Circles

8

General Outcome

- Use direct or indirect measurement to solve problems.

Specific Outcomes

SS1 Demonstrate an understanding of circles by:
- describing the relationships among radius, diameter and circumference of circles
- relating circumference to pi
- determining the sum of the central angles
- constructing circles with a given radius or diameter
- solving problems involving the radii, diameters and circumferences of circles.

SS2 Develop and apply a formula for determining the area of:
- circles.

SP3 Construct, label and interpret circle graphs to solve problems.

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
8.1	✓ draw a circle with a given radius
	✓ draw a circle with a given diameter
	✓ determine the diameter of a circle given its radius
	✓ determine the radius of a circle given its diameter
8.2	✓ estimate and calculate the circumference of a circle given its diameter or radius
	✓ solve problems involving the circumference of circles
8.3	✓ explain how to determine the area of a circle
	✓ estimate and calculate the area of a circle
	✓ solve problems involving the area of a circle
8.4	✓ read circle graphs
	✓ use circle graphs to solve problems
8.5	✓ construct a circle graph with technology
	✓ construct a circle graph without technology

Assessment *as* Learning	Supported Learning
Use the Before column of **BLM 8–1 Chapter 8 Self-Assessment** to provide students with the big picture for this chapter and to help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• As students complete each section of the chapter or complete the Chapter 8 Review, have them review the related parts of **BLM 8–1 Chapter 8 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Chapter 8 Planning Chart

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter Opener • 20–30 minutes		BLM 8–1 Chapter 8 Self-Assessment BLM 8–2 Chapter Opener Math Link BLM 8–3 Perimeter and Area	• grid paper, scissors • compass, stapler
8.1 Construct Circles • 45–50 minutes	**Essential:** 1, 2, 6, 8, 10, Math Link **Typical:** 1–12, Math Link **Extension/Enrichment:** 1, 2, 13–15	Master 8 Centimetre Grid Paper BLM 8–1 Chapter 8 Self-Assessment BLM 8–4 Circles BLM 8–5 Drawing Circles Using a Computer and Drawing Software BLM 8–6 Section 8.1 Extra Practice BLM 8–7 Section 8.1 Math Link	• push pin • ruler • 12-cm long string • compass
8.2 Circumference of a Circle • 80–100 minutes	**Essential:** 1–3, 5, 7, 9, Math Link **Typical:** 1–14, Math Link **Extension/Enrichment:** 1, 2, 15–19	Master 2 Two Stars and One Wish BLM 8–1 Chapter 8 Self-Assessment BLM 8–8 Section 8.2 Extra Practice BLM 8–9 Section 8.2 Math Link	• minimum of 10 circular objects • string • metre stick • ruler • calculator
8.3 Area of a Circle • 80–100 minutes	**Essential:** 1–4, 6, 8, 10, Math Link **Typical:** 1–16, Math Link **Extension/Enrichment:** 1–3, 17–20	Master 8 Centimetre Grid Paper BLM 8–1 Chapter 8 Self-Assessment BLM 8–10 Section 8.3 Extra Practice BLM 8–11 Section 8.3 Math Link	• centimetre grid paper • ruler • compass • scissors • calculator
8.4 Interpret Circle Graphs • 40–50 minutes	**Essential:** 1–3, 5 **Typical:** 1–8 **Extension/Enrichment:** 1, 2, 9	BLM 8–1 Chapter 8 Self-Assessment BLM 8–12 Section 8.4 Extra Practice	• samples of circle graphs from newspapers, magazines, or web sites showing real-world applications
8.5 Create Circle Graphs • 80–100 minutes	**Essential:** 1–4, 7 **Typical:** 1–10 **Extension/Enrichment:** 1–3, 11–13	Master 12 Percent Circles BLM 8–1 Chapter 8 Self-Assessment BLM 8–13 Draw a Circle Graph Using a Percent Circle BLM 8–14 Creating Circle Graphs in Microsoft® Excel BLM 8–15 Creating Circle Graphs in Corel® Quattro® Pro BLM 8–16 Creating Circle Graphs in Appleworks® 6.2 BLM 8–17 Section 8.5 Extra Practice	• calculator • compass • ruler • protractor • coloured pencils • computer with spreadsheet software
Chapter 8 Review • 40–50 minutes	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Master 8 Centimetre Grid Paper BLM 8–1 Chapter 8 Self-Assessment BLM 8–6 Section 8.1 Extra Practice BLM 8–8 Section 8.2 Extra Practice BLM 8–10 Section 8.3 Extra Practice BLM 8–12 Section 8.4 Extra Practice BLM 8–17 Section 8.5 Extra Practice	• compass • protractor • ruler • calculator • computer with spreadsheet software
Chapter 8 Practice Test • 40–50 minutes	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1–7, 9, 10, 12, 13	Master 8 Centimetre Grid Paper BLM 8–1 Chapter 8 Self-Assessment BLM 8–18 Chapter 8 Test	• compass • protractor • ruler • calculator
Chapter 8 Wrap It Up! • 60–75 minutes (Method 2)		Master 1 Project Rubric BLM 8–7 Section 8.1 Math Link BLM 8–9 Section 8.2 Math Link BLM 8–11 Section 8.3 Math Link BLM 8–19 Chapter 8 Wrap It Up!	• research on drums in print material and from Internet sites (optional) • designs from Section 8.1 Math Link (optional) • calculator

Chapter 8 Planning Chart (continued)

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter 8 Math Games • 60–75 minutes			• stiff paper or cardboard • compass, protractor, scissors • coloured pencils, paper clip
Chapter 8 Challenge in Real Life • 60–75 minutes		Master 1 Project Rubric	• sample medallions and rosettes • compass, ruler, coloured pencils
Chapters 5–8 Review • 60–75 minutes	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1, 2, 4–7, 9–11, 13, 16, 17	Master 8 Centimetre Grid Paper Master 12 Percent Circles	• compass • protractor • ruler
Task • 60–75 minutes		Master 1 Project Rubric BLM 8–20 Chapter 8 *Math Links 7* Student Resource Answers BLM 8–21 Chapter 8 BLM Answers	• paper • compass or pin and string • ruler • circle template (optional)

Chapter 8 Assessment Planner

Assessment Options	Type of Assessment	Assessment Tool
Chapter Opener	Assessment *as* Learning (TR pages i, 267)	BLM 8–1 Chapter 8 Self-Assessment Chapter 8 Foldable
8.1 Construct Circles	Assessment *for* Learning (TR pages 270, 271, 272) Assessment *as* Learning (TR pages 271, 272)	Math Learning Log (TR page 272) BLM 8–1 Chapter 8 Self-Assessment
8.2 Circumference of a Circle	Assessment *as* Learning (TR pages 274, 277, 279) Assessment *for* Learning (TR pages 276, 278, 279)	Master 2 Two Stars and One Wish Math Learning Log (TR page 279) BLM 8–1 Chapter 8 Self-Assessment
8.3 Area of a Circle	Assessment *as* Learning (TR pages 282, 284, 286) Assessment *for* Learning (TR pages 283, 285, 286)	Math Learning Log (TR page 286) BLM 8–1 Chapter 8 Self-Assessment
8.4 Interpret Circle Graphs	Assessment *as* Learning (TR pages 288, 290, 291) Assessment *for* Learning (TR page 291)	Math Learning Log (TR page 291) BLM 8–1 Chapter 8 Self-Assessment
8.5 Create Circle Graphs	Assessment *as* Learning (TR pages 294, 295, 297) Assessment *for* Learning (TR pages 294, 295, 296)	Math Learning Log (TR page 297) BLM 8–1 Chapter 8 Self-Assessment
Chapter 8 Review	Assessment *for* Learning (TR page 298) Assessment *as* Learning (TR page 299)	Math Learning Log (TR page 299) BLM 8–1 Chapter 8 Self-Assessment
Chapter 8 Practice Test	Assessment *as* Learning (TR page 300) Assessment *of* Learning (TR page 301)	BLM 8–1 Chapter 8 Self-Assessment BLM 8–18 Chapter 8 Test
Chapter 8 Wrap It Up!	Assessment *of* Learning (TR page 300a)	Master 1 Project Rubric
Chapter 8 Math Games	Assessment *for* Learning (TR page 302)	
Chapter 8 Challenge in Real Life	Assessment *for* Learning (TR page 302a) Assessment *of* Learning (TR page 302a)	Master 1 Project Rubric
Chapters 5–8 Review	Assessment *for* Learning (TR page 306) Assessment *as* Learning (TR page 306)	Math Learning Log (TR page 306)
Task	Assessment *of* Learning (TR page 306a)	Master 1 Project Rubric

You may wish to use one or more of the following materials to help you assess student readiness for Chapter 8.

Assessment *for* Learning	Supported Learning
Method 1: Have students develop a journal to explain what they personally know about the topics and how they use perimeter, area, and metric units in their lives. **Method 2:** Have students complete **BLM 8–3 Perimeter and Area** to check their conceptual understanding. Remind students that you are looking for the scope of their knowledge.	• Students who require reinforcement of prerequisite skills may wish to complete the Get Ready materials available in the *MathLinks 7 Workbook* and at the **www.mathlinks7.ca** book site.

Chapter Opener

Suggested Timing
20–30 minutes

Materials
- grid paper
- scissors
- compass
- stapler

Blackline Masters
BLM 8–1 Chapter 8 Self-Assessment
BLM 8–2 Chapter Opener Math Link

Key Words
radius
diameter
circumference
pi
circle graph
sector
central angle

What's the Math?

In this chapter, students work with the circumference and area of circles, and with circle graphs. They start by learning to draw circles and identify the different parts of circles. Students investigate how to find the circumference and the area of different circles, using estimation and a formula. Students then see how circle graphs are used to represent data before creating circle graphs of their own.

Activity Planning Notes

Start by having students identify circles in their environment and in the visuals in the student resource. Discuss the significance of circles to other cultures. Have students brainstorm the characteristics of circles and how to define a circle.

Math Link

Highlight the importance of drums to other cultures, and discuss the characteristics of circular drums. Ask students what they need to know about the circle before they can design and make a drum. You may wish to use **BLM 8–2 Chapter Opener Math Link** for this purpose.

You may wish to read the Wrap It Up! for this chapter problem, which is on page 301. Students could start to collect information about different drums while they work on the rest of the chapter.

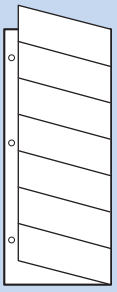
FOLDABLES™
Study Tool

Have students make the Foldable in the student resource to keep track of the information in the chapter. If you have a large stapler, you could have students open up their folded papers and place three staples on the middle fold.

You may prefer to have students keep track of Key Words using a design specifically for that purpose. Students can make the following Foldable and write vocabulary terms on the front of each tab. Have them use the space beneath the tab to write definitions and provide examples.

Step 1 Fold a sheet of notebook paper in half along the long axis with the crease to the right.

Step 2 Measure the height of the page and draw lines to divide the height into seven equal parts. Cut every part as far as the fold, creating tabs as you go. This will create seven tabs.



Step 3 Label each tab with a math term. Write definitions and give examples underneath the tabs.

Key Words Foldables could be slipped into the student resource as a bookmark to help students keep track of the words.

Assessment *as* Learning	Supported Learning
Chapter 8 Foldable As students work on each section in Chapter 8, have them keep track of any problems they are having under the What I Need to Work On tab in their chapter Foldable.	• As students complete each section, have them review the list of items they need to work on and then have them check off any that have been handled.

Supported Learning

Learning Style, ESL, Language, and Memory

- Encourage students to think of examples of circles in their cultural environment. They might sketch circles or bring in photos.

Learning Style and Motor

- Create a Foldable ahead of time to use as a model.

ESL and Language

- Consider displaying Key Words on a math word wall. Students may also choose to create their own vocabulary/picture dictionary. Matching a picture or symbol with each definition helps students consolidate their understanding of key terms.

Meeting the Needs of All Learners

- You may wish to invite someone with a special interest in drums to talk to the class about ceremonial drums. One or more people who make drums may be willing to assist your students in making an appropriate hand drum. Check with your local community to see which cultural groups are involved in drum-making and how to approach the members of these groups. Also research what special drums may be made and/or owned locally. For example, the Inuit have quite a unique drum design. You may wish to invite someone familiar with this to demonstrate it to the class. If possible, have students measure the drum and then use its measurements for some of the questions in the chapter.

Common Errors

- When students talk about circles, they often confuse the terminology.

R_x Encourage them to use descriptions and then apply the vocabulary.

8.1 Construct Circles

Suggested Timing
40–50 minutes

Materials
- push pin
- ruler
- 12-cm long string
- compass

Related Resources
Master 8 Centimetre Grid Paper

BLM 8–1 Chapter 8 Self-Assessment

BLM 8–4 Circles

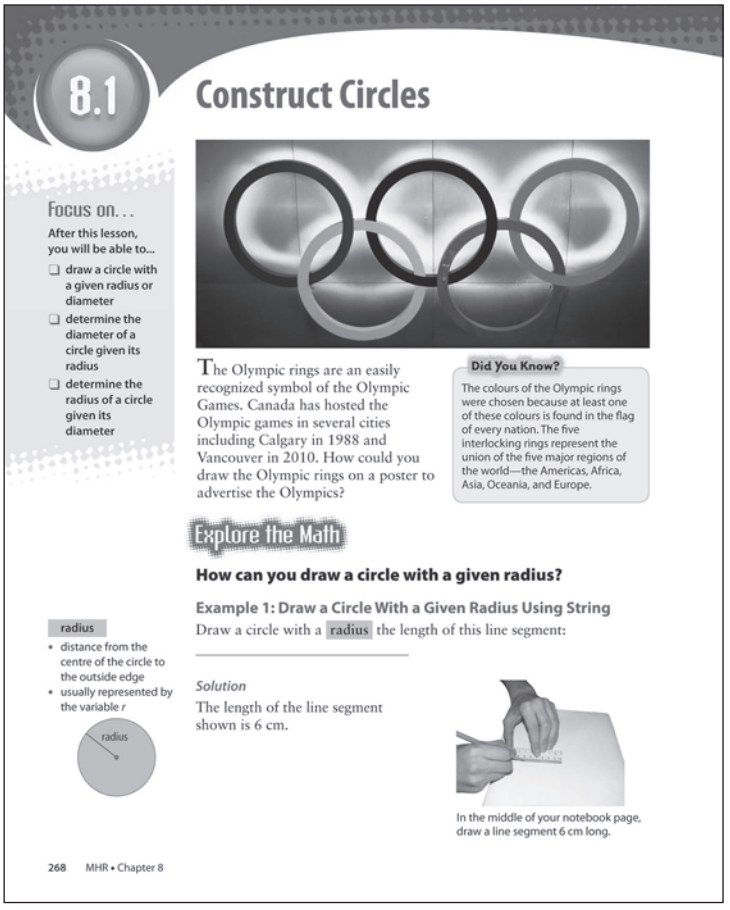
BLM 8–5 Drawing Circles Using a Computer and Drawing Software

BLM 8–6 Section 8.1 Extra Practice

BLM 8–7 Section 8.1 Math Link

Mathematical Processes

- ☑ Communication
- ☑ Connections
- ☑ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☑ Technology
- ☑ Visualization



8.1 Construct Circles

Focus on...
After this lesson, you will be able to...
- draw a circle with a given radius or diameter
- determine the diameter of a circle given its radius
- determine the radius of a circle given its diameter

The Olympic rings are an easily recognized symbol of the Olympic Games. Canada has hosted the Olympic games in several cities including Calgary in 1988 and Vancouver in 2010. How could you draw the Olympic rings on a poster to advertise the Olympics?

Did You Know?
The colours of the Olympic rings were chosen because at least one of these colours is found in the flag of every nation. The five interlocking rings represent the union of the five major regions of the world—the Americas, Africa, Asia, Oceania, and Europe.

Explore the Math

How can you draw a circle with a given radius?

Example 1: Draw a Circle With a Given Radius Using String
Draw a circle with a radius the length of this line segment:

radius
- distance from the centre of the circle to the outside edge
- usually represented by the variable *r*

Solution
The length of the line segment shown is 6 cm.

In the middle of your notebook page, draw a line segment 6 cm long.

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Specific Outcomes

SS1 Demonstrate an understanding of circles by:
- describing the relationships among radius, diameter and circumference of circles
- constructing circles with a given radius or diameter
- solving problems involving the radii, diameters and circumferences of circles.

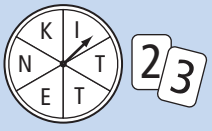
Warm-Up

1. $2.3 + 4.67$ **2.** $2\frac{7}{8} - 1\frac{1}{4}$

3. Draw two line segments that are parallel to each other. Explain how you know they are parallel.

4. Calculate 30% of 80.

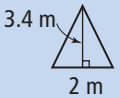
5. The spinner is spun once. The card is flipped. Show the sample space.



Mental Math

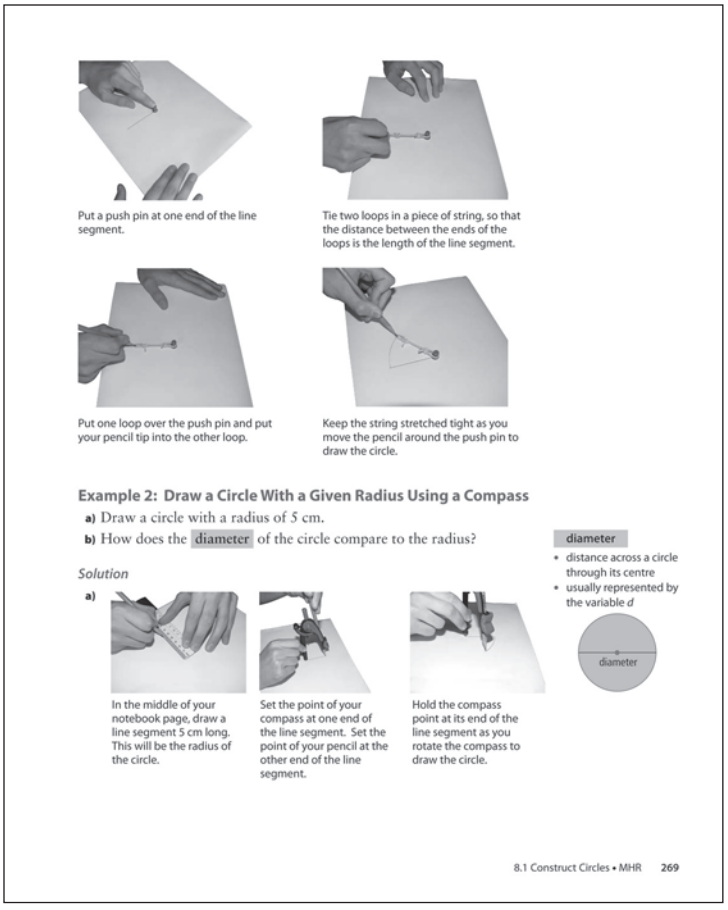
6. Paving stones cost \$52/m^2. Estimate how much it would cost to pave a patio that is 11 m^2.

7. Estimate the area of the triangle.



8. Mentally calculate 75% of 120.

9. $\frac{1}{6} + \frac{3}{6}$ **10.** $\frac{3}{6} - \frac{1}{3}$



Activity Planning Notes

Have students consider the multiple circles in the Olympic Games symbol and different ways to draw them. Encourage students to discuss ways used to develop the symbol. For example, the original artist possibly used a compass. Since then, people may have used various techniques, such as the one with string used in this section.

Explore the Math

Students draw a circle with a given radius first using string and then a compass.

Method 1: Have students try the string investigation. Consider providing the string with one loop already made at one end. Demonstrate how to make a loop with the other end. Follow with the compass activity.

Answers

Warm-Up

1. 6.97 **2.** $1\frac{5}{8}$

3. Check that the lines are parallel. Look for the idea that parallel lines never meet and are the same distance apart all the way along their length.

4. 24

5. Students could use a table, a tree diagram, or some other form of organizer.

	2	3
K	K, 2	K, 3
I	I, 2	I, 3
T	T, 2	T, 3
T	T, 2	T, 3
E	E, 2	E, 3
N	N, 2	N, 3

6. Look for about \$520 (\$52 × 10) or \$500 (\$50 × 10)

7. Look for 3 m^2 (2 m × 3 m ÷ 2).

8. 90

9. $\frac{4}{6}$ or $\frac{2}{3}$ **10.** $\frac{2}{6}$ or $\frac{1}{3}$

Supported Learning

ESL

- English language learners may have difficulty with terms such as *symbol*, *twice*, *accurate*, *spiral*, and *optical illusion*. Have students add new terms to their dictionary.

Motor

- Some students need additional practice with using a compass. Have them use a compass to trace circles on **BLM 8–4 Circles**. Alternatively, consider inviting students to trace 2-D circles they find in the classroom.
- After attempting both strategies, allow students to choose the method they feel more comfortable with to answer #4 to #8.
- If you have access to computers, allow students to draw circles using computer software. **BLM 8–5 Drawing Circles Using a Computer and Drawing Software** provides general instructions for drawing circles.

Common Errors

- Students may experience difficulties using the compass or string apparatus.

R_x Provide students with **BLM 8–4 Circles**. Using the compass or the string, have them practise tracing over the circles.

Answers

Show You Know: Example 2

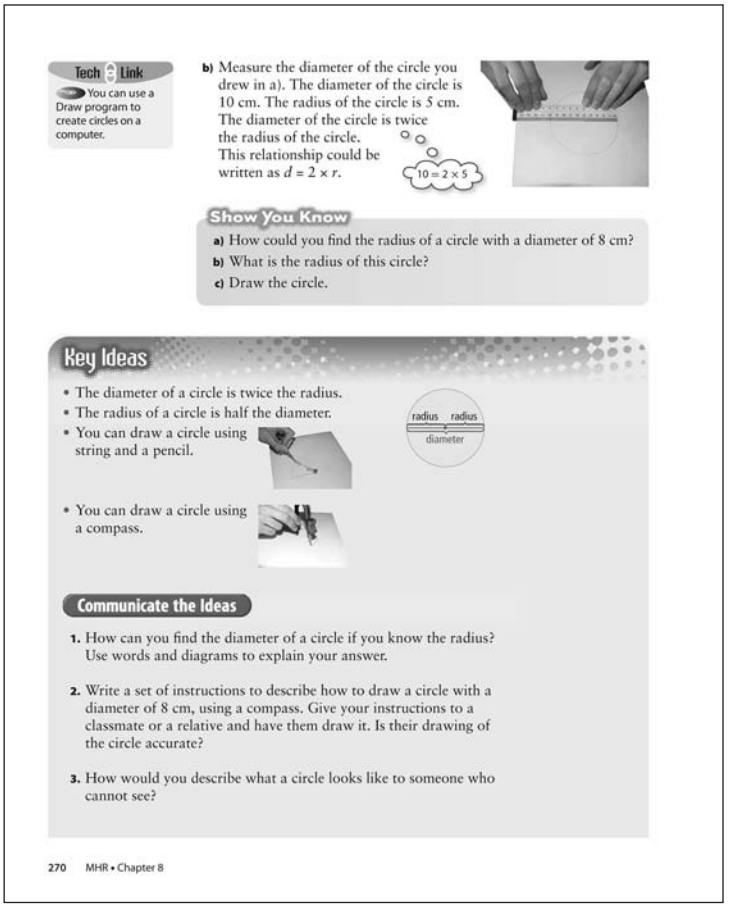
a) Divide the diameter by 2.

b) 4 cm

c) Students should draw a circle with a diameter of 8 cm.

Communicate the Ideas

1. Answers may vary. For example: The radius is one half of the diameter. To find the diameter, multiply the radius by 2.
2. Answers may vary. For example: In the middle of your notebook page, draw a line segment 4 cm long. Set the point of your compass at one end of the line segment. Set the point of your pencil at the other end of the line segment. Holding the compass point at its end of the line segment, rotate the compass to draw the circle.
3. Answers will vary. For example: When you trace around a circle, there are no corners.



Tech Link
You can use a Draw program to create circles on a computer.

b) Measure the diameter of the circle you drew in a). The diameter of the circle is 10 cm. The radius of the circle is 5 cm. The diameter of the circle is twice the radius of the circle. This relationship could be written as $d = 2 \times r$.

$10 = 2 \times 5$

Show You Know
a) How could you find the radius of a circle with a diameter of 8 cm?
b) What is the radius of this circle?
c) Draw the circle.

Key Ideas
- The diameter of a circle is twice the radius.
- The radius of a circle is half the diameter.
- You can draw a circle using string and a pencil.
- You can draw a circle using a compass.

radius radius diameter

Communicate the Ideas
1. How can you find the diameter of a circle if you know the radius? Use words and diagrams to explain your answer.
2. Write a set of instructions to describe how to draw a circle with a diameter of 8 cm, using a compass. Give your instructions to a classmate or a relative and have them draw it. Is their drawing of the circle accurate?
3. How would you describe what a circle looks like to someone who cannot see?

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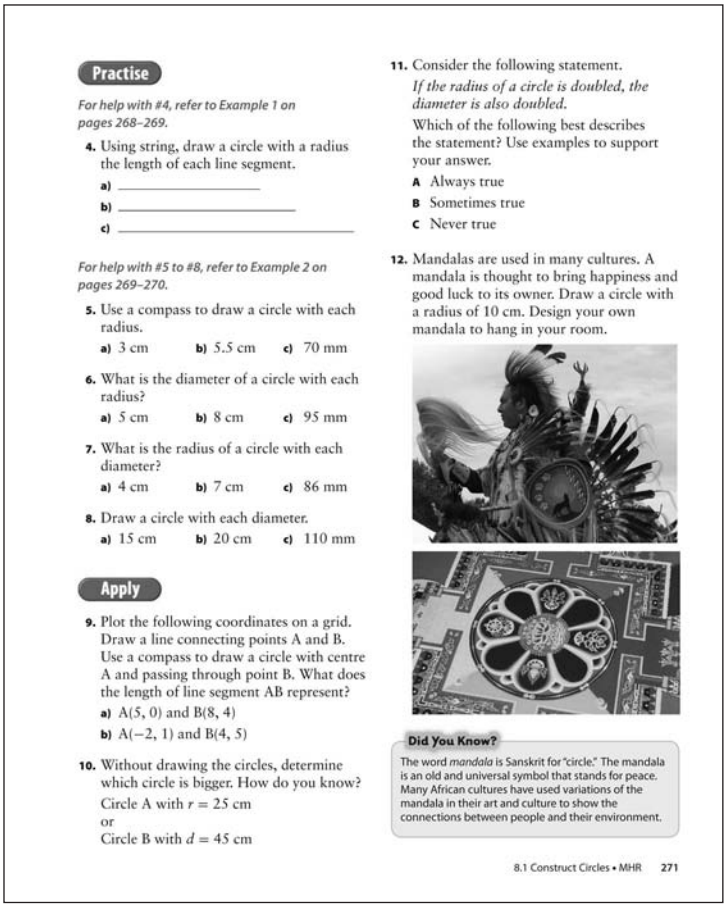
Method 2: Post large sheets of paper around the classroom. Put a push pin in each sheet, and have markers attached with different (loops already in) string lengths. Have students come to the sheets to practise string drawing. Follow with the compass activity.

