frac{5}{12}$ **c)** $\frac{5}{8}$

Communicate the Ideas

1. Answers will vary. For example: They are alike because they are equivalent fractions. They are different because they have different numerators and denominators.

2. a) $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$ **b)** Answers will vary.

3. Answers will vary. For example: The fractions do not have common denominators so it is difficult to subtract parts of a whole that are not equal in size.

Supported Learning

Learning Style

- Allow students to continue to use manipulatives until they feel comfortable using a common denominator.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Students should answer all three questions. You may wish to discuss #1 and #3 as a class. Have students complete #2a) on their own and then compare their answer with a classmate's.	• The answer to #2 could be entered into students' chapter Foldable as a reference for review. • Have students critique each other's work in #2 using **Master 2 Two Stars and One Wish**, which allows them to write two things they like about a piece and one thing they would improve. • Work with the class to develop criteria for judging students' answers. For example, criteria for #2 might include – uses diagrams – includes a clear explanation – provides an answer to the addition

Supported Learning

Learning Style

- Allow students to work with tangram shapes to analyse the relative size of each piece. You may wish to provide **Master 5 Tangram**. Working with various types of manipulatives will help reinforce that fractions are relative to the whole.

ESL and Language

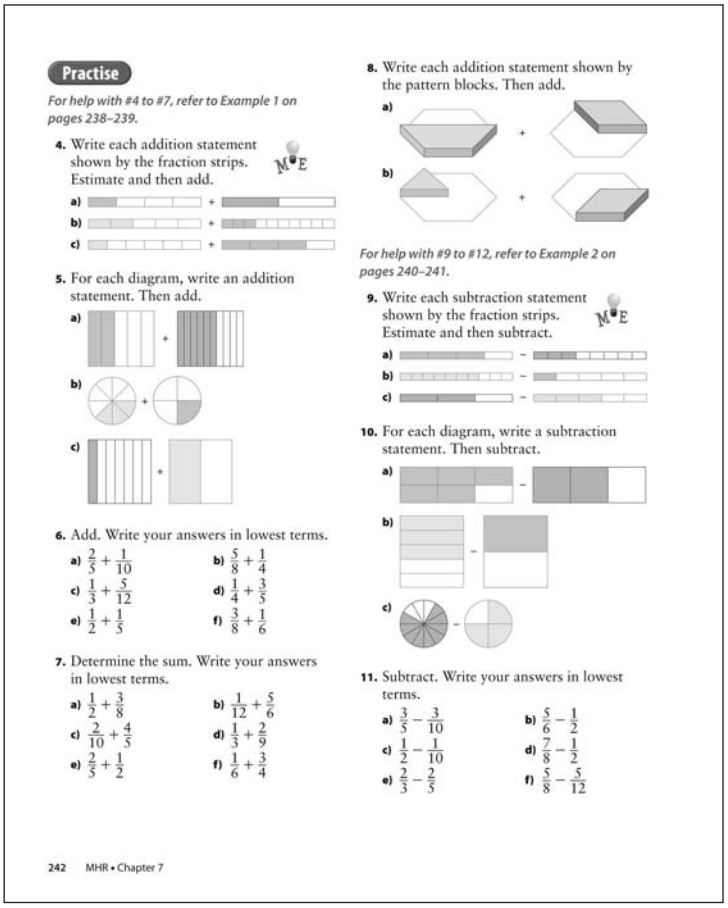
- Have students who are struggling with the vocabulary of this section use diagrams to assist them.

Common Errors

- Students may have difficulty writing equivalent fractions with a specific denominator.

R_x Review the concept of equivalent fractions and how to determine the numerator and denominator.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 9, 11, 16, 17, Math Link
Typical	1–4, 6, 9, 11, 14, 16, 17, 18 *or* 19, 22, Math Link
Extension/Enrichment	1–3, 15, 18 *or* 19, 20–23



Practise

For help with #4 to #7, refer to Example 1 on pages 238–239.

4. Write each addition statement shown by the fraction strips. Estimate and then add.

a)

b)

c)

5. For each diagram, write an addition statement. Then add.

a)

b)

c)

6. Add. Write your answers in lowest terms.

a) $\frac{2}{5}+\frac{1}{10}$ b) $\frac{5}{8}+\frac{1}{4}$

c) $\frac{1}{3}+\frac{5}{12}$ d) $\frac{1}{4}+\frac{3}{5}$

e) $\frac{1}{2}+\frac{1}{5}$ f) $\frac{3}{8}+\frac{1}{6}$

7. Determine the sum. Write your answers in lowest terms.

a) $\frac{1}{2}+\frac{3}{8}$ b) $\frac{1}{12}+\frac{5}{6}$

c) $\frac{2}{10}+\frac{4}{5}$ d) $\frac{1}{3}+\frac{2}{9}$

e) $\frac{2}{5}+\frac{1}{2}$ f) $\frac{1}{6}+\frac{3}{4}$

8. Write each addition statement shown by the pattern blocks. Then add.

a)

b)

For help with #9 to #12, refer to Example 2 on pages 240–241.

9. Write each subtraction statement shown by the fraction strips. Estimate and then subtract.

a)

b)

c)

10. For each diagram, write a subtraction statement. Then subtract.

a)

b)

c)

11. Subtract. Write your answers in lowest terms.

a) $\frac{3}{5}-\frac{3}{10}$ b) $\frac{5}{6}-\frac{1}{2}$

c) $\frac{1}{2}-\frac{1}{10}$ d) $\frac{7}{8}-\frac{1}{2}$

e) $\frac{2}{3}-\frac{2}{5}$ f) $\frac{5}{8}-\frac{5}{12}$

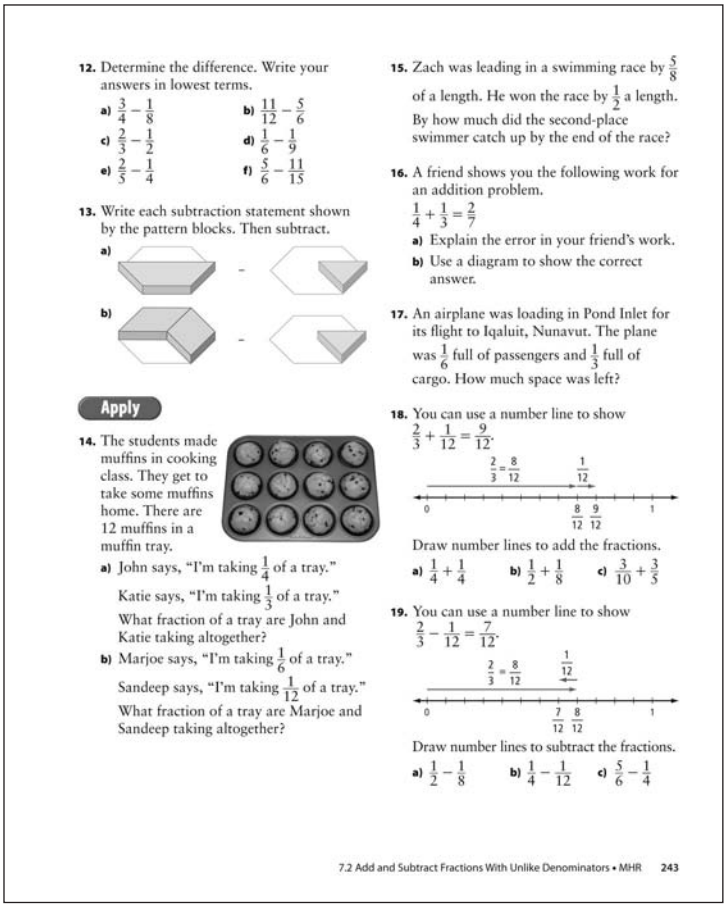
242 MHR • Chapter 7

Practise

Students at the concrete and semi-concrete stages may continue to use manipulatives for addition and subtraction of fractions. Fraction strips, diagrams, and pattern blocks are used in Examples 1 and 2. For the Practise, encourage students to use other methods of adding and subtracting fractions, such as a number line. You may wish to provide students with **Master 4 Vertical and Horizontal Number Lines**. Encourage them to explain how they could use a number line to show addition or subtraction of fractions. They will be asked to use numbers lines in #18 and #19 on page 243.

Remind students that the Mental Math and Estimation symbol beside #4 and #9 applies to most questions. Students will benefit from estimating an answer before performing any calculations.

Assessment *for* Learning	Supported Learning
Practise Have students do #4, #6, #9, and #11. Students who have no problems with these questions can go on to the Apply questions.	• Have students who have difficulty with #4 and #6 talk through Example 1. Clarify any misunderstandings. Coach students through #5a), #7a), and #8a), and then have them complete these questions on their own. • Have students who have difficulty with #9 and #11 talk through Example 2. Clarify any misunderstandings. Coach students through #10a), #12a), and #13a), and then have them complete these questions on their own.



12. Determine the difference. Write your answers in lowest terms.

a) $\frac{3}{4} - \frac{1}{8}$ b) $\frac{11}{12} - \frac{5}{6}$

c) $\frac{2}{3} - \frac{1}{2}$ d) $\frac{1}{6} - \frac{1}{9}$

e) $\frac{2}{5} - \frac{1}{4}$ f) $\frac{5}{6} - \frac{11}{15}$

13. Write each subtraction statement shown by the pattern blocks. Then subtract.

a)

b)

Apply

14. The students made muffins in cooking class. They get to take some muffins home. There are 12 muffins in a muffin tray.

a) John says, "I'm taking $\frac{1}{4}$ of a tray."
Katie says, "I'm taking $\frac{1}{3}$ of a tray."
What fraction of a tray are John and Katie taking altogether?

b) Marjoe says, "I'm taking $\frac{1}{6}$ of a tray."
Sandeep says, "I'm taking $\frac{1}{12}$ of a tray."
What fraction of a tray are Marjoe and Sandeep taking altogether?

15. Zach was leading in a swimming race by $\frac{5}{8}$ of a length. He won the race by $\frac{1}{2}$ a length. By how much did the second-place swimmer catch up by the end of the race?

16. A friend shows you the following work for an addition problem.
$\frac{1}{4} + \frac{1}{3} = \frac{2}{7}$

a) Explain the error in your friend's work.

b) Use a diagram to show the correct answer.

17. An airplane was loading in Pond Inlet for its flight to Iqaluit, Nunavut. The plane was $\frac{1}{6}$ full of passengers and $\frac{1}{3}$ full of cargo. How much space was left?

18. You can use a number line to show $\frac{2}{3} + \frac{1}{12} = \frac{9}{12}$.

Draw number lines to add the fractions.

a) $\frac{1}{4} + \frac{1}{4}$ b) $\frac{1}{2} + \frac{1}{8}$ c) $\frac{3}{10} + \frac{3}{5}$

19. You can use a number line to show $\frac{2}{3} - \frac{1}{12} = \frac{7}{12}$.

Draw number lines to subtract the fractions.

a) $\frac{1}{2} - \frac{1}{8}$ b) $\frac{1}{4} - \frac{1}{12}$ c) $\frac{5}{6} - \frac{1}{4}$

7.2 Add and Subtract Fractions With Unlike Denominators • MHR 243

Apply and Extend

In #14 to #17, students add or subtract fractions with unlike denominators using real-world scenarios. As an extension for #14, you may wish to ask students what number of muffins John, Katie, Marjoe, and Sandeep each take. (John takes 3 muffins; Katie takes 4 muffins; Marjoe takes 2 muffins; and Sandeep takes 1 muffin.)

Question 15 could be solved using a number line for subtraction. You may wish to have students complete #18 and/or #19 before working through this problem since these questions use a number line to show addition and subtraction, respectively.

In the Extend, #20 refers to the Eye of Horus seen in the Math Links on pages 228 and 236. Some students may have already determined that the fractions do not add to 1. You may wish to have these students present their solution to the class.

For #21, students will need to realize that the question involves repeated addition.

Supported Learning

Learning Style

- For #18 and #19, you may wish to provide students with **Master 4 Vertical and Horizontal Number Lines**.
- For #23 on page 244, you may wish to provide students with **Master 5 Tangram**.

Answers

Math Link

Diagrams may vary.

a) $\frac{3}{8}$ + $\frac{2}{8}$ = $\frac{5}{8}$ $\quad\frac{4}{9}$ + $\frac{3}{9}$ = $\frac{7}{9}$

b) $\frac{4}{9}$ is greater than $\frac{3}{8}$ by $\frac{5}{72}$.

c) $\frac{1}{6} + \frac{1}{4}$ or $\frac{1}{12} + \frac{1}{3}$

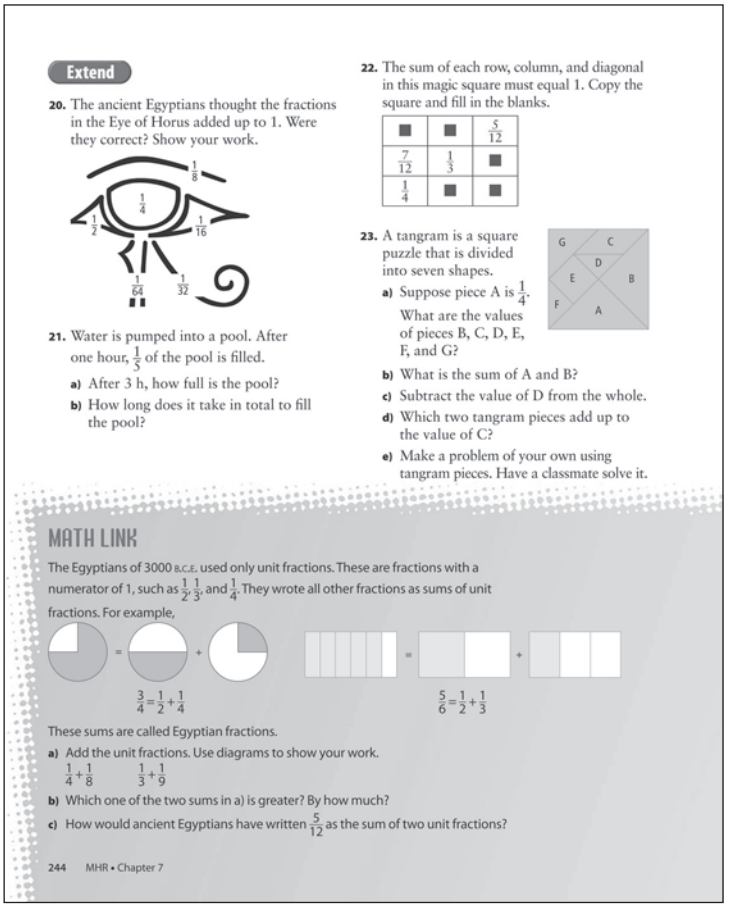
Supported Learning

Learning Style and Memory

- You may wish to give students **BLM 7–5 Section 7.2 Extra Practice**, which provides additional reinforcement.

Learning Style, ESL, Language, and Memory

- Have students complete the Math Link to help them prepare for the Wrap It Up! on page 263.



Extend

20. The ancient Egyptians thought the fractions in the Eye of Horus added up to 1. Were they correct? Show your work.

21. Water is pumped into a pool. After one hour, $\frac{1}{5}$ of the pool is filled.
   a) After 3 h, how full is the pool?
   b) How long does it take in total to fill the pool?

22. The sum of each row, column, and diagonal in this magic square must equal 1. Copy the square and fill in the blanks.

		$\frac{5}{12}$
$\frac{7}{12}$	$\frac{1}{3}$	
$\frac{1}{4}$		

23. A tangram is a square puzzle that is divided into seven shapes.
   a) Suppose piece A is $\frac{1}{4}$. What are the values of pieces B, C, D, E, F, and G?
   b) What is the sum of A and B?
   c) Subtract the value of D from the whole.
   d) Which two tangram pieces add up to the value of C?
   e) Make a problem of your own using tangram pieces. Have a classmate solve it.

MATH LINK

The Egyptians of 3000 B.C.E. used only unit fractions. These are fractions with a numerator of 1, such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. They wrote all other fractions as sums of unit fractions. For example,

$\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$ $\quad\frac{5}{6} = \frac{1}{2} + \frac{1}{3}$

These sums are called Egyptian fractions.

a) Add the unit fractions. Use diagrams to show your work.
$\frac{1}{4} + \frac{1}{8}$ $\quad\frac{1}{3} + \frac{1}{9}$

b) Which one of the two sums in a) is greater? By how much?

c) How would ancient Egyptians have written $\frac{5}{12}$ as the sum of two unit fractions?

