Add and Subtract Integers

General Outcome

- Develop number sense.

Specific Outcomes

N6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
9.1	✓ add integers using integer chips
9.2	✓ add integers using a number line
9.3	✓ subtract integers using integer chips
9.4	✓ use addition to subtract integers
9.5	✓ decide when to add and subtract integers in solving problems

Assessment *as* Learning	Supported Learning
Use the Before column of **BLM 9–1 Chapter 9 Self-Assessment** to provide students with the big picture for this chapter and to help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• As students complete each section of the chapter or complete the Chapter 9 Review, have them review the related parts of **BLM 9–1 Chapter 9 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Chapter 9 Planning Chart

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter Opener • 20–30 minutes		BLM 9–1 Chapter 9 Self-Assessment BLM 9–2 Adding and Subtracting Integers	• notebook paper • scissors • stapler
9.1 Explore Integer Addition • 60–75 minutes	**Essential:** 1–5, 7, 9, 11, 13 **Typical:** 1–3, 5, 7, 9, 11–16 **Extension/Enrichment:** 1–3, 16–20	Master 2 Two Stars and One Wish BLM 9–1 Chapter 9 Self-Assessment BLM 9–3 Section 9.1 Extra Practice	• red and blue integer chips • red and blue coloured pencils (optional) • scissors (optional) • transparent chips (optional)
9.2 Add Integers • 60–75 minutes	**Essential:** 1, 3–5, 7, 9, 10, Math Link **Typical:** 2–10, 13–15, 17–19, Math Link **Extension/Enrichment:** 2–4, 20–23	Master 3 Integer Number Lines Master 4 Vertical and Horizontal Number Lines BLM 9–1 Chapter 9 Self-Assessment BLM 9–4 Section 9.2 Extra Practice BLM 9–5 Section 9.2 Math Link	• red and blue integer chips • red and blue coloured pencils • red and blue paper (optional) • scissors (optional) • transparent plastic strips (optional) • red and blue markers (optional)
9.3 Explore Integer Subtraction • 60–75 minutes	**Essential:** 1–5, 7, 9, 11, 13–15, 18 **Typical:** 1–5, 7, 9, 11, 13–18 **Extension/Enrichment:** 1–4, 19–21	BLM 9–1 Chapter 9 Self-Assessment BLM 9–6 Time Zone Map BLM 9–7 Section 9.3 Extra Practice	• red and blue integer chips • red and blue coloured pencils (optional) • transparent chips (optional)
9.4 Subtract Integers • 40–50 minutes	**Essential:** 1–5, 7, 9, 11, 13, 15, Math Link **Typical:** 1–5, 7, 9, 11, 13, 15–22, Math Link **Extension/Enrichment:** 1–4, 16, 17, 21–24	Master 3 Integer Number Lines Master 4 Vertical and Horizontal Number Lines BLM 9–1 Chapter 9 Self-Assessment BLM 9–8 Section 9.4 Explore the Math BLM 9–9 Section 9.4 Extra Practice BLM 9–10 Section 9.4 Math Link	• red and blue integer chips • transparent chips (optional)
9.5 Apply Integer Operations • 60–75 minutes	**Essential:** 1–5, Science Link **Typical:** 1–5, 9–15, Science Link **Extension/Enrichment:** 1, 2, 13, 15–18	Master 3 Integer Number Lines Master 4 Vertical and Horizontal Number Lines BLM 9–1 Chapter 9 Self-Assessment BLM 9–6 Time Zone Map BLM 9–11 Faces BLM 9–12 Section 9.5 Extra Practice BLM 9–13 Section 9.5 Science Link	• red and blue integer chips (optional) • scissors
Chapter 9 Review • 40–50 minutes	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Master 3 Integer Number Lines Master 4 Vertical and Horizontal Number Lines BLM 9–1 Chapter 9 Self-Assessment BLM 9–3 Section 9.1 Extra Practice BLM 9–4 Section 9.2 Extra Practice BLM 9–6 Time Zone Map BLM 9–7 Section 9.3 Extra Practice BLM 9–9 Section 9.4 Extra Practice BLM 9–12 Section 9.5 Extra Practice	• red and blue integer chips
Chapter 9 Practice Test • 40–50 minutes	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1–7, 9, 10, 12	BLM 9–1 Chapter 9 Self-Assessment BLM 9–14 Chapter 9 Test	• red and blue integer chips

Chapter 9 Planning Chart (continued)

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter 9 Wrap It Up! • 40–50 minutes for the Wrap It Up! (allow more time if students are conducting their own research)		Master 1 Project Rubric BLM 9–5 Section 9.2 Math Link BLM 9–10 Section 9.4 Math Link BLM 9–15 Chapter 9 Wrap It Up!	• research materials or a computer with Internet access
Chapter 9 Math Games • 40–50 minutes		Master 3 Integer Number Lines	
Chapter 9 Challenge in Real Life • 60–75 minutes		Master 1 Project Rubric Master 4 Vertical and Horizontal Number Lines Master 9 0.5 Centimetre Grid Paper BLM 9–16 Coordinate Grid BLM 9–17 Chapter 9 *MathLinks 7* Student Resource Answers BLM 9–18 Chapter 9 BLM Answers	• ruler

Chapter 9 Assessment Planner

Assessment Options	Type of Assessment	Assessment Tool
Chapter Opener	Assessment *as* Learning (TR pages i, 309)	BLM 9–1 Chapter 9 Self-Assessment Chapter 9 Foldable
9.1 Explore Integer Addition	Assessment *as* Learning (TR pages 312, 313, 315) Assessment *for* Learning (TR pages 312, 314)	Master 2 Two Stars and One Wish Math Learning Log (TR page 315) BLM 9–1 Chapter 9 Self-Assessment
9.2 Add Integers	Assessment *as* Learning (TR pages 318, 320, 322) Assessment *for* Learning (TR pages 318, 320, 322)	Math Learning Log (TR page 322) BLM 9–1 Chapter 9 Self-Assessment
9.3 Explore Integer Subtraction	Assessment *as* Learning (TR pages 325, 327, 329) Assessment *for* Learning (TR pages 325, 328)	Math Learning Log (TR page 329) BLM 9–1 Chapter 9 Self-Assessment
9.4 Subtract Integers	Assessment *as* Learning (TR pages 332, 333, 335) Assessment *for* Learning (TR pages 332, 334, 335)	Math Learning Log (TR page 335) BLM 9–1 Chapter 9 Self-Assessment
9.5 Apply Integer Operations	Assessment *as* Learning (TR pages 338, 339, 341) Assessment *for* Learning (TR page 340)	Math Learning Log (TR page 341) BLM 9–1 Chapter 9 Self-Assessment
Chapter 9 Review	Assessment *for* Learning (TR page 342) Assessment *as* Learning (TR page 343)	Math Learning Log (TR page 343) BLM 9–1 Chapter 9 Self-Assessment
Chapter 9 Practice Test	Assessment *as* Learning (TR page 344) Assessment *of* Learning (TR page 345)	BLM 9–1 Chapter 9 Self-Assessment BLM 9–14 Chapter 9 Test
Chapter 9 Wrap It Up!	Assessment *of* Learning (TR page 344a)	Master 1 Project Rubric
Chapter 9 Math Games	Assessment *for* Learning (TR page 346)	
Chapter 9 Challenge in Real Life	Assessment *for* Learning (TR page 346a) Assessment *of* Learning (TR page 346a)	Master 1 Project Rubric

You may wish to use one or more of the following materials to help you assess student readiness for Chapter 9.

Assessment *for* Learning	Supported Learning
Method 1: Have students develop a journal to explain what they personally know about integers and how they use them in their lives. **Method 2:** Have students complete **BLM 9–2 Adding and Subtracting Integers** to check their conceptual understanding. Remind students that you are looking for the scope of their knowledge. Provide them with blank paper to answer the questions.	• Students who require reinforcement of prerequisite skills may wish to complete the Get Ready materials available in the *MathLinks 7 Workbook* and at the **www.mathlinks7.ca** book site.

Chapter Opener

Suggested Timing
20–30 minutes

Materials
- notebook paper
- scissors
- stapler

Blackline Masters
BLM 9–1 Chapter 9 Self-Assessment

Key Words
zero pair
opposite integers

What's the Math?

In this chapter, students learn how to add and subtract integers. They start by adding pairs of integers, including opposite integers, using integer chips and then a number line. Students subtract integers by using integer chips and then adding the opposite integer on a number line. Throughout the chapter, students record the results of adding and subtracting integers using concrete materials, diagrams, and symbols, and apply integer operations to solve problems.

Activity Planning Notes

As a class, read the information about hockey. You may need to define the positions on a hockey team for students who are unfamiliar with hockey. You might ask students to identify their favourite hockey players and to explain their choices. Some students may mention players' achievements (e.g., top goal scorer, best goalkeeper) that are relevant to the discussion in the Math Link.

Math Link

Use the Math Link to initiate a student-centred discussion about ways to measure a hockey player's performance. For example, a forward's performance might be measured by the number of goals scored. A goalie's performance might be measured by the number of shots saved. Some students may mention that average number of goals scored per game or average shots saved per game is a better means of comparison. Students with more knowledge of hockey will likely mention assists and total points scored. Some students who are very knowledgeable may mention hockey statistics such as a player's shooting percentage (i.e., percent of a player's shots on goal that result in a goal).

You may wish to read the Wrap It Up! for this chapter problem, which is on page 345. Consider having students complete the related Math Links in Sections 9.2 and 9.4, as these will assist students who experience difficulty with the Wrap It Up! Alternatively, if you assign only the Wrap It Up! problem, ensure that students are aware of the background information on plus/minus scores from the Math Link in Section 9.2.

FOLDABLES™
Study Tool

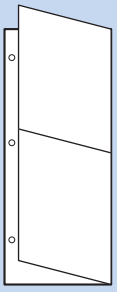
Have students make the Foldable in the student resource to keep track of the information in the chapter. Filling in the What I Need to Work On tab as they progress through the chapter will assist them in identifying and solving any difficulties with concepts, skills, and processes.

You may prefer to have students keep track of Key Words using a design specifically for that purpose. Students can make the following Foldable and write vocabulary terms on the front of each tab. Have them use the space beneath the tab to write definitions and provide examples. Because these tabs have a lot of space, students might also wish to explain how the information about the Key Words is useful when working with integers.

Step 1 Fold a sheet of notebook paper in half along the long axis with the crease to the right.

Step 2 On one side, cut the paper in half as far as the fold, creating two large tabs.

Step 3 Label each tab with a math term. Write definitions and give examples underneath the tabs.



Remind students to take notes about key ideas, examples, Key Words, and what they need to work on under the appropriate tab of the chapter Foldable.

Key Words Foldables could be slipped into the student resource as a bookmark. That will help students keep track of the words.