Method 3: Demonstrate the string activity. Then have students do the compass activity.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• Make sure that students understand that the diameter of any circle is equal to two times the radius. You may wish to use a large circle to illustrate this. Have students identify the radius and the diameter, and then use the radius to measure the diameter. • Allow students to use the method of their choice to draw the circle. Their circle should be within 2 mm of the correct diameter. Students may need to sharpen their pencils to get this level of accuracy. • Provide an additional question for students who would benefit from it: **a)** Draw a circle with a radius of 4 cm. (Students may need to review Example 2 to do this. Make sure that they can differentiate between radius and diameter.) **b)** Predict the diameter of the circle you drew. (Listen to student explanations.) **c)** Measure the diameter of your circle. Were you correct? Explain why or why not. (The diameter is 8 cm. Students who do not remember that the diameter is twice the radius might put a note to themselves in their chapter Foldable under What I Need to Work On.)



Practise

For help with #4, refer to Example 1 on pages 268–269.

4. Using string, draw a circle with a radius the length of each line segment.

a) _____

b) _____

c) _____

For help with #5 to #8, refer to Example 2 on pages 269–270.

5. Use a compass to draw a circle with each radius.

a) 3 cm b) 5.5 cm c) 70 mm

6. What is the diameter of a circle with each radius?

a) 5 cm b) 8 cm c) 95 mm

7. What is the radius of a circle with each diameter?

a) 4 cm b) 7 cm c) 86 mm

8. Draw a circle with each diameter.

a) 15 cm b) 20 cm c) 110 mm

Apply

9. Plot the following coordinates on a grid. Draw a line connecting points A and B. Use a compass to draw a circle with centre A and passing through point B. What does the length of line segment AB represent?

a) A(5, 0) and B(8, 4)

b) A(−2, 1) and B(4, 5)

10. Without drawing the circles, determine which circle is bigger. How do you know?

Circle A with $r = 25$ cm

or

Circle B with $d = 45$ cm

11. Consider the following statement.

If the radius of a circle is doubled, the diameter is also doubled.

Which of the following best describes the statement? Use examples to support your answer.

A Always true

B Sometimes true

C Never true

12. Mandalas are used in many cultures. A mandala is thought to bring happiness and good luck to its owner. Draw a circle with a radius of 10 cm. Design your own mandala to hang in your room.

Did You Know?

The word *mandala* is Sanskrit for "circle." The mandala is an old and universal symbol that stands for peace. Many African cultures have used variations of the mandala in their art and culture to show the connections between people and their environment.

8.1 Construct Circles • MHR 271

Key Ideas

This section summarizes definitions of *diameter* and *radius* and ways to draw a circle.

Communicate the Ideas

These questions allow students to describe diameter and radius, and draw a circle. They apply what they know about circles in a real-world scenario.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 2, 6, 8, 10, Math Link
Typical	1–12, Math Link
Extension/Enrichment	1, 2, 13–15

Practise and Apply

Students will need **Master 8 Centimetre Grid Paper** in order to do #9. In #11, you may wish to discuss with students the difference between "Sometimes true" (true in only specific situations) and "Always true" (can be generalized). In #12, consider allowing students to design a crop circle instead.

Supported Learning

Learning Style and Memory

- Provide **BLM 8–6 Section 8.1 Extra Practice** to students who require more practice.

ESL and Language

- Question 13 has difficult language. Make sure students understand each step in order to complete the question.

Motor

- Suggest students use a program such as Draw or GSP to create circles on a computer. If you decide to do this, book the computer lab in advance and provide **BLM 8–5 Drawing Circles Using a Computer and Drawing Software**. See the Tech Link on student resource page 270.

Gifted and Enrichment

- Challenge students to reason through #11 using algebra.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students do #1. Questions 2 and 3 are good communication questions.	• Check each student's answer to #1. This is a key question. Look for the following explanations: – The diameter is twice the radius. – The radius is half the diameter. – radius + radius = diameter • For #2, allow students to explain the instructions orally.

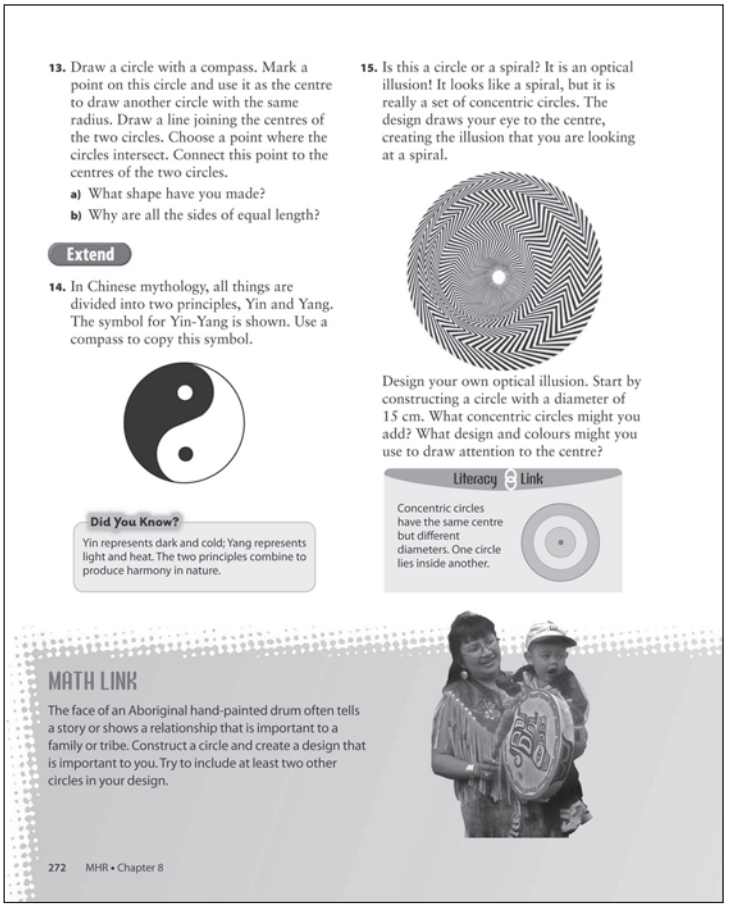
Assessment *for* Learning	Supported Learning
Practise and Apply Have students do #6, #8, and #10. Students who have no problems with these questions can go on to other Apply questions.	• Students who have problems with #6, #8, and #10 will need additional coaching. Have students explain their thinking on these questions; clarify any misunderstandings. Coach students through #7a) and #8a), and then have them complete parts b) and c) of the questions on their own. Have students refer back to examples in the student resource. Check back with them several times to make sure that they understand the concepts.

Answers

Math Link

Designs will vary. Check that the design includes at least two circles.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • How well do you understand the concepts of radius and diameter? • How are radius and diameter related?	• Have students explain the relationship between the radius and diameter of circles. Encourage concrete and kinesthetic learners to use a 2-D circle and a ruler to help explain the relationship. • Depending on students' learning style, have them provide verbal or written answers. • Have students check the What I Need to Work On page of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • You may wish to have students review the part related to Section 8.1 in **BLM 8–1 Chapter 8 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.



13. Draw a circle with a compass. Mark a point on this circle and use it as the centre to draw another circle with the same radius. Draw a line joining the centres of the two circles. Choose a point where the circles intersect. Connect this point to the centres of the two circles.
    a) What shape have you made?
    b) Why are all the sides of equal length?

Extend

14. In Chinese mythology, all things are divided into two principles, Yin and Yang. The symbol for Yin-Yang is shown. Use a compass to copy this symbol.

Did You Know?
Yin represents dark and cold; Yang represents light and heat. The two principles combine to produce harmony in nature.

15. Is this a circle or a spiral? It is an optical illusion! It looks like a spiral, but it is really a set of concentric circles. The design draws your eye to the centre, creating the illusion that you are looking at a spiral.

Design your own optical illusion. Start by constructing a circle with a diameter of 15 cm. What concentric circles might you add? What design and colours might you use to draw attention to the centre?

Literacy Link
Concentric circles have the same centre but different diameters. One circle lies inside another.

MATH LINK
The face of an Aboriginal hand-painted drum often tells a story or shows a relationship that is important to a family or tribe. Construct a circle and create a design that is important to you. Try to include at least two other circles in your design.

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Web Link

For additional examples of hand-painted drums that tell a story or show a relationship, go to **www.mathlinks7.ca** and follow the links.

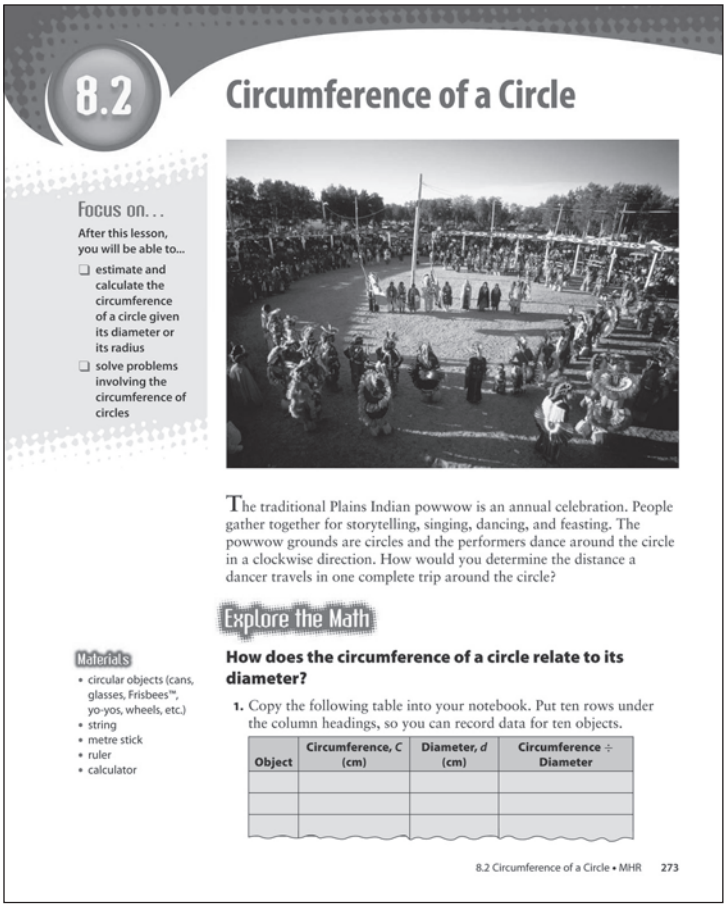
Math Link

Encourage students to develop individual designs that have meaning to them. This could be a cross-cultural activity or one that encourages students to assess their own or their family's strengths and interests.

You may wish to have students bring in examples of circular logos, pieces of art, and other designs before attempting this activity. The work could be included as part of students' art mark. Alternatively, students might design circular buttons for a school music or physical education activity.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 272 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 301.	• You may wish to have students do this Math Link in order to provide them with additional practice drawing circles. As they work on the Math Link, observe and have them self-observe how well they can draw circles using a string and/or compass. • Students who are having difficulty getting started could use **BLM 8–7 Section 8.1 Math Link**, which provides scaffolding for this activity. • Observe students as they work on the Math Link and have them clarify the design. Make sure they include at least two other circles in their design.

8.2 Circumference of a Circle



8.2

Circumference of a Circle

Focus on...

After this lesson, you will be able to...

- estimate and calculate the circumference of a circle given its diameter or its radius
- solve problems involving the circumference of circles

The traditional Plains Indian powwow is an annual celebration. People gather together for storytelling, singing, dancing, and feasting. The powwow grounds are circles and the performers dance around the circle in a clockwise direction. How would you determine the distance a dancer travels in one complete trip around the circle?

Explore the Math

Materials

- circular objects (cans, glasses, Frisbees™, yo-yos, wheels, etc.)
- string
- metre stick
- ruler
- calculator

How does the circumference of a circle relate to its diameter?

1. Copy the following table into your notebook. Put ten rows under the column headings, so you can record data for ten objects.

Object	Circumference, C (cm)	Diameter, d (cm)	Circumference ÷ Diameter

8.2 Circumference of a Circle • MHR 273

Suggested Timing
80–100 minutes

Materials
- minimum of 10 circular objects (e.g., CDs, cans, bicycle tires, Frisbees™, yo-yos, buttons, cups). Choose objects that have diameters that are easily measured.
- string (precut lengths that are at least as long as the circumference of the largest circular object)
- metre stick
- ruler
- calculator

Blackline Masters
Master 2 Two Stars and One Wish
BLM 8–1 Chapter 8 Self-Assessment
BLM 8–8 Section 8.2 Extra Practice
BLM 8–9 Section 8.2 Math Link

Mathematical Processes

- ☑ Communication
- ☑ Connections
- ☑ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☑ Technology
- ☐ Visualization

Specific Outcomes

SS1 Demonstrate an understanding of circles by:
- describing the relationships among radius, diameter and circumference of circles
- relating circumference to pi
- solving problems involving the radii, diameters and circumferences of circles.

Warm-Up

1. Draw a circle with a radius of 4 cm.
2. What is the diameter of the circle you drew? Explain how you know.
3. Draw two line segments that are perpendicular to each other.
4. Write in descending order: 53%, 0.48, $\frac{1}{2}$.
5. List the factors of 36.

Mental Math

6. Find 50% of $42.30. Show your thinking.

For #7 to #10, place a decimal point in the correct position. Show your thinking.

7. $546.5 + 1069.9 + 17.68 = 163408$
8. $15.6 \times 5.9 = 9204$
9. $46.92 \div 9.2 = 510$
10. $4.5 \div 1.5 = 300$

Answers

Warm-Up

1. Measure the diameter of students' circles. Make sure it is 8 cm.

2. 8 cm. One of two answers is possible:
- I measured the diameter so I know it is 8 cm.
- If the radius is 4 cm, the diameter will be twice that.

3. Check that students draw two perpendicular lines.

4. $53\% = 0.53$, 0.48, $\frac{1}{2} = 0.50$. $0.53 > 0.50 > 0.48$; therefore, $53\% > \frac{1}{2} > 0.48$.

5. 1, 2, 3, 4, 6, 9, 12, 18, 36

6. $50\% = \frac{1}{2}$; divide by 2. $\$42 \div 2 = \$21$. $\$0.30 \div 2 = \$0.15$. The answer is $21.15.

7. $500 + 1000 + 20 = 1520$; 1634.08

8. $15 \times 5 = 75$; $16 \times 6 = 96$; The answer is between these; 92.04

9. $45 \div 9 = 5$; 5.10

10. $4 \div 1 = 4$; 3.00

Explore the Math

1.-3. Answers will vary depending on the objects chosen.

4. a) Answers will vary depending on the data. Make sure that the values are calculated to the nearest hundredth.
b) Answers will vary. For example: The calculated values are all close to the same value.
c) Answers will vary. For example: 3
d) Differences may arise due to inexact measurements.

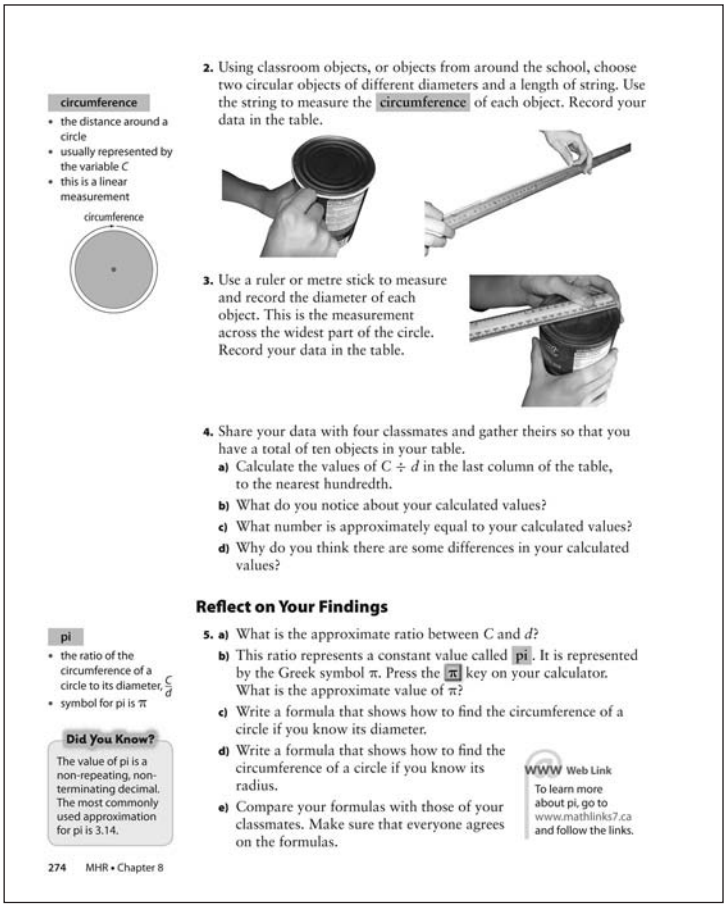
5. a) $C : d = 3 : 1$ **b)** 3.14 **c)** $C = \pi \times d$
d) $C = 2 \times \pi \times r$ **e)** Answers may vary.

Common Errors

- Some students may have difficulty measuring the diameters of objects accurately.

R_x Suggest that students work in pairs and take independent measurements before comparing the measurements.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss the Explore the Math activity. Ask them to share the values for the ratio between C and *d*. Guide a discussion about approximate value. Check responses to #5c) and d) for comprehension of the relationships among circumference, diameter, and radius. Have students conclude their findings.	• For #5b), explain that *constant* means not changing. A constant value always stays the same. • For #5c) and d), students may need some prompting to develop the formulas. • Ask students who are having difficulty with these questions to use the class responses as springboards to similar ones of their own. • The relationships among circumference, diameter, and radius will come up often in Chapter 8, including during the Wrap It Up!



circumference
- the distance around a circle
- usually represented by the variable *C*
- this is a linear measurement

circumference

2. Using classroom objects, or objects from around the school, choose two circular objects of different diameters and a length of string. Use the string to measure the **circumference** of each object. Record your data in the table.

3. Use a ruler or metre stick to measure and record the diameter of each object. This is the measurement across the widest part of the circle. Record your data in the table.

4. Share your data with four classmates and gather theirs so that you have a total of ten objects in your table.
a) Calculate the values of $C \div d$ in the last column of the table, to the nearest hundredth.
b) What do you notice about your calculated values?
c) What number is approximately equal to your calculated values?
d) Why do you think there are some differences in your calculated values?

Reflect on Your Findings

pi
- the ratio of the circumference of a circle to its diameter, $\frac{C}{d}$
- symbol for pi is π

Did You Know?
The value of pi is a non-repeating, non-terminating decimal. The most commonly used approximation for pi is 3.14.

5. a) What is the approximate ratio between *C* and *d*?
b) This ratio represents a constant value called **pi**. It is represented by the Greek symbol π. Press the π key on your calculator. What is the approximate value of π?
c) Write a formula that shows how to find the circumference of a circle if you know its diameter.
d) Write a formula that shows how to find the circumference of a circle if you know its radius.
e) Compare your formulas with those of your classmates. Make sure that everyone agrees on the formulas.

WWW Web Link
To learn more about pi, go to www.mathlinks7.ca and follow the links.

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Activity Planning Notes

Have students consider how the size of the circle affects how far the dancer travels. Encourage students to use what they learned about circles in Section 8.1 to figure out ways to determine the size of a circle on powwow grounds.

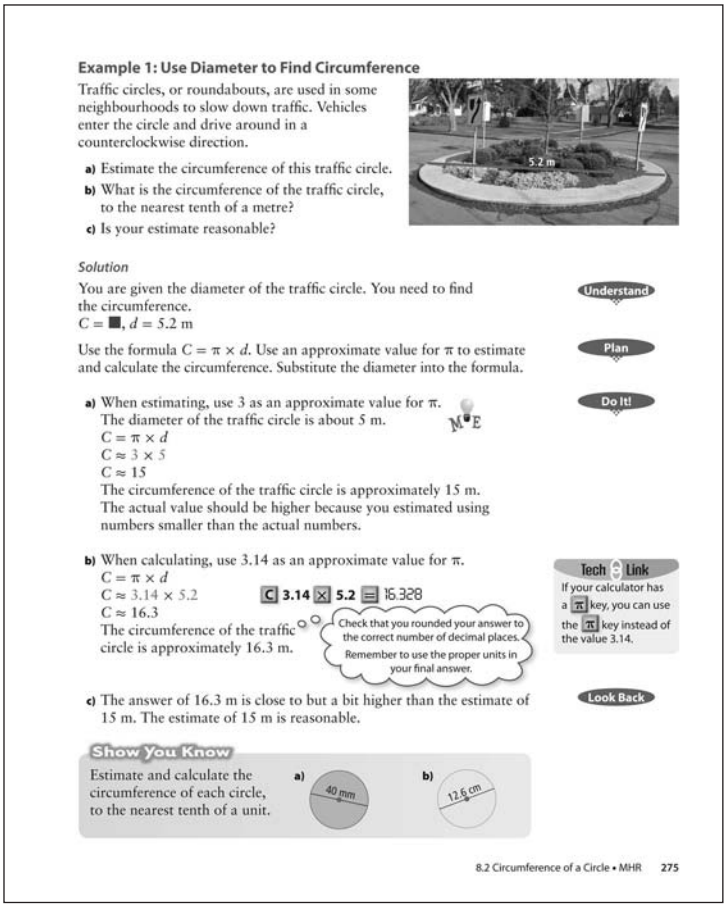
Explore the Math

In this investigation, students relate the circumference of a circle to its diameter.

Method 1: Have each student measure the circumference of two objects and share their data with four classmates.

Method 2: Have pairs of students measure one object and record their data on the chalkboard. Students can use the class-generated data.

Method 3: Demonstrate how to measure the circumference of two objects before asking student volunteers to measure other objects. Students can use the class-generated data.



Example 1: Use Diameter to Find Circumference

Traffic circles, or roundabouts, are used in some neighbourhoods to slow down traffic. Vehicles enter the circle and drive around in a counterclockwise direction.

a) Estimate the circumference of this traffic circle.
b) What is the circumference of the traffic circle, to the nearest tenth of a metre?
c) Is your estimate reasonable?

5.2 m

Solution

You are given the diameter of the traffic circle. You need to find the circumference.
$C = \blacksquare$, $d = 5.2$ m

Understand

Use the formula $C = \pi \times d$. Use an approximate value for π to estimate and calculate the circumference. Substitute the diameter into the formula.

Plan

Do It!

a) When estimating, use 3 as an approximate value for π.
The diameter of the traffic circle is about 5 m.
$C = \pi \times d$
$C \approx 3 \times 5$
$C \approx 15$
The circumference of the traffic circle is approximately 15 m.
The actual value should be higher because you estimated using numbers smaller than the actual numbers.

b) When calculating, use 3.14 as an approximate value for π.
$C = \pi \times d$
$C \approx 3.14 \times 5.2$
$C \approx 16.3$
The circumference of the traffic circle is approximately 16.3 m.

C 3.14 × 5.2 = 16.328

Check that you rounded your answer to the correct number of decimal places.
Remember to use the proper units in your final answer.

Tech Link
If your calculator has a π key, you can use the π key instead of the value 3.14.

c) The answer of 16.3 m is close to but a bit higher than the estimate of 15 m. The estimate of 15 m is reasonable.

Look Back

Show You Know

Estimate and calculate the circumference of each circle, to the nearest tenth of a unit.

a) 40 mm

b) 12.6 cm

8.2 Circumference of a Circle • MHR 275

Example 1 illustrates using diameter to find circumference. Discuss the problem solving steps and the strategy used.

Challenge students to use a different strategy. For example, build a pattern to estimate the circumference of the traffic circle:

- diameter 1 m; circumference about 3.14 m
- diameter 2 m; circumference about 6.28 m
- diameter 3 m; circumference about 9.42 m
- diameter 4 m; circumference about 12.56 m
- diameter 5 m; circumference about 15.7 m
- diameter $\frac{1}{10}$ m, circumference about 0.3 m
- diameter $\frac{2}{10}$ m; circumference about 0.6 m

If the diameter is 5.2 m, the circumference is about 15.7 m + 0.6 m = 16.3 m.

Answers

Show You Know: Example 1

a) Estimate: 120 mm. Calculate: 125.6 mm

b) Estimate: 39 mm. Calculate: 39.6 mm

Supported Learning

Learning Style and Memory

- Suggest to students who have difficulty writing that they discuss the answers orally with a partner.

Learning Style

- Many people use circular formations for a variety of activities, such as group discussions, singing, and games. For example, many First Nations teachings take place when everyone is sitting in a circle. Consider having students sit on chairs in a circle for a class and teach the circle from this perspective. Ask volunteers to use the circle to demonstrate terms such as *circumference* (walk around the perimeter), *radius* (walk from the centre outward), and *diameter* (walk across the middle of the circle).

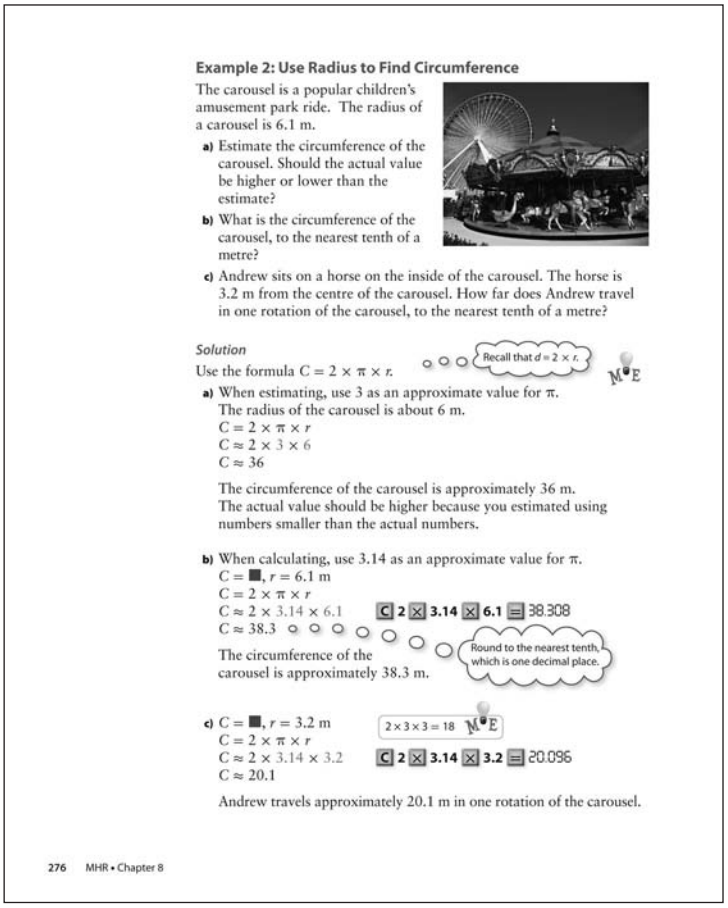
ESL

- English language learners may have difficulty with terms such as *data*, *column*, *circular*, *one rotation*, *crater*, *steel*, *trampoline*, and *circular path*.

Motor

- Give students extra time and a ruler to copy the table in Explore the Math #1. Alternatively, consider allowing them to use a computer to create the table.
- Pair students to measure objects.
- The buttons on most calculators are typically too small and close together for some students to use accurately. Students may benefit from using a calculator with oversized keys.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1 on page 275.	• You may wish to review how to round decimals to the nearest tenth. • Have students read the Tech Link on page 275, which explains how to use the π key. • You may wish to provide additional questions to students who would benefit from them: **a)** A talking circle is 3 m in diameter. Estimate the circumference of the talking circle. ($3 \times 3 = 9$. The circumference is about 9 m. Make sure that students use 3 to estimate pi.) **b)** Calculate the circumference of the talking circle to the nearest tenth of a metre. ($3 \times 3.14 = 9.42$. The circumference is about 9.4 m. Students should correctly round the decimal.) **c)** Is your answer reasonable? (Yes, both answers are close together.) Sit down and coach students through this additional question.