244 MHR • Chapter 7

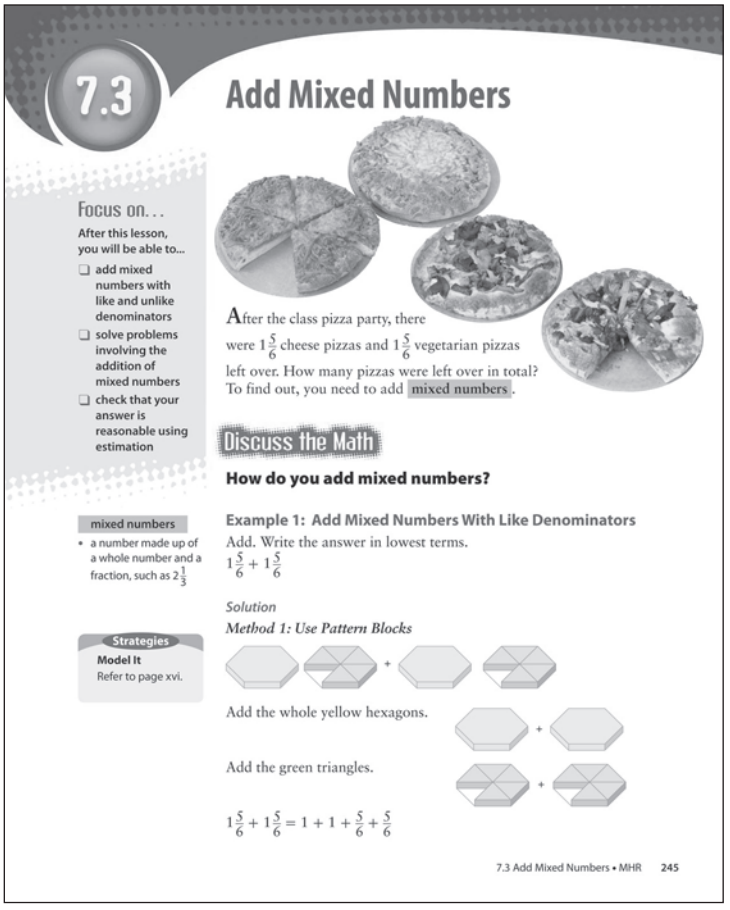
Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • How are the addition and subtraction of fractions with unlike denominators similar? • What is the most important thing you have learned in this section?	• Have students use diagrams to explain each operation involving fractions. • Encourage students to use examples in their explanations. • Depending on students' learning style, you may wish to have them provide oral rather than written answers. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • You may wish to have students fill in the During column for section 7.2 of **BLM 7–1 Chapter 7 Self-Assessment**. Have students indicate how they might improve any items that they have marked either red or yellow.

Math Link

This Math Link about Egyptian fractions continues the exploration of how fractions have been used around the world and in the past. As an extension for part c), you may wish to ask students to show why there cannot be three different unit fractions with a sum of $\frac{5}{12}$.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 244 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 263. It provides them with an opportunity to apply their understanding of adding and subtracting Egyptian fractions using diagrams and mathematical statements.	• Have students describe the meaning of an Egyptian fraction using numbers, words, and diagrams. • Students who are having difficulty getting started could use **BLM 7–6 Section 7.2 Math Link**, which provides scaffolding for the activity.

7.3 Add Mixed Numbers



7.3

Add Mixed Numbers

Focus on...

After this lesson, you will be able to...

- add mixed numbers with like and unlike denominators
- solve problems involving the addition of mixed numbers
- check that your answer is reasonable using estimation

After the class pizza party, there were $1\frac{5}{6}$ cheese pizzas and $1\frac{5}{6}$ vegetarian pizzas left over. How many pizzas were left over in total? To find out, you need to add mixed numbers.

Discuss the Math

How do you add mixed numbers?

mixed numbers
- a number made up of a whole number and a fraction, such as $2\frac{1}{3}$

Example 1: Add Mixed Numbers With Like Denominators

Add. Write the answer in lowest terms.

$1\frac{5}{6} + 1\frac{5}{6}$

Solution

Method 1: Use Pattern Blocks

Strategies
Model It
Refer to page xvi.

Add the whole yellow hexagons.

Add the green triangles.

$1\frac{5}{6} + 1\frac{5}{6} = 1 + 1 + \frac{5}{6} + \frac{5}{6}$

7.3 Add Mixed Numbers • MHR 245

Suggested Timing
80–100 minutes

Materials
- coloured pencils
- pattern blocks

Blackline Masters
Master 13 Pattern Blocks
BLM 7–1 Chapter 7 Self-Assessment
BLM 7–7 Section 7.3 Extra Practice
BLM 7–8 Section 7.3 Math Link

Mathematical Processes
- ☑ Communication
- ☑ Connections
- ☑ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☑ Technology
- ☑ Visualization

Specific Outcomes

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Warm-Up

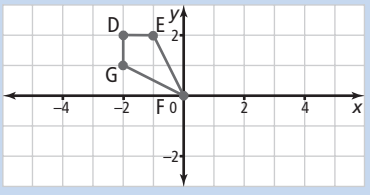
For #1 to #3, calculate and then write your answer in lowest terms.

1. $\frac{1}{3} + \frac{5}{12}$ **2.** $\frac{1}{2} - \frac{3}{10}$ **3.** $\frac{5}{6} - \frac{9}{15}$

4. Estimate and calculate the area of the following parallelogram: $h = 5.4$ cm; $b = 8.3$ cm.

5. a) Copy figure DEFG onto a coordinate grid. Reflect DEFG in the y-axis.

b) What point is in the same location?



Mental Math

6. List the numbers divisible by 6. How do you know?
32, 48, 56, 69, 123, 156, 185, 238

7. List five numbers between 200 and 250 that are divisible by 8. How do you know?

8. Use multiples to find two common multiples for 4 and 5.

9. Write an equivalent fraction using each denominator shown.

a) $\frac{2}{5}$ (10) **b)** $\frac{6}{9}$ (3)

10. Convert each fraction to a decimal number.

a) $\frac{1}{3}$ **b)** $\frac{7}{9}$

Answers

Warm-Up

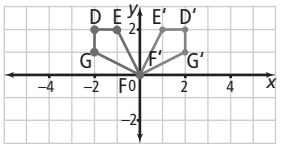
1. $\frac{4}{12} + \frac{5}{12} = \frac{9}{12} = \frac{3}{4}$

2. $\frac{5}{10} - \frac{3}{10} = \frac{2}{10} = \frac{1}{5}$

3. $\frac{25}{30} - \frac{18}{30} = \frac{7}{30}$

4. Estimate: $5 \times 8 = 40$ cm^2
Calculate: $5.4 \times 8.3 = 44.82$ cm^2

5. a)



b) F(0, 0)

6. 48, 156. They are divisible by both 2 and 3.

7. Expect any five of the following: 208, 216, 224, 232, 240, 248. The number is divisible by 2 at least three times.

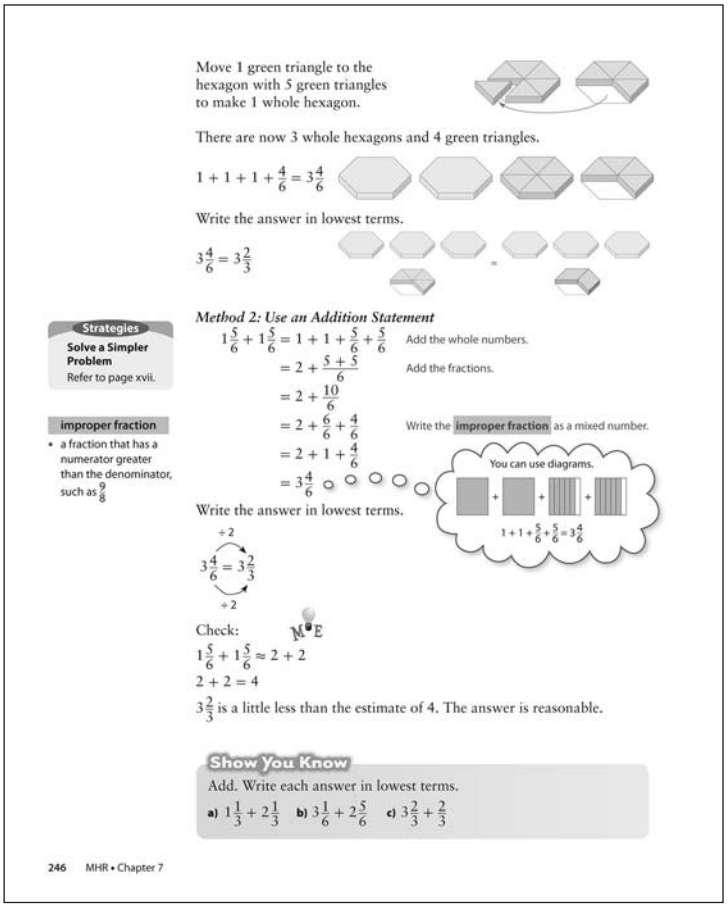
8. 4: 4, 8, 12, 16, (20), 24, 28, 32, 36, (40)
5: 5, 10, 15, (20), 25, 30, 35, (40)
20, 40

9. a) $\frac{4}{10}$ **b)** $\frac{2}{3}$

10. a) $0.\overline{3}$ **b)** $0.\overline{7}$

Show You Know: Example 1

a) $3\frac{2}{3}$ **b)** 6 **c)** $4\frac{1}{3}$



Move 1 green triangle to the hexagon with 5 green triangles to make 1 whole hexagon.

There are now 3 whole hexagons and 4 green triangles.

$1 + 1 + 1 + \frac{4}{6} = 3\frac{4}{6}$

Write the answer in lowest terms.

$3\frac{4}{6} = 3\frac{2}{3}$

Method 2: Use an Addition Statement

Strategies
Solve a Simpler Problem
Refer to page xvii.

$1\frac{5}{6} + 1\frac{5}{6} = 1 + 1 + \frac{5}{6} + \frac{5}{6}$ Add the whole numbers.

$= 2 + \frac{5+5}{6}$ Add the fractions.

$= 2 + \frac{10}{6}$

$= 2 + \frac{6}{6} + \frac{4}{6}$ Write the improper fraction as a mixed number.

$= 2 + 1 + \frac{4}{6}$

$= 3\frac{4}{6}$

improper fraction
- a fraction that has a numerator greater than the denominator, such as $\frac{9}{8}$

You can use diagrams.

$1 + 1 + \frac{5}{6} + \frac{5}{6} = 3\frac{4}{6}$

Write the answer in lowest terms.

$3\frac{4}{6} = 3\frac{2}{3}$ (÷2)

Check:

$1\frac{5}{6} + 1\frac{5}{6} \approx 2 + 2$

$2 + 2 = 4$

$3\frac{2}{3}$ is a little less than the estimate of 4. The answer is reasonable.

Show You Know

Add. Write each answer in lowest terms.

a) $1\frac{1}{3} + 2\frac{1}{3}$ **b)** $3\frac{1}{6} + 2\frac{5}{6}$ **c)** $3\frac{2}{3} + \frac{2}{3}$

246 MHR • Chapter 7

Supported Learning

Meeting the Needs of All Learners

- Use a lot of visual examples and manipulatives to help students learn the concepts.
- You may wish to have students work in pairs on the questions and then display their solutions on the board for the class.

WWW Web Link

Some students may benefit from using technology to help them develop their understanding of adding mixed numbers. Have them go to **www.mathlinks7.ca** and follow the links.

Activity Planning Notes

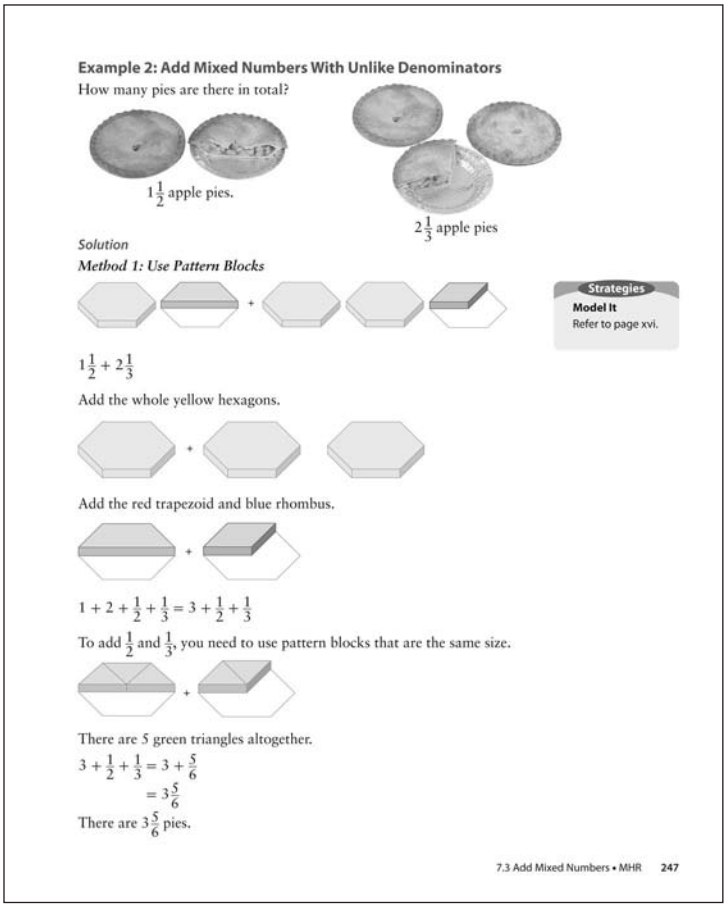
As a class, discuss the section opener. Have students explain how they know that there is $1\frac{5}{6}$ of each type of pizza. Also, discuss how they know $1\frac{5}{6}$ is a mixed number. Have students describe a time that part of something was left after a party or celebration. Ask students how they determined how much was left over.

Discuss the Math

Explain to students that this section involves adding fractions of numbers that are greater than one whole. In Example 1, students add mixed numbers with like denominators, and in Example 2 they add mixed numbers with unlike denominators.

You might want to use one of two approaches for the Discuss the Math. Work through Examples 1 and 2 as a class. Alternatively, work through only Example 1 as a class. Then discuss strategies for adding mixed numbers and have students work in pairs for Example 2.

Example 1 shows two methods of adding mixed numbers. Method 1 involves the use of pattern blocks, and Method 2 involves the use of an addition statement, with students using Mental Math and Estimation to check their sum. Ensure that all students understand the concepts involved in adding mixed numbers with like denominators. If pattern blocks are not available, provide students with **Master 13 Pattern Blocks** for Examples 1 and 2.



Example 2: Add Mixed Numbers With Unlike Denominators

How many pies are there in total?

$1\frac{1}{2}$ apple pies.

$2\frac{1}{3}$ apple pies

Solution

Method 1: Use Pattern Blocks

Strategies
Model It
Refer to page xvi.

$1\frac{1}{2} + 2\frac{1}{3}$

Add the whole yellow hexagons.

Add the red trapezoid and blue rhombus.

$1 + 2 + \frac{1}{2} + \frac{1}{3} = 3 + \frac{1}{2} + \frac{1}{3}$

To add $\frac{1}{2}$ and $\frac{1}{3}$, you need to use pattern blocks that are the same size.

There are 5 green triangles altogether.

$3 + \frac{1}{2} + \frac{1}{3} = 3 + \frac{5}{6}$

$= 3\frac{5}{6}$

There are $3\frac{5}{6}$ pies.

Supported Learning

Motor

- Students with motor challenges may find it difficult to manipulate the pattern blocks. Since this is a good opportunity for them to work on their motor skills, ensure that they are given adequate time to practise.
- You may wish to allow students to use virtual manipulatives instead of the pattern blocks.

Common Errors

- Students may sometimes complete work that shows an answer with a natural number and a mixed fraction, for example, $2\ 1\frac{1}{3}$.

R_x Have students rework the solution by estimating an answer first. Students may need to be reminded to write answers in lowest terms.

- Students add both the numerators and the denominators.

R_x Remind students that fractions can only be added when they are the same size parts of a whole. Have students practise finding common denominators. You may wish to have them review section 7.1.

Assessment *for* Learning	Supported Learning
Example 1 Have students complete the Show You Know on page 246 related to Example 1.	• Allow students to choose the method they use to add mixed numbers. Have students outline their method to the class and explain how they know their method is correct. • You may wish to provide additional questions for students who will benefit from doing them. Have them use pattern blocks or drawings to show the solutions. **a)** $1\frac{1}{4} + 2\frac{3}{4}$ $(1 + 2 + \frac{1}{4} + \frac{3}{4} = 3\frac{4}{4} = 3 + 1 = 4)$ **b)** $2\frac{3}{5} + 4\frac{4}{5}$ $(2 + 4 + \frac{3}{5} + \frac{4}{5} = 6 + \frac{7}{5} = 6 + 1\frac{2}{5} = 7\frac{2}{5}$. Remind students how to work with improper fractions.) Coach students through part a), and then have them do part b) on their own.

In Example 2, students add mixed numbers with unlike denominators using two methods. In Method 1, students use pattern blocks, and in Method 2 they use an addition statement. The use of diagrams is also demonstrated in Method 2. Work through each method as a class. In Method 2, the sum of the mixed numbers is verified using Mental Math and Estimation. Ask students to suggest other methods of checking a calculation. For example, Draw a Diagram.

Have students read the Did You Know? on page 248. Ask them what the fraction bar means and when they have written fractions or seen fractions written using a fraction bar in their daily lives. Have them provide examples.