Assessment *as* Learning	Supported Learning
Chapter 9 Foldable As students work on each section in Chapter 9, have them keep track of any problems they are having under the What I Need to Work On tab in their chapter Foldable.	• As students complete each section, have them review the list of items they need to work on and then have them check off any that have been handled.

Supported Learning

Meeting the Needs of All Learners

- Consider having students play a game to help develop the idea of opposites. In advance, prepare pairs of cards that show opposites (e.g., *hot/cold, up/down, two above/two below, plus/minus, one more than/one less than*). Distribute cards to students and have them find their matching opposite.

Gifted and Enrichment

- You might have students research the Web Link and present an oral report about the origins of lacrosse to the class.

9.1 Explore Integer Addition

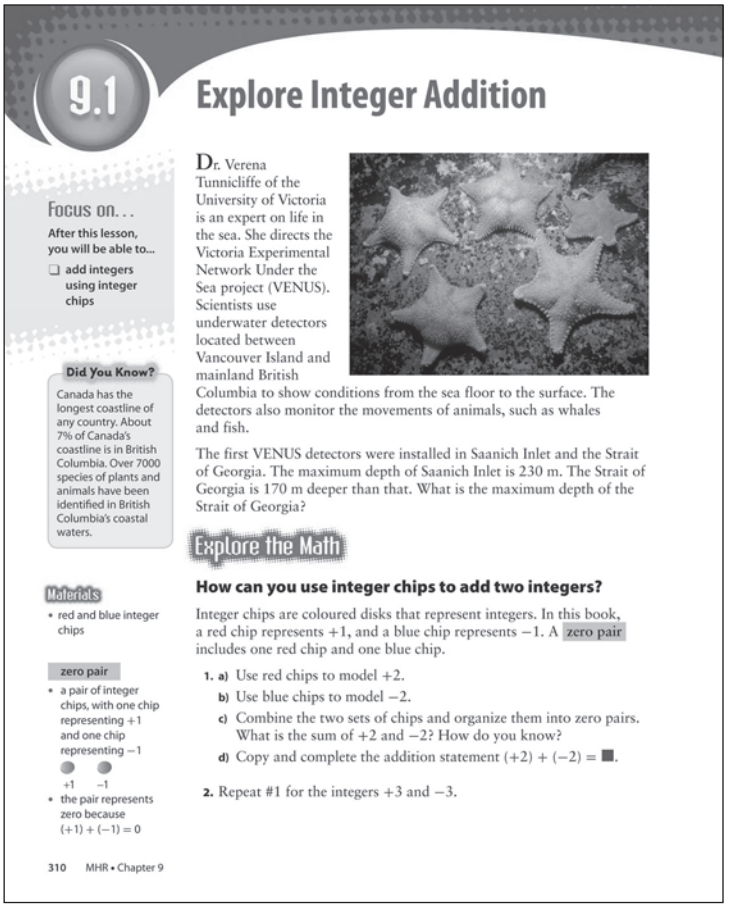
Suggested Timing
60–75 minutes

Materials
- red and blue integer chips
- red and blue coloured pencils (optional)
- scissors (optional)
- transparent chips (optional)

Blackline Masters
Master 2 Two Stars and One Wish
BLM 9–1 Chapter 9 Self-Assessment
BLM 9–3 Section 9.1 Extra Practice

Mathematical Processes

- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



9.1 Explore Integer Addition

Focus on...
After this lesson, you will be able to...
- add integers using integer chips

Did You Know?
Canada has the longest coastline of any country. About 7% of Canada's coastline is in British Columbia. Over 7000 species of plants and animals have been identified in British Columbia's coastal waters.

Dr. Verena Tunnicliffe of the University of Victoria is an expert on life in the sea. She directs the Victoria Experimental Network Under the Sea project (VENUS). Scientists use underwater detectors located between Vancouver Island and mainland British Columbia to show conditions from the sea floor to the surface. The detectors also monitor the movements of animals, such as whales and fish.

The first VENUS detectors were installed in Saanich Inlet and the Strait of Georgia. The maximum depth of Saanich Inlet is 230 m. The Strait of Georgia is 170 m deeper than that. What is the maximum depth of the Strait of Georgia?

Explore the Math

Materials
- red and blue integer chips

zero pair
- a pair of integer chips, with one chip representing +1 and one chip representing −1
- the pair represents zero because (+1) + (−1) = 0

How can you use integer chips to add two integers?

Integer chips are coloured disks that represent integers. In this book, a red chip represents +1, and a blue chip represents −1. A zero pair includes one red chip and one blue chip.

1. a) Use red chips to model +2.
   b) Use blue chips to model −2.
   c) Combine the two sets of chips and organize them into zero pairs. What is the sum of +2 and −2? How do you know?
   d) Copy and complete the addition statement (+2) + (−2) = ■.
2. Repeat #1 for the integers +3 and −3.

310 MHR • Chapter 9

Specific Outcomes

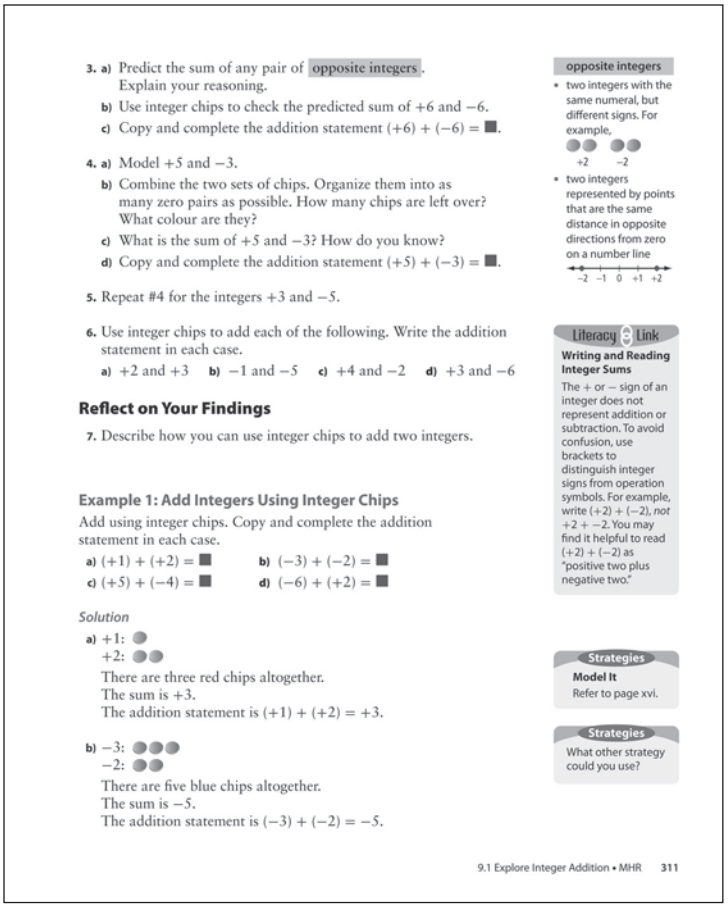
N6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.

Warm-Up

1. A Humane Society has the following animals for adoption: 50% cats, 25% dogs, and 25% other animals. Sketch and label a circle graph to show these numbers.
2. 30% of grade 7 students play soccer. Calculate the central angle needed to show this percent on a circle graph.
3. A circle has a radius of 32 m. Calculate its area.
4. Draw a circle with a radius of 4 cm.
5. Calculate $9\frac{1}{2} - 3\frac{3}{4}$. Show your work.

Mental Math

6. Estimate the diameter of the following circles:
   a) radius = 2.9 m **b)** radius = 9.2 m
7. Estimate the circumference of the following circles:
   a) diameter = 0.9 m **b)** radius = 2.5 m
8. List seven multiples for 5 and 6, then show the least common multiple.
9. Estimate $\frac{3}{4} - \frac{3}{8}$. Is your answer closer to 0 or $\frac{1}{2}$? Explain.
10. List the numbers divisible by 5. How do you know?
    25, 32, 138, 155, 206, 320



3. a) Predict the sum of any pair of opposite integers. Explain your reasoning.
b) Use integer chips to check the predicted sum of +6 and −6.
c) Copy and complete the addition statement (+6) + (−6) = ■.

4. a) Model +5 and −3.
b) Combine the two sets of chips. Organize them into as many zero pairs as possible. How many chips are left over? What colour are they?
c) What is the sum of +5 and −3? How do you know?
d) Copy and complete the addition statement (+5) + (−3) = ■.

5. Repeat #4 for the integers +3 and −5.

6. Use integer chips to add each of the following. Write the addition statement in each case.
a) +2 and +3 b) −1 and −5 c) +4 and −2 d) +3 and −6

Reflect on Your Findings

7. Describe how you can use integer chips to add two integers.

Example 1: Add Integers Using Integer Chips

Add using integer chips. Copy and complete the addition statement in each case.

a) (+1) + (+2) = ■ b) (−3) + (−2) = ■
c) (+5) + (−4) = ■ d) (−6) + (+2) = ■

Solution

a) +1: ●
+2: ●●
There are three red chips altogether.
The sum is +3.
The addition statement is (+1) + (+2) = +3.

b) −3: ●●●
−2: ●●
There are five blue chips altogether.
The sum is −5.
The addition statement is (−3) + (−2) = −5.

opposite integers
- two integers with the same numeral, but different signs. For example, ●● ●● (+2, −2)
- two integers represented by points that are the same distance in opposite directions from zero on a number line (−2, −1, 0, +1, +2)

Literacy Link
Writing and Reading Integer Sums
The + or − sign of an integer does not represent addition or subtraction. To avoid confusion, use brackets to distinguish integer signs from operation symbols. For example, write (+2) + (−2), *not* +2 + −2. You may find it helpful to read (+2) + (−2) as "positive two plus negative two."

Strategies
Model It
Refer to page xvi.

Strategies
What other strategy could you use?

9.1 Explore Integer Addition • MHR 311

Answers

Warm-Up

1. The circle graph should show cats in half the circle, and each of dogs and other animals in one quarter of the circle. Check that the labels are included and correct.
2. $0.3 \times 360 = 108°$ *or* $10\% = 36°$; $30\% = 3 \times 36° = 108°$
3. Area $= 32^2 \times 3.14 = 3215.36 \text{ m}^2$
4. Check the diameter of the circle. It should measure 8 cm.
5. Methods may vary. $5\frac{3}{4}$
6. **a)** $3 \times 2 = 6$ m **b)** $9 \times 2 = 18$ m
7. **a)** $1 \times 3 = 3$ m **b)** $2.5 \times 2 \times 3 = 15$ m, or, if they used front-end estimation, 12 m.
8. 5 = 5, 10, 15, 20, 25, 30, 35; 6 = 6, 12, 18, 24, 30, 36, 42
Least common multiple = 30
9. Closer to 0 because $\frac{3}{8}$ is almost $\frac{1}{2}$ and you're subtracting it from $\frac{3}{4}$.
10. 25, 155, 320. They end in 5 or 0.