Example 2: Use Radius to Find Circumference

The carousel is a popular children's amusement park ride. The radius of a carousel is 6.1 m.

a) Estimate the circumference of the carousel. Should the actual value be higher or lower than the estimate?

b) What is the circumference of the carousel, to the nearest tenth of a metre?

c) Andrew sits on a horse on the inside of the carousel. The horse is 3.2 m from the centre of the carousel. How far does Andrew travel in one rotation of the carousel, to the nearest tenth of a metre?

Solution

Use the formula $C = 2 \times \pi \times r$.

Recall that $d = 2 \times r$.

a) When estimating, use 3 as an approximate value for π.
The radius of the carousel is about 6 m.
$C = 2 \times \pi \times r$
$C \approx 2 \times 3 \times 6$
$C \approx 36$

The circumference of the carousel is approximately 36 m.
The actual value should be higher because you estimated using numbers smaller than the actual numbers.

b) When calculating, use 3.14 as an approximate value for π.
$C = \blacksquare$, $r = 6.1$ m
$C = 2 \times \pi \times r$
$C \approx 2 \times 3.14 \times 6.1$
C 2 × 3.14 × 6.1 = 38.308
$C \approx 38.3$

Round to the nearest tenth, which is one decimal place.

The circumference of the carousel is approximately 38.3 m.

c) $C = \blacksquare$, $r = 3.2$ m
$C = 2 \times \pi \times r$
$C \approx 2 \times 3.14 \times 3.2$
$C \approx 20.1$

2 × 3 × 3 = 18
C 2 × 3.14 × 3.2 = 20.096

Andrew travels approximately 20.1 m in one rotation of the carousel.

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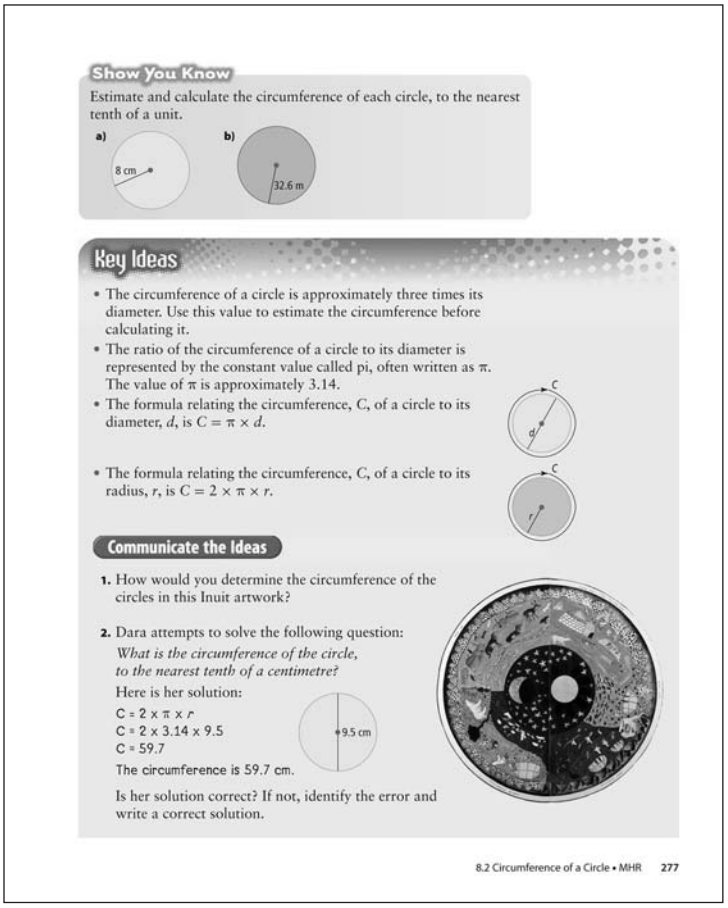
Supported Learning

ESL and Language

- For Example 2, explain what a *carousel* is to English language learners who may not be familiar with the term. Ask students who have taken a carousel ride to describe it.

Example 2 illustrates using radius to find circumference. Review information from Section 8.1. If students know the radius of a circle, they can calculate the diameter and then evaluate the circumference. Point out that the relationship between radius and diameter is exact, whereas the relationship between circumference and pi is approximate because 3.14 is an estimate of pi.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2 on page 277.	• You may wish to provide additional questions to students who would benefit from them: Estimate and calculate the circumference of each circle to the nearest tenth of a metre: **a)** radius = 3.25 m (diameter = 3.25 + 3.25 = 6.5 m. Estimate: $6 \times 3 = 18$ m. To calculate the circumference, students will need to use their multiplying skills from Chapter 2, and then use their estimation to put in the decimal point. $6.5 \times 3.14 = 2041$. Since the answer is about 18, the decimal should be between 0 and 4. $C \approx 20.41$ m. Rounded to tenths, the answer is 20.4 m.) **b)** radius = 9.1 m (diameter = 9.1 + 9.1 = 18.2 m. Estimate: $18 \times 3 = 54$ m. To calculate the circumference, students will need to multiply, and then use their estimation to put in the decimal point. $18.2 \times 3.14 = 57148$. Since the answer is about 54, the decimal should be between 7 and 1. $C \approx 57.148$ m. Rounded to tenths, the answer is 57.1 m.) Sit down and coach students through a) and then have them try b) on their own.



Show You Know

Estimate and calculate the circumference of each circle, to the nearest tenth of a unit.

a) 8 cm

b) 32.6 m

Key Ideas

- The circumference of a circle is approximately three times its diameter. Use this value to estimate the circumference before calculating it.
- The ratio of the circumference of a circle to its diameter is represented by the constant value called pi, often written as π. The value of π is approximately 3.14.
- The formula relating the circumference, C, of a circle to its diameter, d, is $C = \pi \times d$.
- The formula relating the circumference, C, of a circle to its radius, r, is $C = 2 \times \pi \times r$.

Communicate the Ideas

1. How would you determine the circumference of the circles in this Inuit artwork?
2. Dara attempts to solve the following question:
What is the circumference of the circle, to the nearest tenth of a centimetre?
Here is her solution:
C = 2 x π x r
C = 2 x 3.14 x 9.5
C = 59.7
The circumference is 59.7 cm.
9.5 cm
Is her solution correct? If not, identify the error and write a correct solution.

8.2 Circumference of a Circle • MHR 277

Answers

Show You Know: Example 2

a) Estimate: 48 cm. Calculate: 50.2 cm

b) Estimate: 180 m. Calculate: 204.7 m

Communicate the Ideas

1. Answers may vary. Measure the diameter of each circle and then multiply the diameter of each circle by pi.
2. No. Dara used the formula $C = 2 \times \pi \times r$ and substituted 9.5 for the radius. The diameter of the circle is 9.5 cm. Dara should have used the formula $C = \pi \times d$. The correct solution is $C = \pi \times d = \pi \times 9.5 \approx 29.8$ cm.

Key Ideas

This section summarizes the ratio of the circumference of a circle to its diameter and the formula used to calculate circumference.

Discuss why three times the diameter is a good estimate of circumference. Reinforce the idea that estimating before calculating is good practice. Have students write the formula into their chapter Foldable.

Communicate the Ideas

These questions allow students to describe the circumference of Inuit circles and review a solution for a problem related to circumference.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Most students should do both questions. Encourage students to communicate their analysis of both questions with neighbouring students and listen to each other's explanations.	• Check each student's answer to #1. This is a key question; make sure that they have the concept. • Tell students that Dara's error in #2 (multiplying diameter by 2) is common. Encourage students to recognize that Dara confused the formulas for radius and diameter. Brainstorm with students how to avoid making this error. For example, they could always change radius to diameter and then use the diameter formula. That way, they would have to remember only one formula. • Use **Master 2 Two Stars and One Wish** to have students critique other students' writing pieces. This master allows them to write two things they like about a piece and one thing they would like to see improved.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 7, 9, Math Link
Typical	1–14, Math Link
Extension/Enrichment	1, 2, 15–19

Common Errors

- Students may have difficulty knowing which formula for circumference to use in a word problem.

R_x Suggest that students read the problem, including the visual, carefully; then list what they know. Have students estimate the answer.

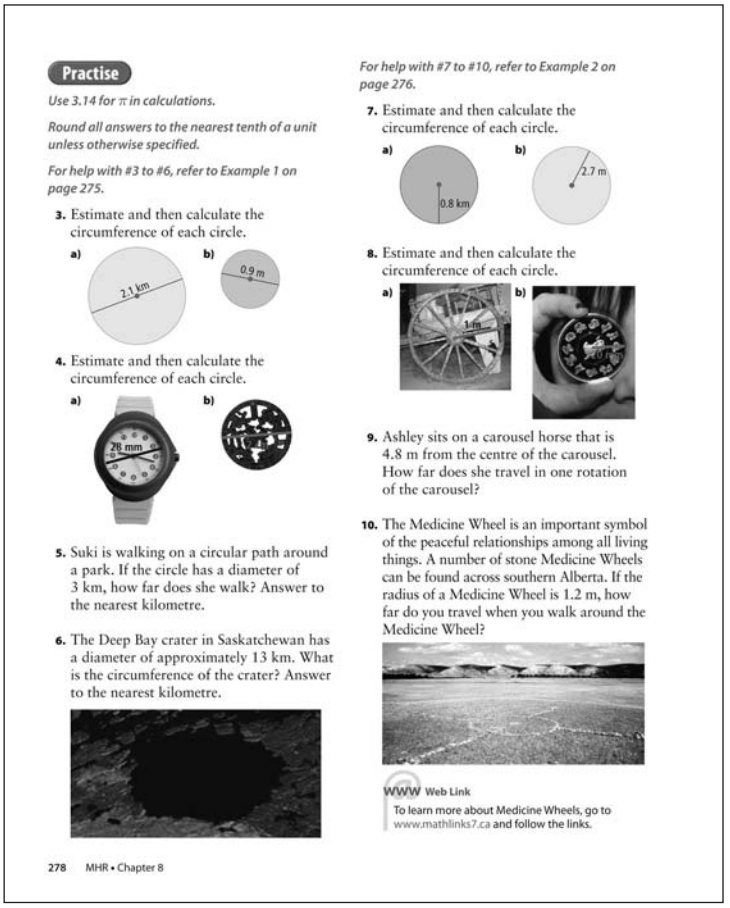
Supported Learning

Learning Style, ESL, Language, and Memory

- Have students add the formulas for circumference to their Foldable.
- Some students may have difficulty processing the steps in word problems and need some coaching.
- For #11, which refers to skating and hockey, ensure students know what a face-off circle is.

Learning Style and Memory

- Provide **BLM 8–8 Section 8.2 Extra Practice** to students who require more practice.



Practise

Use 3.14 for π in calculations.

Round all answers to the nearest tenth of a unit unless otherwise specified.

For help with #3 to #6, refer to Example 1 on page 275.

3. Estimate and then calculate the circumference of each circle.

a) 2.1 km b) 0.9 m

4. Estimate and then calculate the circumference of each circle.

a) 28 mm b)

5. Suki is walking on a circular path around a park. If the circle has a diameter of 3 km, how far does she walk? Answer to the nearest kilometre.

6. The Deep Bay crater in Saskatchewan has a diameter of approximately 13 km. What is the circumference of the crater? Answer to the nearest kilometre.

For help with #7 to #10, refer to Example 2 on page 276.

7. Estimate and then calculate the circumference of each circle.

a) 0.8 km b) 2.7 m

8. Estimate and then calculate the circumference of each circle.

a) 1 m b)

9. Ashley sits on a carousel horse that is 4.8 m from the centre of the carousel. How far does she travel in one rotation of the carousel?

10. The Medicine Wheel is an important symbol of the peaceful relationships among all living things. A number of stone Medicine Wheels can be found across southern Alberta. If the radius of a Medicine Wheel is 1.2 m, how far do you travel when you walk around the Medicine Wheel?

WWW Web Link
To learn more about Medicine Wheels, go to www.mathlinks7.ca and follow the links.

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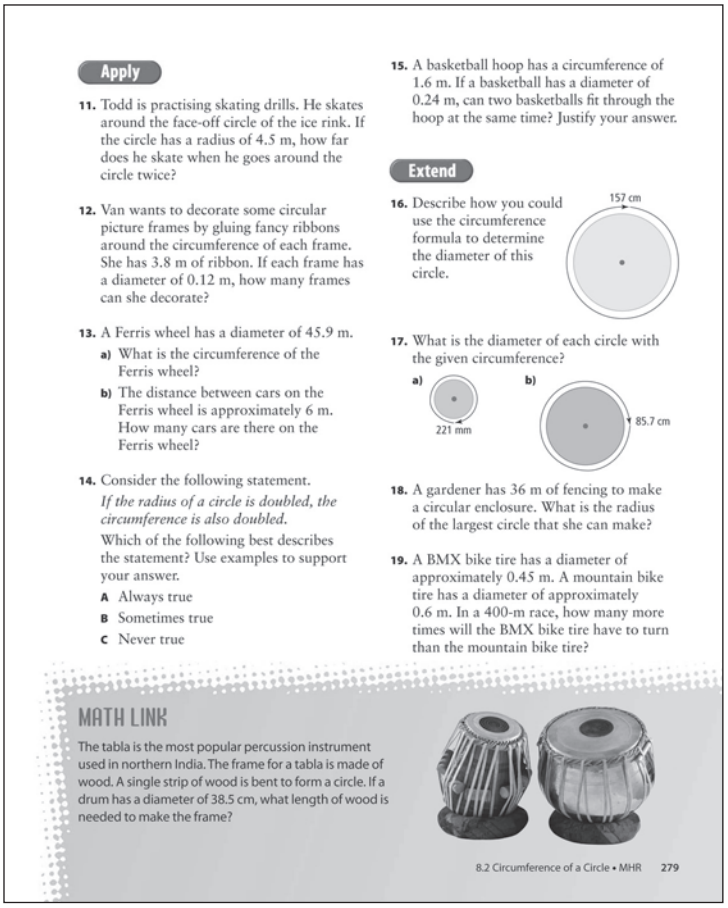
Practise

Assessment *for* Learning	Supported Learning
Practise Have students do #3, #5, #7, and #9. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #3, #5, #7, and #9 will need additional coaching. Have students explain their thinking on these questions; clarify any misunderstandings. Coach students through #4a) and #8a), and then have them complete part b) of these questions on their own. Have students refer back to examples in the student resource. Check back with them several times to make sure that they understand the concepts. • Once they understand the basic concepts, have them do #6 and #10 on their own.

Apply and Extend

In #14, you may wish to remind students of the difference between "Sometimes true" (true in only specific situations) and "Always true" (can be generalized). Challenge enriched students to use algebra to help reason through the statement. In #16, have students use words to describe the steps, and then algebraic symbols. In #19, have students consider the distance each wheel travels in one rotation. Then guide them to consider how many rotations would be needed to cover 400 m. Ask how they could compare distances in centimetres and metres.



Apply

11. Todd is practising skating drills. He skates around the face-off circle of the ice rink. If the circle has a radius of 4.5 m, how far does he skate when he goes around the circle twice?

12. Van wants to decorate some circular picture frames by gluing fancy ribbons around the circumference of each frame. She has 3.8 m of ribbon. If each frame has a diameter of 0.12 m, how many frames can she decorate?

13. A Ferris wheel has a diameter of 45.9 m.
a) What is the circumference of the Ferris wheel?
b) The distance between cars on the Ferris wheel is approximately 6 m. How many cars are there on the Ferris wheel?

14. Consider the following statement.
If the radius of a circle is doubled, the circumference is also doubled.
Which of the following best describes the statement? Use examples to support your answer.
A Always true
B Sometimes true
C Never true

15. A basketball hoop has a circumference of 1.6 m. If a basketball has a diameter of 0.24 m, can two basketballs fit through the hoop at the same time? Justify your answer.

Extend

16. Describe how you could use the circumference formula to determine the diameter of this circle.

17. What is the diameter of each circle with the given circumference?

a)

b)

18. A gardener has 36 m of fencing to make a circular enclosure. What is the radius of the largest circle that she can make?

19. A BMX bike tire has a diameter of approximately 0.45 m. A mountain bike tire has a diameter of approximately 0.6 m. In a 400-m race, how many more times will the BMX bike tire have to turn than the mountain bike tire?

MATH LINK

The tabla is the most popular percussion instrument used in northern India. The frame for a tabla is made of wood. A single strip of wood is bent to form a circle. If a drum has a diameter of 38.5 cm, what length of wood is needed to make the frame?

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • What do you know about the circumference of a circle?	• Have students explain the relationship between circumference and the radius and diameter of a circle. • Encourage concrete and kinesthetic learners to use a circular object and string to help explain the relationship between circumference and the radius and diameter of a circle. • Depending on students' learning styles, have them provide verbal or written answers. • You may wish to have students review the part related to Section 8.2 in **BLM 8–1 Chapter 8 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 279 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 301.	• As students work on the Math Link, observe and have them self-observe how well they solve the circumference problem. • Students who are having difficulty getting started could use **BLM 8–9 Section 8.2 Math Link**, which provides scaffolding for this activity.

Answers

Math Link

$C = \pi \times d = \pi \times 38.5 \approx 121.0$ cm

MATH LINK

The Math Link introduces students to another cultural application of circles.

Students may wonder what part of the tabla is being measured. Discuss that the diameter refers to the drum face. The strip of wood is "o"-shaped and circles the drum face. It appears to be covered with rope in the picture.

You may wish to have students discuss their experiences with various types of drums and their sizes.

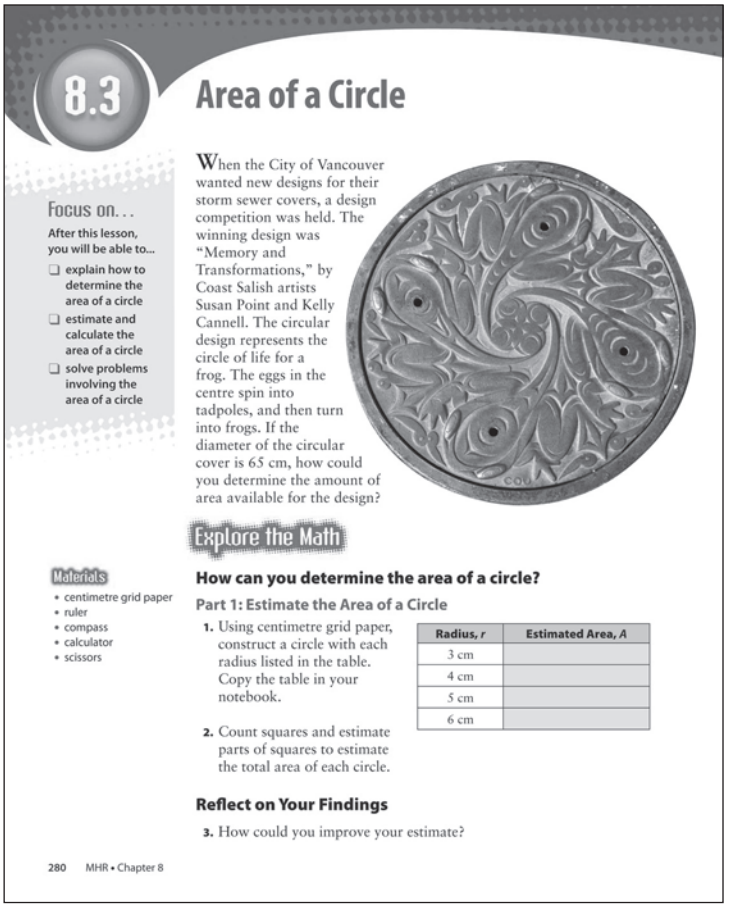
8.3 Area of a Circle

Suggested Timing
80–100 minutes

Materials
- centimetre grid paper
- ruler
- compass
- scissors
- calculator

Blackline Masters
Master 8 Centimetre Grid Paper
BLM 8–1 Chapter 8 Self-Assessment
BLM 8–10 Section 8.3 Extra Practice
BLM 8–11 Section 8.3 Math Link

Mathematical Processes
- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [x] Technology
- [ ] Visualization



8.3 Area of a Circle

Focus on...
After this lesson, you will be able to...
- explain how to determine the area of a circle
- estimate and calculate the area of a circle
- solve problems involving the area of a circle

When the City of Vancouver wanted new designs for their storm sewer covers, a design competition was held. The winning design was "Memory and Transformations," by Coast Salish artists Susan Point and Kelly Cannell. The circular design represents the circle of life for a frog. The eggs in the centre spin into tadpoles, and then turn into frogs. If the diameter of the circular cover is 65 cm, how could you determine the amount of area available for the design?

Explore the Math

Materials
- centimetre grid paper
- ruler
- compass
- calculator
- scissors

How can you determine the area of a circle?

Part 1: Estimate the Area of a Circle

1. Using centimetre grid paper, construct a circle with each radius listed in the table. Copy the table in your notebook.

Radius, r	Estimated Area, A
3 cm	
4 cm	
5 cm	
6 cm	

2. Count squares and estimate parts of squares to estimate the total area of each circle.

Reflect on Your Findings

3. How could you improve your estimate?

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Specific Outcomes

SS1 Demonstrate an understanding of circles by:
- describing the relationships among radius, diameter and circumference of circles
- solving problems involving the radii, diameters and circumferences of circles.

SS2 Develop and apply a formula for determining the area of:
- circles.

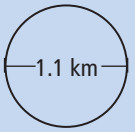
Warm-Up

1. What is the diameter of the circle shown?
2. Estimate the circumference of the circle.
3. Calculate the circumference.

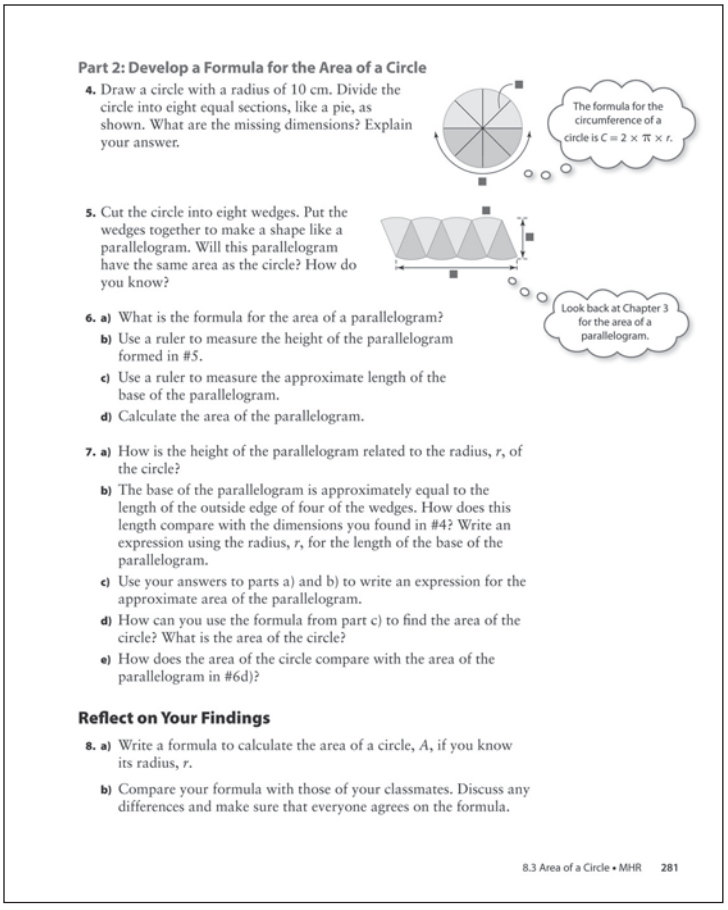
4. Determine the common denominator for $\frac{5}{8}$ and $\frac{2}{3}$, and then the equivalent fractions.
5. Determine the common denominator for $\frac{2}{5}$ and $\frac{1}{2}$. Which is the larger fraction?

Mental Math

6. Estimate the circumference of the circle.



7. Mentally calculate 80% of 42. Show your thinking.
8. Estimate 43 out of 54 as a percent.
9. A spinner is 50% red. Draw two versions of the spinner.



Part 2: Develop a Formula for the Area of a Circle

4. Draw a circle with a radius of 10 cm. Divide the circle into eight equal sections, like a pie, as shown. What are the missing dimensions? Explain your answer.

The formula for the circumference of a circle is $C = 2 \times \pi \times r$.

5. Cut the circle into eight wedges. Put the wedges together to make a shape like a parallelogram. Will this parallelogram have the same area as the circle? How do you know?

Look back at Chapter 3 for the area of a parallelogram.

6. a) What is the formula for the area of a parallelogram?

b) Use a ruler to measure the height of the parallelogram formed in #5.

c) Use a ruler to measure the approximate length of the base of the parallelogram.

d) Calculate the area of the parallelogram.

7. a) How is the height of the parallelogram related to the radius, *r*, of the circle?

b) The base of the parallelogram is approximately equal to the length of the outside edge of four of the wedges. How does this length compare with the dimensions you found in #4? Write an expression using the radius, *r*, for the length of the base of the parallelogram.

c) Use your answers to parts a) and b) to write an expression for the approximate area of the parallelogram.

d) How can you use the formula from part c) to find the area of the circle? What is the area of the circle?

e) How does the area of the circle compare with the area of the parallelogram in #6d)?

Reflect on Your Findings

8. a) Write a formula to calculate the area of a circle, *A*, if you know its radius, *r*.

b) Compare your formula with those of your classmates. Discuss any differences and make sure that everyone agrees on the formula.

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Activity Planning Notes

Have students consider the multiple circles and part circles in the sewer cover design. Encourage students to discuss the techniques that the artist may have used to create the designs, and how the circles in the design help to express the theme.

Explore the Math

It is important that students estimate the area of a circle using grid paper and then develop a formula for the area of a circle using concrete materials. Students could work individually, in pairs, or as a class.

Part 1 illustrates using grid paper to estimate area. Part 2 illustrates developing a formula for the area of a circle.

Method 1: Have students do Part 1 and Part 2.

Method 2: Have students complete Part 1. Consider using the overhead to demonstrate Part 2.

Answers

Warm-Up

1. 7 cm × 2 = 14 cm **2.** 3 × 14 cm ≈ 42 cm

3. 3.14 × 14 cm ≈ 43.96 cm

4. Common denominator = 24; $\frac{5}{8} = \frac{15}{24}$, $\frac{2}{3} = \frac{16}{24}$

5. Common denominator = 10; $\frac{2}{5} = \frac{4}{10}$, $\frac{1}{2} = \frac{5}{10}$. $\frac{1}{2}$ is larger.

6. 3 × 1.1 km ≈ 3.3 km

7. 10% of 42 = 4.2; 80% = 4.2 × 8 = 33.6. Methods may vary.

8. 50% of 54 = 27; 25% = 13.5; 10% = 5.4
75% = 27 + 13.5 = 40.5
85% = 27 + 13.5 + 5.4 = 45.9
The answer is between 75% and 85%. Students might get closer to 80% if they notice that 8 × 5.4 = 43.2.

9. Look for a spinner cut in half with half red, or a spinner cut in quarters with two of the quarters red. Other designs are possible.

Explore the Math

1., 2. Answers will vary. For example:

Radius (*r*)	Estimated Area (*A*)
3 cm	*30 cm²*
4 cm	*50 cm²*
5 cm	*80 cm²*
6 cm	*115 cm²*

3. Answers may vary. For example: Find a way to minimize the rounded-off squares to make the count more accurate.

4. Radius: 10 cm. Answers may vary. For example: The circumference of the circle is 62.8 cm. The missing dimension is half of the circumference of the circle, which is 31.4 cm.