Answers

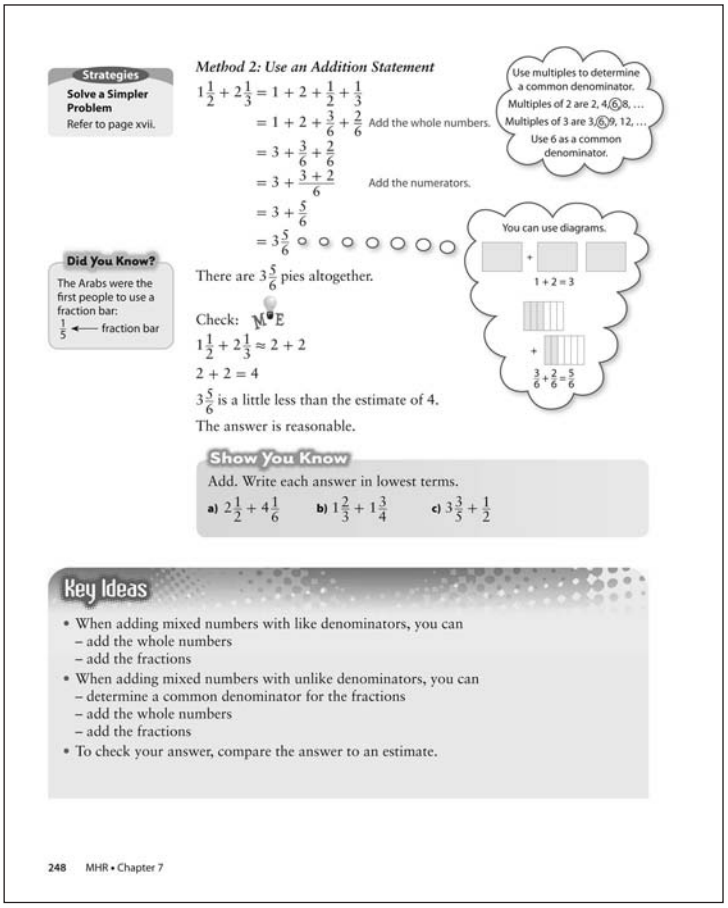
Show You Know: Example 2

a) $6\frac{2}{3}$ **b)** $3\frac{5}{12}$ **c)** $4\frac{1}{10}$

Supported Learning

Learning Styles

- Allow kinesthetic, concrete, and semi-concrete learners to continue to use diagrams, pattern blocks, and other manipulatives.



Strategies
Solve a Simpler Problem
Refer to page xvii.

Method 2: Use an Addition Statement

$1\frac{1}{2} + 2\frac{1}{3} = 1 + 2 + \frac{1}{2} + \frac{1}{3}$
$= 1 + 2 + \frac{3}{6} + \frac{2}{6}$ Add the whole numbers.
$= 3 + \frac{3}{6} + \frac{2}{6}$
$= 3 + \frac{3+2}{6}$ Add the numerators.
$= 3 + \frac{5}{6}$
$= 3\frac{5}{6}$

Use multiples to determine a common denominator.
Multiples of 2 are 2, 4, 6, 8, ...
Multiples of 3 are 3, 6, 9, 12, ...
Use 6 as a common denominator.

You can use diagrams.
1 + 2 = 3
$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

Did You Know?
The Arabs were the first people to use a fraction bar:
$\frac{1}{5}$ ← fraction bar

There are $3\frac{5}{6}$ pies altogether.

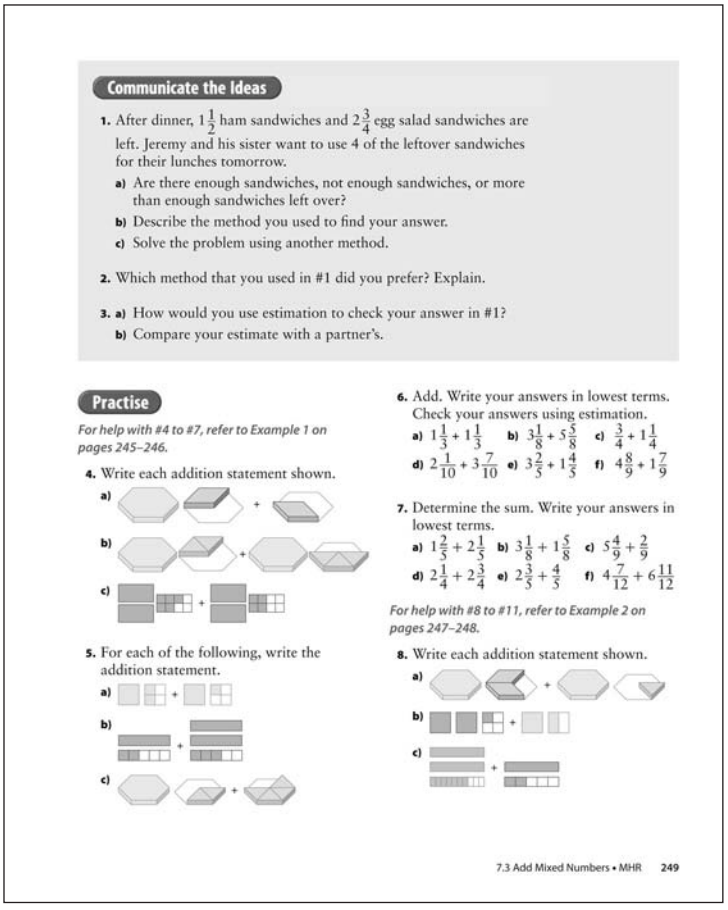
Check:
$1\frac{1}{2} + 2\frac{1}{3} \approx 2 + 2$
$2 + 2 = 4$
$3\frac{5}{6}$ is a little less than the estimate of 4.
The answer is reasonable.

Show You Know
Add. Write each answer in lowest terms.
a) $2\frac{1}{2} + 4\frac{1}{6}$ **b)** $1\frac{2}{3} + 1\frac{3}{4}$ **c)** $3\frac{3}{5} + \frac{1}{2}$

Key Ideas
- When adding mixed numbers with like denominators, you can
  - add the whole numbers
  - add the fractions
- When adding mixed numbers with unlike denominators, you can
  - determine a common denominator for the fractions
  - add the whole numbers
  - add the fractions
- To check your answer, compare the answer to an estimate.

248 MHR • Chapter 7

Assessment *for* Learning	Supported Learning
Example 2 Have students complete the Show You Know related to Example 2.	• Encourage students to write down their estimate, so that they can compare it to their calculated answer. Have them show how they know their answer is reasonable. • When students are writing their answer in lowest terms, remind them to list the factors of the numerator and the denominator separately; identify a common factor; and then divide the numerator and the denominator by this common factor. If they suspect that their solution is still not in lowest terms, have them repeat the steps on the solution until they are confident that it is in lowest terms. • You may wish to provide additional questions for students who will benefit from them. Have them use concrete materials or diagrams to show the solutions. **a)** $1\frac{1}{4} + 2\frac{1}{2}$ $(1 + 2 + \frac{1}{4} + \frac{2}{4} = 3\frac{3}{4}$. Remind students to add using a common denominator.) **b)** $2\frac{4}{5} + 4\frac{2}{3}$ $(2 + 4 + \frac{12}{15} + \frac{10}{15} = 6 + \frac{22}{15} = 6 + 1\frac{7}{15} = 7\frac{7}{15})$ Coach students through part a), and then have them do part b) on their own.



Communicate the Ideas

1. After dinner, $1\frac{1}{2}$ ham sandwiches and $2\frac{3}{4}$ egg salad sandwiches are left. Jeremy and his sister want to use 4 of the leftover sandwiches for their lunches tomorrow.
   a) Are there enough sandwiches, not enough sandwiches, or more than enough sandwiches left over?
   b) Describe the method you used to find your answer.
   c) Solve the problem using another method.
2. Which method that you used in #1 did you prefer? Explain.
3. a) How would you use estimation to check your answer in #1?
   b) Compare your estimate with a partner's.

Practise

For help with #4 to #7, refer to Example 1 on pages 245–246.

4. Write each addition statement shown.
   a)
   b)
   c)

5. For each of the following, write the addition statement.
   a)
   b)
   c)

6. Add. Write your answers in lowest terms. Check your answers using estimation.
   a) $1\frac{1}{3}+1\frac{1}{3}$ b) $3\frac{1}{8}+5\frac{5}{8}$ c) $\frac{3}{4}+1\frac{1}{4}$
   d) $2\frac{1}{10}+3\frac{7}{10}$ e) $3\frac{2}{5}+1\frac{4}{5}$ f) $4\frac{8}{9}+1\frac{7}{9}$

7. Determine the sum. Write your answers in lowest terms.
   a) $1\frac{2}{5}+2\frac{1}{5}$ b) $3\frac{1}{8}+1\frac{5}{8}$ c) $5\frac{4}{9}+\frac{2}{9}$
   d) $2\frac{1}{4}+2\frac{3}{4}$ e) $2\frac{3}{5}+\frac{4}{5}$ f) $4\frac{7}{12}+6\frac{11}{12}$

For help with #8 to #11, refer to Example 2 on pages 247–248.

8. Write each addition statement shown.
   a)
   b)
   c)

7.3 Add Mixed Numbers • MHR 249

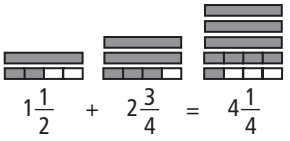
Answers

Communicate the Ideas

1. a) There are more than enough sandwiches left over.

b) Answers may vary. For example: Add the whole numbers: $1 + 2 = 3$. Find a common denominator for the fractions and then add: $\frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4}$. Change the improper fraction to a mixed number: $\frac{5}{4} = 1\frac{1}{4}$. Add the whole number to the mixed number: $3 + 1\frac{1}{4} = 4\frac{1}{4}$. Compare $4\frac{1}{4}$ to 4.

c) Answers may vary. For example: Use fraction strips. $4\frac{1}{4}$ is more than 4.



2. Answers may vary.

3. a) Answer may vary. For example: $1\frac{1}{2} + 2\frac{3}{4} \approx 1 + 3 = 4$

b) Answers may vary.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8, 10, 14, 15, 17, Math Link
Typical	1–4, 6, 8, 10, 12–15, 17, 19, Math Link
Extension/Enrichment	1–3, 17–20

Key Ideas

Have students read and review the Key Ideas. Make sure they understand how to find a common denominator and how to convert an improper fraction to a mixed number. You may wish to have students develop a flowchart to help them determine how to add mixed numbers with like or unlike denominators.

Communicate the Ideas

These questions give students an opportunity to work through a real-world problem using two methods. Students then describe their solution and how they would use estimation to check their answer. Use this opportunity to assess student readiness for the Practise questions and to clarify any misconceptions.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students answer the three questions. Have students complete these questions individually and then discuss their solutions with a partner. Allow students to modify their solutions. Then, discuss them as a class.	• You may wish to have students at the concrete stage use paper folding to model the sandwich fractions. • Students may find it difficult to identify which method they prefer to solve the problem. Encourage them to think about each method they have used to add mixed numbers and to determine which one they are comfortable with and find easy to use. • Encourage students to estimate in #3 by rounding the mixed numbers to whole numbers or by drawing the sandwiches.

Supported Learning

Memory

- Consider allowing students to use the summary from the Key Ideas as a step-by-step checklist to help them complete related questions.

Supported Learning

Learning Style

- Allow students to use the method of adding mixed numbers that they are most comfortable with.
- Encourage students to draw diagrams as necessary to help them work through each word problem.

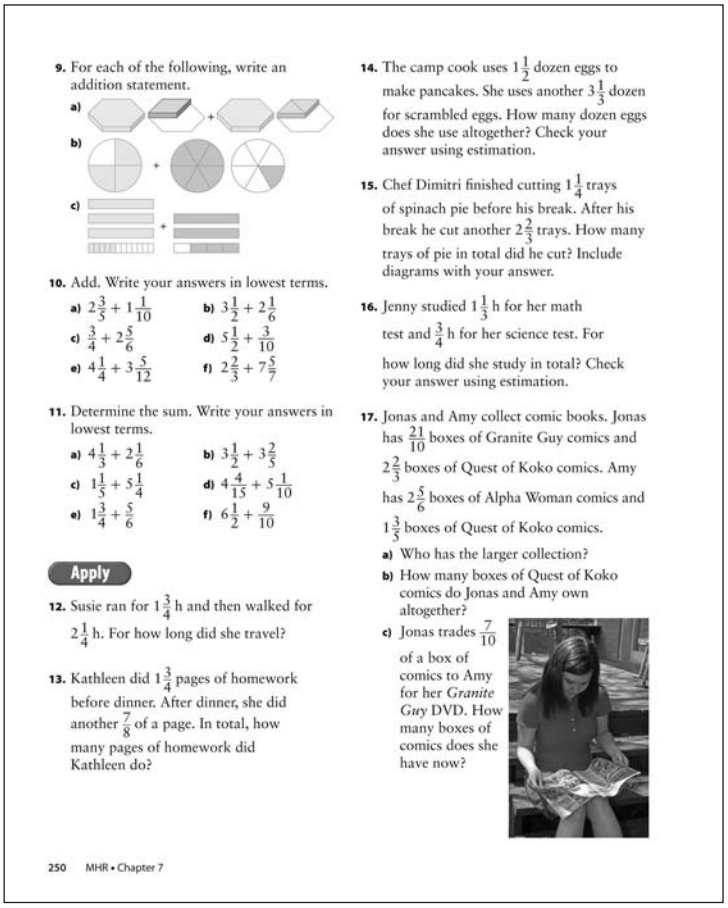
Gifted and Enrichment

- As an extension to #17, you may wish to have students determine how many boxes of comics Jonas has after the trade in part c). The answer is $4\frac{1}{5}$ boxes.

Learning Style and Memory

- **BLM 7–7 Section 7.3 Extra Practice** provides additional reinforcement for students who need it.

Assessment *for* Learning	Supported Learning
Practise and Apply Have students do #4, #6, #8, and #10. Students who have no problems with these questions can move on to #12 and then the other Apply questions.	• If students have difficulty with #4, #6, #8, and #10, give them additional coaching with the examples. Then work through #5a), #7a), #9a), and #11a) with them. Once you are sure they understand the concepts, have them complete these questions on their own. • Assess student learning by having them demonstrate one question of their choice and one question of your choice.



9. For each of the following, write an addition statement.

a)

b)

c)

10. Add. Write your answers in lowest terms.

a) $2\frac{3}{5} + 1\frac{1}{10}$ b) $3\frac{1}{2} + 2\frac{1}{6}$

c) $\frac{3}{4} + 2\frac{5}{6}$ d) $5\frac{1}{2} + \frac{3}{10}$

e) $4\frac{1}{4} + 3\frac{5}{12}$ f) $2\frac{2}{3} + 7\frac{5}{7}$

11. Determine the sum. Write your answers in lowest terms.

a) $4\frac{1}{3} + 2\frac{1}{6}$ b) $3\frac{1}{2} + 3\frac{2}{5}$

c) $1\frac{1}{5} + 5\frac{1}{4}$ d) $4\frac{4}{15} + 5\frac{1}{10}$

e) $1\frac{3}{4} + \frac{5}{6}$ f) $6\frac{1}{2} + \frac{9}{10}$

Apply

12. Susie ran for $1\frac{3}{4}$ h and then walked for $2\frac{1}{4}$ h. For how long did she travel?

13. Kathleen did $1\frac{3}{4}$ pages of homework before dinner. After dinner, she did another $\frac{7}{8}$ of a page. In total, how many pages of homework did Kathleen do?

14. The camp cook uses $1\frac{1}{2}$ dozen eggs to make pancakes. She uses another $3\frac{1}{3}$ dozen for scrambled eggs. How many dozen eggs does she use altogether? Check your answer using estimation.

15. Chef Dimitri finished cutting $1\frac{1}{4}$ trays of spinach pie before his break. After his break he cut another $2\frac{2}{3}$ trays. How many trays of pie in total did he cut? Include diagrams with your answer.

16. Jenny studied $1\frac{1}{3}$ h for her math test and $\frac{3}{4}$ h for her science test. For how long did she study in total? Check your answer using estimation.

17. Jonas and Amy collect comic books. Jonas has $\frac{21}{10}$ boxes of Granite Guy comics and $2\frac{2}{3}$ boxes of Quest of Koko comics. Amy has $2\frac{5}{6}$ boxes of Alpha Woman comics and $1\frac{3}{5}$ boxes of Quest of Koko comics.

a) Who has the larger collection?

b) How many boxes of Quest of Koko comics do Jonas and Amy own altogether?

c) Jonas trades $\frac{7}{10}$ of a box of comics to Amy for her *Granite Guy* DVD. How many boxes of comics does she have now?

250 MHR • Chapter 7

Practise

In #4, #5, #8, and #9, students are shown pattern blocks, fraction strips, or diagrams representing the addition of mixed numbers with like or unlike denominators. They are asked to write the addition statement only. Most students will not need to do all of these questions.

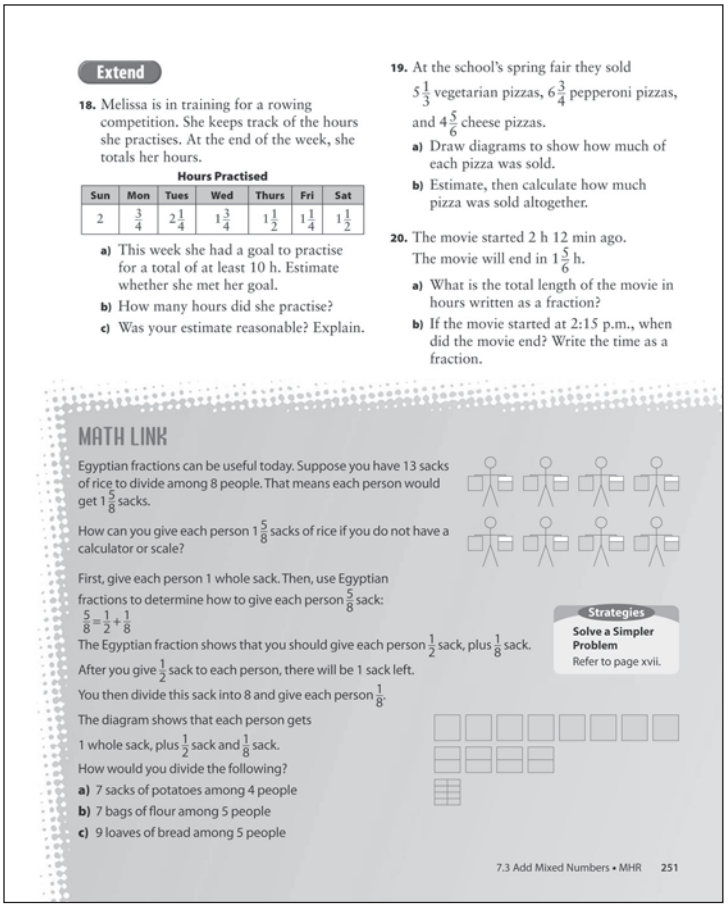
In #6 and #7, students add mixed numbers with like denominators. Students are asked to write their answers in lowest terms. Ensure that students show how they have checked each answer, especially in #6, where they are asked to estimate an answer.