Explore the Math

1. **c)** 0. When the zero pairs are organized there are no chips left over.
d) (+2) + (−2) = 0
2. **c)** 0 **d)** (+3) + (−3) = 0
3. **a)** 0
b) 0. Six zero pairs can be created using +6 and −6.
c) (+6) + (−6) = 0

Activity Planning Notes

Have students determine the depth of the Strait of Georgia individually and then compare their value and the method they used with a partner. After the investigation, have students reconsider the problem presented in the opener. Have students think of the addition as (−230) + (−170) = −400.
Note: Zero pairs can also be called additive inverse pairs.

Explore the Math

In this investigation, students use integer chips to add two integers.

Method 1: Have students work with a partner. Provide a set of red and blue integer chips (at least six of each colour). Alternatively, use other manipulatives, such as pennies or red and blue square cutouts.

Method 2: Have students draw and colour circles to represent integers. Have students cross out or cover up any zero pairs.

Method 3: Demonstrate adding integers by using transparent chips on an overhead projector. Have students draw and colour circles for each example you show, and write the corresponding addition statement.

WWW Web Link

Some students may be interested in the scientific aspects of the VENUS project. For more information about the project, go to **www.mathlinks7.ca** and follow the links.

Common Errors

- Some students may not rearrange the integer chips to clearly determine the number of zero pairs.

Rx Encourage students to arrange the chips so that the zero pairs stand out (e.g., aligning rows of chips one above the other as shown in Example 1 parts c) and d) on page 312.

Answers

4. a) +5: Ⓡ Ⓡ Ⓡ Ⓡ Ⓡ
−3: Ⓑ Ⓑ Ⓑ
b) 2; red
c) +2
d) (+5) + (−3) = +2

5. a) +3: Ⓡ Ⓡ Ⓡ
−5: Ⓑ Ⓑ Ⓑ Ⓑ Ⓑ
b) 2; blue
c) −2
d) (+3) + (−5) = −2

6. a) (+2) + (+3) = +5
b) (−1) + (−5) = −6
c) (+4) + (−2) = +2
d) (+3) + (−6) = −3

7. Answers may vary. For example: Create as many zero pairs as possible. The sum of the two integers is equal to the sum of the integer chips that are left over. If the leftover chips are red the sum is positive. If the leftover chips are blue the sum is negative.

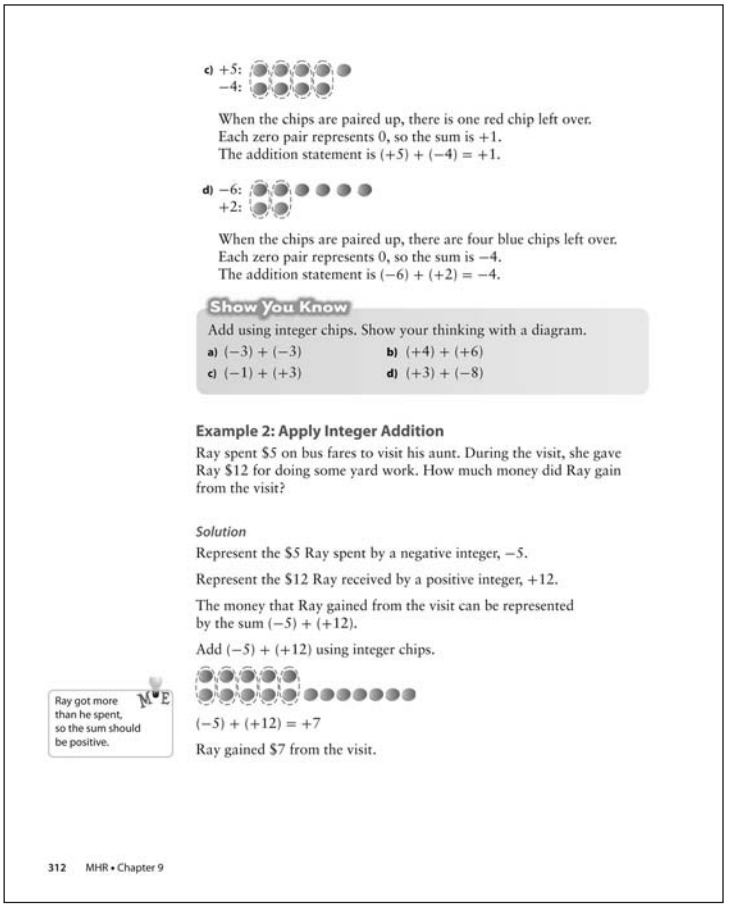
Show You Know: Example 1

a) −6
−3: Ⓑ Ⓑ Ⓑ
−3: Ⓑ Ⓑ Ⓑ

b) +10
+4: Ⓡ Ⓡ Ⓡ Ⓡ
+6: Ⓡ Ⓡ Ⓡ Ⓡ Ⓡ Ⓡ

c) +2
−1: Ⓑ
+3: Ⓡ Ⓡ Ⓡ

d) −5
+3: Ⓡ Ⓡ Ⓡ
−8: Ⓑ Ⓑ Ⓑ Ⓑ Ⓑ Ⓑ Ⓑ Ⓑ



c) +5:
−4:

When the chips are paired up, there is one red chip left over.
Each zero pair represents 0, so the sum is +1.
The addition statement is (+5) + (−4) = +1.

d) −6:
+2:

When the chips are paired up, there are four blue chips left over.
Each zero pair represents 0, so the sum is −4.
The addition statement is (−6) + (+2) = −4.

Show You Know

Add using integer chips. Show your thinking with a diagram.

a) (−3) + (−3)
b) (+4) + (+6)
c) (−1) + (+3)
d) (+3) + (−8)

Example 2: Apply Integer Addition

Ray spent $5 on bus fares to visit his aunt. During the visit, she gave Ray $12 for doing some yard work. How much money did Ray gain from the visit?

Solution

Represent the $5 Ray spent by a negative integer, −5.

Represent the $12 Ray received by a positive integer, +12.

The money that Ray gained from the visit can be represented by the sum (−5) + (+12).

Add (−5) + (+12) using integer chips.

Ray got more than he spent, so the sum should be positive.

(−5) + (+12) = +7

Ray gained $7 from the visit.

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Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students describe how to use integer chips to add integers. Check student responses to #7 for comprehension.	• Ask students who are having difficulty with these questions to use the class responses as springboards to similar ones of their own.

Example 1 illustrates adding integers using integer chips. Reinforce that the example uses a familiar problem solving strategy (i.e., Model It). Ask students to think of another strategy (e.g., Draw a Diagram).

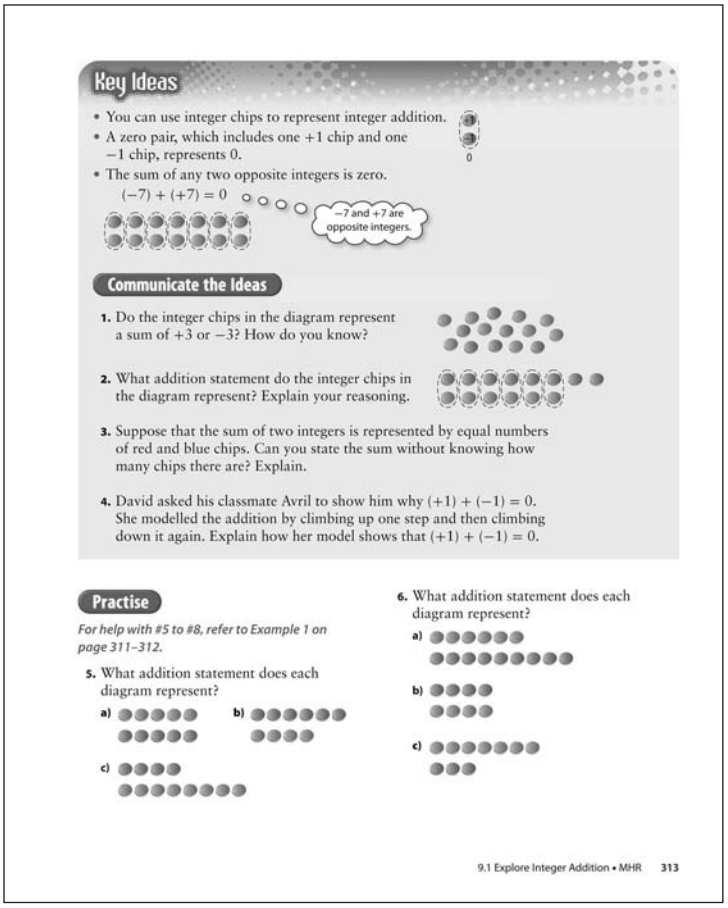
Example 2 illustrates an application of integer addition. Make sure that students understand the use of positive and negative integers to represent the quantities in the example. If they do not, consider discussing #5 and #6 on page 311 again.

Common Errors

- Some students may remove pairs of chips that are not zero pairs (i.e., pairs of red chips or pairs of blue chips).

R_x Emphasize that a zero pair includes one chip of each colour. A zero pair represents two opposite integers such as +1 and −1, which has a value of 0. Remind students that subtracting zero from a number does not change the value of a number, and that therefore removing zero pairs from a model does not affect the value that the model represents.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• Have students talk through their thinking with a partner. • Have students continue to use integer chips or a similar manipulative until they feel comfortable with using zero pairs. • You may wish to provide additional questions very close to those in the Show You Know for students who would benefit from them: **a)** (− 4) + (− 4) (−8. When the integers have the same sign, add the number of chips and maintain the sign. Have students use integer chips to reinforce this concept.) **b)** (−2) + (+4) (+2. Zero pairs come into effect when the integers combine positive and negative signs. Have students use integer chips and show the zero pairs.) **c)** (+2) + (+3) (+5. Again, these integers have the same sign.) **d)** (−3) + (+2) (−1. This question combines positive and negative integers. Have students show the zero pairs.) Sit down and coach students through a) and b), then have them try c) and d) on their own.



Key Ideas

- You can use integer chips to represent integer addition.
- A zero pair, which includes one +1 chip and one −1 chip, represents 0.
- The sum of any two opposite integers is zero.

$(-7) + (+7) = 0$

−7 and +7 are opposite integers.

Communicate the Ideas

1. Do the integer chips in the diagram represent a sum of +3 or −3? How do you know?
2. What addition statement do the integer chips in the diagram represent? Explain your reasoning.
3. Suppose that the sum of two integers is represented by equal numbers of red and blue chips. Can you state the sum without knowing how many chips there are? Explain.
4. David asked his classmate Avril to show him why $(+1) + (-1) = 0$. She modelled the addition by climbing up one step and then climbing down it again. Explain how her model shows that $(+1) + (-1) = 0$.

Practise

For help with #5 to #8, refer to Example 1 on page 311–312.

5. What addition statement does each diagram represent?

a) b) c)

6. What addition statement does each diagram represent?

a) b) c)

9.1 Explore Integer Addition • MHR 313

Point out that since the sum is represented by more red than blue chips, it should be positive. On a more abstract level, you might encourage students to compare the two integers $(-5) + (+12)$ and explain why the sum is positive.