5. Answers may vary. For example: Yes, the parallelogram will have approximately the same area as the circle. The parallelogram is formed from the wedge shapes that form the circle. (The missing dimension is the height of the parallelogram.)

6. a) $A = b \times h$

b) Answers may vary. For example: 10 cm

c) Answers may vary. For example: 31.4 cm

d) 10 × 31.4 ≈ 314 cm²

7. a) The height of the parallelogram is approximately equal to the radius of the circle.

b) Answers may vary. For example: The length should be approximately equal to the dimensions found in #4. The length of the base of the parallelogram is $b = \pi \times r$.

c) For the parallelogram, $A = b \times h = \pi \times r \times r = \pi \times r^2$

d) Multiply the square of the radius times pi. The formula for the area of a circle is $A = \pi \times r^2$. The area of the given circle is $A = \pi \times 102 \approx 314.2$ cm².

e) Answers will vary. For example: The area of the circle is approximately equal to the area of the parallelogram in #6d).

8. a) $A = \pi r^2$ **b)** Answers will vary.

Supported Learning

Learning Style, Language, and Memory

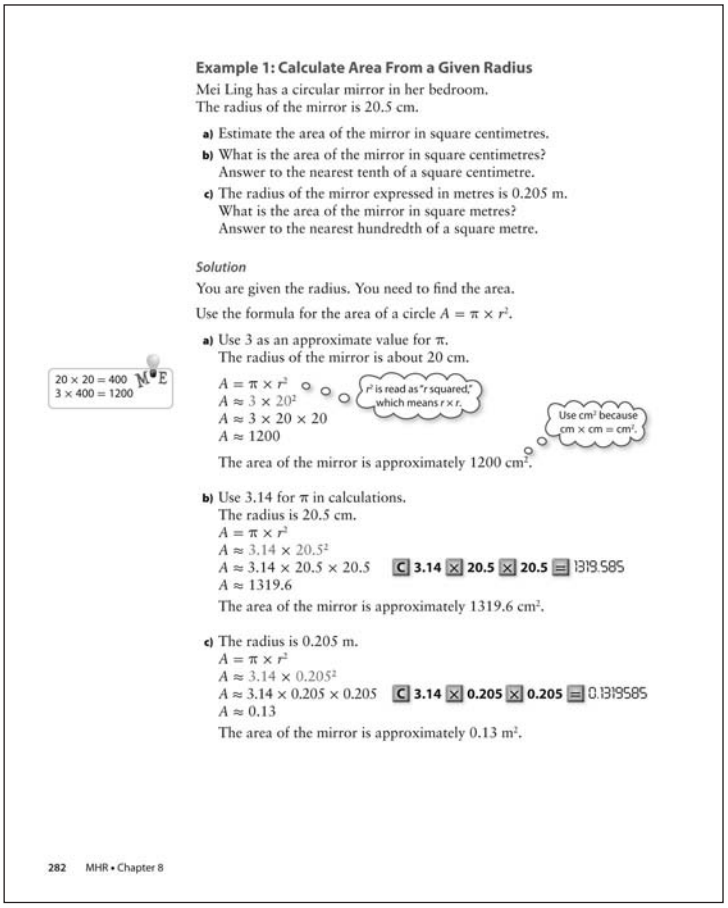
- Allow students to construct their circles on grid paper using the method they are most comfortable with.

ESL

- English language learners may have difficulty with terms such as *frog, tadpole, height, base, ice fishing, square* (repeated multiplication), *sprinkler, porthole, archery, target, always, sometimes, never, outer ring,* and *identical.*
- Some students may need to review how to determine the area of a parallelogram and the formula. The language in Explore the Math is dense and the reasoning difficult to follow. Partner students to reinforce vocabulary.

Motor

- Give students extra time and a ruler to copy the table in #1. Alternatively, consider allowing students to use a computer to create the table.



Example 1: Calculate Area From a Given Radius

Mei Ling has a circular mirror in her bedroom. The radius of the mirror is 20.5 cm.

a) Estimate the area of the mirror in square centimetres.

b) What is the area of the mirror in square centimetres? Answer to the nearest tenth of a square centimetre.

c) The radius of the mirror expressed in metres is 0.205 m. What is the area of the mirror in square metres? Answer to the nearest hundredth of a square metre.

Solution

You are given the radius. You need to find the area.

Use the formula for the area of a circle $A = \pi \times r^2$.

a) Use 3 as an approximate value for π.
The radius of the mirror is about 20 cm.

$20 \times 20 = 400$
$3 \times 400 = 1200$

$A = \pi \times r^2$
$A \approx 3 \times 20^2$
$A \approx 3 \times 20 \times 20$
$A \approx 1200$

r^2 is read as "r squared," which means $r \times r$.

Use cm^2 because $cm \times cm = cm^2$.

The area of the mirror is approximately 1200 cm^2.

b) Use 3.14 for π in calculations.
The radius is 20.5 cm.
$A = \pi \times r^2$
$A \approx 3.14 \times 20.5^2$
$A \approx 3.14 \times 20.5 \times 20.5$ C 3.14 × 20.5 × 20.5 = 1319.585
$A \approx 1319.6$
The area of the mirror is approximately 1319.6 cm^2.

c) The radius is 0.205 m.
$A = \pi \times r^2$
$A \approx 3.14 \times 0.205^2$
$A \approx 3.14 \times 0.205 \times 0.205$ C 3.14 × 0.205 × 0.205 = 0.1319585
$A \approx 0.13$
The area of the mirror is approximately 0.13 m^2.

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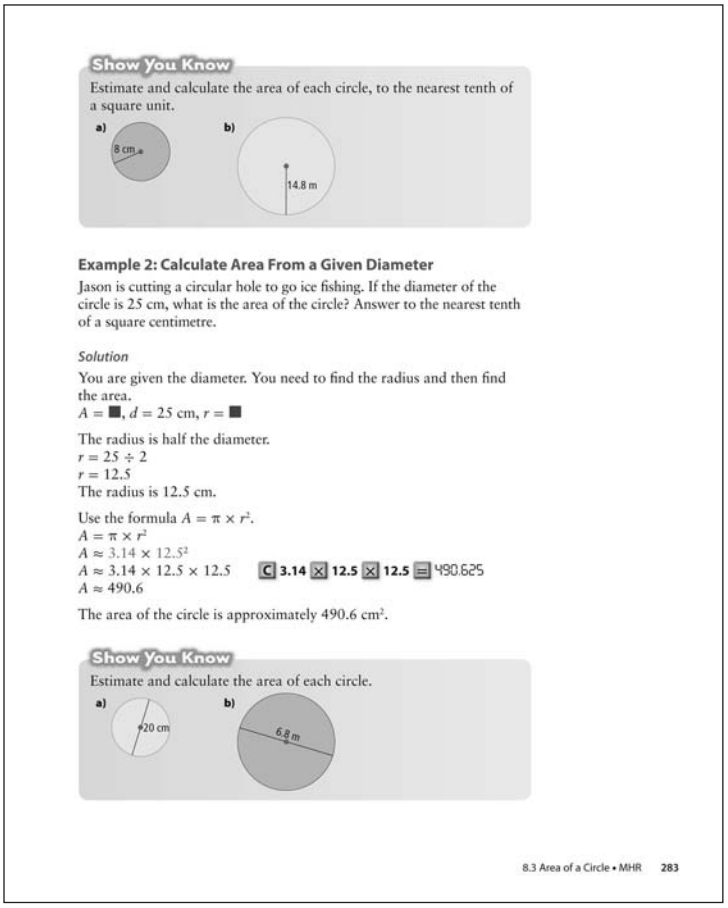
Assessment *as* Learning	Supported Learning
Reflect on Your Findings **Part 1** Watch as students count the squares in #2. Note how they estimate the area of partial squares. Have students answer #3. **Part 2** Listen as students discuss what they discovered during the Explore the Math activity. In #8, have students work in groups to develop a formula for the area of a circle.	**Part 1** • Encourage students to consider how they could more accurately join parts of squares to improve their estimates. • Ask students who are having difficulty to use the class responses as springboards to prepare similar answers of their own. **Part 2** • Review the formula for the area of a parallelogram. • You will need to help students relate to the idea that the base of the parallelogram is half of the circumference of a circle, and help them develop the formula.

Method 3: Provide pre-drawn pictures of circles on centimetre grid paper and have students estimate the area of each circle for Part 1. Consider using the overhead to demonstrate Part 2.

In Example 1, students calculate the area of a circle using the radius. Note that some students may not be familiar with the idea of squaring a number. Point out that r^2 means the same thing as $r \times r$.

Point out the thought bubbles that model how some people think through the solutions to questions like these.

Example 2 illustrates calculating area from a given diameter. Students must use the diameter to find the radius in order to use the formula for area of a circle.



Show You Know

Estimate and calculate the area of each circle, to the nearest tenth of a square unit.

a)

b)

Example 2: Calculate Area From a Given Diameter

Jason is cutting a circular hole to go ice fishing. If the diameter of the circle is 25 cm, what is the area of the circle? Answer to the nearest tenth of a square centimetre.

Solution

You are given the diameter. You need to find the radius and then find the area.

$A = \blacksquare$, $d = 25$ cm, $r = \blacksquare$

The radius is half the diameter.

$r = 25 \div 2$

$r = 12.5$

The radius is 12.5 cm.

Use the formula $A = \pi \times r^2$.

$A = \pi \times r^2$

$A \approx 3.14 \times 12.5^2$

$A \approx 3.14 \times 12.5 \times 12.5$ C 3.14 × 12.5 × 12.5 = 490.625

$A \approx 490.6$

The area of the circle is approximately 490.6 cm^2.

Show You Know

Estimate and calculate the area of each circle.

a)

b)

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• Have students talk through their thinking in a group. • You may wish to provide additional questions to students who would benefit from them: Estimate and calculate the area of the following circles to the nearest tenth of a square centimetre: **a)** radius = 12 cm (Estimate: $3 \times 10^2 \approx 300$ cm^2. Calculate: $3.14 \times 122 \approx 452.16$ cm^2. Round up because the number after tenths is greater than 5. The area is approximately 452.2 cm^2. Make sure that students square the radius, show the answer in cm^2, and remember that the answer is an approximation.) **b)** radius = 23.3 cm (Estimate: $3 \times 202 \approx 1200$ cm^2. Calculate: $3.14 \times 23.32 = 1704.6746$ cm^2. Round up because the number after tenths is greater than 5. The area is approximately 1704.7 cm^2.) Sit down and coach students through part a), and then have them try part b) on their own.

Answers

Show You Know: Example 1

a) Estimate: 192 cm^2. Calculate: 201.0 cm^2

b) Estimate: 675 m^2. Calculate: 687.8 m^2

Show You Know: Example 2

a) Estimate: 300 m^2. Calculate: 314 m^2

b) Estimate: 27 m^2. Calculate: 36.3 m^2

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• Have students talk through their thinking in a group. • You may wish to provide additional questions to students who would benefit from them: Estimate and calculate the area of the following circles to the nearest hundredth of a square centimetre. Predict whether each estimate is high or low: **a)** diameter = 25 cm (radius = 25 cm ÷ 2 = 12.5 cm. Estimate: $3 \times 122 \approx 432$ cm^2. Too low because radius and pi were rounded down. Calculate: $3.14 \times 12.52 \approx 490.625$ cm^2. Round up because the number after hundredths is 5. The area is approximately 490.63 cm^2. Make sure that students use the diameter to determine the radius, multiply the radius times itself, show the answer in cm^2, and remember that the answer should be in hundredths.) **b)** diameter = 4.8 cm (radius = 4.8 cm ÷ 2 = 2.4 cm. Estimate: $3 \times 22 \approx 12$ cm^2. Too low because radius and pi were rounded down. Calculate: $3.14 \times 2.42 \approx 18.0864$ cm^2. Round up because the number after hundredths is more than 5. The area is approximately 18.09 cm^2. Sit down and coach students through a), and then have them try b) on their own.

Answers

Communicate the Ideas

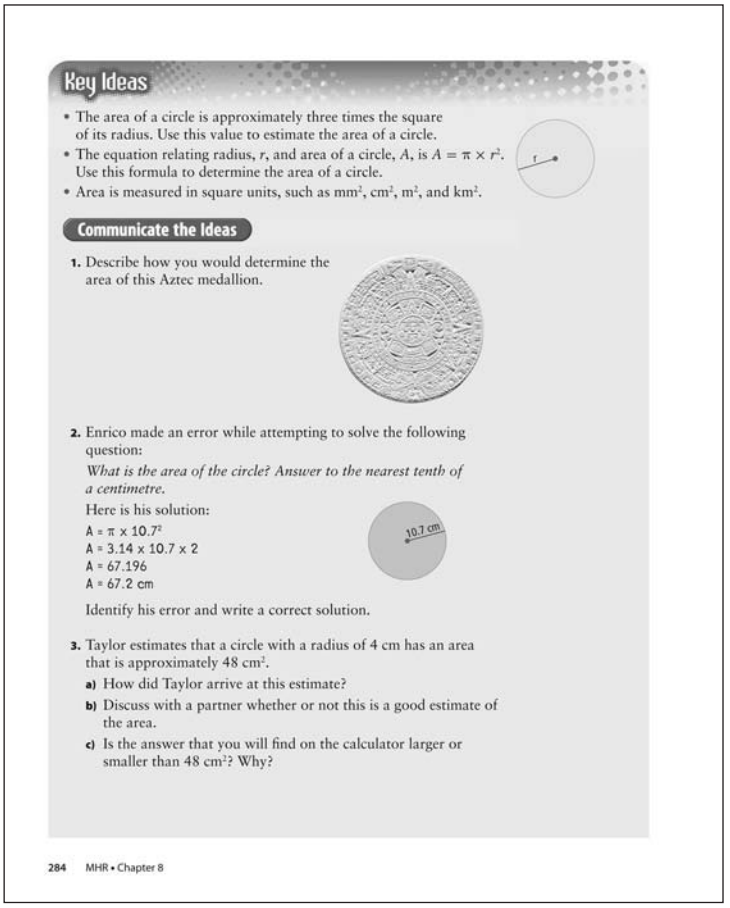
1. Measure the diameter. Divide the diameter by 2 to find the radius. Substitute the radius into the formula $A = \pi \times r^2$.

2. In his calculations, Enrico doubled the radius instead of squaring the radius.
The correct solution is
$A = \pi \times r^2$
$A \approx 3.14 \times 10.72$
$A \approx 359.5 \text{ cm}^2$

3. a) Taylor estimated that $\pi \approx 3$, used $r = 4$ cm, and did the following calculations:
$A = 42 \times 3$
$A \approx 16 \times 3$
$A \approx 48 \text{ cm}^2$

b) Answers will vary.

c) The answer found on the calculator will be larger. For Taylor's estimate he has rounded pi down to 3.



Key Ideas

- The area of a circle is approximately three times the square of its radius. Use this value to estimate the area of a circle.
- The equation relating radius, r, and area of a circle, A, is $A = \pi \times r^2$. Use this formula to determine the area of a circle.
- Area is measured in square units, such as mm², cm², m², and km².

Communicate the Ideas

1. Describe how you would determine the area of this Aztec medallion.

2. Enrico made an error while attempting to solve the following question:
What is the area of the circle? Answer to the nearest tenth of a centimetre.
Here is his solution:
$A = \pi \times 10.7^2$
$A = 3.14 \times 10.7 \times 2$
$A = 67.196$
$A = 67.2 \text{ cm}$
Identify his error and write a correct solution.

3. Taylor estimates that a circle with a radius of 4 cm has an area that is approximately 48 cm².
a) How did Taylor arrive at this estimate?
b) Discuss with a partner whether or not this is a good estimate of the area.
c) Is the answer that you will find on the calculator larger or smaller than 48 cm²? Why?

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Common Errors

- Students may have difficulty with unit conversions (when to move the decimal to the left, and when to move it to the right).

R_x Review with students how to multiply and divide by multiples of ten by moving the decimal point to the right or left.

- Students may forget to use square units for area.

R_x Reinforce that area is always in squared units. Show counting squares for determining area and therefore using square units.

Key Ideas

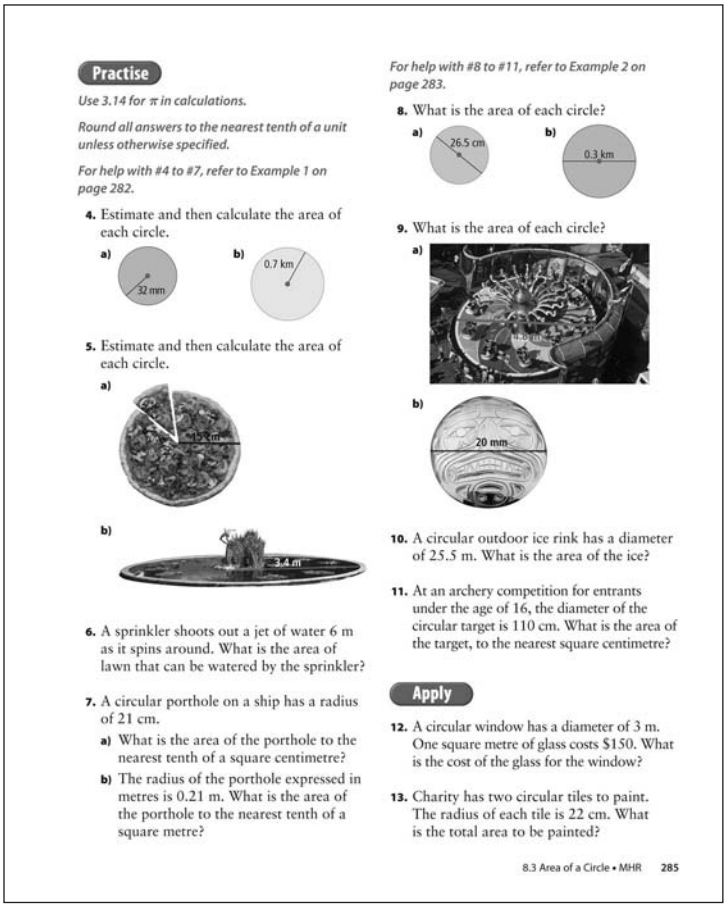
The Key Ideas summarize how to estimate and calculate the area of a circle.

Discuss why the units are squared for area and not for circumference. Reinforce that estimating before calculating is good practice. Have students write the formula for area into their chapter Foldable.

Communicate the Ideas

These questions allow students to apply their understanding of area. In #1, students determine the area of a medallion. In #2, students review a solution for a problem related to area. In #3, students solve an area problem.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students complete #1, and then have pairs of students work together. Have one partner do #2 and the other partner do #3. Once they have finished, have them communicate their analysis of the problem with their partner and listen to each other's explanations.	• Check each student's answer to #1. This is a key question; make sure that they have the concept. • Tell students that Enrico's error in #2 is common (multiplying by 2 instead of multiplying a number by itself). Remind students that *squared* means a number multiplied by itself. Brainstorm with students how to avoid making this error. • Discuss Taylor's estimation method. (He multiplied 4 by itself and then multiplied the answer by 3.) This is a useful way to estimate area of a circle if you know the radius.



Practise

Use 3.14 for π in calculations.

Round all answers to the nearest tenth of a unit unless otherwise specified.

For help with #4 to #7, refer to Example 1 on page 282.

4. Estimate and then calculate the area of each circle.

a) 32 mm

b) 0.7 km

5. Estimate and then calculate the area of each circle.

a) 11.5 cm

b) 3.4 m

6. A sprinkler shoots out a jet of water 6 m as it spins around. What is the area of lawn that can be watered by the sprinkler?

7. A circular porthole on a ship has a radius of 21 cm.

a) What is the area of the porthole to the nearest tenth of a square centimetre?

b) The radius of the porthole expressed in metres is 0.21 m. What is the area of the porthole to the nearest tenth of a square metre?

For help with #8 to #11, refer to Example 2 on page 283.

8. What is the area of each circle?

a) 26.5 cm

b) 0.3 km

9. What is the area of each circle?

a) 12.6 in.

b) 20 mm

10. A circular outdoor ice rink has a diameter of 25.5 m. What is the area of the ice?

11. At an archery competition for entrants under the age of 16, the diameter of the circular target is 110 cm. What is the area of the target, to the nearest square centimetre?

Apply

12. A circular window has a diameter of 3 m. One square metre of glass costs $150. What is the cost of the glass for the window?

13. Charity has two circular tiles to paint. The radius of each tile is 22 cm. What is the total area to be painted?

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Practise and Apply

Assessment *for* Learning	Supported Learning
Practise Have students do #4, #6, #8, and #10. Students who have no problems with these questions can go on to the Apply questions	• Students who have problems with #4, #6, #8, and #10 will need additional coaching. Have students explain their thinking on these questions. Clarify any misunderstandings. Coach students through #5a) and #9a), and then have them complete part b) of these questions on their own. Have students refer back to examples in the student resource. Check back with them several times to make sure that they understand the concepts.

In #12, consider relating to a real-world situation by asking students who might use such information. In #15 and #16, guide students to visualize a circle within a larger circle; the area of the ring is the difference between the areas of the large circle and the small circle. In #17, ask students to write both formulas and determine when they will be equal. In #19, have students think about how to determine the area of a semi-circle. In #20, remind students about how to find the radius from a given circumference.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8, 10, Math Link
Typical	1–16, Math Link
Extension/Enrichment	1–3, 17–20

Supported Learning

Learning Style, Language, and Memory

- Some students may have difficulty processing the steps in word problems and need some coaching.

Learning Style and Memory

- Provide **BLM 8–10 Section 8.3 Extra Practice** to students who require more practice.

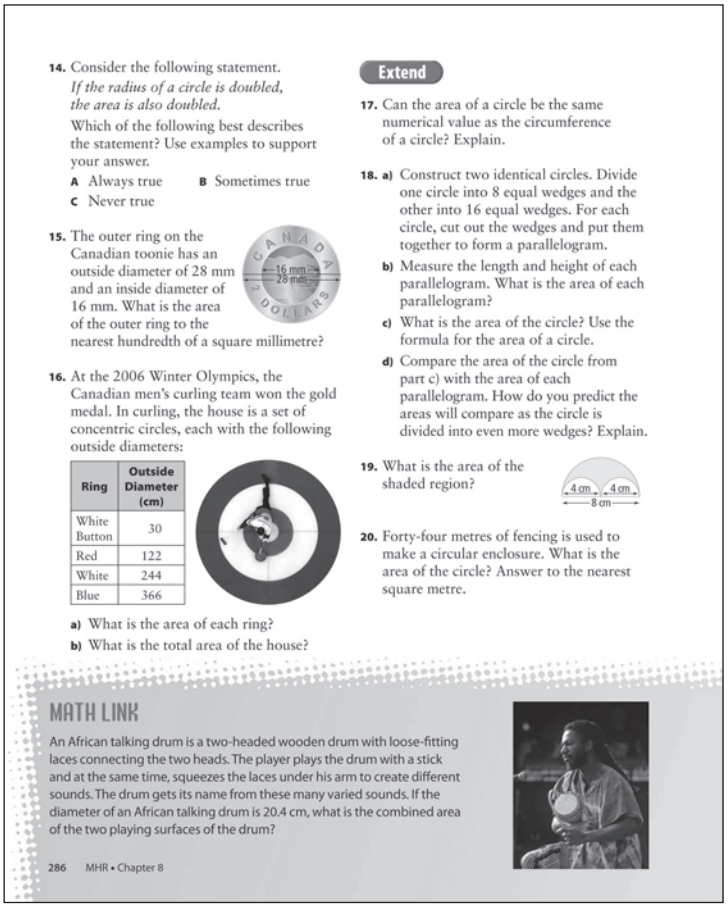
ESL and Language

- Team students with those who have a good understanding of terminology.

Answers

Math Link

$A = 653.4 \text{ cm}^2$



14. Consider the following statement.
If the radius of a circle is doubled, the area is also doubled.
Which of the following best describes the statement? Use examples to support your answer.
A Always true B Sometimes true
C Never true

15. The outer ring on the Canadian toonie has an outside diameter of 28 mm and an inside diameter of 16 mm. What is the area of the outer ring to the nearest hundredth of a square millimetre?

16. At the 2006 Winter Olympics, the Canadian men's curling team won the gold medal. In curling, the house is a set of concentric circles, each with the following outside diameters:

Ring	Outside Diameter (cm)
White Button	30
Red	122
White	244
Blue	366

a) What is the area of each ring?
b) What is the total area of the house?

Extend

17. Can the area of a circle be the same numerical value as the circumference of a circle? Explain.

18. a) Construct two identical circles. Divide one circle into 8 equal wedges and the other into 16 equal wedges. For each circle, cut out the wedges and put them together to form a parallelogram.
b) Measure the length and height of each parallelogram. What is the area of each parallelogram?
c) What is the area of the circle? Use the formula for the area of a circle.
d) Compare the area of the circle from part c) with the area of each parallelogram. How do you predict the areas will compare as the circle is divided into even more wedges? Explain.

19. What is the area of the shaded region?

20. Forty-four metres of fencing is used to make a circular enclosure. What is the area of the circle? Answer to the nearest square metre.

MATH LINK

An African talking drum is a two-headed wooden drum with loose-fitting laces connecting the two heads. The player plays the drum with a stick and at the same time, squeezes the laces under his arm to create different sounds. The drum gets its name from these many varied sounds. If the diameter of an African talking drum is 20.4 cm, what is the combined area of the two playing surfaces of the drum?

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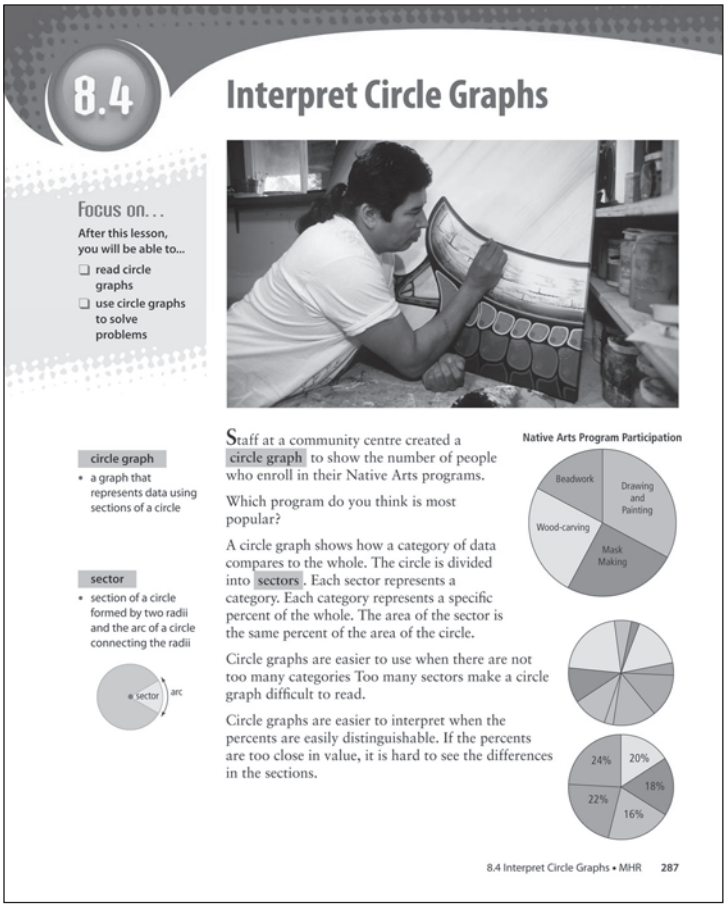
Math Link

This Math Link introduces students to another cultural application of circles. You may need to reinforce that the African wooden drum has two drum heads. To make this concrete, you might show a tin can and discuss how both ends of the can could be used to drum on.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • What is the easiest thing about calculating area? • What is the hardest thing about calculating area? • What strategies help you know how to calculate area?	• Use students' responses to help them analyse where they may be having problems. Work with them to develop strategies for solving these problems. • Depending on students' learning style, have them provide verbal or written answers. • You may wish to have students review the part related to Section 8.3 in **BLM 8–1 Chapter 8 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 286 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 301.	• You may wish to have students do this Math Link in order to apply their understanding of calculating the area of a circle from a given diameter. As they work on the Math Link, observe and have them self-observe how well they solve the problem. • Students who are having difficulty getting started could use **BLM 8–11 Section 8.3 Math Link**, which provides scaffolding for this activity.