Apply and Extend

Encourage students to show a full solution for each problem. For #14, remind students that a fraction is a part of a whole, and in this case the whole is a dozen. As a challenge, have students determine how many eggs are in the answer of $4\frac{5}{6}$ dozen (58 eggs).

In #17, encourage students to organize their information before performing any calculations. As a class, have students share and compare the methods they used to solve this question.

In #20, students will need to convert a time in hours and minutes to a fraction of an hour. Students may wish to use a number line to help them visualize the problem.



Extend

18. Melissa is in training for a rowing competition. She keeps track of the hours she practises. At the end of the week, she totals her hours.

Hours Practised

Sun	Mon	Tues	Wed	Thurs	Fri	Sat
2	$\frac{3}{4}$	$2\frac{1}{4}$	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{2}$

a) This week she had a goal to practise for a total of at least 10 h. Estimate whether she met her goal.
b) How many hours did she practise?
c) Was your estimate reasonable? Explain.

19. At the school's spring fair they sold $5\frac{1}{3}$ vegetarian pizzas, $6\frac{3}{4}$ pepperoni pizzas, and $4\frac{5}{6}$ cheese pizzas.
a) Draw diagrams to show how much of each pizza was sold.
b) Estimate, then calculate how much pizza was sold altogether.

20. The movie started 2 h 12 min ago. The movie will end in $1\frac{5}{6}$ h.
a) What is the total length of the movie in hours written as a fraction?
b) If the movie started at 2:15 p.m., when did the movie end? Write the time as a fraction.

MATH LINK

Egyptian fractions can be useful today. Suppose you have 13 sacks of rice to divide among 8 people. That means each person would get $1\frac{5}{8}$ sacks.

How can you give each person $1\frac{5}{8}$ sacks of rice if you do not have a calculator or scale?

First, give each person 1 whole sack. Then, use Egyptian fractions to determine how to give each person $\frac{5}{8}$ sack:
$\frac{5}{8}=\frac{1}{2}+\frac{1}{8}$
The Egyptian fraction shows that you should give each person $\frac{1}{2}$ sack, plus $\frac{1}{8}$ sack.
After you give $\frac{1}{2}$ sack to each person, there will be 1 sack left.
You then divide this sack into 8 and give each person $\frac{1}{8}$.
The diagram shows that each person gets 1 whole sack, plus $\frac{1}{2}$ sack and $\frac{1}{8}$ sack.

Strategies
Solve a Simpler Problem
Refer to page xvii.

How would you divide the following?
a) 7 sacks of potatoes among 4 people
b) 7 bags of flour among 5 people
c) 9 loaves of bread among 5 people

7.3 Add Mixed Numbers • MHR 251

Answers

Math Link

a) Give each person 1 sack of potatoes, plus $\frac{1}{2}$ sack of potatoes, plus $\frac{1}{4}$ sack of potatoes.

b) Give each person 1 bag of flour, plus $\frac{1}{5}$ bag of flour, plus $\frac{1}{5}$ bag of flour.

c) Give each person 1 loaf of bread, plus $\frac{1}{2}$ loaf of bread, plus $\frac{1}{4}$ loaf of bread, plus $\frac{1}{20}$ loaf of bread.

MATH LINK

This Math Link provides students with an opportunity to apply their understanding of fraction addition to a real-world application involving unit fractions.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • Which method of adding mixed numbers do you find easiest to understand and use?	• Have students list the similarities and differences between adding mixed numbers with like and unlike denominators. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • You may wish to have students fill in the During column for section 7.3 of **BLM 7–1 Chapter 7 Self-Assessment**. Have students indicate how they might improve any items that they have marked either red or yellow.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 251 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 263.	• Have students use words and numbers to explain what they remember about Egyptian fractions from the previous Math Link on page 244. • Encourage students to estimate, then calculate, their answers. • Have students check each other's calculations for accuracy. • Observe students as they work on the Math Link and have them explain and provide support for their solutions. • Refer students who are having difficulty getting started to **BLM 7–8 Section 7.3 Math Link**, which provides scaffolding for this activity.

Supported Learning

Learning Style

- Encourage concrete and semi-concrete learners to provide in their Math Learning Log diagrams and examples of their favoured method of addition.

Learning Style, ESL, Language, and Memory

- Have students complete the Math Link to help them prepare for the Wrap It Up! at the end of the chapter.

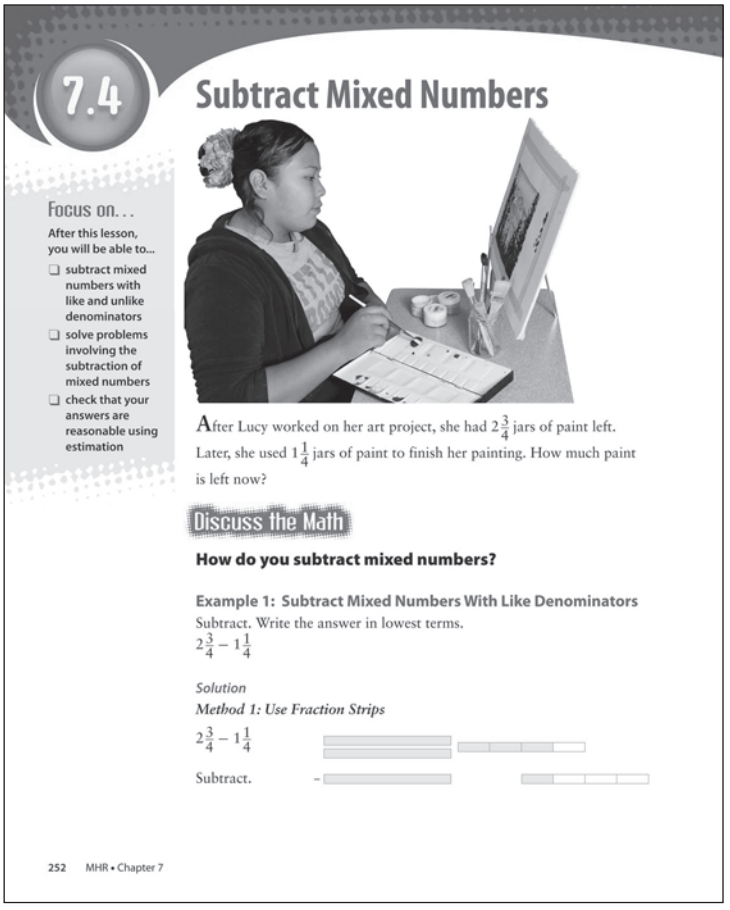
7.4 Subtract Mixed Numbers

Suggested Timing
80–100 minutes

Materials
- coloured pencils
- calculator (optional)

Blackline Masters
Master 15 Fraction Strips
BLM 7–1 Chapter 7 Self-Assessment
BLM 7–9 Section 7.4 Extra Practice
BLM 7–10 Section 7.4 Math Link

Mathematical Processes
- ☑ Communication
- ☑ Connections
- ☑ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☑ Technology
- ☑ Visualization


7.4 Subtract Mixed Numbers

Focus on...
After this lesson, you will be able to...
- subtract mixed numbers with like and unlike denominators
- solve problems involving the subtraction of mixed numbers
- check that your answers are reasonable using estimation

After Lucy worked on her art project, she had $2\frac{3}{4}$ jars of paint left. Later, she used $1\frac{1}{4}$ jars of paint to finish her painting. How much paint is left now?

Discuss the Math

How do you subtract mixed numbers?

Example 1: Subtract Mixed Numbers With Like Denominators
Subtract. Write the answer in lowest terms.
$2\frac{3}{4} - 1\frac{1}{4}$

Solution
Method 1: Use Fraction Strips
$2\frac{3}{4} - 1\frac{1}{4}$
Subtract. –

252 MHR • Chapter 7

Specific Outcomes

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

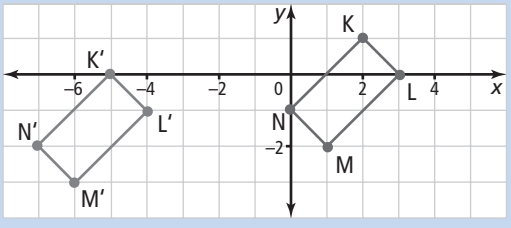
Warm-Up

For #1 to #3, show your answer as a mixed number in lowest terms.

1. $6\frac{2}{5} + 3\frac{1}{4}$ **2.** $2\frac{3}{4} + 4\frac{1}{3}$ **3.** $1\frac{1}{4} + 5\frac{2}{7}$

4. Estimate and calculate the area of the following triangle: $h = 6.2$ cm; $b = 2.8$ cm.

5. **a)** What are the coordinates of K′L′M′N′?
b) Describe the movement from K to K′ and L to L′.



Mental Math

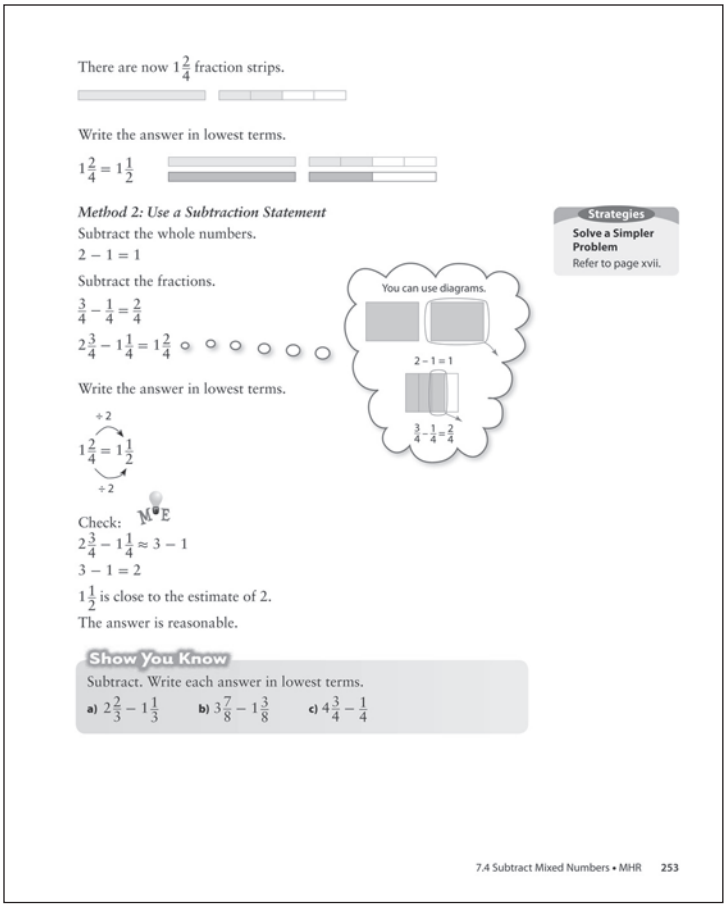
6. List six numbers between 300 and 400 that are divisible by 9. How do you know?

7. List six numbers between 1000 and 1050 that are divisible by 10. How do you know?

8. Write an equivalent fraction using each denominator shown.
a) $\frac{4}{7}$ (14) **b)** $\frac{3}{5}$ (10)

9. Use multiples to show a common denominator for the following fractions. Rewrite each fraction using the common denominator.
$\frac{5}{6}$ and $\frac{4}{9}$

10. Show the following fractions as decimals to the thousandths place.
a) $\frac{3}{5}$ **b)** $\frac{3}{8}$



There are now $1\frac{2}{4}$ fraction strips.

Write the answer in lowest terms.

$1\frac{2}{4} = 1\frac{1}{2}$

Method 2: Use a Subtraction Statement

Subtract the whole numbers.

$2 - 1 = 1$

Subtract the fractions.

$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

$2\frac{3}{4} - 1\frac{1}{4} = 1\frac{2}{4}$

Write the answer in lowest terms.

$1\frac{2}{4} = 1\frac{1}{2}$ (÷2)

Check:

$2\frac{3}{4} - 1\frac{1}{4} \approx 3 - 1$

$3 - 1 = 2$

$1\frac{1}{2}$ is close to the estimate of 2.

The answer is reasonable.

Strategies
Solve a Simpler Problem
Refer to page xvii.

You can use diagrams.

$2 - 1 = 1$

$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

Show You Know

Subtract. Write each answer in lowest terms.

a) $2\frac{2}{3} - 1\frac{1}{3}$ b) $3\frac{7}{8} - 1\frac{3}{8}$ c) $4\frac{3}{4} - \frac{1}{4}$

7.4 Subtract Mixed Numbers • MHR 253

Answers

Warm-Up

1. $9 + \frac{8}{20} + \frac{5}{20} = 9\frac{13}{20}$

2. $6 + \frac{9}{12} + \frac{4}{12} = 6 + \frac{13}{12} = 7\frac{1}{12}$

3. $6 + \frac{7}{28} + \frac{8}{28} = 6\frac{15}{28}$

4. Estimate: $6 \times 3 \div 2 = 9$ cm^2
Calculate: $6.2 \times 2.8 \div 2 = 8.68$ cm^2

5. **a)** K′(–5, 0), L′ (–4, –1), M′ (–6, –3), N′ (–7, –2)
b) K moved 7 units horizontally left and 1 unit vertically down.
L moved 7 units horizontally left and 1 unit vertically down.

6. Accept any six of 306, 315, 324, 333, 342, 351, 360, 369, 378, 387, 396. The sum of the digits is divisible by 9.

7. Accept any six of 1002, 1008, 1014, 1020, 1026, 1032, 1038, 1044. The numbers are divisible by 2 and 3.

8. **a)** $\frac{8}{14}$ **b)** $\frac{6}{10}$

9. $\frac{15}{18}, \frac{8}{18}$

10. **a)** 0.600 **b)** 0.375

Show You Know: Example 1

a) $1\frac{1}{3}$ **b)** $2\frac{1}{2}$ **c)** $4\frac{1}{2}$

Activity Planning Notes

Read and discuss the section opener as a class. Ask students to describe a time in their life when they had to solve a similar problem involving fractions or mixed numbers. Encourage students to explain or show how Lucy's problem can be written as a mathematical statement. Ask them what operation needs to be performed.

Discuss the Math

In this section, students learn how to subtract mixed numbers with like and unlike denominators. Allow students the time they need to develop these skills. Work through Examples 1 and 2 as a class.

Example 1 provides two methods of subtracting mixed numbers with like denominators. In Method 1, students use fraction strips to show subtraction. You may wish to give students **Master 15 Fraction Strips**. Method 2 uses a subtraction statement and shows how a diagram could help students perform the subtraction. Explain to students that, as they learned when adding mixed numbers, in this case the whole numbers can be handled separately from the fractions.

Supported Learning

ESL

- Use an example with a diagram that would explain the phrase "$2\frac{3}{4}$ jars of paint left."

Common Errors

- Students may be confused about the whole number part of the second term in part c) of the Example 1 Show You Know.

R$_x$ Explain to students that there are 0 wholes and $\frac{1}{4}$ of a whole.

Common Errors

- Students may subtract a larger fraction from a smaller fraction without doing the regrouping.

R_x Have students review their work and pay close attention to their subtraction strategies.

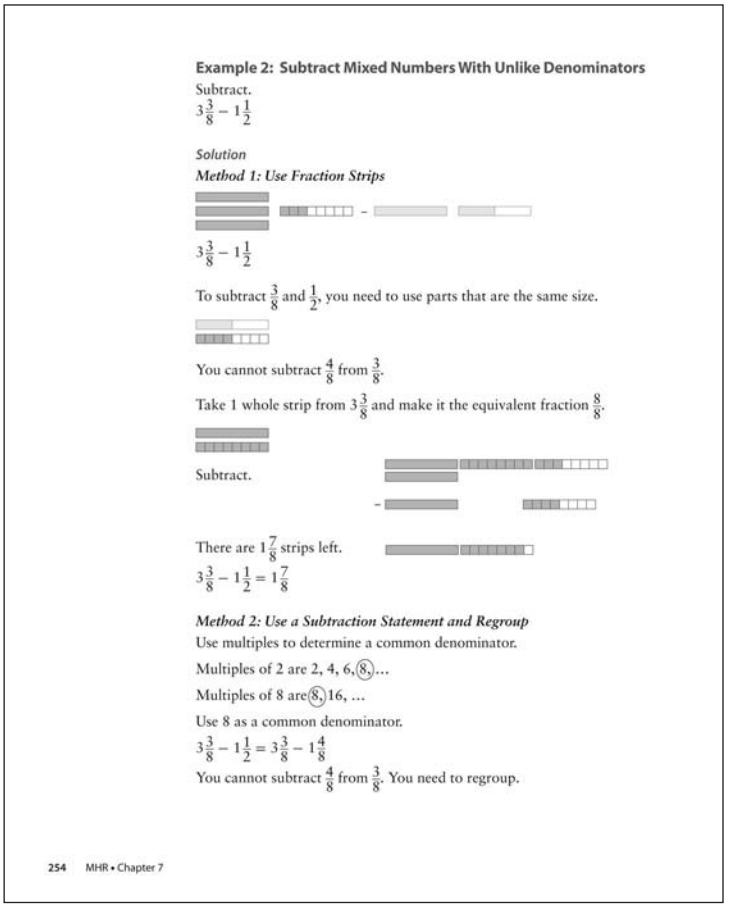
Supported Learning

Learning Style

- Allow students to use fraction strips, diagrams, or a subtraction statement to perform their calculations.