Key Ideas

This section summarizes how to use integer chips to represent addition.

Communicate the Ideas

These questions allow students to apply their understanding of opposite integers and zero pairs using integer chips and symbolic notation. In #1, students check their understanding of opposite integers. In #2 and #3, students check their understanding of zero pairs. In #4, students reinforce their understanding of zero pairs.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students do #1 to #3. Essential level students should also do #4. Other students may not need it. Note that #3 is related to #1. Encourage students to discuss their answer with neighbouring students and listen to each other's explanations.	• Consider having students work in a group. • Check answers to #1 to #3. These are key questions; make sure that students understand the concepts. • Ask students who have difficulty with #4 to think of other ways to model $(+1) + (-1) = 0$. You might give a penny to a student and then ask for it back. Ask what is left in their hand. Then ask the student to tell you how this model shows that $(+1) + (-1) = 0$. • Use **Master 2 Two Stars and One Wish** to have students critique other students' writing pieces. This master allows them to write two things they like about a piece and one thing they would like to improve.

Answers

Communicate the Ideas

1. +3. There are 6 zero pairs with 3 red chips remaining.
2. $(-8) + (+6) = -2$. There are 6 zero pairs with 2 blue chips remaining.
3. Yes, 0. The red chips and the blue chips could be combined to create zero pairs with no chips remaining.
4. Answers will vary. For example: +1 is one step up; −1 is one step down; Avril has not changed her position.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–5, 7, 9, 11, 13
Typical	1–3, 5, 7, 9, 11–16,
Extension/Enrichment	1–3, 16–20

Supported Learning

ESL and Language

- Make sure that students can express addition statements clearly in words by referring them to the Literacy Link on page 311. Encourage students to read +6 as "positive six" not "plus six," and −4 as "negative four" not "minus four."

Motor

- You may wish to allow students who have difficulty manipulating integer chips to use larger chips or virtual manipulatives.

Meeting the Needs of All Learners

- Some students may need additional reinforcement to understand zero pairs and opposite integers. Consider having students use integer chips to explain zero pairs and opposite integers to someone else.

Gifted and Enrichment

- Pose a problem such as the following: Hing started out with a certain amount of money. During the day, she spent $5 on fruit and juice and $20 on a DVD movie. After she went home, she found $3 on the couch. She left the house again and bought a magazine for $4. How much money did she start out with if she had $2 left?

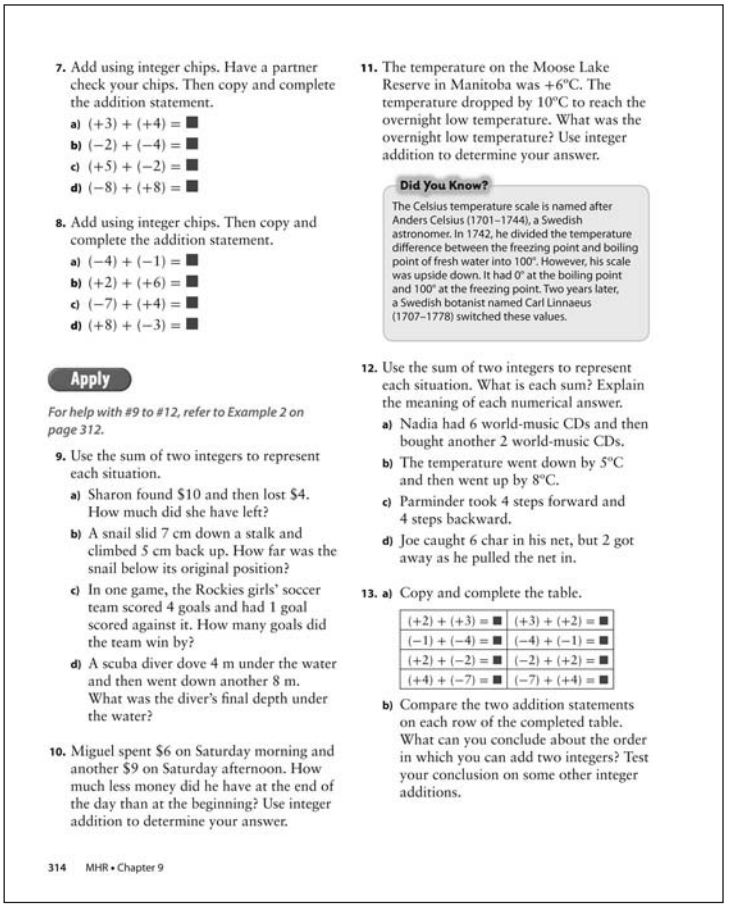
Common Errors

- Some students may ignore the signs of the integers that they are adding.

R_x Reinforce the use of two colours of manipulatives to represent positive and negative integers.

- Some students may be unsure of where to use brackets in writing addition statements.

R_x Reinforce the content of the Literacy Link on page 311. Explain that brackets keep integer signs and operation symbols separate. Therefore, brackets are used only on the side of the addition statement that includes operation symbols. The other side of the statement has no operation symbols and so brackets are unnecessary.



7. Add using integer chips. Have a partner check your chips. Then copy and complete the addition statement.
   a) $(+3) + (+4) =$ ■
   b) $(-2) + (-4) =$ ■
   c) $(+5) + (-2) =$ ■
   d) $(-8) + (+8) =$ ■

8. Add using integer chips. Then copy and complete the addition statement.
   a) $(-4) + (-1) =$ ■
   b) $(+2) + (+6) =$ ■
   c) $(-7) + (+4) =$ ■
   d) $(+8) + (-3) =$ ■

Apply

For help with #9 to #12, refer to Example 2 on page 312.

9. Use the sum of two integers to represent each situation.
   a) Sharon found $10 and then lost $4. How much did she have left?
   b) A snail slid 7 cm down a stalk and climbed 5 cm back up. How far was the snail below its original position?
   c) In one game, the Rockies girls' soccer team scored 4 goals and had 1 goal scored against it. How many goals did the team win by?
   d) A scuba diver dove 4 m under the water and then went down another 8 m. What was the diver's final depth under the water?

10. Miguel spent $6 on Saturday morning and another $9 on Saturday afternoon. How much less money did he have at the end of the day than at the beginning? Use integer addition to determine your answer.

11. The temperature on the Moose Lake Reserve in Manitoba was +6°C. The temperature dropped by 10°C to reach the overnight low temperature. What was the overnight low temperature? Use integer addition to determine your answer.

Did You Know?

The Celsius temperature scale is named after Anders Celsius (1701–1744), a Swedish astronomer. In 1742, he divided the temperature difference between the freezing point and boiling point of fresh water into 100°. However, his scale was upside down. It had 0° at the boiling point and 100° at the freezing point. Two years later, a Swedish botanist named Carl Linnaeus (1707–1778) switched these values.

12. Use the sum of two integers to represent each situation. What is each sum? Explain the meaning of each numerical answer.
   a) Nadia had 6 world-music CDs and then bought another 2 world-music CDs.
   b) The temperature went down by 5°C and then went up by 8°C.
   c) Parminder took 4 steps forward and 4 steps backward.
   d) Joe caught 6 char in his net, but 2 got away as he pulled the net in.

13. a) Copy and complete the table.

$(+2) + (+3) =$ ■	$(+3) + (+2) =$ ■
$(-1) + (-4) =$ ■	$(-4) + (-1) =$ ■
$(+2) + (-2) =$ ■	$(-2) + (+2) =$ ■
$(+4) + (-7) =$ ■	$(-7) + (+4) =$ ■

   b) Compare the two addition statements on each row of the completed table. What can you conclude about the order in which you can add two integers? Test your conclusion on some other integer additions.

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Supported Learning

Learning Style and Memory

- Provide **BLM 9–4 Section 9.3 Extra Practice** to students who require more practice.

Learning Style

- Encourage concrete and kinesthetic learners to model addition statements using integer chips.

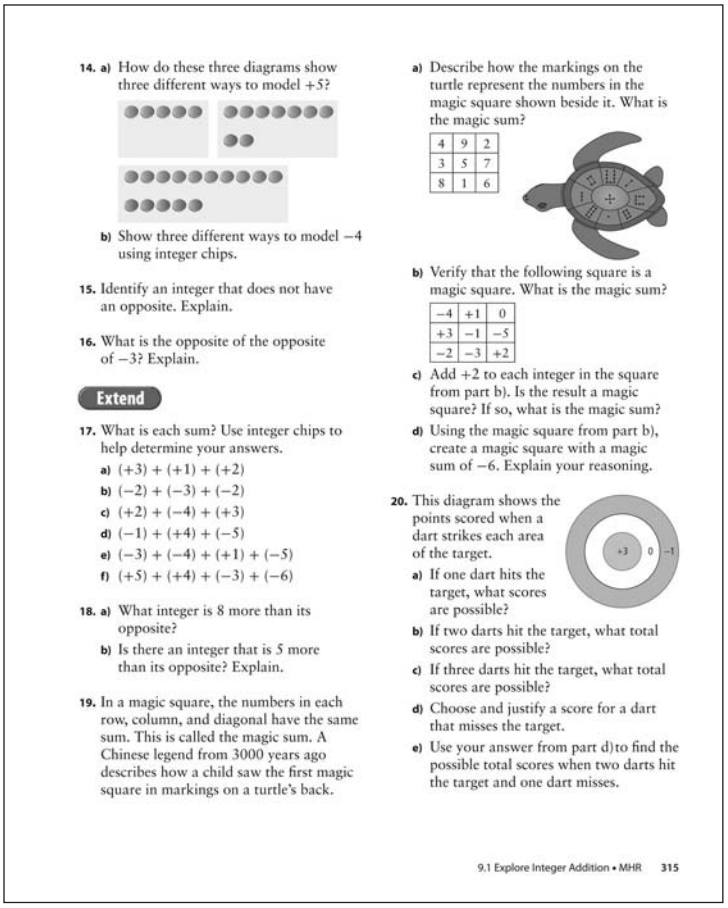
ESL and Language

- English language learners may have difficulty with terms such as *gain, snail, slid, stalk, original position, dove under water, depth under water,* and *turtle*.
- Team students with those who have a good understanding of terminology.

Practise

Students will need integer chips. In #7, allow students to work with a partner. You may decide to have students work with a partner or in a small group for the remaining questions.

Assessment *for* Learning	Supported Learning
Practise and Apply Have students do #5, #7, #9, and #11. Students who have no problems with these questions can go on to the rest of the Apply questions.	• Students who have problems with #5 and #7 will need additional coaching. Have students explain their thinking on these questions; clarify any misunderstandings. Coach students through #6a) and #8a), and then have them complete the remaining parts of the questions on their own. Have students refer back to examples in the student resource. Check back with them several times to make sure that they understand the concepts. • Coach students who have problems with #9 through a) and b), and then have them correct c) and d). When they are more confident, have them do #10. • Coach students who have problems with #11 through #12a), and then have them do the balance of #12.