8.4 Interpret Circle Graphs



8.4 Interpret Circle Graphs

Focus on...

After this lesson, you will be able to...

- read circle graphs
- use circle graphs to solve problems

circle graph
- a graph that represents data using sections of a circle

sector
- section of a circle formed by two radii and the arc of a circle connecting the radii

Staff at a community centre created a circle graph to show the number of people who enroll in their Native Arts programs.

Which program do you think is most popular?

A circle graph shows how a category of data compares to the whole. The circle is divided into sectors. Each sector represents a category. Each category represents a specific percent of the whole. The area of the sector is the same percent of the area of the circle.

Circle graphs are easier to use when there are not too many categories Too many sectors make a circle graph difficult to read.

Circle graphs are easier to interpret when the percents are easily distinguishable. If the percents are too close in value, it is hard to see the differences in the sections.

Native Arts Program Participation

8.4 Interpret Circle Graphs • MHR 287

Suggested Timing
40–50 minutes

Materials
- samples of circle graphs from newspapers, magazines, or web sites showing real-world applications

Blackline Masters
BLM 8–1 Chapter 8 Self-Assessment

BLM 8–12 Section 8.4 Extra Practice

Mathematical Processes

- [x] Communication
- [ ] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [ ] Visualization

Specific Outcomes

SP3 Construct, label and interpret circle graphs to solve problems.

Warm-Up

1. Draw a circle with a diameter of 10 cm.
2. Estimate and calculate the circumference of the circle.
3. Estimate and calculate the area of the circle.
4. Add, and write your answer in lowest terms.
$\frac{9}{10} + \frac{3}{5}$
5. Subtract, and write your answer in lowest terms.
$\frac{7}{8} - \frac{3}{4}$

Mental Math

6. Show $\frac{12}{24}$ in lowest terms.
7. Estimate $1\frac{9}{10} + 2\frac{4}{5}$. Show your thinking.
8. Estimate $5\frac{2}{3} - 1\frac{1}{3}$. Show your thinking.
9. List the numbers from 40 to 50 that are divisible by 3.
10. List the numbers from 35 to 100 that are divisible by 5.

Answers

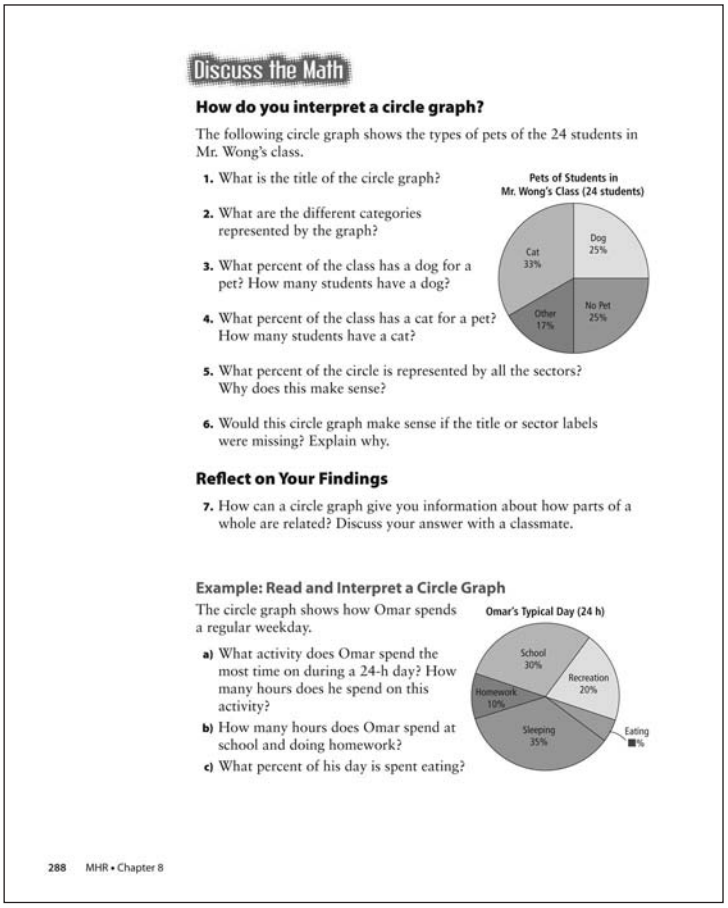
Warm-Up

1. Measure student drawings for a 10-cm diameter circle.
2. Estimate: $10 \text{ cm} \times 3 = 30 \text{ cm}$.
   Calculate: $10 \text{ cm} \times 3.14 = 31.4 \text{ cm}$
3. Estimate: $52 \times 3 = 75 \text{ cm}^2$.
   Calculate: $52 \times 3.14 = 78.5 \text{ cm}^2$
4. $\frac{9}{10} + \frac{6}{10} = 1\frac{1}{2}$
5. $\frac{7}{8} - \frac{6}{8} = \frac{1}{8}$
6. $\frac{1}{2}$
7. $1\frac{9}{10}$ is almost 2. $2\frac{4}{5}$ is almost 3. $2 + 3 = 5$
8. $5\frac{2}{3}$ is almost 6. $1\frac{1}{3}$ is close to 1. $6 - 1 = 5$
9. 42, 45, 48
10. 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100

Discuss the Math

1. Pets of Students in Mr. Wong's Class
2. dog, cat, other, no pet
3. 25%; 6 students
4. 33%; 8 students
5. 100%. Answers will vary. For example: The circle graph includes all of the students in Mr. Wong's class.
6. No. Answers will vary. For example: The sectors and title give the graph meaning.
7. Answers will vary. For example: Sector sizes can be compared to show how parts of the whole are related.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students respond with a partner. Check their responses for understanding of what information a circle graph provides.	• Students may have difficulty understanding the question. Consider using the following prompts to help clarify the meaning: – How does a circle graph show how one part compares to the whole? – How does one part compare to any other part? – What do the percents in a circle graph have to add up to? • Ask students who are having difficulty with this question to use the class responses as springboards to prepare similar responses of their own.



Discuss the Math

How do you interpret a circle graph?

The following circle graph shows the types of pets of the 24 students in Mr. Wong's class.

1. What is the title of the circle graph?
2. What are the different categories represented by the graph?
3. What percent of the class has a dog for a pet? How many students have a dog?
4. What percent of the class has a cat for a pet? How many students have a cat?
5. What percent of the circle is represented by all the sectors? Why does this make sense?
6. Would this circle graph make sense if the title or sector labels were missing? Explain why.

Pets of Students in Mr. Wong's Class (24 students)

Dog 25%
Cat 33%
Other 17%
No Pet 25%

Reflect on Your Findings

7. How can a circle graph give you information about how parts of a whole are related? Discuss your answer with a classmate.

Example: Read and Interpret a Circle Graph

The circle graph shows how Omar spends a regular weekday.

a) What activity does Omar spend the most time on during a 24-h day? How many hours does he spend on this activity?
b) How many hours does Omar spend at school and doing homework?
c) What percent of his day is spent eating?

Omar's Typical Day (24 h)

School 30%
Recreation 20%
Homework 10%
Sleeping 35%
Eating ■%

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Activity Planning Notes

Direct students to study the circle graph and pose some discussion prompts: How many different Native Arts programs are represented? Name them. Which program seems most popular? least popular?

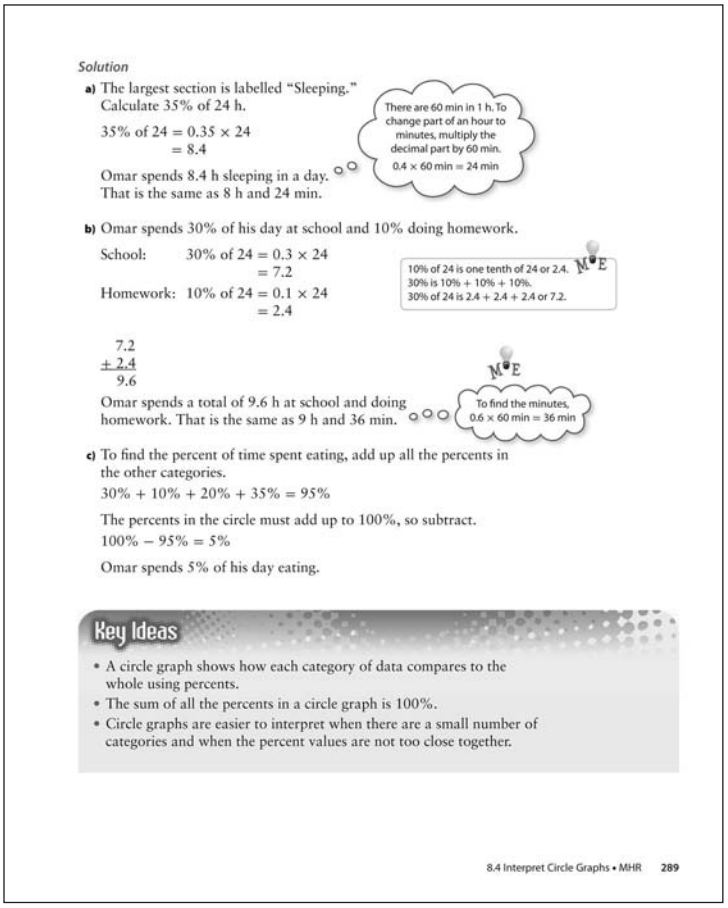
Have students note any difficulties in reading the two other circle graphs on the page.

Discuss the Math

In this investigation, it is important for students to learn how to read and interpret a circle graph.

Method 1: Have students work through the investigation individually or with a partner, and then have a class discussion.

Method 2: Guide the class through the investigation.



Solution

a) The largest section is labelled "Sleeping."
Calculate 35% of 24 h.

$35\% \text{ of } 24 = 0.35 \times 24$
$= 8.4$

Omar spends 8.4 h sleeping in a day.
That is the same as 8 h and 24 min.

There are 60 min in 1 h. To change part of an hour to minutes, multiply the decimal part by 60 min.
$0.4 \times 60 \text{ min} = 24 \text{ min}$

b) Omar spends 30% of his day at school and 10% doing homework.

School: $30\% \text{ of } 24 = 0.3 \times 24$
$= 7.2$

Homework: $10\% \text{ of } 24 = 0.1 \times 24$
$= 2.4$

10% of 24 is one tenth of 24 or 2.4.
30% is 10% + 10% + 10%.
30% of 24 is 2.4 + 2.4 + 2.4 or 7.2.

$7.2 + 2.4 = 9.6$

Omar spends a total of 9.6 h at school and doing homework. That is the same as 9 h and 36 min.

To find the minutes,
$0.6 \times 60 \text{ min} = 36 \text{ min}$

c) To find the percent of time spent eating, add up all the percents in the other categories.

$30\% + 10\% + 20\% + 35\% = 95\%$

The percents in the circle must add up to 100%, so subtract.

$100\% - 95\% = 5\%$

Omar spends 5% of his day eating.

Key Ideas

- A circle graph shows how each category of data compares to the whole using percents.
- The sum of all the percents in a circle graph is 100%.
- Circle graphs are easier to interpret when there are a small number of categories and when the percent values are not too close together.

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Supported Learning

Learning Style, ESL, Language, and Memory

- Some students may need to review some of the terminology, such as *sector*, *arc*, *category*, and *percent*, as well as concepts such as calculating percent. Consider having students make up their own definitions for each word. Record the ones that are most clear on a class chart. You might have students use terminology to label the parts of a circle graph.
- Work through the Discuss the Math as a class and then have students try a second example with a partner.

ESL

- English language learners may have difficulty with terms such as *most popular*, *specific*, *categories*, *distinguishable*, *media*, *schedule*, *dealership*, *legend*, and *key*.
- The graph in the Example uses a category called Recreation. Have students offer suggestions of recreational activities. Note similarities and differences in different cultures.

The Example guides students through reading and interpreting a circle graph. As a group, discuss the following ideas:

- If sectors look approximately the same size on a circle graph, read the labels to determine which sector is larger than the other.
- To convert from a percent to a decimal, divide by 100. You can use the decimal to find out the percent of something. For example, 35% of 24 can be calculated by multiplying 0.35 by 24.
- To change a decimal of an hour to minutes, multiply by 60. (There are 60 min in an hour.) $0.6 \text{ h} = 0.6 \times 60 = 36 \text{ min}$
- The percents in a circle graph add to 100. That's because a circle graph shows parts of a whole.

Key Ideas

Reinforce that circle graphs are more suitable for representing data when there are limited categories and the percent values are spread apart. Have students prepare their own summary of circle graphs.

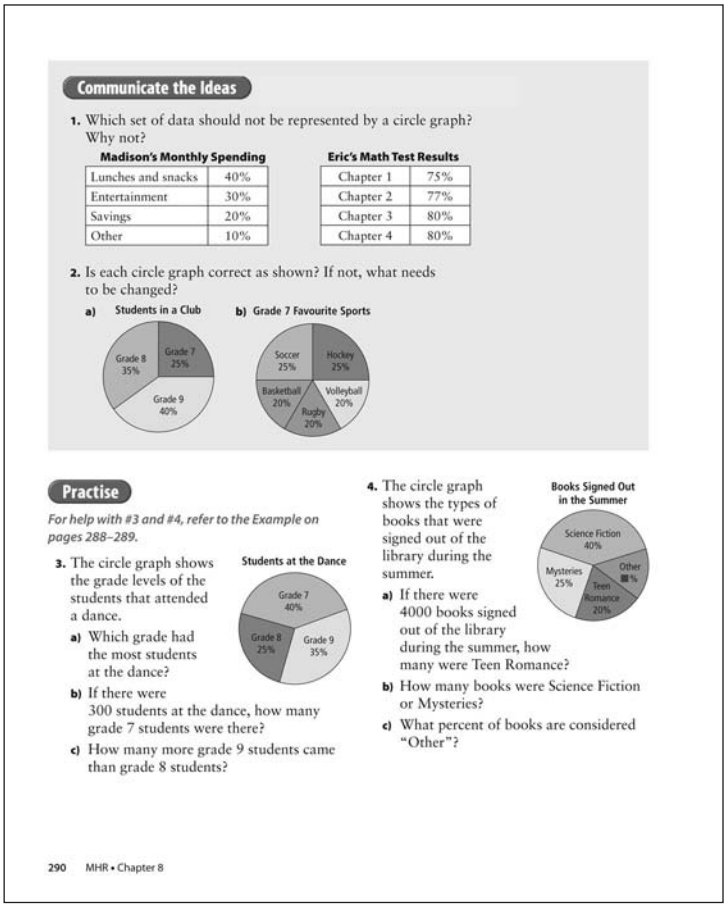
Answers

Communicate the Ideas

1. Eric's Math Test Results. The sum of all percents in the circle graph does not equal 100%.

2. a) Correct

b) Incorrect. The sum of all percents equals 110%. The sum of the percents has to be changed to equal 100%.



Communicate the Ideas

1. Which set of data should not be represented by a circle graph? Why not?

Madison's Monthly Spending	
Lunches and snacks	40%
Entertainment	30%
Savings	20%
Other	10%

Eric's Math Test Results	
Chapter 1	75%
Chapter 2	77%
Chapter 3	80%
Chapter 4	80%

2. Is each circle graph correct as shown? If not, what needs to be changed?

a) Students in a Club

b) Grade 7 Favourite Sports

Practise

For help with #3 and #4, refer to the Example on pages 288–289.

3. The circle graph shows the grade levels of the students that attended a dance.

Students at the Dance

a) Which grade had the most students at the dance?

b) If there were 300 students at the dance, how many grade 7 students were there?

c) How many more grade 9 students came than grade 8 students?

4. The circle graph shows the types of books that were signed out of the library during the summer.

Books Signed Out in the Summer

a) If there were 4000 books signed out of the library during the summer, how many were Teen Romance?

b) How many books were Science Fiction or Mysteries?

c) What percent of books are considered "Other"?

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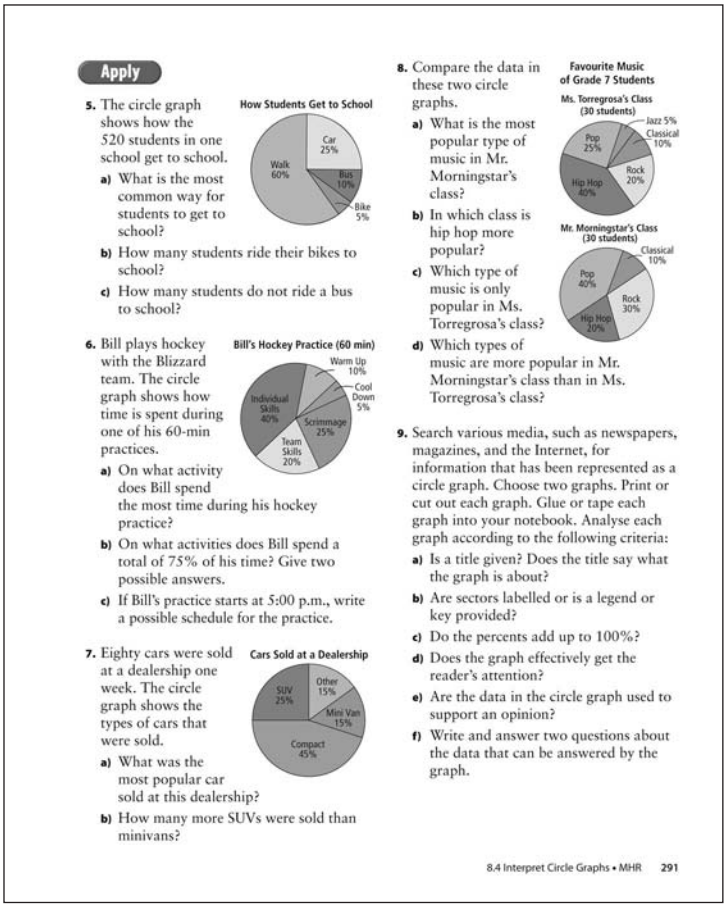
Communicate the Ideas

These questions allow students to apply their understanding of how circle graphs may be appropriately used to represent sets of data. In #1, students decide if sets of data should be represented by a circle graph. In #2, students review and correct two circle graphs.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Encourage students to communicate their solutions to a partner and listen to each other's explanations.	• Check each student's answers to #1 and #2. These are key questions; make sure that they have the concepts. • Tell students that Eric's data in #1 could technically be represented by a circle graph but that it is not the best format.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5
Typical	1–8
Extension/Enrichment	1, 2, 9



Apply

5. The circle graph shows how the 520 students in one school get to school.

How Students Get to School

a) What is the most common way for students to get to school?

b) How many students ride their bikes to school?

c) How many students do not ride a bus to school?

6. Bill plays hockey with the Blizzard team. The circle graph shows how time is spent during one of his 60-min practices.

Bill's Hockey Practice (60 min)

a) On what activity does Bill spend the most time during his hockey practice?

b) On what activities does Bill spend a total of 75% of his time? Give two possible answers.

c) If Bill's practice starts at 5:00 p.m., write a possible schedule for the practice.

7. Eighty cars were sold at a dealership one week. The circle graph shows the types of cars that were sold.

Cars Sold at a Dealership

a) What was the most popular car sold at this dealership?

b) How many more SUVs were sold than minivans?

8. Compare the data in these two circle graphs.

Favourite Music of Grade 7 Students

Ms. Torregrosa's Class (30 students)

Mr. Morningstar's Class (30 students)

a) What is the most popular type of music in Mr. Morningstar's class?

b) In which class is hip hop more popular?

c) Which type of music is only popular in Ms. Torregrosa's class?

d) Which types of music are more popular in Mr. Morningstar's class than in Ms. Torregrosa's class?

9. Search various media, such as newspapers, magazines, and the Internet, for information that has been represented as a circle graph. Choose two graphs. Print or cut out each graph. Glue or tape each graph into your notebook. Analyse each graph according to the following criteria:

a) Is a title given? Does the title say what the graph is about?

b) Are sectors labelled or is a legend or key provided?

c) Do the percents add up to 100%?

d) Does the graph effectively get the reader's attention?

e) Are the data in the circle graph used to support an opinion?

f) Write and answer two questions about the data that can be answered by the graph.

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Practise and Apply

Assessment *for* Learning	Supported Learning
Practise and Apply Have students do #3. Students who have no problems with this question can go on to the Apply questions.	• Students who have problems with #3 will need additional coaching. Help students correct their answers to #3, then coach them through #4, and assign #5. • Have students refer back to the Example in the student resource. Check back with them several times to make sure that they understand the concepts as they work on #5.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • What is the advantage of knowing how to read and interpret circle graphs?	• Bring in examples of circle graphs used in various media and discuss with students the advantages of knowing how to read such graphs. For example, you might show a circle graph that shows the percentage of students who prefer cheese pizza, pepperoni pizza, or vegetarian pizza. Tell students that they might use this information if they were planning to sell pizzas as a fundraiser, by ordering the different kinds of pizza according to their popularity. • Depending on students' learning style, have them provide verbal or written answers. • You may wish to have students review the part related to Section 8.4 in **BLM 8–1 Chapter 8 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Supported Learning

Learning Style and Memory

- Provide **BLM 8–12 Section 8.4 Extra Practice** to students who require more practice.

ESL and Language

- Since the text in the Apply questions is quite dense, consider giving English language learners fewer questions.
- In #5c), prompt English language learners to understand that students not riding a bus refers to students using all of the other modes of transportation.
- Consider showing students some legends or keys before they begin #9.

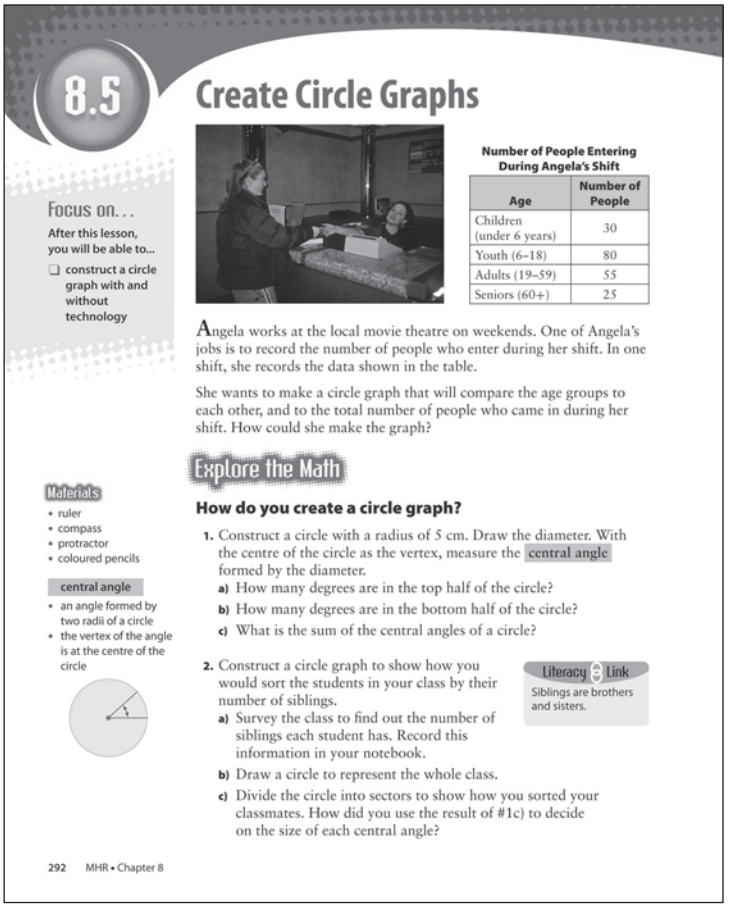
8.5 Create Circle Graphs

Suggested Timing
80–100 minutes

Materials
- calculator
- compass
- ruler
- protractor
- coloured pencils
- computer with spreadsheet software

Blackline Masters
Master 12 Percent Circles
BLM 8–1 Chapter 8 Self-Assessment
BLM 8–13 Draw a Circle Graph Using a Percent Circle
BLM 8–14 Creating Circle Graphs in Microsoft® Excel
BLM 8–15 Creating Circle Graphs in Corel® Quattro® Pro
BLM 8–16 Creating Circle Graphs in Appleworks® 6.2
BLM 8–17 Section 8.5 Extra Practice

Mathematical Processes
- ☑ Communication
- ☑ Connections
- ☑ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☑ Technology
- ☑ Visualization



8.5 Create Circle Graphs

Focus on...
After this lesson, you will be able to...
- ❑ construct a circle graph with and without technology

Number of People Entering During Angela's Shift

Age	Number of People
Children (under 6 years)	30
Youth (6–18)	80
Adults (19–59)	55
Seniors (60+)	25

Angela works at the local movie theatre on weekends. One of Angela's jobs is to record the number of people who enter during her shift. In one shift, she records the data shown in the table.

She wants to make a circle graph that will compare the age groups to each other, and to the total number of people who came in during her shift. How could she make the graph?

Explore the Math

Materials
- ruler
- compass
- protractor
- coloured pencils

central angle
- an angle formed by two radii of a circle
- the vertex of the angle is at the centre of the circle

How do you create a circle graph?

1. Construct a circle with a radius of 5 cm. Draw the diameter. With the centre of the circle as the vertex, measure the central angle formed by the diameter.
   a) How many degrees are in the top half of the circle?
   b) How many degrees are in the bottom half of the circle?
   c) What is the sum of the central angles of a circle?
2. Construct a circle graph to show how you would sort the students in your class by their number of siblings.
   a) Survey the class to find out the number of siblings each student has. Record this information in your notebook.
   b) Draw a circle to represent the whole class.
   c) Divide the circle into sectors to show how you sorted your classmates. How did you use the result of #1c) to decide on the size of each central angle?

Literacy Link
Siblings are brothers and sisters.

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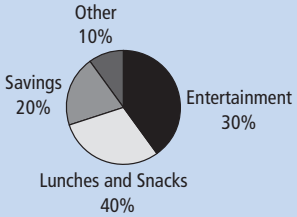
Specific Outcomes

SS1 Demonstrate an understanding of circles by:
- determining the sum of the central angles.

SP3 Construct, label and interpret circle graphs to solve problems.