Motor

- Give students adequate time to work with the fraction strips in Examples 1 and 2.
- Students may benefit from using virtual manipulatives instead of the fraction strips.



Example 2: Subtract Mixed Numbers With Unlike Denominators

Subtract.

$3\frac{3}{8} - 1\frac{1}{2}$

Solution

Method 1: Use Fraction Strips

$3\frac{3}{8} - 1\frac{1}{2}$

To subtract $\frac{3}{8}$ and $\frac{1}{2}$, you need to use parts that are the same size.

You cannot subtract $\frac{4}{8}$ from $\frac{3}{8}$.

Take 1 whole strip from $3\frac{3}{8}$ and make it the equivalent fraction $\frac{8}{8}$.

Subtract.

There are $1\frac{7}{8}$ strips left.

$3\frac{3}{8} - 1\frac{1}{2} = 1\frac{7}{8}$

Method 2: Use a Subtraction Statement and Regroup

Use multiples to determine a common denominator.

Multiples of 2 are 2, 4, 6, (8)...

Multiples of 8 are (8,) 16, ...

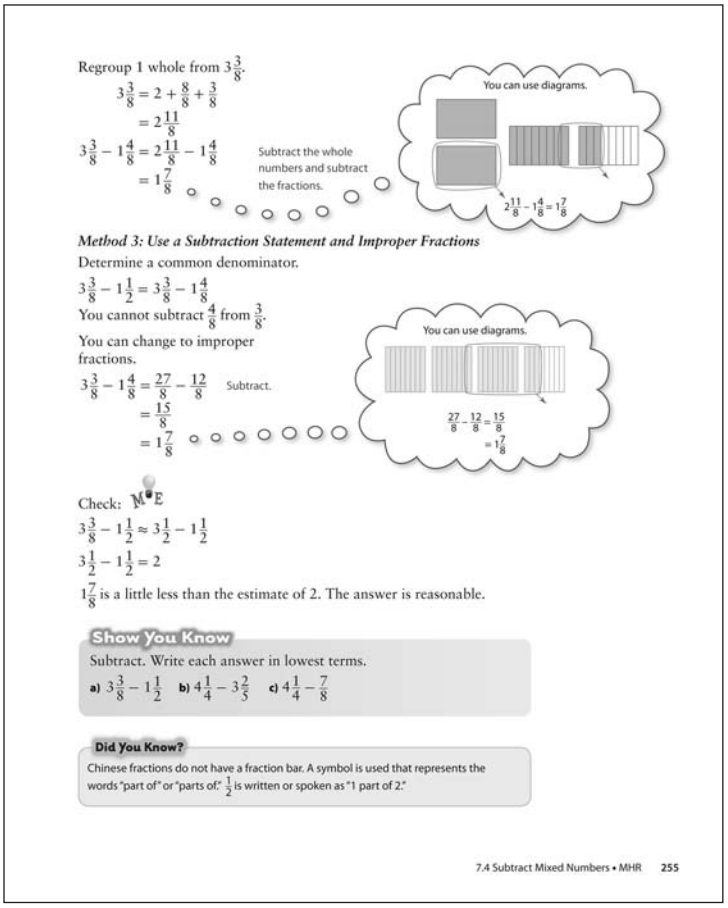
Use 8 as a common denominator.

$3\frac{3}{8} - 1\frac{1}{2} = 3\frac{3}{8} - 1\frac{4}{8}$

You cannot subtract $\frac{4}{8}$ from $\frac{3}{8}$. You need to regroup.

254 MHR • Chapter 7

Assessment *for* Learning	Supported Learning
Example 1 Have students complete the Show You Know on page 253 related to Example 1.	• Encourage students to determine which method might be most effective for each Show You Know question before they attempt it. • Have students share their solutions with a partner. • As a class, discuss the methods students used and their rationale for using them. • Provide additional questions for students who may benefit from them. Have them use diagrams to find the answers, and then write down what they did to find the solution. **a)** $2\frac{2}{5} - 1\frac{4}{5}$ ($1\frac{7}{5} - 1\frac{4}{5} = \frac{3}{5}$. Make sure that students understand how $2\frac{2}{5}$ becomes $1\frac{7}{5}$). **b)** $3\frac{1}{6} - 1\frac{5}{6}$ ($2\frac{7}{6} - 1\frac{5}{6} = 1\frac{2}{6} = 1\frac{1}{3}$. Remind students to write $\frac{2}{6}$ in lowest terms by dividing the numerator and denominator by 2.) Coach students through part a). Have them try part b) on their own.



Regroup 1 whole from $3\frac{3}{8}$.

$3\frac{3}{8} = 2 + \frac{8}{8} + \frac{3}{8}$

$= 2\frac{11}{8}$

$3\frac{3}{8} - 1\frac{4}{8} = 2\frac{11}{8} - 1\frac{4}{8}$

$= 1\frac{7}{8}$

Subtract the whole numbers and subtract the fractions.

You can use diagrams.

$2\frac{11}{8} - 1\frac{4}{8} = 1\frac{7}{8}$

Method 3: Use a Subtraction Statement and Improper Fractions

Determine a common denominator.

$3\frac{3}{8} - 1\frac{1}{2} = 3\frac{3}{8} - 1\frac{4}{8}$

You cannot subtract $\frac{4}{8}$ from $\frac{3}{8}$.

You can change to improper fractions.

$3\frac{3}{8} - 1\frac{4}{8} = \frac{27}{8} - \frac{12}{8}$ Subtract.

$= \frac{15}{8}$

$= 1\frac{7}{8}$

You can use diagrams.

$\frac{27}{8} - \frac{12}{8} = \frac{15}{8}$

$= 1\frac{7}{8}$

Check:

$3\frac{3}{8} - 1\frac{1}{2} \approx 3\frac{1}{2} - 1\frac{1}{2}$

$3\frac{1}{2} - 1\frac{1}{2} = 2$

$1\frac{7}{8}$ is a little less than the estimate of 2. The answer is reasonable.

Show You Know

Subtract. Write each answer in lowest terms.

a) $3\frac{3}{8} - 1\frac{1}{2}$ b) $4\frac{1}{4} - 3\frac{2}{5}$ c) $4\frac{1}{4} - \frac{7}{8}$

Did You Know?

Chinese fractions do not have a fraction bar. A symbol is used that represents the words "part of" or "parts of." $\frac{1}{2}$ is written or spoken as "1 part of 2."

7.4 Subtract Mixed Numbers • MHR 255

In Example 2, students subtract mixed numbers with unlike denominators using three methods.

Method 1 uses fraction strips. Method 2 uses a subtraction statement and regrouping of one whole. Method 3 uses a subtraction statement and improper fractions. Methods 2 and 3 start similarly, with finding a common denominator. Then, Method 2 regroups one whole from the mixed number, whereas Method 3 changes the mixed number to an improper fraction. Some students may feel that finding a common denominator first and attempting to subtract the numerators creates a step that could easily be avoided by directly changing to improper fractions. You may wish to allow students to use this method, but make sure they understand and can perform subtractions of mixed numbers using each method shown.

Have students read the Did You Know? at the bottom of page 255. If you have students in your class who are familiar with Chinese fractions, you may wish to ask them to show the symbol and/or to explain the wording of fractions further.

Answers

Show You Know: Example 2

a) $1\frac{7}{8}$ b) $\frac{17}{20}$ c) $3\frac{3}{8}$

Supported Learning

Learning Style

- Some students may need to use a calculator to work with improper fractions so that they can focus on developing the strategies related to subtracting mixed numbers.

ESL and Language

- Make sure students understand the terminology used in this section. They may need to be reminded of the meaning of *improper fraction*.

Answers

Communicate the Ideas

1. $\frac{1}{2}$ bottle of pop is left. Answers may vary.

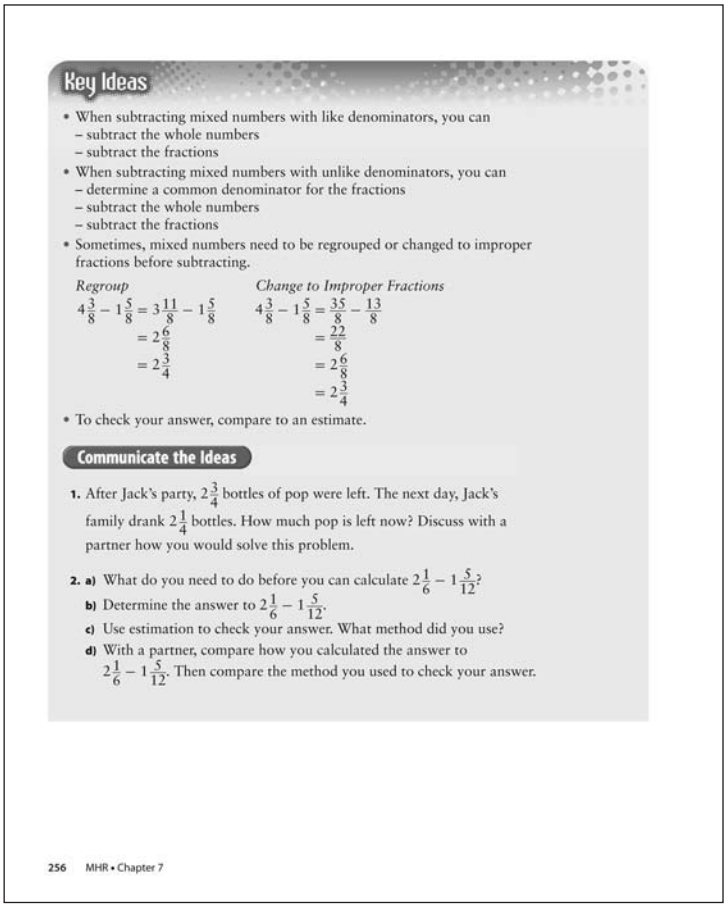
2. a) Answers may vary. For example: Regroup the mixed numbers or change the mixed numbers to improper fractions and determine a common denominator.

b) $\frac{3}{4}$

c) Answers may vary. For example:

$2\frac{1}{6} - 1\frac{5}{12} \approx 2 - 1\frac{1}{2} = \frac{1}{2}$

d) Answers may vary.



Key Ideas

- When subtracting mixed numbers with like denominators, you can
  - subtract the whole numbers
  - subtract the fractions
- When subtracting mixed numbers with unlike denominators, you can
  - determine a common denominator for the fractions
  - subtract the whole numbers
  - subtract the fractions
- Sometimes, mixed numbers need to be regrouped or changed to improper fractions before subtracting.

Regroup

$4\frac{3}{8} - 1\frac{5}{8} = 3\frac{11}{8} - 1\frac{5}{8}$
$= 2\frac{6}{8}$
$= 2\frac{3}{4}$

Change to Improper Fractions

$4\frac{3}{8} - 1\frac{5}{8} = \frac{35}{8} - \frac{13}{8}$
$= \frac{22}{8}$
$= 2\frac{6}{8}$
$= 2\frac{3}{4}$

- To check your answer, compare to an estimate.

Communicate the Ideas

1. After Jack's party, $2\frac{3}{4}$ bottles of pop were left. The next day, Jack's family drank $2\frac{1}{4}$ bottles. How much pop is left now? Discuss with a partner how you would solve this problem.

2. a) What do you need to do before you can calculate $2\frac{1}{6} - 1\frac{5}{12}$?

b) Determine the answer to $2\frac{1}{6} - 1\frac{5}{12}$.

c) Use estimation to check your answer. What method did you use?

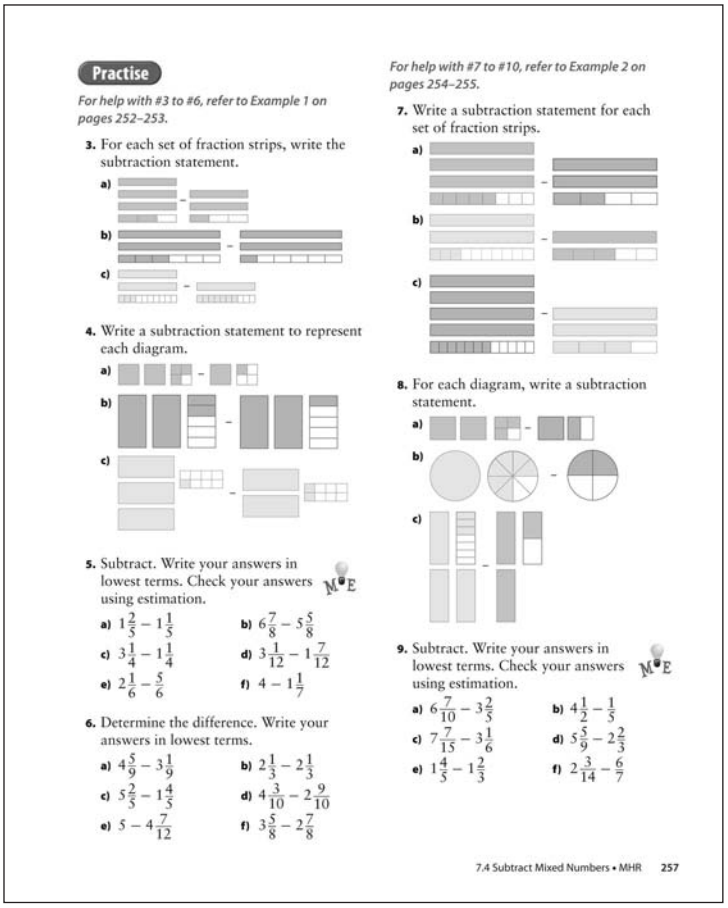
d) With a partner, compare how you calculated the answer to $2\frac{1}{6} - 1\frac{5}{12}$. Then compare the method you used to check your answer.

256 MHR • Chapter 7

Assessment *for* Learning	Supported Learning
Example 2 Have students complete the Show You Know on page 255 related to Example 2.	• Have students talk through their thinking in a group. Listen as they discuss various methods of subtracting mixed numbers. • When students need to regroup from a whole, pay special attention to how they accomplish this task. • Encourage students to show their work using diagrams. • Remind students that to subtract or add fractions and mixed numbers they need to use a common denominator. • Discuss the solutions and the methods used as a class. • Provide additional questions for students who may benefit from them. Have them use diagrams to find the answers, and then write down their solution. **a)** $2\frac{2}{5} - 1\frac{1}{2}$ ($2\frac{4}{10} - 1\frac{5}{10} = 1\frac{14}{10} - 1\frac{5}{10} = \frac{9}{10}$. Make sure that students understand how $2\frac{2}{5}$ becomes $1\frac{14}{10}$.) **b)** $3\frac{1}{6} - 1\frac{3}{4}$ ($3\frac{2}{12} - 1\frac{9}{12} = 2\frac{14}{12} - 1\frac{9}{12} = 1\frac{5}{12}$) Coach students through part a). Have them try part b) on their own.

Key Ideas

Have students read the Key Ideas, which summarize the steps of subtracting mixed numbers. Make sure students understand and can apply these steps to subtraction involving like denominators and unlike denominators.



Practise

For help with #3 to #6, refer to Example 1 on pages 252–253.

3. For each set of fraction strips, write the subtraction statement.

a)

b)

c)

4. Write a subtraction statement to represent each diagram.

a)

b)

c)

5. Subtract. Write your answers in lowest terms. Check your answers using estimation.

a) $1\frac{2}{5} - 1\frac{1}{5}$

b) $6\frac{7}{8} - 5\frac{5}{8}$

c) $3\frac{1}{4} - 1\frac{1}{4}$

d) $3\frac{1}{12} - 1\frac{7}{12}$

e) $2\frac{1}{6} - \frac{5}{6}$

f) $4 - 1\frac{1}{7}$

6. Determine the difference. Write your answers in lowest terms.

a) $4\frac{5}{9} - 3\frac{1}{9}$

b) $2\frac{1}{3} - 2\frac{1}{3}$

c) $5\frac{2}{5} - 1\frac{4}{5}$

d) $4\frac{3}{10} - 2\frac{9}{10}$

e) $5 - 4\frac{7}{12}$

f) $3\frac{5}{8} - 2\frac{7}{8}$

For help with #7 to #10, refer to Example 2 on pages 254–255.