14. a) How do these three diagrams show three different ways to model +5?

b) Show three different ways to model −4 using integer chips.

15. Identify an integer that does not have an opposite. Explain.

16. What is the opposite of the opposite of −3? Explain.

Extend

17. What is each sum? Use integer chips to help determine your answers.
a) (+3) + (+1) + (+2)
b) (−2) + (−3) + (−2)
c) (+2) + (−4) + (+3)
d) (−1) + (+4) + (−5)
e) (−3) + (−4) + (+1) + (−5)
f) (+5) + (+4) + (−3) + (−6)

18. a) What integer is 8 more than its opposite?
b) Is there an integer that is 5 more than its opposite? Explain.

19. In a magic square, the numbers in each row, column, and diagonal have the same sum. This is called the magic sum. A Chinese legend from 3000 years ago describes how a child saw the first magic square in markings on a turtle's back.
a) Describe how the markings on the turtle represent the numbers in the magic square shown beside it. What is the magic sum?

4	9	2
3	5	7
8	1	6

b) Verify that the following square is a magic square. What is the magic sum?

−4	+1	0
+3	−1	−5
−2	−3	+2

c) Add +2 to each integer in the square from part b). Is the result a magic square? If so, what is the magic sum?
d) Using the magic square from part b), create a magic square with a magic sum of −6. Explain your reasoning.

20. This diagram shows the points scored when a dart strikes each area of the target.
a) If one dart hits the target, what scores are possible?
b) If two darts hit the target, what total scores are possible?
c) If three darts hit the target, what total scores are possible?
d) Choose and justify a score for a dart that misses the target.
e) Use your answer from part d) to find the possible total scores when two darts hit the target and one dart misses.

9.1 Explore Integer Addition • MHR 315

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • What do you understand about zero pairs? • What do you find difficult about zero pairs?	• Depending on students' learning style, have them provide verbal or written answers. • You may wish to have students review the part related to Section 9.1 in **BLM 9–1 Chapter 9 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Apply and Extend

In #12, students explain the meaning of numerical answers in addition of integers. Being able to do so is especially important in subtraction of integers because changing the order in which two different integers are subtracted yields different results.

In #13, students discover that the order of the integers does not matter when writing integers in an addition statement.

In #14, students determine that adding or subtracting any number of zero pairs has no effect on the value represented in an integer model.

In #20, you might suggest using a table to organize how the three darts can hit each target. Tell students to remember that each target can be hit by 0, 1, 2, or all 3 darts.

In #20d), students can choose any score for a dart that misses the target, as long as they can justify it. Usually, a dart that misses a dartboard results in a score of 0, but hitting the outer circle on the target in this question results in a score of −1. Therefore, it is expected that a worse shot (i.e., one that misses the target altogether) would result in a score of less than −1.

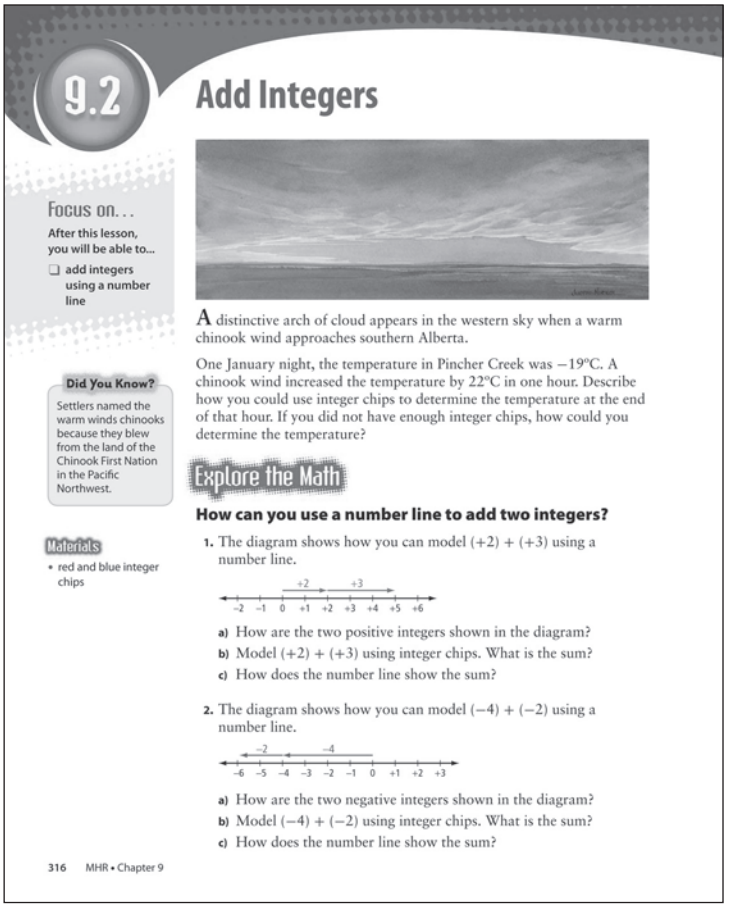
9.2 Add Integers

Suggested Timing
60–75 minutes

Materials
- red and blue integer chips
- red and blue coloured pencils
- red and blue paper (optional)
- scissors (optional)
- transparent plastic strips (optional)
- red and blue markers (optional)

Blackline Masters
Master 3 Integer Number Lines
Master 4 Vertical and Horizontal Number Lines
BLM 9–1 Chapter 9 Self-Assessment
BLM 9–4 Section 9.2 Extra Practice
BLM 9–5 Section 9.2 Math Link

Mathematical Processes
- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



9.2 Add Integers

Focus on...
After this lesson, you will be able to...
- add integers using a number line

A distinctive arch of cloud appears in the western sky when a warm chinook wind approaches southern Alberta.

One January night, the temperature in Pincher Creek was −19°C. A chinook wind increased the temperature by 22°C in one hour. Describe how you could use integer chips to determine the temperature at the end of that hour. If you did not have enough integer chips, how could you determine the temperature?

Did You Know?
Settlers named the warm winds chinooks because they blew from the land of the Chinook First Nation in the Pacific Northwest.

Explore the Math

How can you use a number line to add two integers?

Materials
- red and blue integer chips

1. The diagram shows how you can model (+2) + (+3) using a number line.

a) How are the two positive integers shown in the diagram?
b) Model (+2) + (+3) using integer chips. What is the sum?
c) How does the number line show the sum?

2. The diagram shows how you can model (−4) + (−2) using a number line.

a) How are the two negative integers shown in the diagram?
b) Model (−4) + (−2) using integer chips. What is the sum?
c) How does the number line show the sum?

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Specific Outcomes

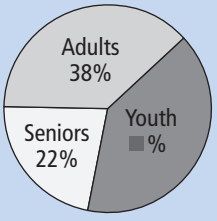
N6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.

Warm-Up

1. Draw integer chips to show the following addition statement: (+3) + (+5).

2. Calculate (+2) + (−2). Explain your answer.

3. What percent of people attending the Friday movie were youths? Show your thinking.



4. Draw a circle with a diameter of 3 cm.

5. Calculate $4\frac{1}{3} + 2\frac{1}{6}$. Show your work.

Mental Math

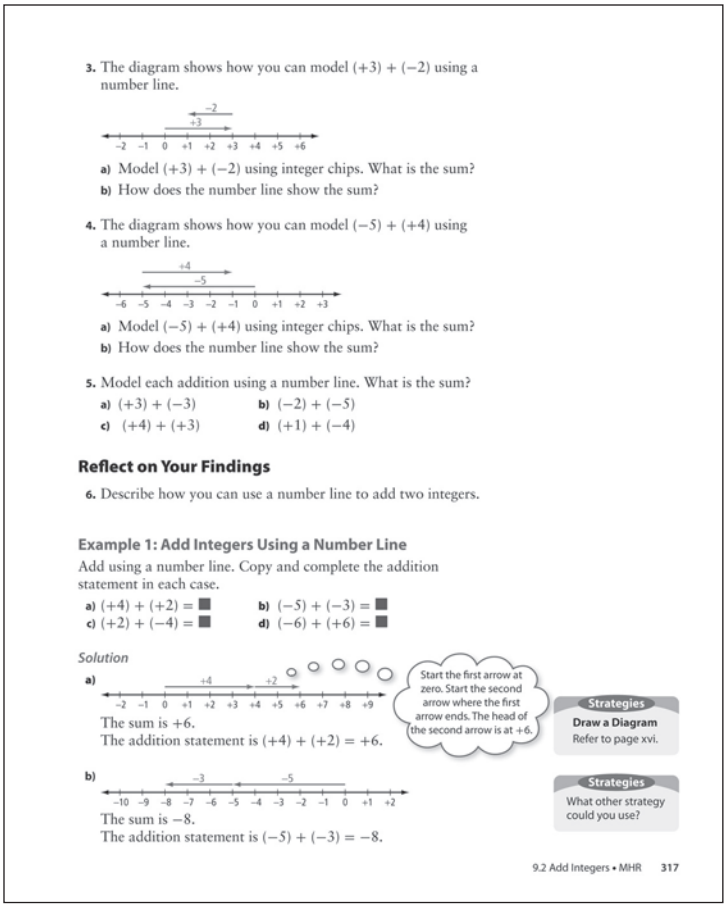
6. Estimate the area of the following circles:
a) radius = 2.9 m **b)** radius = 5.1 m

7. List 6 multiples for 4 and 5, then show the least common multiple.

8. List the numbers divisible by 4. How do you know?
1020, 2041, 3056, 4008, 4258

9. Show how to mentally find 20% of 750.

10. Estimate 275 out of 500 as a percent. Show your thinking.



3. The diagram shows how you can model (+3) + (−2) using a number line.

a) Model (+3) + (−2) using integer chips. What is the sum?
b) How does the number line show the sum?

4. The diagram shows how you can model (−5) + (+4) using a number line.

a) Model (−5) + (+4) using integer chips. What is the sum?
b) How does the number line show the sum?

5. Model each addition using a number line. What is the sum?
a) (+3) + (−3) b) (−2) + (−5)
c) (+4) + (+3) d) (+1) + (−4)

Reflect on Your Findings

6. Describe how you can use a number line to add two integers.

Example 1: Add Integers Using a Number Line

Add using a number line. Copy and complete the addition statement in each case.

a) (+4) + (+2) = ■ b) (−5) + (−3) = ■
c) (+2) + (−4) = ■ d) (−6) + (+6) = ■

Solution

a)

The sum is +6.
The addition statement is (+4) + (+2) = +6.

Start the first arrow at zero. Start the second arrow where the first arrow ends. The head of the second arrow is at +6.

Strategies
Draw a Diagram
Refer to page xvi.

b)

The sum is −8.
The addition statement is (−5) + (−3) = −8.

Strategies
What other strategy could you use?