Warm-Up

1. The graph shows Corey's weekly spending. What does he spend the most on?

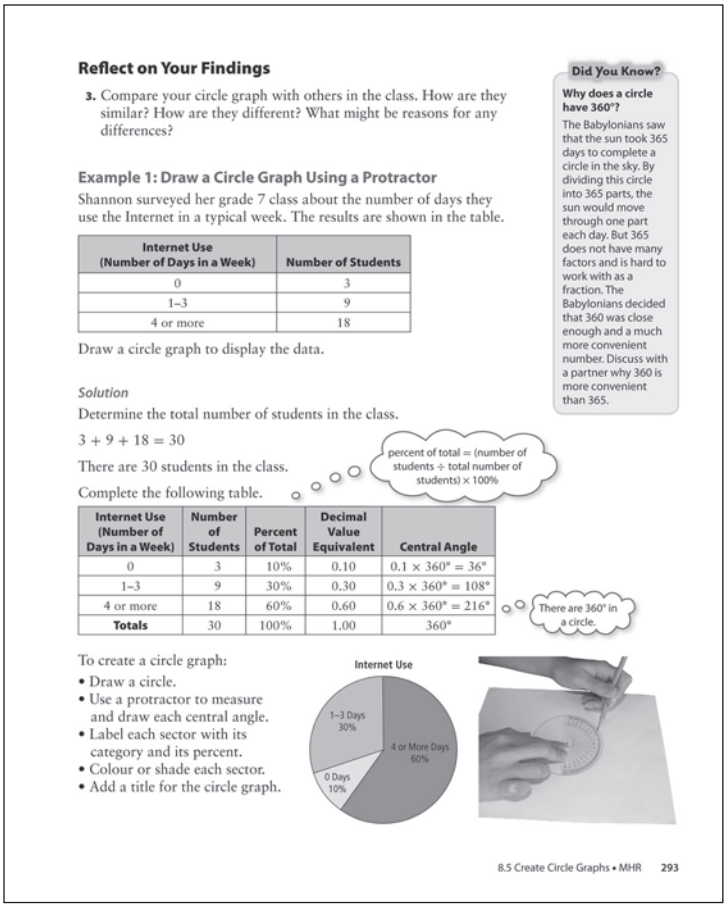


2. Corey gets $20 per week. How much does he save?
3. Sketch a circle graph showing Corey's spending. Tell how you estimate the size of each sector.
4. A meat pie has a diameter of 22.5 cm. What is the area of the top pie crust? Show your answer to the nearest tenth of a cm^2.
5. A riding ring has a radius of 20 m. What is the circumference of the riding ring?

Mental Math

Show your thinking for each of the following.

6. Estimate $\frac{4}{5} + \frac{4}{5}$.
7. Estimate $\frac{5}{6} - \frac{3}{4}$.
8. The team star usually makes a basket on 4 out of 5 tries. She tries for 15 baskets. About how many is she likely to make?
9. Another player usually makes 5 out of 8 baskets. She tries for 24 baskets. About how many is she likely to make?



Reflect on Your Findings

3. Compare your circle graph with others in the class. How are they similar? How are they different? What might be reasons for any differences?

Example 1: Draw a Circle Graph Using a Protractor

Shannon surveyed her grade 7 class about the number of days they use the Internet in a typical week. The results are shown in the table.

Internet Use (Number of Days in a Week)	Number of Students
0	3
1–3	9
4 or more	18

Draw a circle graph to display the data.

Solution

Determine the total number of students in the class.

$3 + 9 + 18 = 30$

There are 30 students in the class.

Complete the following table.

percent of total = (number of students ÷ total number of students) × 100%

Internet Use (Number of Days in a Week)	Number of Students	Percent of Total	Decimal Value Equivalent	Central Angle
0	3	10%	0.10	$0.1 \times 360° = 36°$
1–3	9	30%	0.30	$0.3 \times 360° = 108°$
4 or more	18	60%	0.60	$0.6 \times 360° = 216°$
Totals	30	100%	1.00	360°

There are 360° in a circle.

To create a circle graph:
- Draw a circle.
- Use a protractor to measure and draw each central angle.
- Label each sector with its category and its percent.
- Colour or shade each sector.
- Add a title for the circle graph.

Did You Know?

Why does a circle have 360°?

The Babylonians saw that the sun took 365 days to complete a circle in the sky. By dividing this circle into 365 parts, the sun would move through one part each day. But 365 does not have many factors and is hard to work with as a fraction. The Babylonians decided that 360 was close enough and a much more convenient number. Discuss with a partner why 360 is more convenient than 365.

Activity Planning Notes

Have students study the chart and ask them to explain why they would use a circle graph to represent the data.

Explore the Math

Students create a circle graph, first using a protractor and then using technology. Students could work individually, in pairs, or as a class.

Method 1: Have students do #1 and #2.

Method 2: Have students complete #1. As an alternative to #2, create a human circle graph by having students line up around the circumference of a circle according to number of siblings (e.g., 0, 1, 2, 3, 4, or more). For instance, all students with no siblings will stand together and hold a specific colour of paper to reinforce that they make up one part of the graph. Stand in the centre of the circle and pass a string around each category to complete the circle graph.

Answers

Warm-Up

1. lunches and snacks
2. savings = 20%; 10% of \$20 = \$2; 20% = \$4
   $\$20 \times 0.2 = \$4$
3. Look for an explanation that $30\% + 20\% = \frac{1}{2}$ and that $40\% + 10\% = \frac{1}{2}$. Students might make 30% a little larger than one quarter of a circle, and 20% a little smaller. They might mentally divide the other half of the circle into five equal pieces and put four together to make 40%. The rest would be 10%. Calculations would suggest the following measurements for the sector angles: 40% = 144°; 30% = 108°; 20% = 72°; 10% = 36°. Check that student sketches are within this range.
4. Area $= 11.25^2 \times 3.14 = 397.4 \text{ cm}^2$
5. Circumference $= 40 \text{ m} \times 3.14 = 125.6 \text{ m}$
6. $\frac{5}{4}$ is almost 1. The answer is close to 2.
7. Both of these numbers are almost 1. The answer is close to 0.
8. $\frac{4}{5} = \frac{12}{15}$
9. $\frac{5}{8} = \frac{15}{24}$

Explore the Math

1. **a)** 180° **b)** 180° **c)** 360°
2. **a)** Answers will vary.
   b) Answers will vary.
   c) Answers will vary.
3. Answers will vary.

Supported Learning

Meeting the Needs of All Learners

- Some students may need additional reinforcement to process the information and the instructions. You might work through the Explore the Math as a whole class and then have students try a second example with a partner.
- You might have students practise measuring angles using a protractor before beginning this section.
- Work with students to create a checklist of the steps required to create a circle graph. Allow them to use the checklist to answer questions.
- Consider having students work with a partner to use computer spreadsheets.
- Remind students to refer to their Foldable when they are confused about key terms. Discuss the Literacy Link that describes *siblings*.

Supported Learning

Learning Style and Motor

- After students have tried both methods for constructing circle graphs, allow them to choose either method to answer the questions.

ESL

- English language learners may have difficulty with terms such as *shift* (job), *construct*, *convenient*, *errors*, *lunch specials*, *articles*, and *advertising*.

Motor

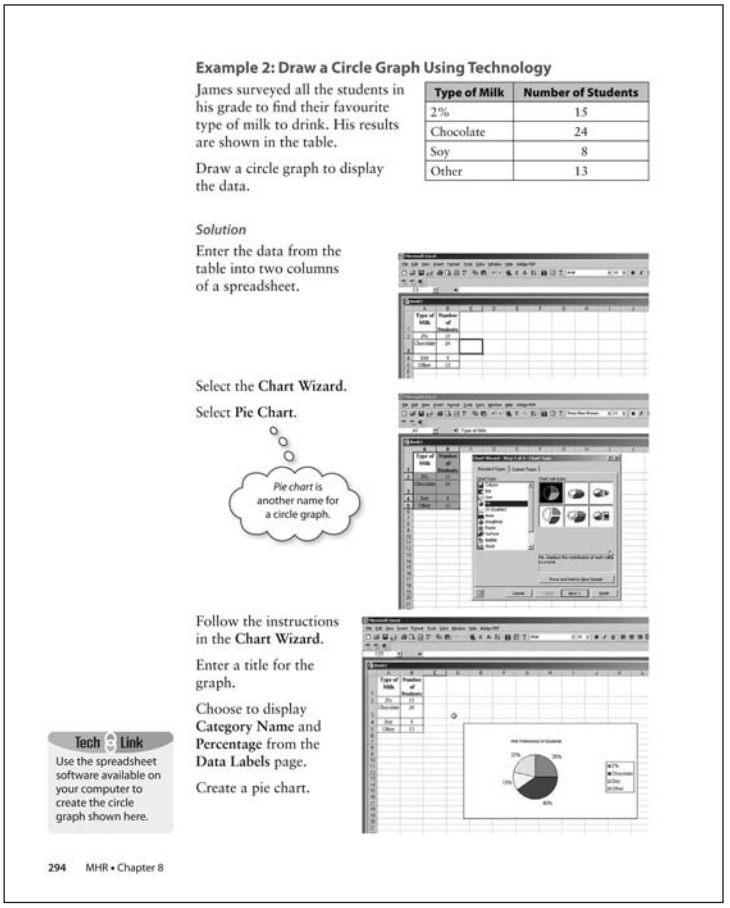
- Consider allowing students to use a virtual protractor.

Meeting the Needs of All Learners

- Encourage students to collect local data for their circle graphs. For example, they might wish to collect data about the populations of different First Nations or Inuit communities in Western and Northern Canada. Alternatively, they might collect data about the local immigrant populations or the proportion of various cultural groups in their community. Have students display the data on a circle graph and interpret the graph for the class.

Gifted and Enrichment

- Refer students to the Did You Know? on page 293 and have them find the factors of 360 and 365 to show how 360 is a more convenient number. The factors of 360 are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, and 360. The factors of 365 are 1, 5, 73, and 365.



Example 2: Draw a Circle Graph Using Technology

James surveyed all the students in his grade to find their favourite type of milk to drink. His results are shown in the table.

Type of Milk	Number of Students
2%	15
Chocolate	24
Soy	8
Other	13

Draw a circle graph to display the data.

Solution

Enter the data from the table into two columns of a spreadsheet.

Select the **Chart Wizard**.

Select **Pie Chart**.

Pie chart is another name for a circle graph.

Follow the instructions in the **Chart Wizard**.

Enter a title for the graph.

Choose to display **Category Name** and **Percentage** from the **Data Labels** page.

Create a pie chart.

Tech Link
Use the spreadsheet software available on your computer to create the circle graph shown here.

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Common Errors

- Students graph the number of days instead of the number of students as sectors on the circle graph in Example 1.

R$_x$ Emphasize that the first column is the category and the second column is how many fit in that category.

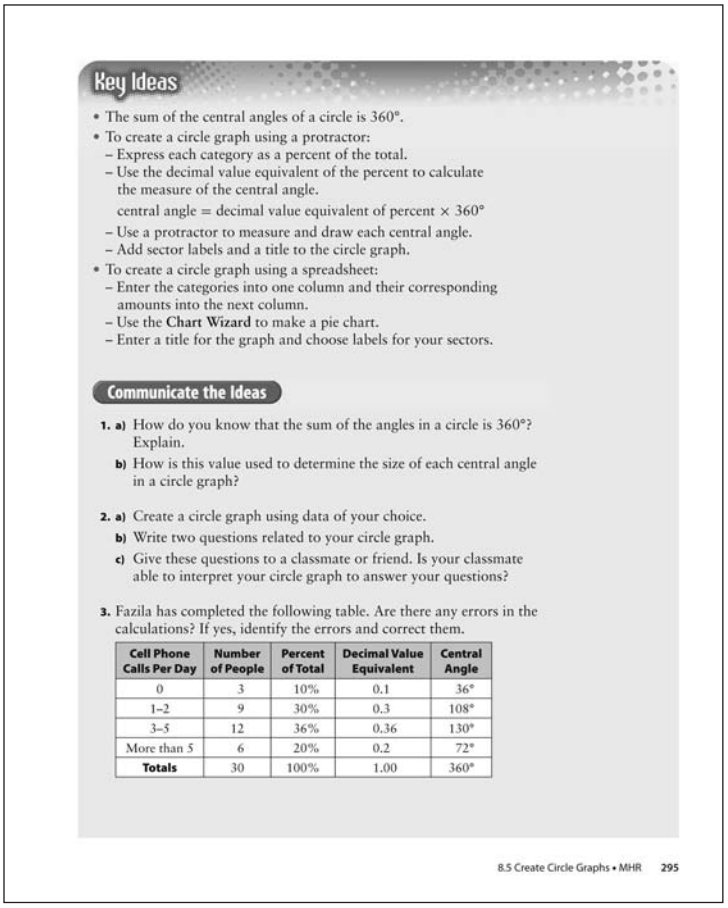
Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students compare their circle graphs. In a class discussion, have students generalize conclusions about the similarities and differences, and the reasons for the differences.	• Reinforce that the sum of the angles in a circle is 360°. • Ask students who are having difficulty with this question to use the class responses as springboards to prepare similar responses of their own.

Example 1 explains how to draw a circle graph using a protractor.

Method 1: Have students draw the circle graph using a protractor.

Method 2: Consider having students draw the circle graph using a percent circle. Provide them with **BLM 8–13 Draw a Circle Graph Using a Percent Circle**. Explain that a percent circle is divided into 100 equal sectors. Each sector represents 1%. Use an overhead to demonstrate shading the sector for each category. Shade a 10% sector to represent the first category. Show students how to mark off the remaining sectors before having them start their own circle graph.

Assessment *for* Learning	Supported Learning
Example 1 Have students draw the circle graph related to Example 1 on page 293.	• Have students talk through their thinking in a group. • Coach students who need help in handling a protractor. Students with motor difficulties may need to use **Master 12 Percent Circles** or technology to produce circle graphs.



Key Ideas

- The sum of the central angles of a circle is 360°.
- To create a circle graph using a protractor:
  - Express each category as a percent of the total.
  - Use the decimal value equivalent of the percent to calculate the measure of the central angle.
    central angle = decimal value equivalent of percent × 360°
  - Use a protractor to measure and draw each central angle.
  - Add sector labels and a title to the circle graph.
- To create a circle graph using a spreadsheet:
  - Enter the categories into one column and their corresponding amounts into the next column.
  - Use the **Chart Wizard** to make a pie chart.
  - Enter a title for the graph and choose labels for your sectors.

Communicate the Ideas

1. a) How do you know that the sum of the angles in a circle is 360°? Explain.

b) How is this value used to determine the size of each central angle in a circle graph?

2. a) Create a circle graph using data of your choice.

b) Write two questions related to your circle graph.

c) Give these questions to a classmate or friend. Is your classmate able to interpret your circle graph to answer your questions?

3. Fazila has completed the following table. Are there any errors in the calculations? If yes, identify the errors and correct them.

Cell Phone Calls Per Day	Number of People	Percent of Total	Decimal Value Equivalent	Central Angle
0	3	10%	0.1	36°
1–2	9	30%	0.3	108°
3–5	12	36%	0.36	130°
More than 5	6	20%	0.2	72°
Totals	30	100%	1.00	360°

8.5 Create Circle Graphs • MHR 295

Answers

Communicate the Ideas

1. Answers may vary. For example: There are 180° in the top half of the circle and 180° in the bottom half of the circle. The sum of these two values is 360°.

2. Answers will vary.

3. For the 3–5 cell phone calls per day category,
- the percent of total should be 40%
- the decimal equivalent value should be 0.4
- the central angle should be 144°

Assessment *as* Learning	Supported Learning
Communicate the Ideas Students should answer each of the questions, since they are all key questions.	• Check each student's answers to #1 to #3. Make sure that they have the concepts. • In #2, have students share their graph and questions with a classmate, and then get feedback to make any necessary corrections. • Tell students that Fazila's error in #3 is a computational one. Brainstorm with students how to avoid making this kind of error.

Example 2 explains how to draw a circle graph using technology.

Assessment *for* Learning	Supported Learning
Example 2 Have students draw the circle graph using technology related to Example 2 on page 294.	• You may wish to provide **BLM 8–14 Creating Circle Graphs in Microsoft® Excel**, **BLM 8–15 Creating Circle Graphs in Corel® Quattro® Pro**, or **BLM 8–16 Creating Circle Graphs in Appleworks® 6.2** for students who are having difficulty.

Key Ideas

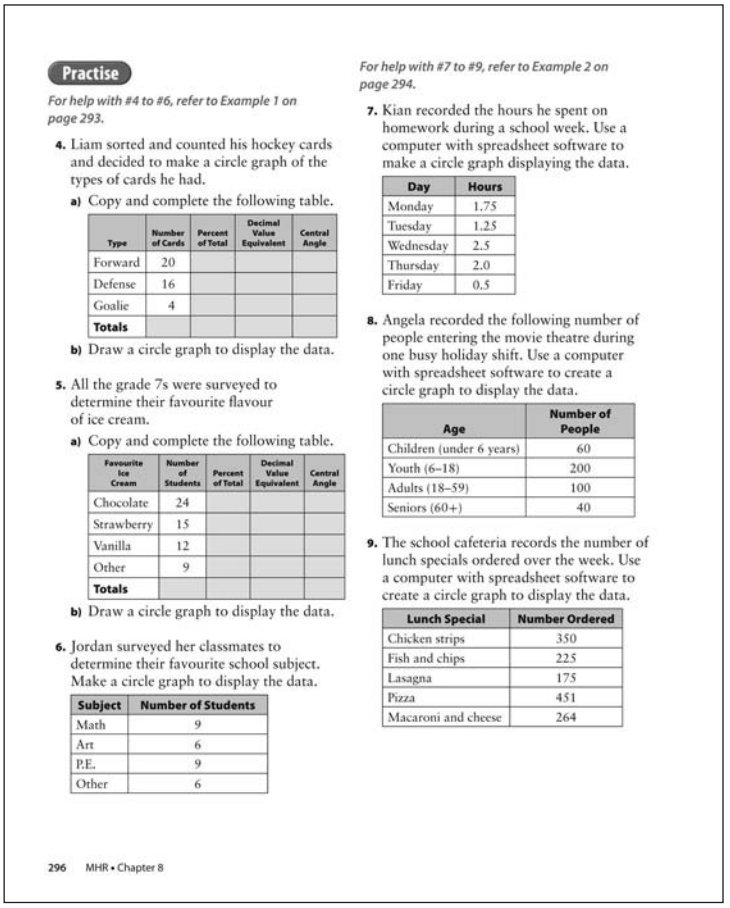
This section summarizes how to create a circle graph using a protractor and using technology.

Reinforce that the central angle is calculated by multiplying the decimal value equivalent of the percent by the number of degrees in a circle (i.e., 360°). Have students prepare their own summary of how to make a circle graph using a protractor.

Communicate the Ideas

These questions allow students to apply their understanding of the sum of the angles in a circle. In #1, students explain how they know that the sum of the angles in a circle is 360°. In #2, students create a circle graph and write two questions. In #3, students use their understanding of percent, decimals, and sector angles to identify and correct errors in a data table.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 7
Typical	1–10
Extension/Enrichment	1–3, 11–13



Practise

For help with #4 to #6, refer to Example 1 on page 293.

4. Liam sorted and counted his hockey cards and decided to make a circle graph of the types of cards he had.

a) Copy and complete the following table.

Type	Number of Cards	Percent of Total	Decimal Value Equivalent	Central Angle
Forward	20			
Defense	16			
Goalie	4			
Totals				

b) Draw a circle graph to display the data.

5. All the grade 7s were surveyed to determine their favourite flavour of ice cream.

a) Copy and complete the following table.

Favourite Ice Cream	Number of Students	Percent of Total	Decimal Value Equivalent	Central Angle
Chocolate	24			
Strawberry	15			
Vanilla	12			
Other	9			
Totals				

b) Draw a circle graph to display the data.

6. Jordan surveyed her classmates to determine their favourite school subject. Make a circle graph to display the data.

Subject	Number of Students
Math	9
Art	6
P.E.	9
Other	6

For help with #7 to #9, refer to Example 2 on page 294.

7. Kian recorded the hours he spent on homework during a school week. Use a computer with spreadsheet software to make a circle graph displaying the data.

Day	Hours
Monday	1.75
Tuesday	1.25
Wednesday	2.5
Thursday	2.0
Friday	0.5

8. Angela recorded the following number of people entering the movie theatre during one busy holiday shift. Use a computer with spreadsheet software to create a circle graph to display the data.

Age	Number of People
Children (under 6 years)	60
Youth (6–18)	200
Adults (18–59)	100
Seniors (60+)	40

9. The school cafeteria records the number of lunch specials ordered over the week. Use a computer with spreadsheet software to create a circle graph to display the data.

Lunch Special	Number Ordered
Chicken strips	350
Fish and chips	225
Lasagna	175
Pizza	451
Macaroni and cheese	264

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Common Errors

- Students may draw the wrong central angle in a circle graph.

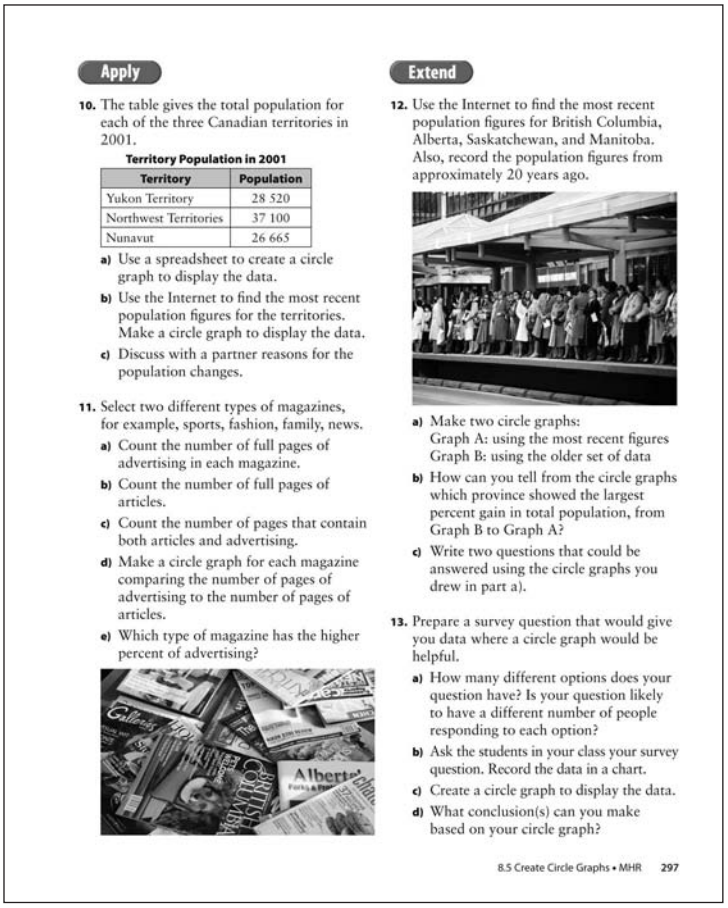
R_x Have students practise using a protractor to read angles (see the Warm-Up in Chapter 3). You may wish to have some students use **Master 12 Percent Circles**.

- When entering data in a spreadsheet, students may mix up what goes in the first column.

R_x Emphasize that the first column is the category and the second column is how many fit in that category.

Practise

Assessment *for* Learning	Supported Learning
Practise Have students do #4 and #7. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #4 and #7 will need additional coaching. Have students explain their thinking on these questions; clarify any misunderstandings. Coach students through #5 and #8, and then have them try #6 and #9 on their own. Have students refer back to the examples in the student resource. Check back with them several times to make sure that they understand the concepts.



Apply

10. The table gives the total population for each of the three Canadian territories in 2001.

Territory Population in 2001

Territory	Population
Yukon Territory	28 520
Northwest Territories	37 100
Nunavut	26 665

a) Use a spreadsheet to create a circle graph to display the data.

b) Use the Internet to find the most recent population figures for the territories. Make a circle graph to display the data.

c) Discuss with a partner reasons for the population changes.

11. Select two different types of magazines, for example, sports, fashion, family, news.

a) Count the number of full pages of advertising in each magazine.

b) Count the number of full pages of articles.

c) Count the number of pages that contain both articles and advertising.

d) Make a circle graph for each magazine comparing the number of pages of advertising to the number of pages of articles.

e) Which type of magazine has the higher percent of advertising?

Extend

12. Use the Internet to find the most recent population figures for British Columbia, Alberta, Saskatchewan, and Manitoba. Also, record the population figures from approximately 20 years ago.

a) Make two circle graphs:
Graph A: using the most recent figures
Graph B: using the older set of data

b) How can you tell from the circle graphs which province showed the largest percent gain in total population, from Graph B to Graph A?

c) Write two questions that could be answered using the circle graphs you drew in part a).

13. Prepare a survey question that would give you data where a circle graph would be helpful.

a) How many different options does your question have? Is your question likely to have a different number of people responding to each option?

b) Ask the students in your class your survey question. Record the data in a chart.

c) Create a circle graph to display the data.

d) What conclusion(s) can you make based on your circle graph?

8.5 Create Circle Graphs • MHR 297

Apply and Extend

In #11, you may wish to provide appropriate magazines for students to use. In #13, have students get your approval for any survey question before they survey the class.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • Why are percents important for creating circle graphs?	• Depending on students' learning style, have them provide verbal or written answers. • You may wish to have students review the part related to Section 8.5 in **BLM 8–1 Chapter 8 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Supported Learning

Learning Style and Memory

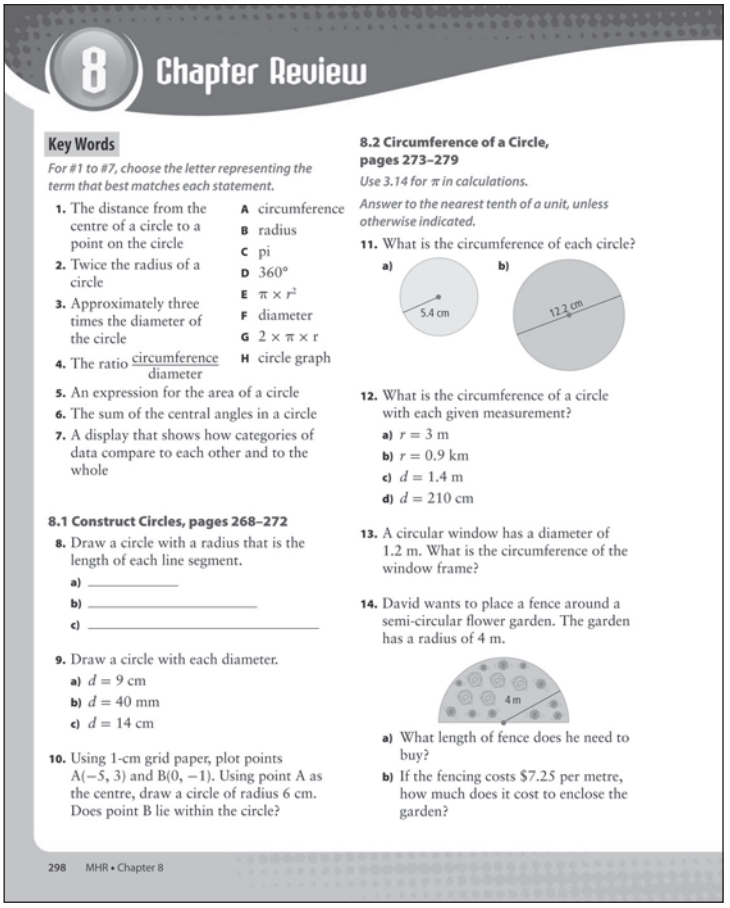
- Provide **BLM 8–17 Section 8.5 Extra Practice** to students who require more practice.