7. Write a subtraction statement for each set of fraction strips.

a)

b)

c)

8. For each diagram, write a subtraction statement.

a)

b)

c)

9. Subtract. Write your answers in lowest terms. Check your answers using estimation.

a) $6\frac{7}{10} - 3\frac{2}{5}$

b) $4\frac{1}{2} - \frac{1}{5}$

c) $7\frac{7}{15} - 3\frac{1}{6}$

d) $5\frac{5}{9} - 2\frac{2}{3}$

e) $1\frac{4}{5} - 1\frac{2}{3}$

f) $2\frac{3}{14} - \frac{6}{7}$

7.4 Subtract Mixed Numbers • MHR 257

Assessment *as* Learning	Supported Learning
Communicate the Ideas These questions allow students to demonstrate their knowledge of subtracting mixed numbers and to justify and explain their methods. Have students work in pairs or groups to complete both of the Communicate the Ideas questions.	• The concept of regrouping to subtract fractions is likely new to students. Allow students to use manipulatives until they fully understand the concepts in this section.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 7, 9, 11, 12, Math Link
Typical	1–3, 5, 7, 9, 11–13, 15–17, 21, 22, Math Link
Extension/Enrichment	1, 2, 13, 14, 16, 18, 20–23

Communicate the Ideas

In #1, students subtract mixed numbers with like denominators to solve the problem. Encourage students to use manipulatives to help them visualize and work through the problem. You may wish to have students model this problem using bottles of water.

In #2, students subtract mixed numbers with unlike denominators. Then students are asked to describe the method of estimation that they used. You may wish to review some methods of estimating, such as front-end estimation and relative size. Both of these types of estimation are discussed in Chapter 2.

Practise

Allow students to use the methods they feel most comfortable with when answering the Practise questions. Have each student show a solution they feel proud of, and then choose one solution that you would like to see from each student. In #3, #4, #7, and #9, students write a subtraction statement for each set of fraction strips or diagrams. Students subtract mixed numbers with like denominators in #5 and #6, and they subtract mixed numbers with unlike denominators in #9 and #10.

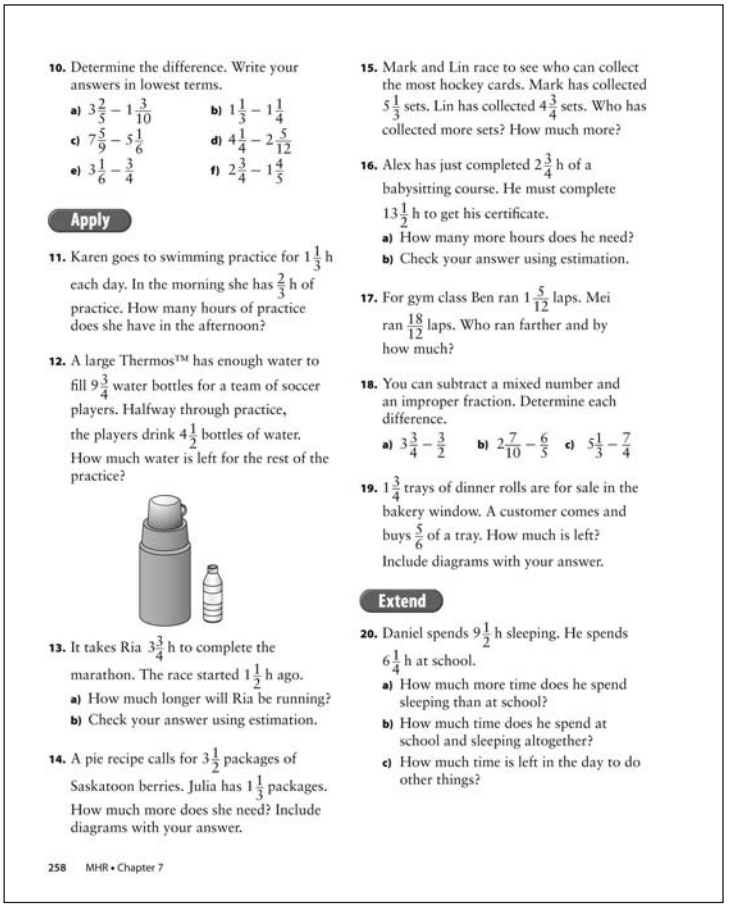
Supported Learning

ESL and Language

- The language used in some of the word problems may be difficult for English language learners. In #12, clarify that if a player drinks the water, it is being taken away from the total. Also, you may need to explain the purpose of a Thermos™.

Learning Style and Memory

- **BLM 7–9 Section 7.4 Extra Practice** provides additional reinforcement for students who need it.



10. Determine the difference. Write your answers in lowest terms.

a) $3\frac{2}{5} - 1\frac{3}{10}$ b) $1\frac{1}{3} - 1\frac{1}{4}$

c) $7\frac{5}{9} - 5\frac{1}{6}$ d) $4\frac{1}{4} - 2\frac{5}{12}$

e) $3\frac{1}{6} - \frac{3}{4}$ f) $2\frac{3}{4} - 1\frac{4}{5}$

Apply

11. Karen goes to swimming practice for $1\frac{1}{3}$ h each day. In the morning she has $\frac{2}{3}$ h of practice. How many hours of practice does she have in the afternoon?

12. A large Thermos™ has enough water to fill $9\frac{3}{4}$ water bottles for a team of soccer players. Halfway through practice, the players drink $4\frac{1}{2}$ bottles of water. How much water is left for the rest of the practice?

13. It takes Ria $3\frac{3}{4}$ h to complete the marathon. The race started $1\frac{1}{2}$ h ago.

a) How much longer will Ria be running?

b) Check your answer using estimation.

14. A pie recipe calls for $3\frac{1}{2}$ packages of Saskatoon berries. Julia has $1\frac{1}{3}$ packages. How much more does she need? Include diagrams with your answer.

15. Mark and Lin race to see who can collect the most hockey cards. Mark has collected $5\frac{1}{3}$ sets. Lin has collected $4\frac{3}{4}$ sets. Who has collected more sets? How much more?

16. Alex has just completed $2\frac{3}{4}$ h of a babysitting course. He must complete $13\frac{1}{2}$ h to get his certificate.

a) How many more hours does he need?

b) Check your answer using estimation.

17. For gym class Ben ran $1\frac{5}{12}$ laps. Mei ran $\frac{18}{12}$ laps. Who ran farther and by how much?

18. You can subtract a mixed number and an improper fraction. Determine each difference.

a) $3\frac{3}{4} - \frac{3}{2}$ b) $2\frac{7}{10} - \frac{6}{5}$ c) $5\frac{1}{3} - \frac{7}{4}$

19. $1\frac{3}{4}$ trays of dinner rolls are for sale in the bakery window. A customer comes and buys $\frac{5}{6}$ of a tray. How much is left? Include diagrams with your answer.

Extend

20. Daniel spends $9\frac{1}{2}$ h sleeping. He spends $6\frac{1}{4}$ h at school.

a) How much more time does he spend sleeping than at school?

b) How much time does he spend at school and sleeping altogether?

c) How much time is left in the day to do other things?

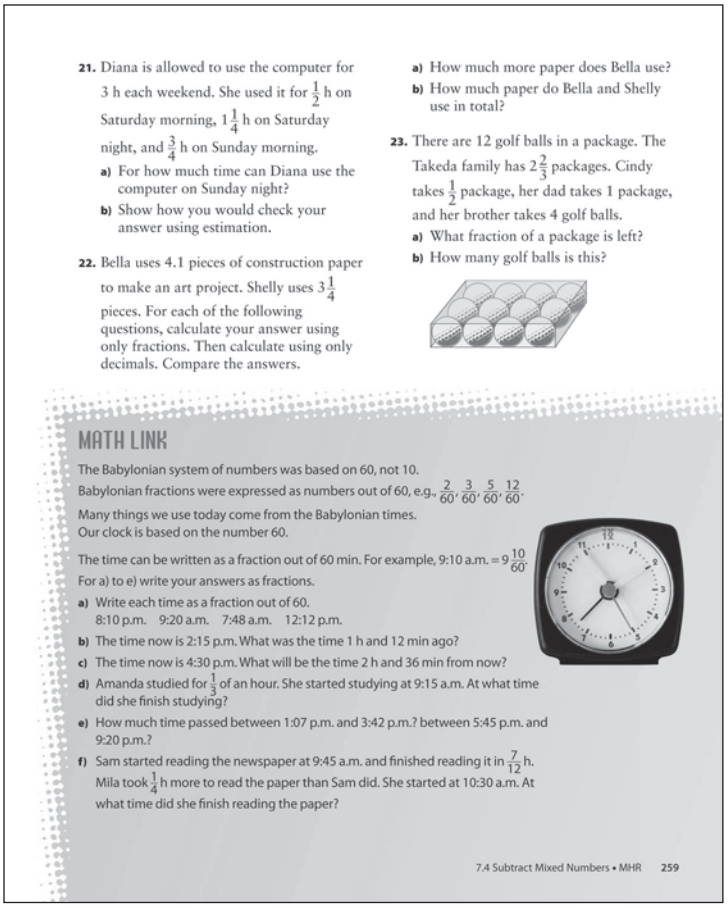
258 MHR • Chapter 7

Assessment *for* Learning	Supported Learning
Practise Have students do #3, #5, #7, and #9. Students who have no problems with these questions can go on to the Apply questions.	• If students have difficulty with these questions, have them review the examples and clarify any misconceptions. Make sure students are able to convert between mixed numbers and improper fractions. • Work through #4a), #6a) and b), #8a), and #10a) and b) with students. Then have them try the remaining parts of these questions on their own. Make sure students show their work.

Apply and Extend

All but one of the questions in the Apply and Extend categories provide real-world contexts to help students relate to the math in this section. The exception is #18, which gives students an opportunity to subtract an improper fraction from a mixed number. Two of the questions in this set involve like denominators: #11 and #17. You may wish to assign these questions to students who first need practise solving problems that do not involve finding a common denominator. The balance of the Apply and Extend questions involve unlike denominators. Encourage students to work through them at their own pace and to make sure they understand each problem.

A number of problems require students to use their regrouping or improper fractions strategies. Refer students who are having difficulty to Example 2. In #22, students convert between decimal numbers and fractions as they solve a subtraction and an addition problem.



21. Diana is allowed to use the computer for 3 h each weekend. She used it for $\frac{1}{2}$ h on Saturday morning, $1\frac{1}{4}$ h on Saturday night, and $\frac{3}{4}$ h on Sunday morning.

a) For how much time can Diana use the computer on Sunday night?

b) Show how you would check your answer using estimation.

22. Bella uses 4.1 pieces of construction paper to make an art project. Shelly uses $3\frac{1}{4}$ pieces. For each of the following questions, calculate your answer using only fractions. Then calculate using only decimals. Compare the answers.

a) How much more paper does Bella use?

b) How much paper do Bella and Shelly use in total?

23. There are 12 golf balls in a package. The Takeda family has $2\frac{2}{3}$ packages. Cindy takes $\frac{1}{2}$ package, her dad takes 1 package, and her brother takes 4 golf balls.

a) What fraction of a package is left?

b) How many golf balls is this?

MATH LINK

The Babylonian system of numbers was based on 60, not 10.
Babylonian fractions were expressed as numbers out of 60, e.g., $\frac{2}{60}, \frac{3}{60}, \frac{5}{60}, \frac{12}{60}$.
Many things we use today come from the Babylonian times.
Our clock is based on the number 60.
The time can be written as a fraction out of 60 min. For example, 9:10 a.m. = $9\frac{10}{60}$.
For a) to e) write your answers as fractions.

a) Write each time as a fraction out of 60.
8:10 p.m. 9:20 a.m. 7:48 a.m. 12:12 p.m.

b) The time now is 2:15 p.m. What was the time 1 h and 12 min ago?

c) The time now is 4:30 p.m. What will be the time 2 h and 36 min from now?

d) Amanda studied for $\frac{1}{3}$ of an hour. She started studying at 9:15 a.m. At what time did she finish studying?

e) How much time passed between 1:07 p.m. and 3:42 p.m.? between 5:45 p.m. and 9:20 p.m.?

f) Sam started reading the newspaper at 9:45 a.m. and finished reading it in $\frac{7}{12}$ h. Mila took $\frac{1}{4}$ h more to read the paper than Sam did. She started at 10:30 a.m. At what time did she finish reading the paper?

7.4 Subtract Mixed Numbers • MHR 259

Assessment *as* Learning	Supported Learning
Math Learning Log Have students respond to the following prompts: • Describe how to regroup a mixed number. • In what situations do you need to regroup? Give an example.	• Depending on students' learning style, have them provide oral or written answers. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • You may wish to have students fill in the During column for section 7.4 of **BLM 7–1 Chapter 7 Self-Assessment**. Have students indicate how they might improve any items they have marked either red or yellow.

Assessment *for* Learning	Supported Learning
Math Link Have students complete the Math Link on page 259 to help them prepare for the chapter problem wrap-up titled Wrap It Up! on page 263.	• Allow students extra time to work through the conversions and calculations in the Math Link. • If students have difficulty subtracting times, allow them to use a clock or a time line to help with their calculations. • Students should be encouraged to share their solutions with a partner. • You may wish to provide **BLM 7–10 Section 7.4 Math Link** to students who require scaffolding for this activity.

Answers

Math Link

a) $8\frac{10}{60}$ p.m., $9\frac{20}{60}$ a.m., $7\frac{48}{60}$ a.m., $12\frac{12}{60}$ p.m.

b) The time was $1\frac{3}{60}$ p.m.

c) The time will be $7\frac{6}{60}$ p.m.

d) Amanda finished studying at $9\frac{35}{60}$ a.m.

e) $2\frac{35}{60}$ h, $3\frac{35}{60}$ h

f) 11:20 a.m.

MATH LINK

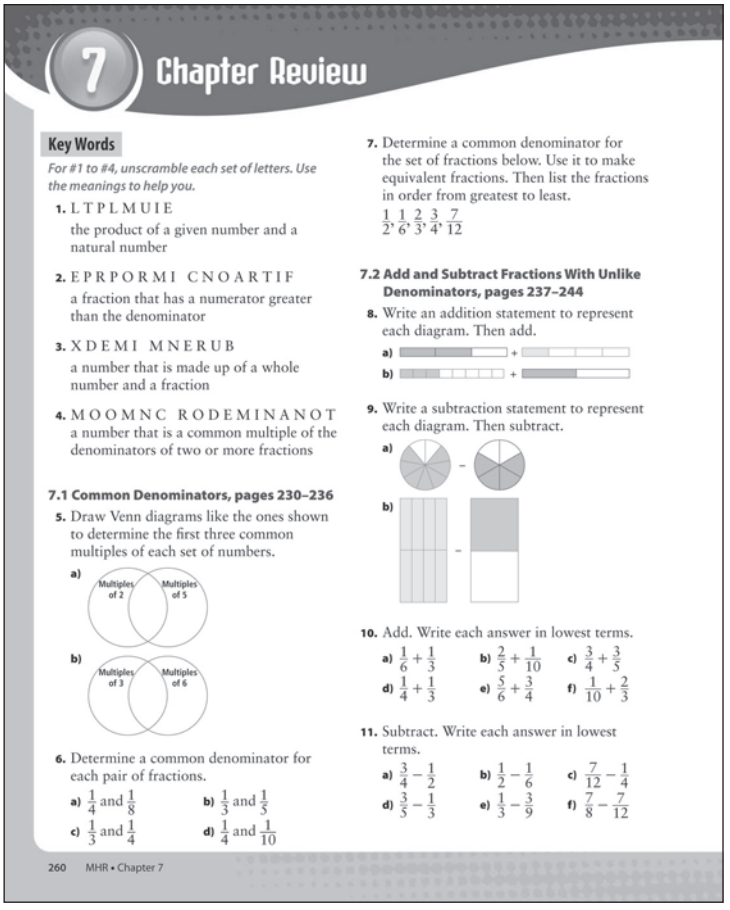
This Math Link is intended to help students apply what they have learned about fraction addition and subtraction to real-world applications involving time. Throughout this chapter, the Math Links demonstrate how fractions have been applied in different cultures, and in this section, the Babylonian system of numbers is related to our clock. Have students discuss or research other ways that the Babylonian system of numbers might have influenced the ways in which we do things today.

7 Chapter Review

Suggested Timing
40–50 minutes

Materials
- coloured pencils

Blackline Masters
Master 9 0.5 Centimetre Grid Paper
Master 13 Pattern Blocks
Master 15 Fraction Strips
BLM 7–1 Chapter 7 Self-Assessment
BLM 7–3 Section 7.1 Extra Practice
BLM 7–5 Section 7.7 Extra Practice
BLM 7–7 Section 7.3 Extra Practice
BLM 7–9 Section 7.4 Extra Practice



7 Chapter Review

Key Words

For #1 to #4, unscramble each set of letters. Use the meanings to help you.