9.2 Add Integers • MHR 317

Answers

Warm-Up

1. Students should draw eight red chips.
2. (+2) + (−2) − 0. This is a zero pair.
3. 38% + 22% = 60%. The whole circle is 100%. 100% − 60% = 40%; Youths = 40%.
4. Check that students use a 1.5 cm radius to make this circle.
5. Method will vary. $6\frac{3}{6} = 6\frac{1}{2}$
6. **a)** $3^2 \times 3 = 9 \times 3 = 27$ m^2
   b) $5^2 \times 3 = 25 \times 3 = 75$ m^2
7. 4 = 4, 8, 12, 16, 20, 24; 5 = 5, 10, 15, 20, 25
   Least common multiple = 20
8. 1020, 3056, 4008. The last two digits of each number are divisible by 2 and then 2 again.
9. 10% = 750 ÷ 10 = 75; 20% = 2 × 75 = 150 or 20% = $\frac{1}{5}$; 750 ÷ 5 = 150
10. 50% of 500 = 250 10% of 500 = 50
    5% of 500 = 25 275 = 250 + 25, or 55%

Explore the Math

1. **a)** Answers may vary. For example: The positive integers are shown moving to the right.
   b) +5
   c) Answers may vary. For example: (+2) + (+3) = (right 2) + (right 3) = (right 5) = +5

Activity Planning Notes

For the first question, students could use integer chips. For the second question, ask for alternatives to using integer chips.

Explore the Math

Students develop a model for adding integers using number lines.

Method 1: Have students work in pairs or small groups and supply them with integer chips (at least seven of each colour) or other appropriate manipulatives. For #5, students can draw coloured diagrams to model the integer additions. Alternatively, have students use **Master 3 Integer Number Lines**.

Method 2: Have students work in pairs or small groups and supply them with integer chips and **Master 3 Integer Number Lines**. Have students use red and blue paper strips of appropriate lengths that represent integers to model the additions.

Method 3: Demonstrate adding integers by using coloured transparent plastic strips on a number line on an overhead projector. Have students draw coloured diagrams and write the corresponding addition statements.

Supported Learning

Meeting the Needs of All Learners

- Some students may have difficulty using a number line and need some coaching. Continue to model solutions using number lines. Have partners take turns to model solutions.
- Some students may find it easier to use a vertical number line. Provide **Master 4 Vertical and Horizontal Number Lines**.

Gifted and Enrichment

- Use the Did You Know? on page 316 to spark interest in words that have Aboriginal origins. Students might use the library or the Internet to research other such words.

Answers

2. a) Answers may vary. For example: The negative integers are shown moving to the left.

b) -6

c) Answers may vary. For example: $(-4) + (-2) =$ (left 4) + (left 2) = (left 6) $= -6$

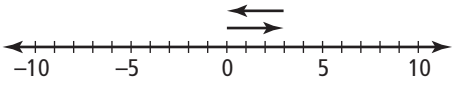
3. a) $+1$

b) $(+3) + (-2) =$ (right 3) + (left 2) = (right 1) $= +1$

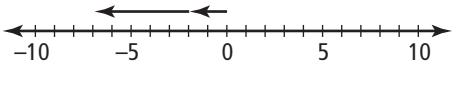
4. a) -1

b) $(-5) + (+4) =$ (left 5) + (right 4) = (left 1) $= -1$

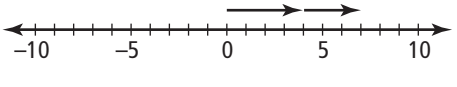
5. a) 0



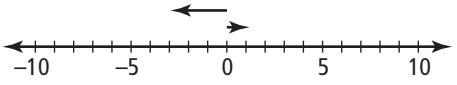
b) -7



c) $+7$



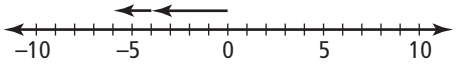
d) -3



6. Answers will vary.

Show You Know: Example 1

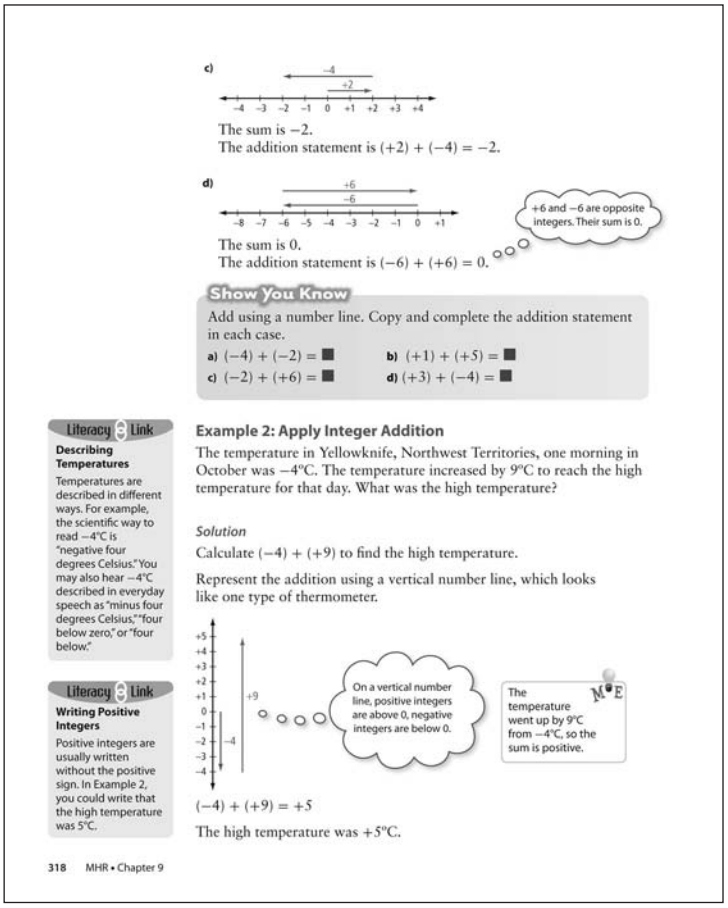
a) $(-4) + (-2) = -6$



Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss what they did during the Explore the Math, or read their explanations in #6.	• Number lines are integral to Chapter 9. Make sure that students have a clear understanding of how to use them for addition. • Some students may be able to demonstrate how they can use number lines to add integers, but have difficulty describing the process. As students demonstrate, verbalize what they are doing. Help students verbalize what they did and then ask them to record their explanation either verbally or in writing.

Web Link

For more words related to chinook jargon, go to **www.mathlinks7.ca** and follow the links.



c)

The sum is −2.
The addition statement is (+2) + (−4) = −2.

d)

+6 and −6 are opposite integers. Their sum is 0.

The sum is 0.
The addition statement is (−6) + (+6) = 0.

Show You Know

Add using a number line. Copy and complete the addition statement in each case.

a) $(-4) + (-2) =$ ■ b) $(+1) + (+5) =$ ■
c) $(-2) + (+6) =$ ■ d) $(+3) + (-4) =$ ■

Literacy Link

Describing Temperatures

Temperatures are described in different ways. For example, the scientific way to read −4°C is "negative four degrees Celsius." You may also hear −4°C described in everyday speech as "minus four degrees Celsius," "four below zero," or "four below."

Example 2: Apply Integer Addition

The temperature in Yellowknife, Northwest Territories, one morning in October was −4°C. The temperature increased by 9°C to reach the high temperature for that day. What was the high temperature?

Solution

Calculate (−4) + (+9) to find the high temperature.

Represent the addition using a vertical number line, which looks like one type of thermometer.

(−4) + (+9) = +5

The high temperature was +5°C.

Literacy Link

Writing Positive Integers

Positive integers are usually written without the positive sign. In Example 2, you could write that the high temperature was 5°C.

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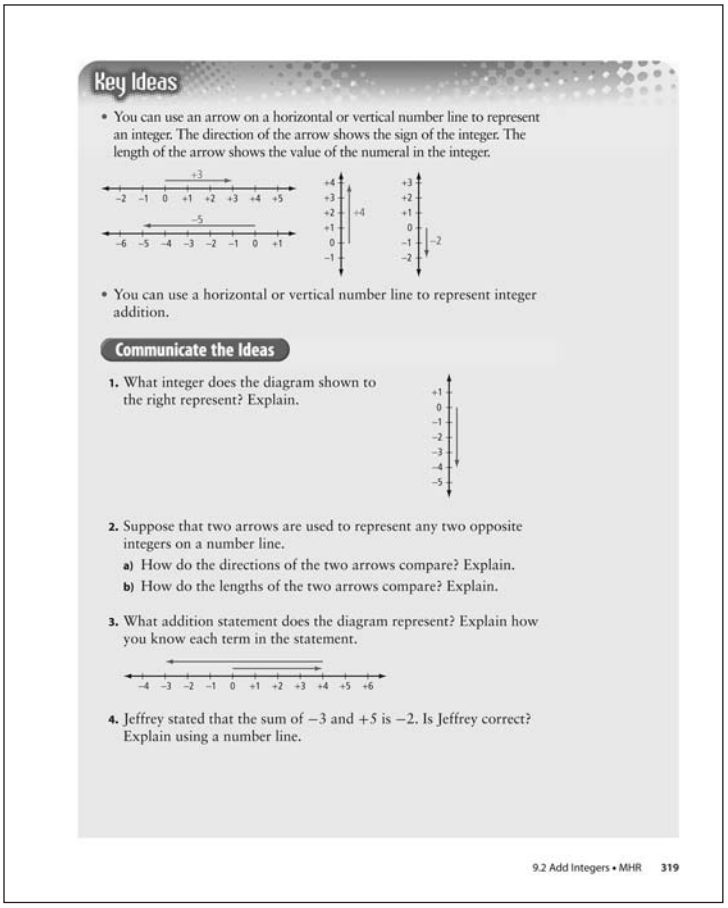
Example 1 illustrates how to add integers using a number line. The example uses a familiar problem solving strategy (i.e., Draw a Diagram). Ask students if they can think of another strategy they could use. For example, they could model the addition using integer chips (Model It). Alternatively, they could also walk up stairs for positive integers and down stairs for negative integers. If they do this, they will need to decide what stair represents zero.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• Have students talk through their thinking with a partner. • You may wish to provide additional questions very close to those in the Show You Know for students who would benefit from them: **a)** (−6) + (−6) (−12. When the integers have the same sign, add the number of chips and maintain the sign.) **b)** (+3) + (−4) (−1. Zero pairs come into effect when the integers combine positive and negative signs.) **c)** (+5) + (+4) (+9. Again, these integers have the same sign.) **d)** (−5) + (+5) (0. This reinforces the concept of zero pairs from Section 9.1.) Sit down and coach students through a) and b), and then have them try c) and d) on their own.



Key Ideas

- You can use an arrow on a horizontal or vertical number line to represent an integer. The direction of the arrow shows the sign of the integer. The length of the arrow shows the value of the numeral in the integer.

- You can use a horizontal or vertical number line to represent integer addition.