8 Chapter Review

Suggested Timing
40–50 minutes

Materials
- compass
- protractor
- ruler
- calculator
- computer with spreadsheet software

Blackline Masters
Master 8 Centimetre Grid Paper
Master 12 Percent Circles
BLM 8–1 Chapter 8 Self-Assessment
BLM 8–6 Section 8.1 Extra Practice
BLM 8–8 Section 8.2 Extra Practice
BLM 8–10 Section 8.3 Extra Practice
BLM 8–12 Section 8.4 Extra Practice
BLM 8–17 Section 8.5 Extra Practice



8 Chapter Review

Key Words

For #1 to #7, choose the letter representing the term that best matches each statement.

1. The distance from the centre of a circle to a point on the circle
2. Twice the radius of a circle
3. Approximately three times the diameter of the circle
4. The ratio $\frac{\text{circumference}}{\text{diameter}}$
5. An expression for the area of a circle
6. The sum of the central angles in a circle
7. A display that shows how categories of data compare to each other and to the whole

A circumference
B radius
C pi
D 360°
E $\pi \times r^2$
F diameter
G $2 \times \pi \times r$
H circle graph

8.1 Construct Circles, pages 268–272

8. Draw a circle with a radius that is the length of each line segment.
   a) _____
   b) _____
   c) _____
9. Draw a circle with each diameter.
   a) $d = 9$ cm
   b) $d = 40$ mm
   c) $d = 14$ cm
10. Using 1-cm grid paper, plot points A(−5, 3) and B(0, −1). Using point A as the centre, draw a circle of radius 6 cm. Does point B lie within the circle?

8.2 Circumference of a Circle, pages 273–279

Use 3.14 for π in calculations.

Answer to the nearest tenth of a unit, unless otherwise indicated.

11. What is the circumference of each circle?
    a) 5.4 cm
    b) 12.2 cm
12. What is the circumference of a circle with each given measurement?
    a) $r = 3$ m
    b) $r = 0.9$ km
    c) $d = 1.4$ m
    d) $d = 210$ cm
13. A circular window has a diameter of 1.2 m. What is the circumference of the window frame?
14. David wants to place a fence around a semi-circular flower garden. The garden has a radius of 4 m.
    a) What length of fence does he need to buy?
    b) If the fencing costs $7.25 per metre, how much does it cost to enclose the garden?

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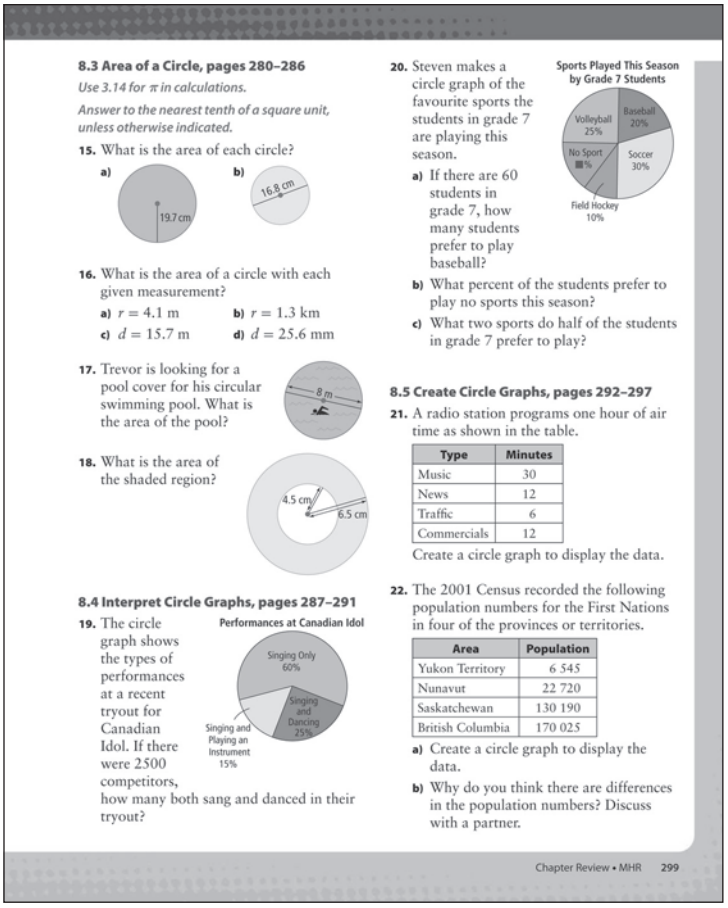
Activity Planning Notes

Have students work independently to complete the review questions. If students encounter difficulties, they could discuss strategies with other students. Encourage them to refer to the information in their chapter Foldable and then to the specific section in the student resource and/or their notebooks. Once they have found a suitable strategy, students should include it in the appropriate section of their Foldable.

Students will need **Master 8 Centimetre Grid Paper** in order to complete #10.

Assessment *for* Learning	Supported Learning
Chapter 8 Review The chapter review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On tab of their chapter Foldable. Have students do at least one question related to each item in that tab. • Have students revisit any section they are having difficulty with prior to working on the Chapter 8 Practice Test.



8.3 Area of a Circle, pages 280–286

Use 3.14 for π in calculations.

Answer to the nearest tenth of a square unit, unless otherwise indicated.

15. What is the area of each circle?

a) 19.7 cm

b) 16.8 cm

16. What is the area of a circle with each given measurement?

a) $r = 4.1$ m b) $r = 1.3$ km

c) $d = 15.7$ m d) $d = 25.6$ mm

17. Trevor is looking for a pool cover for his circular swimming pool. What is the area of the pool?

8 m

18. What is the area of the shaded region?

4.5 cm 6.5 cm

8.4 Interpret Circle Graphs, pages 287–291

19. The circle graph shows the types of performances at a recent tryout for Canadian Idol. If there were 2500 competitors, how many both sang and danced in their tryout?

Performances at Canadian Idol

Singing Only 60%

Singing and Dancing 25%

Singing and Playing an Instrument 15%

20. Steven makes a circle graph of the favourite sports the students in grade 7 are playing this season.

a) If there are 60 students in grade 7, how many students prefer to play baseball?

b) What percent of the students prefer to play no sports this season?

c) What two sports do half of the students in grade 7 prefer to play?

Sports Played This Season by Grade 7 Students

Baseball 20%

Soccer 30%

Field Hockey 10%

No Sport ■%

Volleyball 25%

8.5 Create Circle Graphs, pages 292–297

21. A radio station programs one hour of air time as shown in the table.

Type	Minutes
Music	30
News	12
Traffic	6
Commercials	12

Create a circle graph to display the data.

22. The 2001 Census recorded the following population numbers for the First Nations in four of the provinces or territories.

Area	Population
Yukon Territory	6 545
Nunavut	22 720
Saskatchewan	130 190
British Columbia	170 025

a) Create a circle graph to display the data.

b) Why do you think there are differences in the population numbers? Discuss with a partner.

Chapter Review • MHR 299

Supported Learning

Learning Style and Memory

- Students who require more practice on a particular topic may refer to **BLM 8–6 Section 8.1 Extra Practice**, **BLM 8–8 Section 8.2 Extra Practice**, **BLM 8–10 Section 8.3 Extra Practice**, **BLM 8–12 Section 8.4 Extra Practice**, and **BLM 8–17 Section 8.5 Extra Practice**.

Learning Style and Motor

- Allow students to use **Master 12 Percent Circles** to help them construct the circle graphs for #21 and #22. Alternatively, allow students to use technology for these questions.

Learning Style

- Allow students to complete the chapter review using any combination of oral and written answers.

ESL, Language, and Memory

- Encourage students to use their chapter Foldable during the Chapter 8 Review and to add any notes into the pertinent sections.

Gifted and Enrichment

- Students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge for gifted students, go to **www.mathlinks7.ca** and follow the links.

Assessment *as* Learning	Supported Learning
Math Learning Log Once students have completed the chapter review, have them reflect on their progress and complete a journal entry for each statement: – I am comfortable with the following parts of the chapter … – I can calculate circumference and area by … – I can explain a sector angle by … – I am having difficulty with … – Here's how I plan to address the areas I am having difficulty with …	• Have students refer back to the What I Need to Work On section of their chapter Foldable and answer these questions from the contents of that section. • You may wish to have students refer to **BLM 8–1 Chapter 8 Self-Assessment** when they report on what they are comfortable with, what they continue to have difficulty with, and what they plan to do about it. • There are many different ways to estimate. Encourage students to use the one they feel most comfortable with, but to try other methods. They might find that some methods work better for some types of questions and others for other types.

8 Practice Test

Suggested Timing
40–50 minutes

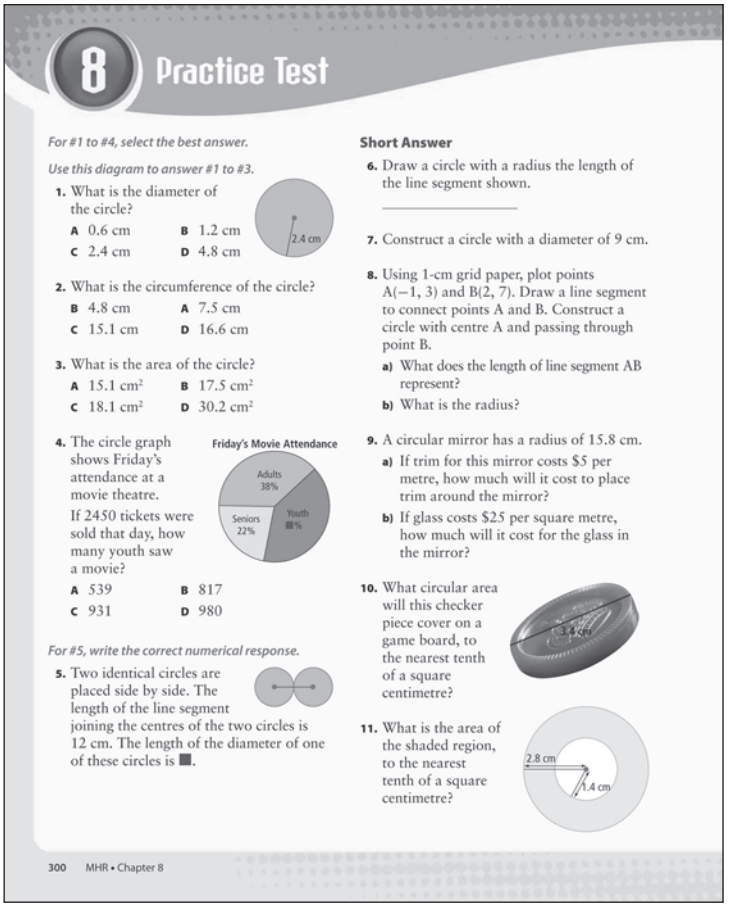
Materials
- compass
- protractor
- ruler
- calculator

Blackline Masters
Master 8 Centimetre Grid Paper

BLM 8–1 Chapter 8 Self-Assessment

BLM 8–18 Chapter 8 Test

Assessment *as* Learning	Supported Learning
Chapter 8 Self-Assessment Have students review their earlier responses on **BLM 8–1 Chapter 8 Self-Assessment**.	• Have students use their responses on the practice test and work they completed earlier in the chapter to complete the After column of this self-assessment. • Before students do the Chapter 8 Test, coach them in the areas in which they are having problems.



8 Practice Test

For #1 to #4, select the best answer.

Use this diagram to answer #1 to #3.

1. What is the diameter of the circle?
A 0.6 cm B 1.2 cm
C 2.4 cm D 4.8 cm

2. What is the circumference of the circle?
B 4.8 cm A 7.5 cm
C 15.1 cm D 16.6 cm

3. What is the area of the circle?
A 15.1 cm² B 17.5 cm²
C 18.1 cm² D 30.2 cm²

4. The circle graph shows Friday's attendance at a movie theatre. If 2450 tickets were sold that day, how many youth saw a movie?
A 539 B 817
C 931 D 980

Friday's Movie Attendance
Adults 38%
Seniors 22%
Youth

For #5, write the correct numerical response.

5. Two identical circles are placed side by side. The length of the line segment joining the centres of the two circles is 12 cm. The length of the diameter of one of these circles is ■.

Short Answer

6. Draw a circle with a radius the length of the line segment shown.

7. Construct a circle with a diameter of 9 cm.

8. Using 1-cm grid paper, plot points A(−1, 3) and B(2, 7). Draw a line segment to connect points A and B. Construct a circle with centre A and passing through point B.
a) What does the length of line segment AB represent?
b) What is the radius?

9. A circular mirror has a radius of 15.8 cm.
a) If trim for this mirror costs $5 per metre, how much will it cost to place trim around the mirror?
b) If glass costs $25 per square metre, how much will it cost for the glass in the mirror?

10. What circular area will this checker piece cover on a game board, to the nearest tenth of a square centimetre?

11. What is the area of the shaded region, to the nearest tenth of a square centimetre?
2.8 cm
1.4 cm

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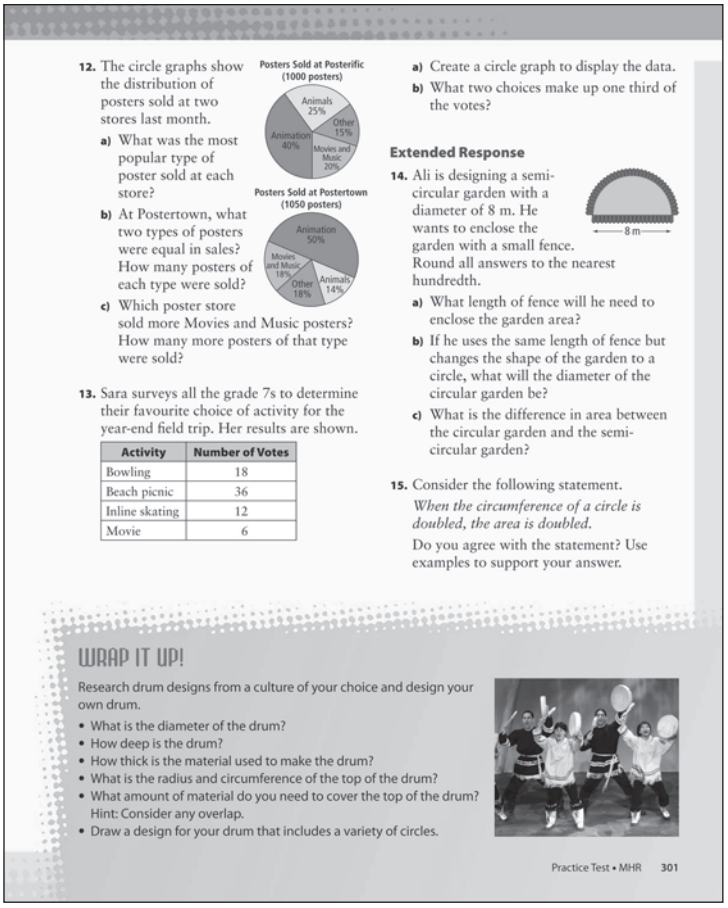
Study Guide

Question(s)	Section(s)	Refer to	I can …
1, 5–8	8.1	Example 2	✓ draw a circle with a given radius or diameter ✓ determine the diameter of a circle given its radius ✓ determine the radius of a circle given its diameter
2	8.2	Examples 1, 2	✓ estimate and calculate the circumference of a circle given its diameter or its radius ✓ solve problems involving the circumference of circles
3, 9–11, 14, 15	5.5	Examples 1, 2	✓ explain how to determine the area of a circle ✓ estimate and calculate the area of a circle ✓ solve problems involving the area of a circle
4, 12	8.4	Example	✓ read circle graphs ✓ use circle graphs to solve problems
13	8.5	Examples 1, 2	✓ construct a circle graph with and without technology



12. The circle graphs show the distribution of posters sold at two stores last month.

Posters Sold at Posterific (1000 posters)

Posters Sold at Postertown (1050 posters)

a) What was the most popular type of poster sold at each store?

b) At Postertown, what two types of posters were equal in sales? How many posters of each type were sold?

c) Which poster store sold more Movies and Music posters? How many more posters of that type were sold?

13. Sara surveys all the grade 7s to determine their favourite choice of activity for the year-end field trip. Her results are shown.

Activity	Number of Votes
Bowling	18
Beach picnic	36
Inline skating	12
Movie	6

a) Create a circle graph to display the data.

b) What two choices make up one third of the votes?

Extended Response

14. Ali is designing a semi-circular garden with a diameter of 8 m. He wants to enclose the garden with a small fence. Round all answers to the nearest hundredth.

a) What length of fence will he need to enclose the garden area?

b) If he uses the same length of fence but changes the shape of the garden to a circle, what will the diameter of the circular garden be?

c) What is the difference in area between the circular garden and the semi-circular garden?

15. Consider the following statement.

When the circumference of a circle is doubled, the area is doubled.

Do you agree with the statement? Use examples to support your answer.

WRAP IT UP!

Research drum designs from a culture of your choice and design your own drum.

- What is the diameter of the drum?
- How deep is the drum?
- How thick is the material used to make the drum?
- What is the radius and circumference of the top of the drum?
- What amount of material do you need to cover the top of the drum? Hint: Consider any overlap.
- Draw a design for your drum that includes a variety of circles.

Activity Planning Notes

The practice test can be assigned as an in-class or take-home assignment. These are the minimum questions that will meet the related curriculum outcomes: #1–#7, #9, #10, #12, and #13.

Answers to the Chapter 8 Practice Test are provided on **BLM 8–20 Chapter 8 *MathLinks 7* Student Resource Answers**.

Assessment *of* Learning	Supported Learning
Chapter 8 Test After students complete the practice test, you may wish to use **BLM 8–18 Chapter 8 Test** as a summative assessment.	• Consider allowing students to use their chapter Foldable and/or a calculator. • Consider using the Math Games on page 302 or the Challenge in Real Life on page 303 to assess the knowledge and skills of students who have difficulty with tests.

Supported Learning

ESL, Language, and Memory

- Consider allowing students to use their chapter Foldable during the practice test.

Wrap It Up!

Suggested Timing
60–75 minutes (Method 2)

Materials
- research on drums in print material and from Internet sites (optional)
- designs from Section 8.1 Math Link (optional)
- calculator

Blackline Masters
Master 1 Project Rubric
BLM 8–7 Section 8.1 Math Link
BLM 8–9 Section 8.2 Math Link
BLM 8–11 Section 8.3 Math Link
BLM 8–19 Chapter 8 Wrap It Up

Common Errors

- When determining the amount of material needed, students may need to consider overlapping material.

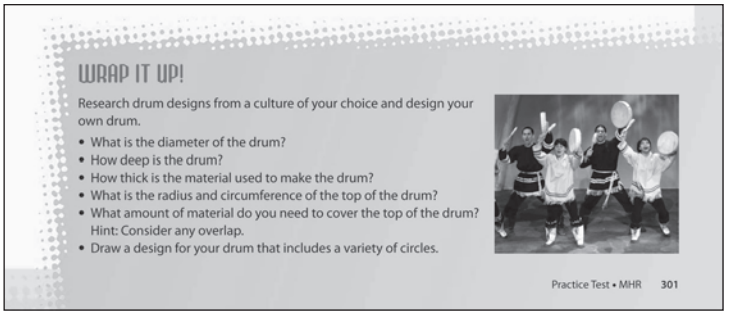
R_x As part of their research, they can try to find out how much overlap is needed (probably 2.54 cm) and increase their radius accordingly.

Supported Learning

Motor

- Allow students to complete the Wrap it Up! using a computer program, such as Draw.

Assessment *of* Learning	Supported Learning
Wrap It Up! Encourage students to research a culture of their choice. It is important for students to present their plan with mathematical justification in each of the areas of the problem. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this Wrap It Up! Page 301a provides notes on how to use the rubric for this Wrap It Up!	• Encourage strong students to research a culture that has not been introduced in the Math Links. Suggest to students requiring more support that they do further research on a culture that has already been identified. • If students have not completed the Math Links earlier in the chapter, you may wish to provide them with **BLM 8–7 Section 8.1 Math Link**, **BLM 8–9 Section 8.2 Math Link**, and **BLM 8–11 Section 8.3 Math Link**. • Some students may benefit from using **BLM 8–19 Chapter 8 Wrap It Up!**, which provides scaffolding for the chapter problem wrap-up. • Observe how accurately students design, explain, and justify the plan they have created.


WRAP IT UP!

Research drum designs from a culture of your choice and design your own drum.

- What is the diameter of the drum?
- How deep is the drum?
- How thick is the material used to make the drum?
- What is the radius and circumference of the top of the drum?
- What amount of material do you need to cover the top of the drum? Hint: Consider any overlap.
- Draw a design for your drum that includes a variety of circles.

Practice Test • MHR 301

Specific Outcomes

SS1 Demonstrate an understanding of circles by:
- constructing circles with a given radius or diameter
- solving problems involving the radii, diameters and circumferences of circles.

SS2 Develop and apply a formula for determining the area of:
- circles.

Activity Planning Notes

This chapter problem can be scaled up or down to meet the needs of your class.

Method 1: If students are particularly keen on drums, you may wish to have students design a drum that they actually build. This could involve inviting community members into the classroom to coach and assist students. Part of this project might be done in Art class.

Method 2: To minimize the time spent on this chapter problem, have students start working on their research as time allows throughout the chapter. Work that they did in the Math Link for Section 8.1 could be used here.

Method 3: Consider a cross-topic investigation with Social Studies. Have students research the drums of a culture they are studying in Social Studies class. They can do the math calculations related to the drum they research.

The chart below shows **Master 1 Project Rubric** for tasks such as that in the Wrap It Up!, and provides notes that specify how to identify the level of specific answers for this project.

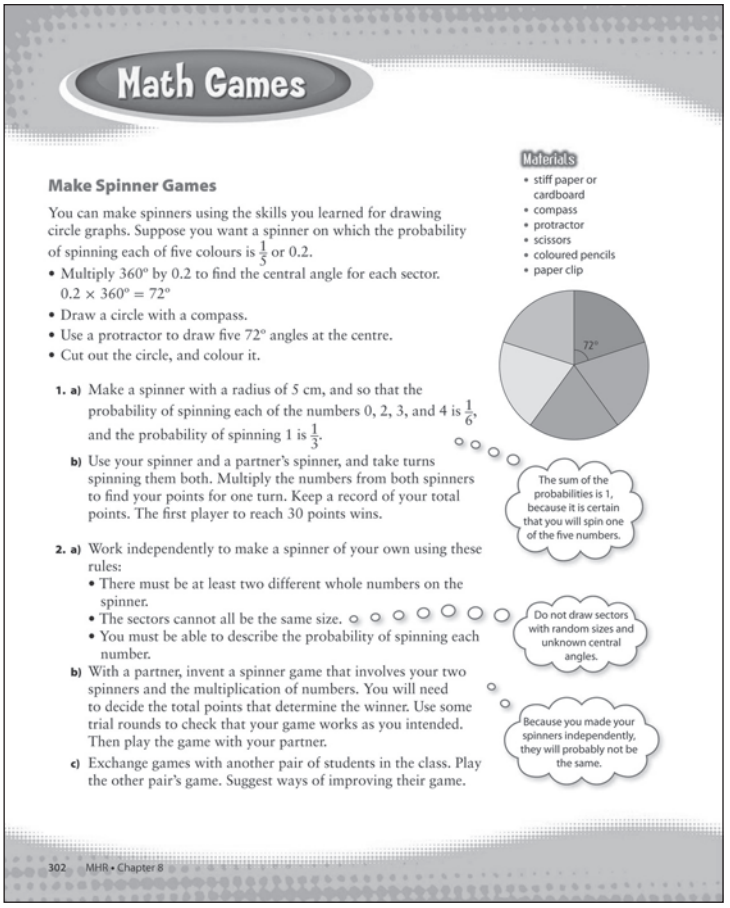
Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete solution with a dimension omitted or incorrect, or an aspect of the drum description (radius, diameter, circumference, type of material, circle designs) missing
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides a diagram that is accurately labelled with the dimensions of the drum and identifies the circle design, but fails to specify the materials used and makes no attempt to find the radius and the circumference *or* • provides a diagram that is accurately labelled for all dimensions and calculations, but the work does not proceed beyond this
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a description of the drum and the material used *or* • provides a drawing with the dimensions of the drum without giving the circumference or radius
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides a basic description of the drum with no drawing or dimensions *or* • provides a diagram with only initial labelling

Math Games

Suggested Timing
60–75 minutes

Materials
- stiff paper or cardboard
- compass
- protractor
- scissors
- coloured pencils
- paper clip

Assessment *for* Learning	Supported Learning
Make Spinner Games Have students work with a partner to create their spinners. Students may wish to create spinners of different sizes other than the 5 cm specified. In #2, restrict students to using only whole numbers on their spinners. Have students play the game with a partner of similar math ability.	• Encourage students to use mental math and keep a record of their points in a table. You may wish to allow some students to use a calculator to double-check totals. • Have students play with one partner, consider how they might increase their chances of winning, then play with another partner of similar math ability. • After students have played a game several times, ask them if the game is a fair game. If it is not, ask how they would make it fair.


Math Games

Make Spinner Games

You can make spinners using the skills you learned for drawing circle graphs. Suppose you want a spinner on which the probability of spinning each of five colours is $\frac{1}{5}$ or 0.2.

- Multiply 360° by 0.2 to find the central angle for each sector.
  $0.2 \times 360° = 72°$
- Draw a circle with a compass.
- Use a protractor to draw five 72° angles at the centre.
- Cut out the circle, and colour it.

Materials
- stiff paper or cardboard
- compass
- protractor
- scissors
- coloured pencils
- paper clip

72°

1. a) Make a spinner with a radius of 5 cm, and so that the probability of spinning each of the numbers 0, 2, 3, and 4 is $\frac{1}{6}$, and the probability of spinning 1 is $\frac{1}{3}$.

b) Use your spinner and a partner's spinner, and take turns spinning them both. Multiply the numbers from both spinners to find your points for one turn. Keep a record of your total points. The first player to reach 30 points wins.

The sum of the probabilities is 1, because it is certain that you will spin one of the five numbers.

2. a) Work independently to make a spinner of your own using these rules:
- There must be at least two different whole numbers on the spinner.
- The sectors cannot all be the same size.
- You must be able to describe the probability of spinning each number.

Do not draw sectors with random sizes and unknown central angles.

b) With a partner, invent a spinner game that involves your two spinners and the multiplication of numbers. You will need to decide the total points that determine the winner. Use some trial rounds to check that your game works as you intended. Then play the game with your partner.

Because you made your spinners independently, they will probably not be the same.

c) Exchange games with another pair of students in the class. Play the other pair's game. Suggest ways of improving their game.

302 MHR • Chapter 8

WWW Web Link

For a site in which students generate a spinner having equal sectors and try it out, go to **www.mathlinks7.ca** and follow the links.

Supported Learning

Learning Style and Memory

- Some students may need to review the term *central angle* and how to use a protractor and a compass to construct their spinners. Consider showing them how to make the spinner shown on page 302.

Gifted and Enrichment

- Encourage students to try using decimal numbers.
- Challenge students to develop spinners that show 100% probability and 0% probability for some events.