1. L T P L M U I E
the product of a given number and a natural number

2. E P R P O R M I C N O A R T I F
a fraction that has a numerator greater than the denominator

3. X D E M I M N E R U B
a number that is made up of a whole number and a fraction

4. M O O M N C R O D E M I N A N O T
a number that is a common multiple of the denominators of two or more fractions

7.1 Common Denominators, pages 230–236

5. Draw Venn diagrams like the ones shown to determine the first three common multiples of each set of numbers.
a) Multiples of 2, Multiples of 5
b) Multiples of 3, Multiples of 6

6. Determine a common denominator for each pair of fractions.
a) $\frac{1}{4}$ and $\frac{1}{8}$ b) $\frac{1}{3}$ and $\frac{1}{5}$
c) $\frac{1}{3}$ and $\frac{1}{4}$ d) $\frac{1}{4}$ and $\frac{1}{10}$

7. Determine a common denominator for the set of fractions below. Use it to make equivalent fractions. Then list the fractions in order from greatest to least.
$\frac{1}{2}, \frac{1}{6}, \frac{2}{3}, \frac{3}{4}, \frac{7}{12}$

7.2 Add and Subtract Fractions With Unlike Denominators, pages 237–244

8. Write an addition statement to represent each diagram. Then add.
a) +
b) +

9. Write a subtraction statement to represent each diagram. Then subtract.
a) −
b) −

10. Add. Write each answer in lowest terms.
a) $\frac{1}{6} + \frac{1}{3}$ b) $\frac{2}{5} + \frac{1}{10}$ c) $\frac{3}{4} + \frac{3}{5}$
d) $\frac{1}{4} + \frac{1}{3}$ e) $\frac{5}{6} + \frac{3}{4}$ f) $\frac{1}{10} + \frac{2}{3}$

11. Subtract. Write each answer in lowest terms.
a) $\frac{3}{4} - \frac{1}{2}$ b) $\frac{1}{2} - \frac{1}{6}$ c) $\frac{7}{12} - \frac{1}{4}$
d) $\frac{3}{5} - \frac{1}{3}$ e) $\frac{1}{3} - \frac{3}{9}$ f) $\frac{7}{8} - \frac{7}{12}$

260 MHR • Chapter 7

Supported Learning

Learning Style

- Allow students to complete the chapter review using any combination of oral descriptions, diagrams, and written answers.
- You may wish to differentiate for students the questions you feel they *must* know, the questions they *should* know, and those questions that are *nice* to know.
- Allow students to use **Master 9 0.5 Centimetre Grid Paper**, **Master 13 Pattern Blocks**, and/or **Master 15 Fraction Strips** as they complete the chapter review.

ESL, Language, and Memory

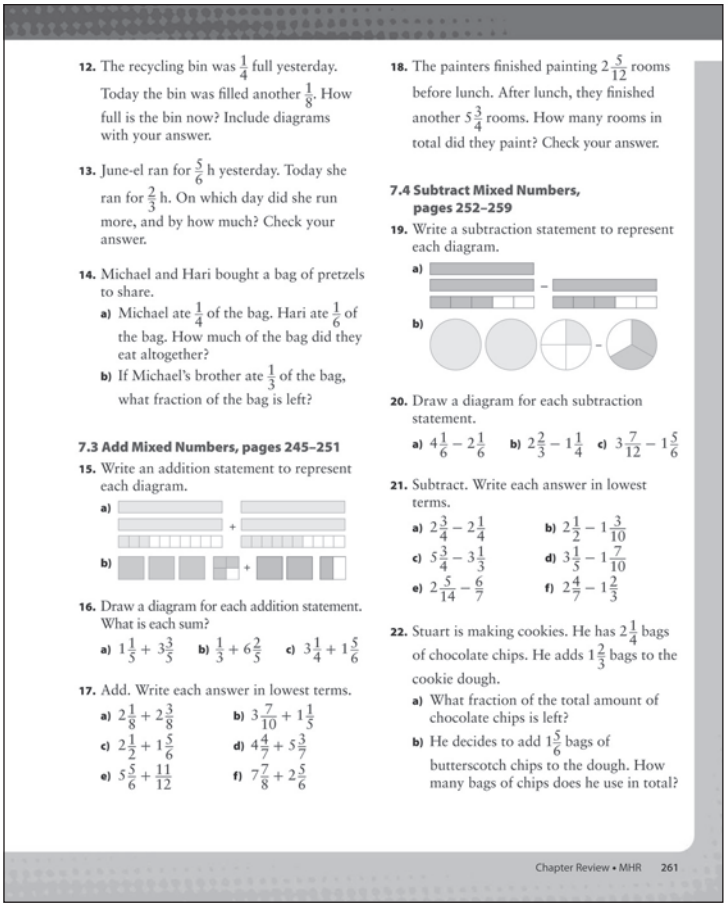
- Encourage students to use their chapter Foldable during the chapter review and to add any notes into the relevant sections.

Activity Planning Notes

Encourage students to work independently to complete the review. If students encounter difficulties, suggest that they discuss strategies with other students. Also, have them refer to the information in their chapter Foldable and then to the specific section in the student resource and/or their notebooks. Students may wish to work in pairs to review the words in #1 to #4.

Have students write the numbers from 5 to 22 in two columns in their notebooks. As they read each question, have students indicate any questions they may need a little help with, a lot of help with, or no help with.

Assessment *for* Learning	Supported Learning
Chapter 7 Review The chapter review provides an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On tab of their chapter Foldable. • Have students do at least one question related to any concept, skill, or process that has been giving them trouble. • Have students revisit any section they are having difficulty with prior to beginning the chapter test.



12. The recycling bin was $\frac{1}{4}$ full yesterday. Today the bin was filled another $\frac{1}{8}$. How full is the bin now? Include diagrams with your answer.

13. June-el ran for $\frac{5}{6}$ h yesterday. Today she ran for $\frac{2}{3}$ h. On which day did she run more, and by how much? Check your answer.

14. Michael and Hari bought a bag of pretzels to share.
a) Michael ate $\frac{1}{4}$ of the bag. Hari ate $\frac{1}{6}$ of the bag. How much of the bag did they eat altogether?
b) If Michael's brother ate $\frac{1}{3}$ of the bag, what fraction of the bag is left?

7.3 Add Mixed Numbers, pages 245–251

15. Write an addition statement to represent each diagram.
a)
b)

16. Draw a diagram for each addition statement. What is each sum?
a) $1\frac{1}{5} + 3\frac{3}{5}$ b) $\frac{1}{3} + 6\frac{2}{5}$ c) $3\frac{1}{4} + 1\frac{5}{6}$

17. Add. Write each answer in lowest terms.
a) $2\frac{1}{8} + 2\frac{3}{8}$ b) $3\frac{7}{10} + 1\frac{1}{5}$
c) $2\frac{1}{2} + 1\frac{5}{6}$ d) $4\frac{4}{7} + 5\frac{3}{7}$
e) $5\frac{5}{6} + \frac{11}{12}$ f) $7\frac{7}{8} + 2\frac{5}{6}$

18. The painters finished painting $2\frac{5}{12}$ rooms before lunch. After lunch, they finished another $5\frac{3}{4}$ rooms. How many rooms in total did they paint? Check your answer.

7.4 Subtract Mixed Numbers, pages 252–259

19. Write a subtraction statement to represent each diagram.
a)
b)

20. Draw a diagram for each subtraction statement.
a) $4\frac{1}{6} - 2\frac{1}{6}$ b) $2\frac{2}{3} - 1\frac{1}{4}$ c) $3\frac{7}{12} - 1\frac{5}{6}$

21. Subtract. Write each answer in lowest terms.
a) $2\frac{3}{4} - 2\frac{1}{4}$ b) $2\frac{1}{2} - 1\frac{3}{10}$
c) $5\frac{3}{4} - 3\frac{1}{3}$ d) $3\frac{1}{5} - 1\frac{7}{10}$
e) $2\frac{5}{14} - \frac{6}{7}$ f) $2\frac{4}{7} - 1\frac{2}{3}$

22. Stuart is making cookies. He has $2\frac{1}{4}$ bags of chocolate chips. He adds $1\frac{2}{3}$ bags to the cookie dough.
a) What fraction of the total amount of chocolate chips is left?
b) He decides to add $1\frac{5}{6}$ bags of butterscotch chips to the dough. How many bags of chips does he use in total?

Chapter Review • MHR 261

Supported Learning

Learning Style and Memory

- Refer students who require additional reinforcement to **BLM 7–3 Section 7.1 Extra Practice**, **BLM 7–5 Section 7.2 Extra Practice**, **BLM 7–7 Section 7.3 Extra Practice**, and **BLM 7–9 Section 7.4 Extra Practice**.
- Create and display in the classroom a list reminding students of things they can do to help themselves. This list might include the following points:
  – Check my notebook.
  – Check my journal.
  – Check my Foldables.
  – Check the student resource.
  – Check with a friend or two.
  – Check with my teacher/parent/guardian.

Gifted and Enrichment

- Students may already be familiar with the skills handled in this review. To provide additional questions, go to **www.mathlinks7.ca** and follow the links.

Assessment *as* Learning	Supported Learning
Math Learning Log Once students have completed the Chapter 7 Review and prior to the chapter test, have students reflect on their progress and write a journal entry that completes each statement: – I am comfortable with the following parts of the chapter ... – The method I prefer to add or subtract fractions is ... – I am having difficulty with ... – Here's how I plan to address the areas I am having difficult with ...	• Have students refer back to the What I Need to Work On tab of their chapter Foldable to provide information about what they continue to have problems with and what problems they had that have now been resolved. • You may wish to have students refer to **BLM 7–1 Chapter 7 Self-Assessment** when they report on what they are comfortable with, what they continue to have difficulty with, and what they plan to do to resolve their difficulty. • Students have learned different ways to add and subtract fractions and mixed numbers. Encourage students to use the method they feel most comfortable with, but to try other methods. They might find that some methods work better for some types of questions and others for other types. Have them record their findings in the appropriate sections of the chapter Foldable. • You may wish to have students revisit the webs they worked on in the Chapter Opener. Encourage them to add any new concepts that they have learned.

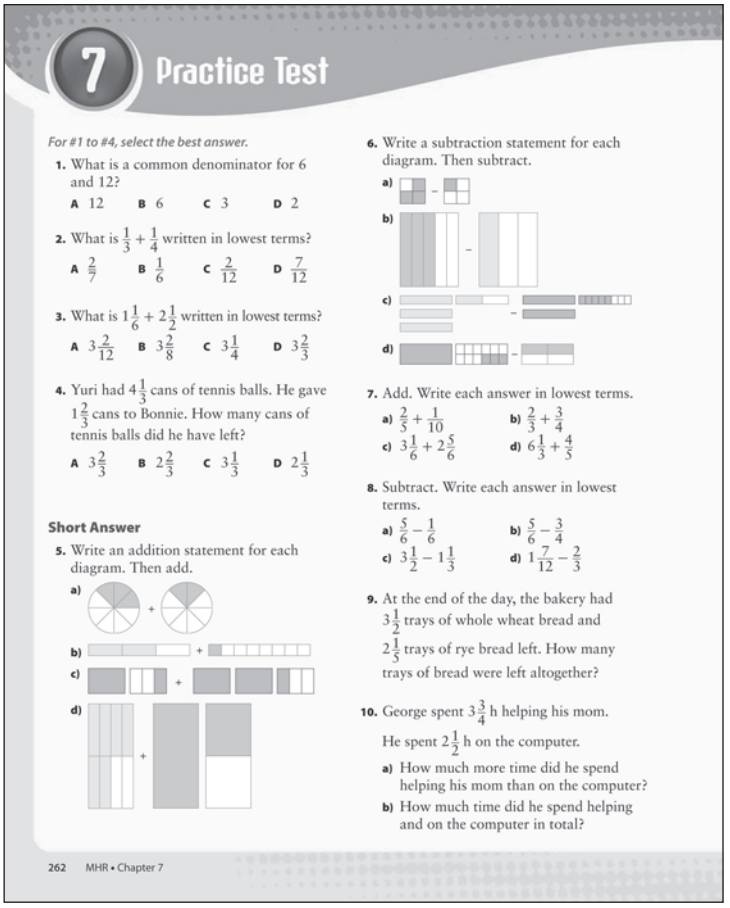
7 Practice Test

Suggested Timing
40–50 minutes

Materials
- coloured pencils
- calculator (optional)

Blackline Masters
Master 9 0.5 Centimetre Grid Paper
Master 13 Pattern Blocks
Master 15 Fraction Strips
BLM 7–1 Chapter 7 Self-Assessment
BLM 7–11 Chapter 7 Test

Assessment *as* Learning	Supported Learning
Chapter 7 Self-Assessment Have students review their earlier responses on **BLM 7–1 Chapter 7 Self-Assessment**.	• Have students use their responses on the practice test and work they completed earlier in the chapter to complete the After column of this self-assessment. Before the chapter test, coach them in the areas in which they are having difficulty.



7 Practice Test

For #1 to #4, select the best answer.

1. What is a common denominator for 6 and 12?
A 12 B 6 C 3 D 2

2. What is $\frac{1}{3} + \frac{1}{4}$ written in lowest terms?
A $\frac{2}{7}$ B $\frac{1}{6}$ C $\frac{2}{12}$ D $\frac{7}{12}$

3. What is $1\frac{1}{6} + 2\frac{1}{2}$ written in lowest terms?
A $3\frac{2}{12}$ B $3\frac{2}{8}$ C $3\frac{1}{4}$ D $3\frac{2}{3}$

4. Yuri had $4\frac{1}{3}$ cans of tennis balls. He gave $1\frac{2}{3}$ cans to Bonnie. How many cans of tennis balls did he have left?
A $3\frac{2}{3}$ B $2\frac{2}{3}$ C $3\frac{1}{3}$ D $2\frac{1}{3}$

Short Answer

5. Write an addition statement for each diagram. Then add.
a) b) c) d)

6. Write a subtraction statement for each diagram. Then subtract.
a) b) c) d)

7. Add. Write each answer in lowest terms.
a) $\frac{2}{5} + \frac{1}{10}$ b) $\frac{2}{3} + \frac{3}{4}$
c) $3\frac{1}{6} + 2\frac{5}{6}$ d) $6\frac{1}{3} + \frac{4}{5}$

8. Subtract. Write each answer in lowest terms.
a) $\frac{5}{6} - \frac{1}{6}$ b) $\frac{5}{6} - \frac{3}{4}$
c) $3\frac{1}{2} - 1\frac{1}{3}$ d) $1\frac{7}{12} - \frac{2}{3}$

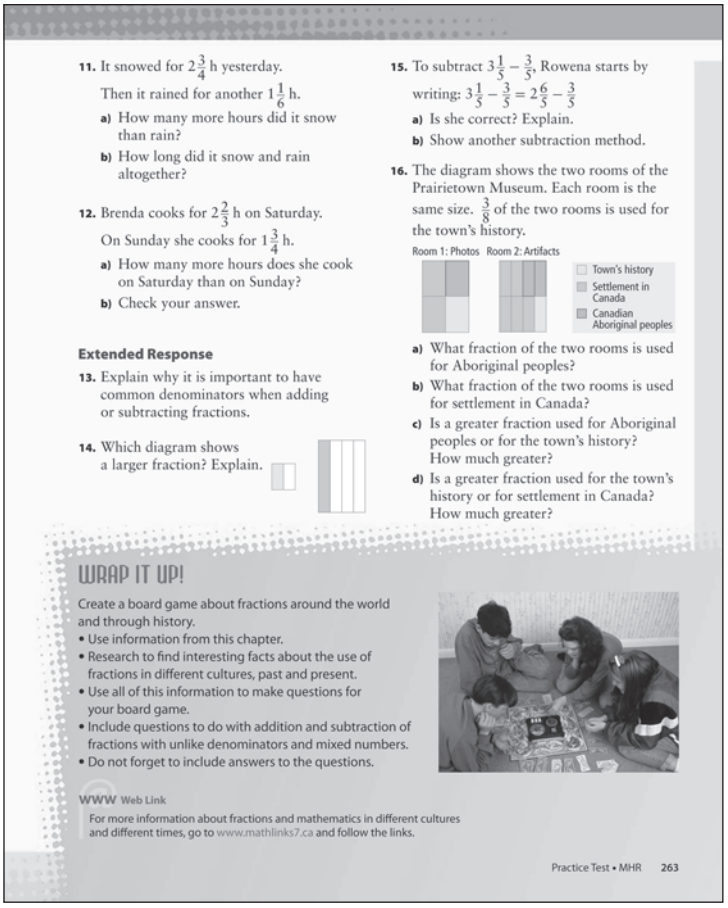
9. At the end of the day, the bakery had $3\frac{1}{2}$ trays of whole wheat bread and $2\frac{1}{5}$ trays of rye bread left. How many trays of bread were left altogether?

10. George spent $3\frac{3}{4}$ h helping his mom. He spent $2\frac{1}{2}$ h on the computer.
a) How much more time did he spend helping his mom than on the computer?
b) How much time did he spend helping and on the computer in total?

262 MHR • Chapter 7

Study Guide

Question(s)	Section(s)	Refer to	I can …
1, 14	7.1	Example	✓ use paper folding, diagrams, or multiples to find a common denominator
2	7.2	Example 1	✓ add fractions with unlike denominators
3, 9	7.3	Example 2	✓ add mixed numbers with unlike denominators
4, 15	7.4	Example 1	✓ use fraction strips, diagrams, or a subtraction statement to subtract mixed numbers with like denominators
5, 7	7.2	Example 1	✓ use fraction strips, diagrams, or a common denominator to add fractions with unlike denominators
	7.3	Examples 1, 2	✓ use pattern blocks, diagrams, or an addition statement to add mixed numbers with like or unlike denominators
6, 8	7.2	Example 2	✓ use pattern blocks, fraction strips, diagrams, or a common denominator to subtract fractions with unlike denominators
	7.4	Examples 1, 2	✓ use fraction strips, diagrams, or a subtraction statement to subtract mixed numbers with like or unlike denominators
10, 11, 16	7.3, 7.4	Example 2	✓ add and subtract mixed numbers with unlike denominators
12	7.4	Example 2	✓ use fraction strips, diagrams, or a subtraction statement to subtract mixed numbers with unlike denominators
13	7.2	Explore the Math	✓ add and subtract fractions with unlike denominators



11. It snowed for $2\frac{3}{4}$ h yesterday.
Then it rained for another $1\frac{1}{6}$ h.
a) How many more hours did it snow than rain?
b) How long did it snow and rain altogether?