Communicate the Ideas

1. What integer does the diagram shown to the right represent? Explain.

2. Suppose that two arrows are used to represent any two opposite integers on a number line.
   a) How do the directions of the two arrows compare? Explain.
   b) How do the lengths of the two arrows compare? Explain.

3. What addition statement does the diagram represent? Explain how you know each term in the statement.

4. Jeffrey stated that the sum of −3 and +5 is −2. Is Jeffrey correct? Explain using a number line.

Example 2 illustrates an application of integer addition. Have students compare the values of the numerals in the integers to predict the sign of the sum before completing the addition. Make sure that students understand that the integer +9 represents the 9°C temperature increase.

Example 2 introduces vertical number lines. You might ask students to represent the addition in this example on a horizontal number line such as **Master 4 Vertical and Horizontal Number Lines**.

Key Ideas

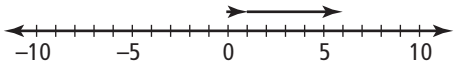
This section summarizes how to use arrows on vertical or horizontal number lines to represent integers. Emphasize that the direction of an arrow on a number line shows the sign of an integer and the length of an arrow shows the value of the numeral in the integer.

Communicate the Ideas

These questions allow students to apply their understanding of addition of integers using number lines and symbolic notation. In #1, students interpret a number line. In #2, students reinforce their understanding of the use of arrows on a number line. In #3, students interpret a number line. In #4, students use a number line to review a given solution for a problem.

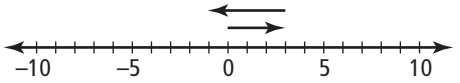
Answers

b) (+1) + (+5) = +6



c) (−2) + (+6) = +4

d) (+3) + (−4) = −1



Communicate the Ideas

1. −4. The direction of the arrow is below zero, the sign of the integer is negative. The length of the arrow is 4 units. The integer represented by the diagram is −4.

2. a) The arrows are in opposite directions from zero on the number line.
b) The arrows are the same length.

3. (+4) + (−7) = −3. The direction of each arrow represents the sign of the integer. The length of each arrow represents the numerical value of the integer. The first integer is represented by the bottom arrow and the second integer is represented by the top arrow.

4. Yes

Supported Learning

ESL and Language

- In the Literacy Link describing temperature on page 318, some students may not know that all of the ways of saying –4°C have the same meaning. Explicitly teach this information.

Gifted and Enrichment

- You might show students how to use technology for part b) of Example 1. The following key sequence is for a TI-30Xa calculator: [C] 5 [+/−] [+] 3 [+/−] [=]

  Students with other calculators may need to explore a way to enter negative integers on their calculators, and may need your assistance. Clarify that for most calculators the integer sign key and the subtraction key are not the same. Encourage students to write out the correct key sequence for their own calculator and to compare with other students. When students can successfully complete b) using their calculators, you might ask them to verify the answers to c) and d).

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 3–5, 7, 9, 10, Math Link
Typical	2–10, 13–15, 17–19, Math Link
Extension/Enrichment	2–4, 20–23

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have essential level students do #1, #3, and #4. These questions include concrete examples that students can use to generalize their understanding. Other students could do #2 to #4. Note that #2 is quite abstract. Encourage students to use a concrete example to help them generalize their understanding.	• In #4, encourage students to share their number lines with neighbouring students and listen to each other's explanations. • If you wish, have students work in small groups. • For #4, reinforce that the sum of opposite integers is zero. Ask students how the number line shows that this is true for any pair of opposite integers. (Moving in one direction and then moving an equal distance in the opposite direction always returns you to your original position. There is no overall change.)

Supported Learning

Learning Style, ESL, Language, and Memory

- Some students may have difficulty processing the steps in word problems and need some coaching.

Motor

- Provide students with a copy of **Master 4 Vertical and Horizontal Number Lines**.

Common Errors

- Some students may think that the arrow representing an integer on a number line must start at zero.

R_x Emphasize that the direction of an arrow shows the sign of an integer and the length of an arrow shows the value of the number in the integer. An arrow of a given direction and length represents the same integer no matter where the arrow starts.

- Some students may make mistakes in recording the signs of integers or omit the signs altogether.

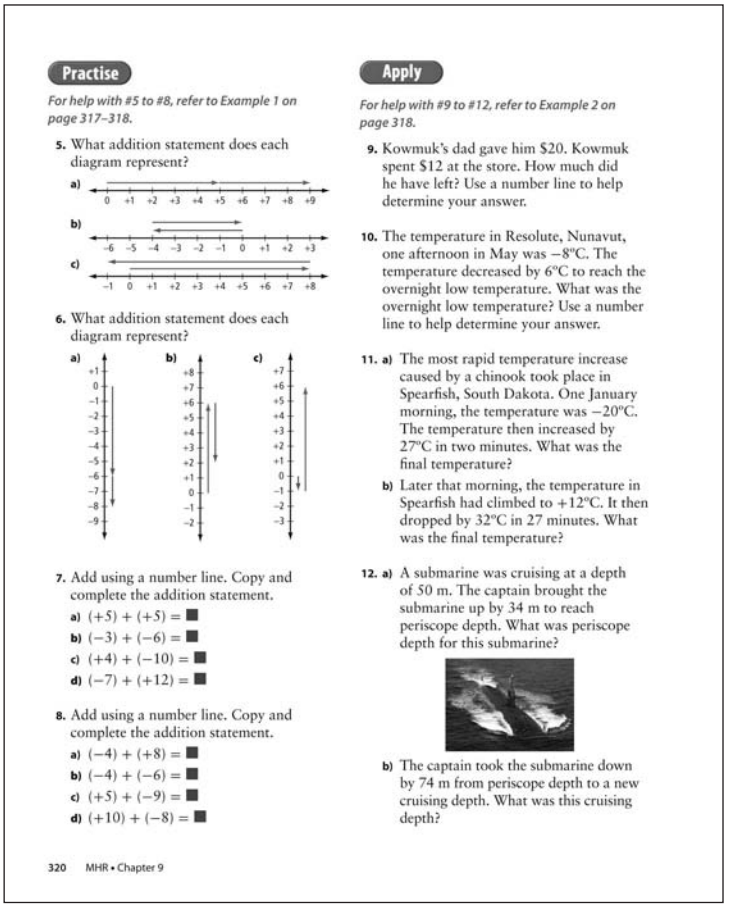
R_x Show how manipulatives or a diagram indicate the sign of a sum and stress the importance of recording the signs. For the addition of a positive integer and a negative integer, have students compare the values of the numerals in the integers and predict the sign of the sum.

- Some students may have difficulty with word problems.

R_x Suggest that students read each problem carefully, and then list what they know. Have students estimate the answer.

- Some students may try to move too quickly to the symbolic method for adding integers and make mistakes as a result.

R_x Encourage students to continue using concrete or semi-concrete methods until they have a clear understanding of how to add integers.



Practise

For help with #5 to #8, refer to Example 1 on page 317–318.

5. What addition statement does each diagram represent?

a)

b)

c)

6. What addition statement does each diagram represent?

a) b) c)

7. Add using a number line. Copy and complete the addition statement.

a) $(+5) + (+5) =$ ■
b) $(-3) + (-6) =$ ■
c) $(+4) + (-10) =$ ■
d) $(-7) + (+12) =$ ■

8. Add using a number line. Copy and complete the addition statement.

a) $(-4) + (+8) =$ ■
b) $(-4) + (-6) =$ ■
c) $(+5) + (-9) =$ ■
d) $(+10) + (-8) =$ ■

Apply

For help with #9 to #12, refer to Example 2 on page 318.

9. Kowmuk's dad gave him $20. Kowmuk spent $12 at the store. How much did he have left? Use a number line to help determine your answer.

10. The temperature in Resolute, Nunavut, one afternoon in May was −8°C. The temperature decreased by 6°C to reach the overnight low temperature. What was the overnight low temperature? Use a number line to help determine your answer.

11. a) The most rapid temperature increase caused by a chinook took place in Spearfish, South Dakota. One January morning, the temperature was −20°C. The temperature then increased by 27°C in two minutes. What was the final temperature?

b) Later that morning, the temperature in Spearfish had climbed to +12°C. It then dropped by 32°C in 27 minutes. What was the final temperature?

12. a) A submarine was cruising at a depth of 50 m. The captain brought the submarine up by 34 m to reach periscope depth. What was periscope depth for this submarine?

b) The captain took the submarine down by 74 m from periscope depth to a new cruising depth. What was this cruising depth?

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Practise and Apply

Assessment *for* Learning	Supported Learning
Practise and Apply Have students do #5, #7, #9, and #10. Students who have no problems with these questions can go on to the balance of the Apply questions.	• Students who have problems with #5 and #7 will need additional coaching. Have students explain their thinking on these questions; clarify any misunderstandings. Coach students through #6a) and #8a), and then have them complete parts b) and c) on their own. Have students refer back to examples in the student resource. Check back with them several times to make sure that they understand the concepts. • In #7 and #8, encourage students to use the orientation of number line they prefer and stress that both can be used to add integers. • If students have difficulty with the real-world applications in #9 and #10, refer them to #5 in Get Ready. Give them more practice with questions of this type by doing #11a) and #12a) with them, and then having them do #11b) and #12b) on their own.

13. What is the sum of +1987 and −1987? How do you know?

14. The sum of two integers is −1.
   a) What could the integers be? Give four possible answers.
   b) Are there more possible answers? Explain.

15. a) Copy and complete the pattern.
   (+9) + (−5) = ■
   (+8) + (−4) = ■
   (+7) + (−3) = ■
   (+6) + (−2) = ■
   b) Describe and explain the pattern.
   c) What are the next three lines of the pattern?

16. The integers from +1 to +6 are marked on the faces of a number cube. The integers from −1 to −6 are marked on the faces of another number cube. What possible totals can you score by rolling both number cubes?

17. Suppose a friend knows how to add positive integers but has never added negative integers.
   a) How could you use the following pattern to show your friend how to add +6 and −2?
   (+6) + (+3) = +9
   (+6) + (+2) = +8
   (+6) + (+1) = +7
   (+6) + 0 = +6
   (+6) + (−1) = ■
   (+6) + (−2) = ■
   b) Make up a pattern to show your friend how to add +2 and −4.
   c) Make up a pattern to show your friend how to add −3 and −5.

18. Is each statement always true, sometimes true, or never true? Explain your reasoning.
   a) The sum of two negative integers is a positive integer.
   b) The sum of a positive integer and a negative integer is zero.
   c) The sum of a positive integer and zero is a positive integer.
   d) The sum of a positive integer and a negative integer is a negative integer.