Specific Outcomes

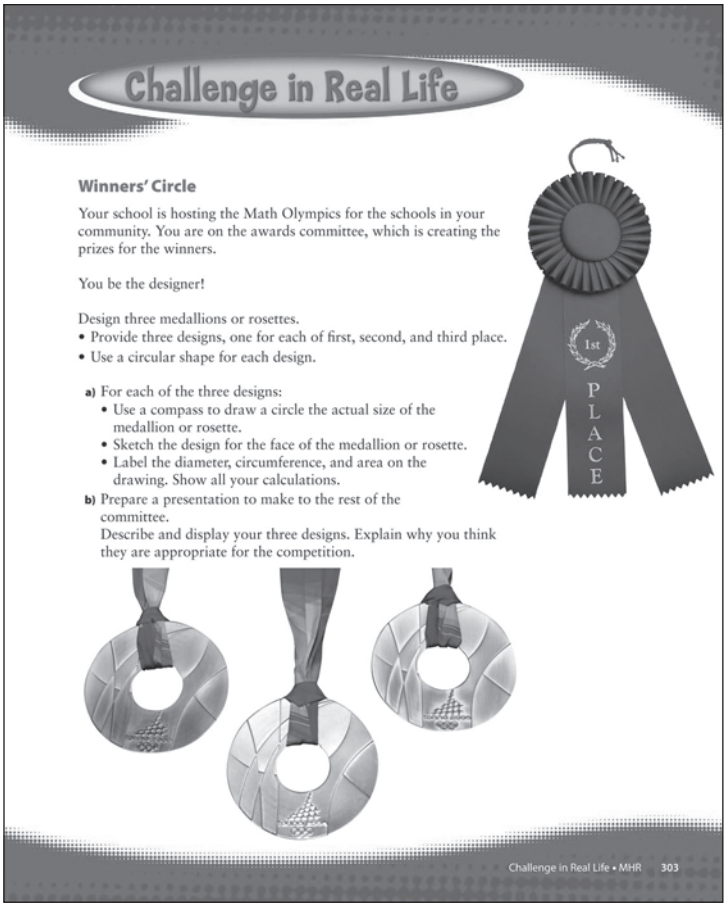
SP3 Construct, label and interpret circle graphs to solve problems.
SP4 Express probabilities as ratios, fractions and percents.
N2 Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.
N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Activity Planning Notes

As a class, consider using a pre-made spinner such as the one on page 302. Spin the spinner several times to find out which colour comes up most often. Assign points for each colour. Use a table to track the results.

After, have students brainstorm other ways to plan spinners (e.g., use a die). Ask what the central angle would be if they tossed one six-sided die ($360° \div 6 = 60°$). Then discuss what central angle would be needed for the following simulations:
- picking a number from 1 to 10 ($360° \div 10 = 36°$)
- selecting a month of the year at random ($360° \div 12 = 30°$)

Challenge in Real Life


Challenge in Real Life

Winners' Circle

Your school is hosting the Math Olympics for the schools in your community. You are on the awards committee, which is creating the prizes for the winners.

You be the designer!

Design three medallions or rosettes.
- Provide three designs, one for each of first, second, and third place.
- Use a circular shape for each design.

a) For each of the three designs:
- Use a compass to draw a circle the actual size of the medallion or rosette.
- Sketch the design for the face of the medallion or rosette.
- Label the diameter, circumference, and area on the drawing. Show all your calculations.

b) Prepare a presentation to make to the rest of the committee.
Describe and display your three designs. Explain why you think they are appropriate for the competition.

1st PLACE

Challenge in Real Life • MHR 303

Suggested Timing
60–75 minutes

Materials
- sample medallions and rosettes
- compass
- ruler
- coloured pencils

Blackline Masters
Master 1 Project Rubric

Mathematical Processes

- [x] Communication
- [x] Connections
- [ ] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [x] Technology
- [x] Visualization

Specific Outcomes

SS1 Demonstrate an understanding of circles by:
- constructing circles with a given radius or diameter
- solving problems involving the radii, diameters and circumferences of circles.

Activity Planning Notes

If you are planning to host an event such as Math Olympics for the school, make this real by having students design medallions for the winners.

You may wish to use the following steps to introduce and complete this challenge:

1. Read through Winners' Circle. Discuss the type of rosettes and medallions that students have seen before. Have a variety of medallions and rosettes available, or borrow some from the school showcase.
2. Discuss how circles are used in designs of medallions and rosettes. Note the sizes of the circles and briefly review diameter, circumference, and area.
3. Have students work individually to design their awards. Encourage them to start by discussing the process they might use (e.g., check out possible designs, draw circles, sketch the design, label, and show all calculations).

Supported Learning

Learning Style, Language, and Memory
- Some students may need additional reinforcement to process the information and the instructions.
- Allow students to present their designs either orally or in written form.

Supported Learning

Motor

- Provide students with templates of different-sized circles on which to draw their designs. Alternatively, they can research pictures of medallions or rosettes and use these pictures for the calculations and presentation.

Meeting the Needs of All Learners

- Consider encouraging students to design awards for the Arctic Winter Games or a similar competition. Students might decorate their medallions and rosettes using appropriate cultural images and words.

Gifted and Enrichment

- Encourage students to include a circular, triangular, or parallelogram cutout in the medallion. They will then have to consider the cutout when they calculate the area.
- Students could research actual medal costs if they are designing awards for a real event. If not, you could provide them with the following instructions: A company charges \$0.55/cm^2 for awards made of pure metal alloy, and \$0.36/cm^2 for awards made of plastic, with a \$0.12/cm^2 surcharge to coat the front and back of each plastic award. For rosettes they charge \$0.08/cm^2 for the cloth material. Calculate the cost of creating your designs.
- Have students research the dimensions and the weight of gold, silver, and bronze Olympic medals.

4. Clarify that the task is to
- plan a circular design for each of first, second, and third prize
- use a compass to draw a circle the actual size of each medallion or rosette
- sketch the design for the face of the medallion or rosette
- label the diameter, circumference, and area, and show all calculations
- present the completed design to classmates, including an explanation of why the design is appropriate

5. Review **Master 1 Project Rubric** with students so that they will know what is expected.

This challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment *for* Learning	Supported Learning
Winners' Circle Discuss the challenge with the class. Have students work together to develop a response, and then provide separate designs.	• Review with students how to calculate area. • Allow students to present their report either in written form or orally. • For a second challenge, complete with teaching notes and student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

Assessment *of* Learning	Supported Learning
Winners' Circle Discuss the challenge with the class. Have students work together to develop a response, and then provide separate designs.	• Use **Master 1 Project Rubric** to assist you in assessing student work. Page 303a provides notes on how to use this rubric for this challenge. • To view student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as the Challenge in Real Life and provides notes that specify how to identify the level of specific answers for this challenge.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct response to part a) and a well-justified explanation to part b) that is mathematically supported
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete and correct response to part a) *or* • provides complete and correct calculations and drawings for two medals, with a significant start to part b)
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides a complete part a), with errors in circumference or area *or* • provides a complete part a), with missing labels to substantiate calculations *or* • provides two complete and correct awards labelled and correct calculations, but makes no start to part b)
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a complete drawing and labels for the awards but incomplete or incorrect calculations for area and circumference
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• initiates a start to part a)

Chapters 5-8 Review

Suggested Timing
60–75 minutes

Materials
- compass
- protractor
- ruler

Blackline Masters
Master 8 Centimetre Grid Paper

Master 12 Percent Circles

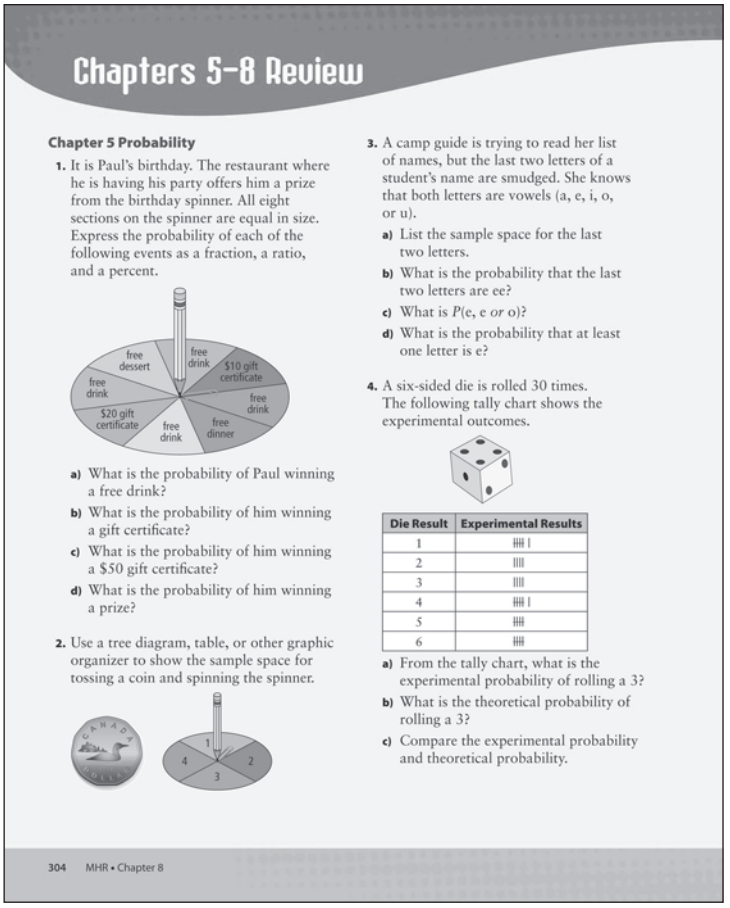
Supported Learning

Learning Style

- Allow students to complete the review using any combination of oral description, diagrams, and written answers.

Motor

- Students may prefer to use **Master 12 Percent Circles** to complete #18.



Chapters 5-8 Review

Chapter 5 Probability

1. It is Paul's birthday. The restaurant where he is having his party offers him a prize from the birthday spinner. All eight sections on the spinner are equal in size. Express the probability of each of the following events as a fraction, a ratio, and a percent.

free dessert, free drink, $10 gift certificate, free drink, free dinner, free drink, $20 gift certificate, free drink

a) What is the probability of Paul winning a free drink?
b) What is the probability of him winning a gift certificate?
c) What is the probability of him winning a $50 gift certificate?
d) What is the probability of him winning a prize?

2. Use a tree diagram, table, or other graphic organizer to show the sample space for tossing a coin and spinning the spinner.

3. A camp guide is trying to read her list of names, but the last two letters of a student's name are smudged. She knows that both letters are vowels (a, e, i, o, or u).
a) List the sample space for the last two letters.
b) What is the probability that the last two letters are ee?
c) What is P(e, e *or* o)?
d) What is the probability that at least one letter is e?

4. A six-sided die is rolled 30 times. The following tally chart shows the experimental outcomes.

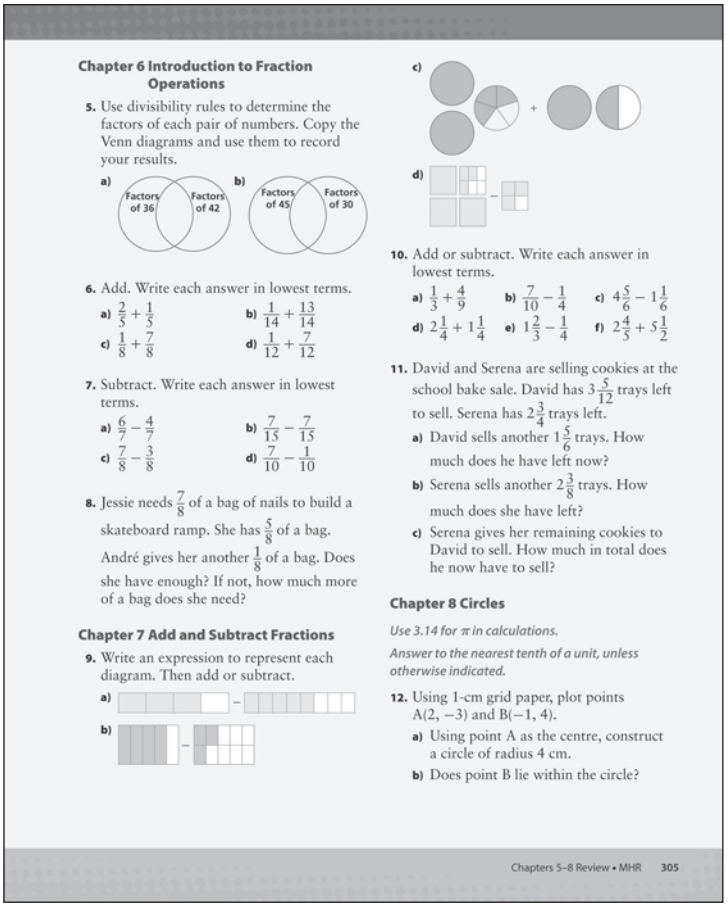
Die Result	Experimental Results
1	𝍸 I
2	𝍸
3	𝍸
4	𝍸 I
5	𝍸
6	𝍸

a) From the tally chart, what is the experimental probability of rolling a 3?
b) What is the theoretical probability of rolling a 3?
c) Compare the experimental probability and theoretical probability.

304 MHR • Chapter 8

Study Guide

Question(s)	Section(s)	Refer to	I can …
1	5.1	Examples 1, 2	✓ find the probability of an event in several different ways
2	5.2	Examples 1, 2	✓ organize the outcomes of two independent events using tables and tree diagrams
3	5.3	Examples 1, 2	✓ solve simple probability problems involving two independent events
4	5.5	Example 1	✓ compare experimental probability with theoretical probability
5	6.1	Example 2	✓ find the factors of a number using divisibility rules
6–8	6.1 6.2 6.3	Example 3 Example 3 Example 3	✓ write a fraction in lowest terms using common factors ✓ add fractions with like denominators ✓ subtract fractions with like denominators
9–11	7.2 7.3 7.4	Examples 1, 2 Examples 1, 2 Examples 1, 2	✓ add and subtract fractions with unlike denominators ✓ add and subtract mixed numbers with like and unlike denominators
12, 15	8.1	Apply	✓ draw a circle with a given radius or diameter
13, 16	8.2 8.3	Examples 1, 2 Examples 1, 2	✓ estimate and calculate the circumference of a circle given its diameter or its radius ✓ estimate and calculate the area of a circle
14	8.2	Example 2	✓ solve problems involving the circumference of circles
17	8.4	Example	✓ read circle graphs ✓ use circle graphs to solve problems
18	8.5	Example 1, 2	✓ construct a circle graph with and without technology



Chapter 6 Introduction to Fraction Operations

5. Use divisibility rules to determine the factors of each pair of numbers. Copy the Venn diagrams and use them to record your results.

a) Factors of 36 / Factors of 42

b) Factors of 45 / Factors of 30

6. Add. Write each answer in lowest terms.

a) $\frac{2}{5} + \frac{1}{5}$ b) $\frac{1}{14} + \frac{13}{14}$

c) $\frac{1}{8} + \frac{7}{8}$ d) $\frac{1}{12} + \frac{7}{12}$

7. Subtract. Write each answer in lowest terms.

a) $\frac{6}{7} - \frac{4}{7}$ b) $\frac{7}{15} - \frac{7}{15}$

c) $\frac{7}{8} - \frac{3}{8}$ d) $\frac{7}{10} - \frac{1}{10}$

8. Jessie needs $\frac{7}{8}$ of a bag of nails to build a skateboard ramp. She has $\frac{5}{8}$ of a bag. André gives her another $\frac{1}{8}$ of a bag. Does she have enough? If not, how much more of a bag does she need?

Chapter 7 Add and Subtract Fractions

9. Write an expression to represent each diagram. Then add or subtract.

a)

b)

c)

d)

10. Add or subtract. Write each answer in lowest terms.

a) $\frac{1}{3} + \frac{4}{9}$ b) $\frac{7}{10} - \frac{1}{4}$ c) $4\frac{5}{6} - 1\frac{1}{6}$

d) $2\frac{1}{4} + 1\frac{1}{4}$ e) $1\frac{2}{3} - \frac{1}{4}$ f) $2\frac{4}{5} + 5\frac{1}{2}$

11. David and Serena are selling cookies at the school bake sale. David has $3\frac{5}{12}$ trays left to sell. Serena has $2\frac{3}{4}$ trays left.

a) David sells another $1\frac{5}{6}$ trays. How much does he have left now?

b) Serena sells another $2\frac{3}{8}$ trays. How much does she have left?

c) Serena gives her remaining cookies to David to sell. How much in total does he now have to sell?

Chapter 8 Circles

Use 3.14 for π in calculations.

Answer to the nearest tenth of a unit, unless otherwise indicated.

12. Using 1-cm grid paper, plot points A(2, −3) and B(−1, 4).

a) Using point A as the centre, construct a circle of radius 4 cm.

b) Does point B lie within the circle?

Chapters 5–8 Review • MHR 305

Supported Learning

Gifted and Enrichment

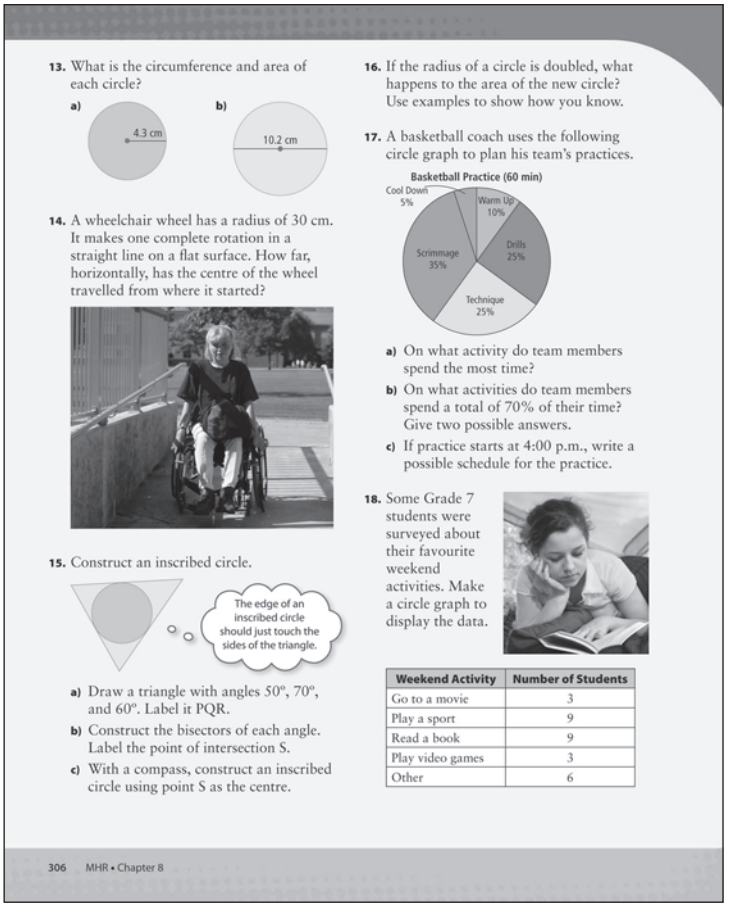
- Extend #11 by asking: How many cookies do you think are on one tray? Explain.
- Students may already be familiar with the skills handled in this review. To provide extra questions, go to **www.mathlinks7.ca** and follow the links.

Activity Planning Notes

Students might work independently to complete the questions, then in pairs to compare solutions. Alternatively, assign the Chapters 5–8 Review to reinforce skills and concepts learned so far. If students encounter difficulties, they could discuss strategies with other students. Encourage them to refer to the information in each chapter Foldable and then to the specific section in the student resource and/or their notebooks. Once they have found a suitable strategy, students should include it in the appropriate section of their chapter Foldable.

Students will need **Master 8 Centimetre Grid Paper** in order to complete #12.

These are the minimum questions which will meet the curriculum requirements: #1, #2, #4–#7, #9–#11, #13, #16, and #17.



13. What is the circumference and area of each circle?

a)

b)

14. A wheelchair wheel has a radius of 30 cm. It makes one complete rotation in a straight line on a flat surface. How far, horizontally, has the centre of the wheel travelled from where it started?

15. Construct an inscribed circle.

a) Draw a triangle with angles 50°, 70°, and 60°. Label it PQR.

b) Construct the bisectors of each angle. Label the point of intersection S.

c) With a compass, construct an inscribed circle using point S as the centre.

16. If the radius of a circle is doubled, what happens to the area of the new circle? Use examples to show how you know.

17. A basketball coach uses the following circle graph to plan his team's practices.

a) On what activity do team members spend the most time?

b) On what activities do team members spend a total of 70% of their time? Give two possible answers.

c) If practice starts at 4:00 p.m., write a possible schedule for the practice.

18. Some Grade 7 students were surveyed about their favourite weekend activities. Make a circle graph to display the data.

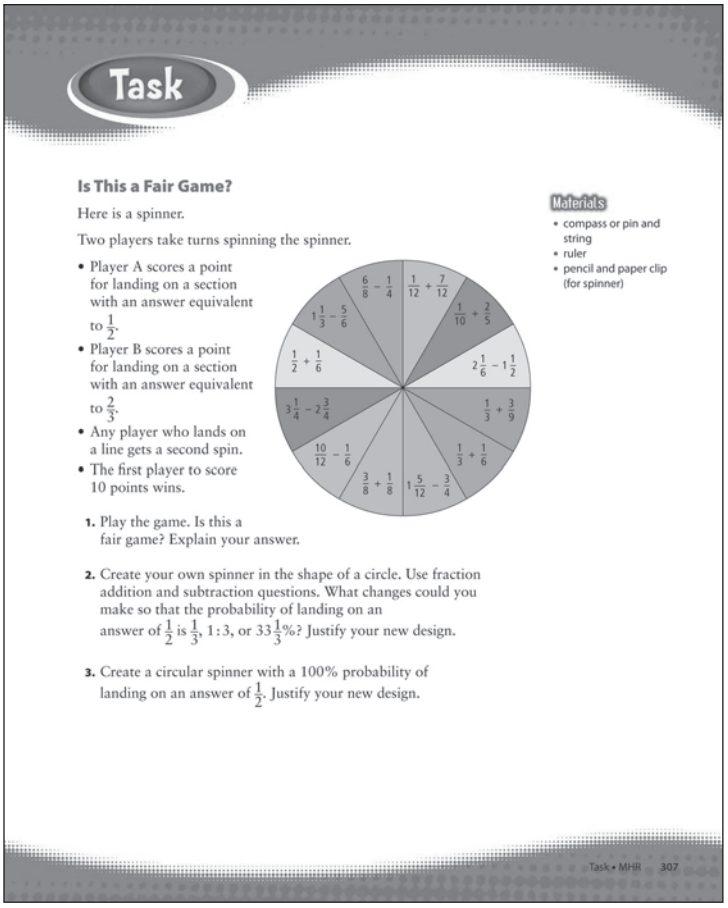
Weekend Activity	Number of Students
Go to a movie	3
Play a sport	9
Read a book	9
Play video games	3
Other	6

306 MHR • Chapter 8

Assessment *for* Learning	Supported Learning
Chapters 5–8 Review This cumulative review provides an opportunity for students to assess themselves by completing selected questions pertaining to each chapter and checking their answers against the answers in the back of the student resource.	• Have students review the tests from each chapter and any challenges related to those chapters, identify the items that they had problems with, and do the questions related to those items. Have students do at least one question that tests skills from each chapter. • Have students revisit any chapter section they are having difficulty with.

Assessment *as* Learning	Supported Learning
Math Learning Log Once students have completed the Chapters 5–8 Review, have them reflect on their progress and complete a journal entry for each statement: – I continue to have difficulty with … – Here's how I plan to address what I am having difficulty with …	• Encourage students to try to clear up any problems they have had during the past four chapters. Work with them to provide the necessary coaching.

Task



Task

Is This a Fair Game?

Here is a spinner.

Two players take turns spinning the spinner.

- Player A scores a point for landing on a section with an answer equivalent to $\frac{1}{2}$.
- Player B scores a point for landing on a section with an answer equivalent to $\frac{2}{3}$.
- Any player who lands on a line gets a second spin.
- The first player to score 10 points wins.

Materials
- compass or pin and string
- ruler
- pencil and paper clip (for spinner)

1. Play the game. Is this a fair game? Explain your answer.

2. Create your own spinner in the shape of a circle. Use fraction addition and subtraction questions. What changes could you make so that the probability of landing on an answer of $\frac{1}{2}$ is $\frac{1}{3}$, 1:3, or $33\frac{1}{3}$%? Justify your new design.

3. Create a circular spinner with a 100% probability of landing on an answer of $\frac{1}{2}$. Justify your new design.

Task • MHR 307

Suggested Timing
60–75 minutes

Materials
- paper
- compass or pin and string
- ruler
- circle template (optional)

Blackline Masters
Master 1 Project Rubric

Mathematical Processes

- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization

Specific Outcomes

SS1 Demonstrate an understanding of circles by:
- constructing circles with a given radius or diameter

SP4 Express probabilities as ratios, fractions and percents.

SP5 Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.

N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Activity Planning Notes

Read through the task as a class. Have students consider the fairness of the game, and discuss their ideas in small groups. They can work individually to develop a report on whether or not the game is fair. Students might discuss their design with a partner, and then complete it individually.

You may wish to use the following steps to introduce and complete this task:

1. a) Have students play the game using a spinner by putting a pencil tip in the centre of the spinner with a paper clip around the pencil tip.

Supported Learning

Learning Style, Language, and Memory

- Consider simplifying #1 by providing a spinner with eight sections, where students need only to reduce fractions to either $\frac{1}{2}$ or $\frac{2}{3}$. (It could be set as unfair by assigning $\frac{1}{2}$ to more than half of the sections.) Without playing the game, have students justify whether it is fair or unfair.
- Some students may need additional reinforcement to process the information and the instructions.

Motor

- Consider enlarging the spinner. Have students keep track of their experimental probability. They could then compare experimental probability to theoretical probability using the fact that answers of $\frac{1}{2}$ and $\frac{2}{3}$ each cover half of the spinner.
- Allow students to use circle templates for their spinner.

b) Have students evaluate the addition and subtraction questions on the given game spinner.

c) Discuss different ways of assessing the probability of hitting an answer of $\frac{1}{2}$ or $\frac{2}{3}$.

2. Many students may design their own spinner with the required probabilities by deciding what fraction of the circle sections should have an answer of $\frac{1}{2}$. Tell students to ignore the rule about getting a second spin when landing on a line when creating their own spinner.

3. Clarify that the task is to

- assess the fairness of the game
- write a report that explains why the game is or is not fair
- create a spinner that uses fraction addition and subtraction questions with an answer of $\frac{1}{2}$ or $\frac{2}{3}$ and gives a 1 in 3 chance of landing on an answer of $\frac{1}{2}$
- create an additional spinner with a 100% probability of landing on an answer of $\frac{1}{2}$

4. Review **Master 1 Project Rubric** with students so that they will know what is expected.

Assessment *of* Learning	Supported Learning
Is This a Fair Game? Discuss the Task with the class. Have students work together to develop a response, and then provide separate designs.	• Use **Master 1 Project Rubric** to assist you in assessing student work. Page 307a provides notes on how to use this rubric for this Task. • To view student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links. • For a second task, complete with teaching notes and student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows **Master 1 Project Rubric** for this Task and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct response
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete response to all questions, with an error in a mathematical equation *or* • provides a weak justification for #1 *or* • provides an incomplete justification for #3
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides complete and correct responses to #1 and #2 *or* • provides complete and correct responses to #1 and #3 *or* • provides incomplete responses to #2 or #3
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides complete responses to #1 and #2 with incorrect mathematical equations to justify fractions *or* • provides a correct response to either #1 or #2 with no equations *or* • provides an incomplete/incorrect response to #2 or #3 with weak/no justification **Note:** To get full credit for #2 and #3, the equations used to generate the fraction answers must be correct.
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides a correct response to #1, #2, or #3 *or* • provides a correct response to one of #1, #2, or #3 with no justification