12. Brenda cooks for $2\frac{2}{3}$ h on Saturday.
On Sunday she cooks for $1\frac{3}{4}$ h.
a) How many more hours does she cook on Saturday than on Sunday?
b) Check your answer.

Extended Response

13. Explain why it is important to have common denominators when adding or subtracting fractions.

14. Which diagram shows a larger fraction? Explain.

15. To subtract $3\frac{1}{5} - \frac{3}{5}$, Rowena starts by writing: $3\frac{1}{5} - \frac{3}{5} = 2\frac{6}{5} - \frac{3}{5}$
a) Is she correct? Explain.
b) Show another subtraction method.

16. The diagram shows the two rooms of the Prairietown Museum. Each room is the same size. $\frac{3}{8}$ of the two rooms is used for the town's history.
Room 1: Photos Room 2: Artifacts
Town's history
Settlement in Canada
Canadian Aboriginal peoples
a) What fraction of the two rooms is used for Aboriginal peoples?
b) What fraction of the two rooms is used for settlement in Canada?
c) Is a greater fraction used for Aboriginal peoples or for the town's history? How much greater?
d) Is a greater fraction used for the town's history or for settlement in Canada? How much greater?

WRAP IT UP!

Create a board game about fractions around the world and through history.
- Use information from this chapter.
- Research to find interesting facts about the use of fractions in different cultures, past and present.
- Use all of this information to make questions for your board game.
- Include questions to do with addition and subtraction of fractions with unlike denominators and mixed numbers.
- Do not forget to include answers to the questions.

WWW Web Link
For more information about fractions and mathematics in different cultures and different times, go to www.mathlinks7.ca and follow the links.

Practice Test • MHR 263

Activity Planning Notes

Have students start the practice test the same way they started the chapter review—by having each student indicate which questions they may need a little help with, a lot of help with, or no help with. Have them first complete the work they know they can do. Then, have them complete the work they know something about. Finally, have students do their best on the work they are still struggling with.

This practice test can be assigned as an in-class or take-home assignment. If it is used to assist you in evaluating student progress, choose which questions will help you make your assessment. These are the minimum questions that will meet the related curriculum outcomes: #1–#8, #10, and #11.

Answers to the Chapter 7 Practice Test are provided on **BLM 7–14 Chapter 7 *MathLinks 7* Student Resource Answers**.

Supported Learning

Learning Style and Motor

- Allow kinesthetic and concrete learners to use **Master 13 Pattern Blocks** and **Master 15 Fraction Strips** as they work on the test.
- Allow visual learners to use **Master 9 0.5 Centimetre Grid Paper** to draw diagrams, if necessary, to help them answer questions on the test

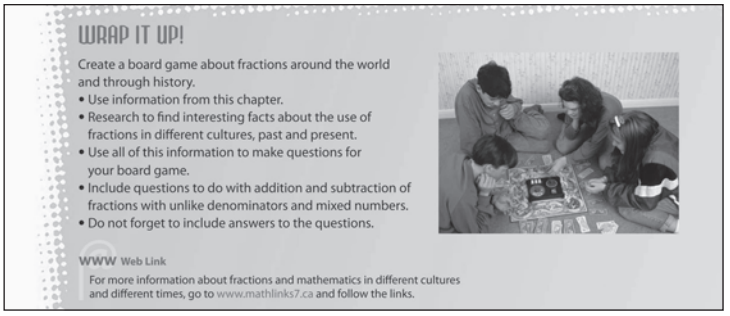
Assessment *of* Learning	Supported Learning
Chapter 7 Test After students complete the practice test, you may wish to use **BLM 7–11 Chapter 7 Test** as a summative assessment.	• Consider allowing students to use their chapter Foldable and/or a calculator. • Consider using the Math Games on page 264 or the Challenge in Real Life on page 265 to assess the knowledge and skills of students who have difficulty with tests.

Wrap It Up!

Suggested Timing
60–75 minutes

Materials
- materials to create a board game, such as scissors, coloured pencils, cardboard, playing pieces, and so on

Blackline Masters
Master 1 Project Rubric
BLM 7–4 Section 7.1 Math Link
BLM 7–6 Section 7.2 Math Link
BLM 7–8 Section 7.3 Math Link
BLM 7–10 Section 7.4 Math Link
BLM 7–12 Chapter 7 Wrap It Up!


WRAP IT UP!

Create a board game about fractions around the world and through history.
- Use information from this chapter.
- Research to find interesting facts about the use of fractions in different cultures, past and present.
- Use all of this information to make questions for your board game.
- Include questions to do with addition and subtraction of fractions with unlike denominators and mixed numbers.
- Do not forget to include answers to the questions.

WWW Web Link
For more information about fractions and mathematics in different cultures and different times, go to www.mathlinks7.ca and follow the links.

Specific Outcomes

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Supported Learning

Meeting the Needs of All Learners
- Allow students to work in groups of two or three. Have these groups play each other's games and assess the games according to the developed criteria.

Common Errors
- Students may not adhere to the guidelines set up by the class to complete the project.

R_x Allow students an opportunity to rework their board games.
- Students may not provide the correct answers for the questions in their game.

R_x Have students play each other's board games according to the guidelines created and discuss any discrepancies that arise.

Activity Planning Notes

Introduce the Wrap It Up! to the class and discuss criteria for the completed project. You may decide to have a minimum and a maximum number of facts from each culture and in total. Students may work individually or in small groups to develop their game.

It is important that students create questions that involve the addition and subtraction of fractions with unlike denominators, and addition and subtraction of mixed numbers with like and unlike denominators. Review these terms with students and make sure they understand the task.

Assessment *of* Learning	Supported Learning
Wrap It Up! This activity gives students the opportunity to research and apply their knowledge of the use of fractions in various cultures to create a game board. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this Wrap It Up! Page 263a provides notes on how to use this rubric for the Wrap It Up!	• Allow students to use some of the research that they might have done throughout the unit to develop their board games. • Remind students to formulate rules for their game and to test them beforehand. • You may wish to have students review the work they completed in the Math Links in sections 7.1, 7.2, 7.3, and 7.4 before they begin. • If students have not completed the Math Links earlier in the chapter, you may wish to provide them with **BLM 7–4 Section 7.1 Math Link**, **BLM 7–6 Section 7.2 Math Link**, **BLM 7–8 Section 7.3 Math Link**, and **BLM 7–10 Section 7.4 Math Link**. • You may wish to have students use **BLM 7–12 Chapter 7 Wrap It Up!**, which provides scaffolding for the chapter problem wrap-up.

The chart below shows **Master 1 Project Rubric** for tasks such as the Wrap It Up! and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• designs a functional game with sufficient evidence of research of past and present applications of fractions • includes accurate and functional question cards that reflect the research done
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• designs a game in which information about other cultures was used (other than just the information in the chapter) • includes question cards that reflect researched information but that focus on either past or present only
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• designs a game based only on the information provided on the cultures in the chapter • includes an insufficient number of question cards (and answers) for their game due to a lack of research *or* • includes questions and answers that are contrived to make the game playable
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• develops a game board using common fractions that do not have any link to the ones presented or to additional ones researched • includes question cards that are accurate and that reflect a functional game
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• attempts to begin a game, but insufficient work is evident for the game to be functional

Math Games

Suggested Timing
40–50 minutes

Materials
Each pair of students will need
- two six-sided dice (one red and one blue)
- two paper clips (to be used with the spinners)
- coloured pencils or coloured paper

Blackline Masters
BLM 7–13 Fraction Race Spinners

Supported Learning

Learning Style

- Have some students play only one version of the game and allow extra time for their calculations.

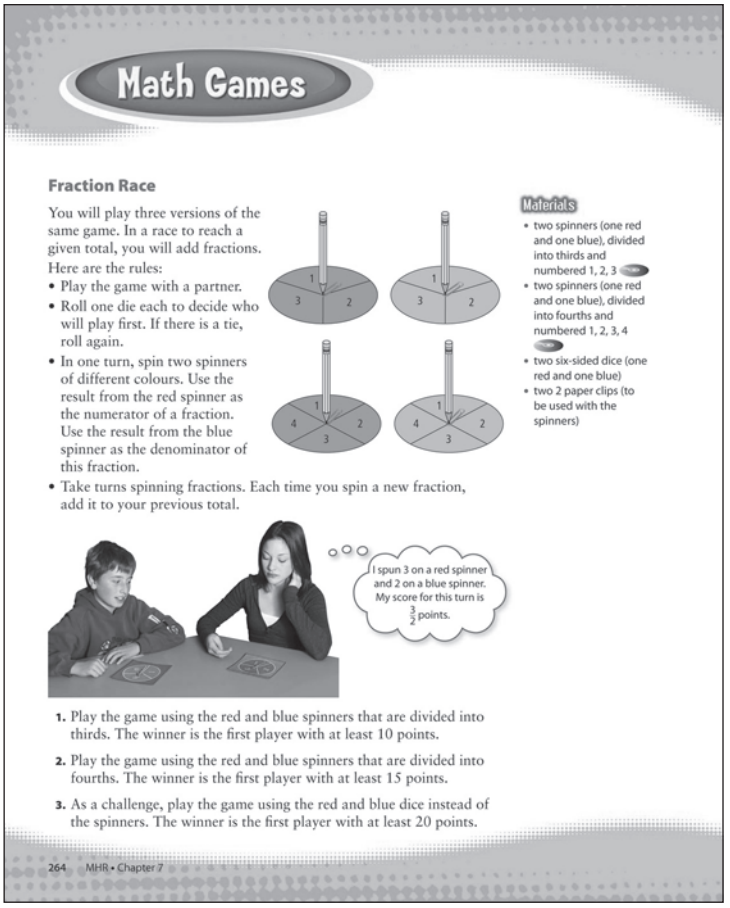
Memory

- Post a sample fraction indicating which spinner colour gives the numerator and which colour gives the denominator.

Common Errors

- Students forget how to find a common denominator when adding fractions.

R_x Review this process with students. You may want to start by having them list multiples of each denominator and then identify a common multiple. Alternatively, they could use paper folding.



Math Games

Fraction Race

You will play three versions of the same game. In a race to reach a given total, you will add fractions. Here are the rules:
- Play the game with a partner.
- Roll one die each to decide who will play first. If there is a tie, roll again.
- In one turn, spin two spinners of different colours. Use the result from the red spinner as the numerator of a fraction. Use the result from the blue spinner as the denominator of this fraction.
- Take turns spinning fractions. Each time you spin a new fraction, add it to your previous total.

Materials
- two spinners (one red and one blue), divided into thirds and numbered 1, 2, 3
- two spinners (one red and one blue), divided into fourths and numbered 1, 2, 3, 4
- two six-sided dice (one red and one blue)
- two 2 paper clips (to be used with the spinners)

I spun 3 on a red spinner and 2 on a blue spinner. My score for this turn is $\frac{3}{2}$ points.

1. Play the game using the red and blue spinners that are divided into thirds. The winner is the first player with at least 10 points.
2. Play the game using the red and blue spinners that are divided into fourths. The winner is the first player with at least 15 points.
3. As a challenge, play the game using the red and blue dice instead of the spinners. The winner is the first player with at least 20 points.

264 MHR • Chapter 7

Specific Outcomes

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Assessment *for* Learning	Supported Learning
Fraction Race Have students play this game with a partner of similar math ability.	• You may wish to have students keep a tracking sheet of the sums of their fractions. • Encourage students to show their work in determining each total. • If students are having trouble adding fractions, review the skills necessary using a manipulative such as fraction strips. Some students may need to use fraction strips for each addition. • After students play the game, brainstorm strategies as a class.

Activity Planning Notes

Read through the game with students. Discuss strategies and/or rules as a class prior to playing the game. You might wish to have pairs of players come up with their own rules; for example, forgetting to add a fraction results in the loss of one point. **BLM 7–13 Fraction Race Spinners** provides spinners that can be cut out and coloured appropriately.

Review these terms: *numerator, denominator, proper fraction, improper fraction, mixed number,* and *common denominator*. Make sure students understand that they can only add fractions if they have a common denominator. Encourage students to record each new fraction that they have generated using the dice or the spinners.

Challenge in Real Life


Challenge in Real Life

Magazine Design

The job of magazine designers is to create eye-catching page spreads, like the one shown.

You be the magazine designer! Design two or more magazine spreads.

15 tips for good travel etiquette

a) First decide where to place the ads. Here is the advertising information:
- Three advertisers have two ads each.
- The advertisers want their two ads to be different sizes and appear on different spreads.
- The table shows the sizes and costs of ads.

Portion of Page	Cost
full page	$299
$\frac{3}{4}$ page	$244
$\frac{1}{2}$ page	$154
$\frac{1}{4}$ page	$99
$\frac{1}{8}$ page	$79

b) Fill the rest of the space with articles and pictures.

c) Make a table that shows the following information for each advertiser:
- the fraction of a page covered by each ad
- the cost of each ad
- the total cost of the two ads

Show your work.

d) Make another table that shows the fraction of space on each page covered by articles, the fraction covered by pictures, and the total space covered by both articles and pictures. Show your work.

Challenge in Real Life • MHR 265

Suggested Timing
60–75 minutes

Materials
- sample magazines
- 11 × 17 paper
- ruler
- coloured pencils or markers
- scissors
- glue or tape

Technology Tools
- drawing software (optional)
- spreadsheet software (optional)

Blackline Masters
Master 1 Project Rubric

Mathematical Processes
- ☑ Communication
- ☑ Connections
- ☑ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☑ Technology
- ☑ Visualization

Specific Outcomes

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Activity Planning Notes

You may wish to use the following steps to introduce and complete this challenge:

1. Read through Magazine Design with students and discuss the sample layout from *Animal Wellness* magazine. Encourage students to share ideas about what makes the sample design good. (For example, the visual and title on the first page take up about $\frac{2}{3}$ of the page and run over into a small fraction of the second page; the text is on the bottom $\frac{1}{3}$ of the first page and about $\frac{2}{3}$ of the second page; the advertisements do not interrupt the flow of the article.)
2. Have students name magazines that they read and tell what they find inside these magazines. Have students look at page spreads in sample small Canadian magazines, such as sports, outdoor, teen, and pet magazines. (Clarify that a page spread is two facing pages.)

Supported Learning

Learning Style, Language, ESL, and Memory
- You may wish to have students work together in groups on one report. Alternatively, allow each student to work on only one aspect of the project. For example, three students might divide up parts a), b), and c).
- Students with strong communication skills may wish to choose a subject for their magazine, such as skateboarding or cooking, along with a name. Based on this subject, they could then draw the ads, draw or take their own pictures, and write the articles.

Supported Learning

Learning Style

- For students who have difficulty working with fractions with unlike denominators, suggest that they create a table of ad sizes with fractions with a common denominator.

Portion of Page	Cost
full page	$299
$\frac{6}{8}$ page	$244
$\frac{4}{8}$ page	$154
$\frac{2}{8}$ page	$99
$\frac{1}{8}$ page	$79

ESL

- Students may need assistance with the terms *magazine designers, eye-catching, page spreads, ads, advertising,* and *articles*.
- You may wish to allow English language learners to write in their first language and still be assessed for their math understanding.
- Alternatively, while other students are writing their own ads and articles, English language learners might either label the spaces as ads and articles or cut and paste ads and articles from magazines.

Motor

- If these students find it difficult to do the physical layout of the design spread, you may wish to allow them to create their design using a drawing tool on a computer.

Ask students to identify the different elements in the spreads: articles, photos, and advertisements.

3. With the class, discuss how the amount of space used for ads, articles, and photos can be expressed as fractions of a page. Talk about how to determine what fraction of a page each element covers. Make sure students understand that the fractions must add up to one, which represents one whole page.
4. Provide students with 11 × 17 paper. Have them turn it horizontally and fold it in half to represent a page spread. Students can work individually to draw their magazine layouts. Note that you may wish to give students the option of using drawing or spreadsheet software for their designs.
5. Clarify that the task is to
   - choose two different ad sizes for each of the three advertisers
   - choose the number of page spreads they will create (two or more)
   - draw the layouts so that they include all six ads, along with articles and photos
   - make two tables that show the fraction of space covered by the ads, the costs of the ads, and the fraction of space covered by articles and pictures
6. Review the rubric with students so that they understand what is expected.

This challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment *for* Learning	Supported Learning
Magazine Design Discuss the challenge with the class. Have students work individually or in groups to create their design. Have students submit individual tables for parts c) and d).	• Review with students how to add and subtract fractions with unlike denominators. • Allow students to print their designs and reports if they used a computer to create their page layouts and/or tables. • For a second challenge, complete with teaching notes and student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

Assessment *of* Learning	Supported Learning
Magazine Design Discuss the challenge with the class. Have students work individually or in groups to create their design. Have students submit individual tables for parts c) and d).	• Use **Master 1 Project Rubric** to assist you in assessing student work. Specific question notes are provided in the final column on the following page to enable you to use **Master 1 Project Rubric** for the Magazine Design challenge. • Have students estimate any calculations before doing them. Then have students justify why their calculations might be reasonable. • To view student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as the Challenge in Real Life and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete magazine spread that meets all requirements and has an appealing look
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• completes parts a), b), c), and d), with either a calculation error or no justification for work
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies, mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• completes parts a), b), and c) *or* • completes parts a), b), and d) *or* • completes parts a) and c) *or* • completes parts a) and d)
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• completes parts a), b), and the first bullet of c) *or* • completes parts a) and b)
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• completes parts a) or b) *or* • completes any other correct step