19. Suppose you had to choose one way to model each of the following additions. Would you choose integer chips or a number line? Explain.
   a) (−6) + (+4)
   b) (+90) + (−140)

Extend

20. Use a number line to add the following integers. Use integer chips to check your answers.
   a) (+4) + (+3) + (+7)
   b) (−5) + (−7) + (−4)
   c) (+6) + (−9) + (+5)
   d) (−6) + (+3) + (−8) + (+2)

21. Copy and complete each statement.
   a) (+4) + (■) = +9
   b) (■) + (+4) = −2
   c) (−2) + (■) = −7
   d) +6 = (■) + (−3)
   e) (+8) + (■) = +2
   f) −1 = (−5) + (■)

Apply and Extend

If students are unsure of the order in which they should write the integers in an addition, remind them of their findings in #13 in Section 9.1.

Have students do #16 as a game by providing two dice of different colours. The red die could be positive and the blue die negative. You might use this game to replace some basic practice questions. If you have already covered Chapter 5 on Probability, you might extend the question by having students consider the probability of rolling each total.

The emphasis of Section 9.2 is on modelling integer addition semi-concretely. However, #17 uses a symbolic approach by starting with the familiar addition of integers and then using patterning to add negative integers. Encourage students to generalize their findings from this question.

In #19b), some students may choose integer chips and suggest the use of one red chip to represent +10 and one blue chip to represent −10. This avoids using large numbers of chips. You might ask students if the answer would be the same for the addition of (+93) + (−137).

Supported Learning

Learning Style and Memory

- Question 16 can be played as a game, thus providing interesting practice for students.
- Provide **BLM 9–4 Section 9.2 Extra Practice** to students who require more practice.

Answers

Math Link

a) −2 **b)** +3 **c)** −5 **d)** −4

Supported Learning

Learning Style and Memory

- Consider having students play the Integer Word Game described on page 346 to reinforce addition of integers.
- Have students play the game outlined in the Web Link.

ESL and Language

- Direct students to the Literacy Link that explains consecutive numbers.
- Write some examples of consecutive numbers on the board, and then ask students for other examples. Have English language learners add *consecutive* to their dictionary.

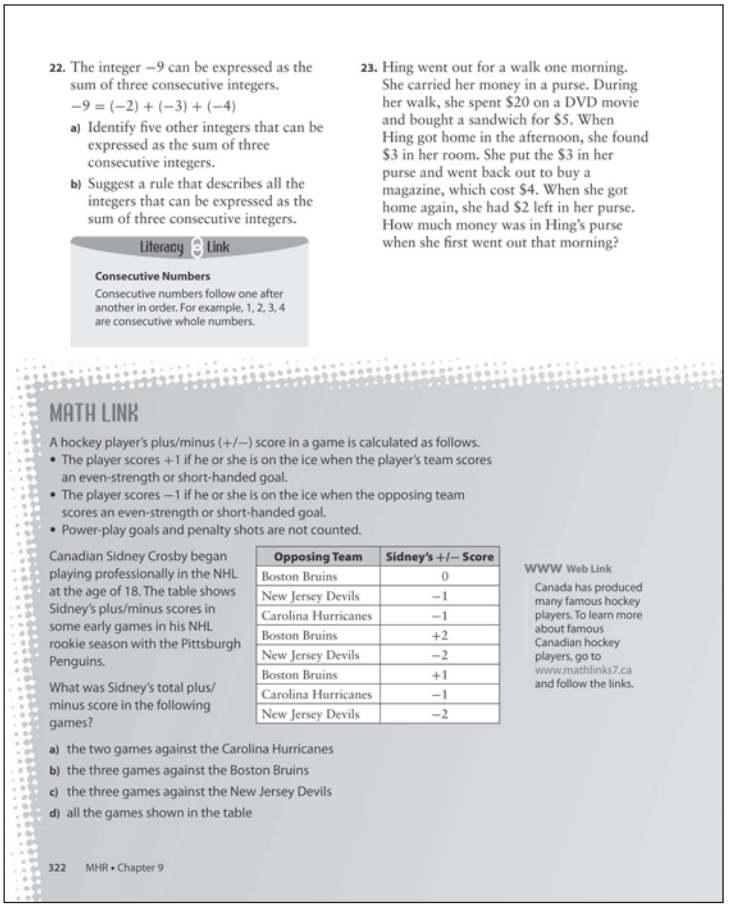
Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • What methods do you know how to use for adding two integers? • What do you like and dislike about each method?	• Encourage concrete learners to use integer chips and number lines to help them answer the questions. • Depending on students' learning style, have them provide verbal or written answers. • You may wish to have students review the part related to Section 9.2 in **BLM 9–1 Chapter 9 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

MATH LINK

This Math Link provides background information on plus/minus scores that students will need to help them do the chapter problem titled Wrap It Up!

WWW Web Link

For an online game that allows students to drag positive or negative chips to the working area and combine them in pairs to see the sum, go to **www.mathlinks7.ca** and follow the links.



22. The integer −9 can be expressed as the sum of three consecutive integers.
$-9 = (-2) + (-3) + (-4)$
a) Identify five other integers that can be expressed as the sum of three consecutive integers.
b) Suggest a rule that describes all the integers that can be expressed as the sum of three consecutive integers.

Literacy Link
Consecutive Numbers
Consecutive numbers follow one after another in order. For example, 1, 2, 3, 4 are consecutive whole numbers.

23. Hing went out for a walk one morning. She carried her money in a purse. During her walk, she spent $20 on a DVD movie and bought a sandwich for $5. When Hing got home in the afternoon, she found $3 in her room. She put the $3 in her purse and went back out to buy a magazine, which cost $4. When she got home again, she had $2 left in her purse. How much money was in Hing's purse when she first went out that morning?

MATH LINK

A hockey player's plus/minus (+/−) score in a game is calculated as follows.
- The player scores +1 if he or she is on the ice when the player's team scores an even-strength or short-handed goal.
- The player scores −1 if he or she is on the ice when the opposing team scores an even-strength or short-handed goal.
- Power-play goals and penalty shots are not counted.

Canadian Sidney Crosby began playing professionally in the NHL at the age of 18. The table shows Sidney's plus/minus scores in some early games in his NHL rookie season with the Pittsburgh Penguins.

Opposing Team	Sidney's +/− Score
Boston Bruins	0
New Jersey Devils	−1
Carolina Hurricanes	−1
Boston Bruins	+2
New Jersey Devils	−2
Boston Bruins	+1
Carolina Hurricanes	−1
New Jersey Devils	−2

What was Sidney's total plus/minus score in the following games?
a) the two games against the Carolina Hurricanes
b) the three games against the Boston Bruins
c) the three games against the New Jersey Devils
d) all the games shown in the table

WWW Web Link
Canada has produced many famous hockey players. To learn more about famous Canadian hockey players, go to www.mathlinks7.ca and follow the links.

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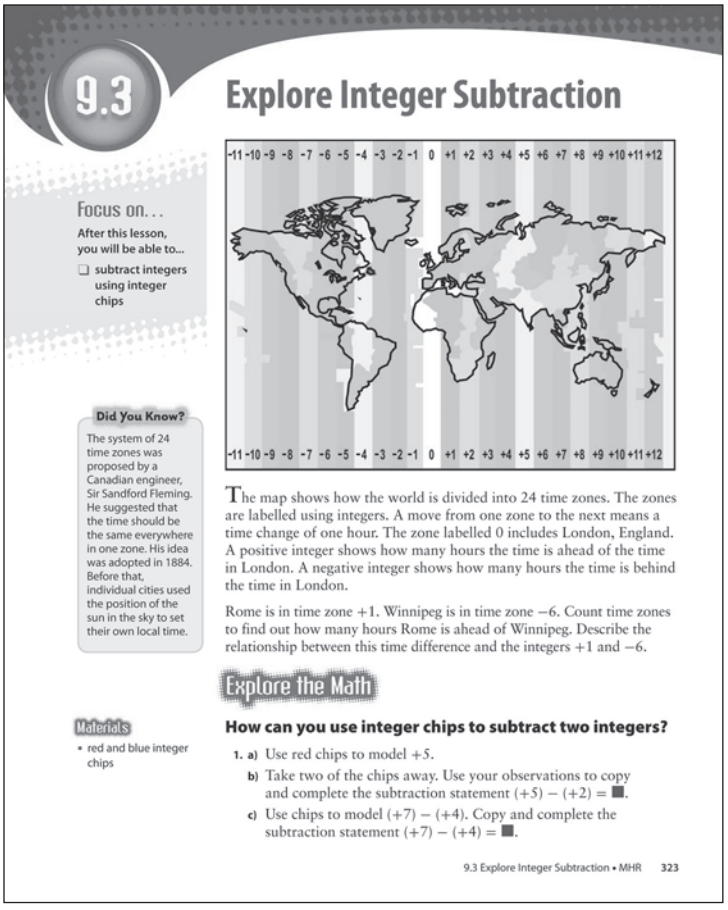
In #23, you may wish to have students compare strategies for solving the problem. Among the possible strategies are to work backward or use guess and check, and possibly include the use of a table. Some students may prefer using concrete materials to act out the problem or to check their answers.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 322 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 345. Alternatively, if you assign only the Wrap It Up! problem, ensure that students are aware of the background information on plus/minus scores from the Math Link in Section 9.2.	• You may wish to have students do this Math Link in order to apply their understanding of adding integers. As they work on the Math Link, observe and have them self-observe for how well they solve the problem. • Students who are having difficulty getting started could use **BLM 9–5 Section 9.2 Math Link**, which provides scaffolding for this activity.

9.3 Explore Integer Subtraction



9.3 Explore Integer Subtraction

Focus on...

After this lesson, you will be able to...

- subtract integers using integer chips

-11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9 +10 +11 +12

Did You Know?

The system of 24 time zones was proposed by a Canadian engineer, Sir Sandford Fleming. He suggested that the time should be the same everywhere in one zone. His idea was adopted in 1884. Before that, individual cities used the position of the sun in the sky to set their own local time.

The map shows how the world is divided into 24 time zones. The zones are labelled using integers. A move from one zone to the next means a time change of one hour. The zone labelled 0 includes London, England. A positive integer shows how many hours the time is ahead of the time in London. A negative integer shows how many hours the time is behind the time in London.

Rome is in time zone +1. Winnipeg is in time zone −6. Count time zones to find out how many hours Rome is ahead of Winnipeg. Describe the relationship between this time difference and the integers +1 and −6.

Explore the Math

Materials

- red and blue integer chips

How can you use integer chips to subtract two integers?

1. a) Use red chips to model +5.
   b) Take two of the chips away. Use your observations to copy and complete the subtraction statement (+5) − (+2) = ■.
   c) Use chips to model (+7) − (+4). Copy and complete the subtraction statement (+7) − (+4) = ■.

9.3 Explore Integer Subtraction • MHR 323

Suggested Timing
60–75 minutes

Materials
- red and blue integer chips
- red and blue coloured pencils (optional)
- transparent chips (optional)

Blackline Masters
BLM 9–1 Chapter 9 Self-Assessment
BLM 9–6 Time Zone Map
BLM 9–7 Section 9.3 Extra Practice

Mathematical Processes

- [x] Communication
- [x] Connections
- [ ] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization

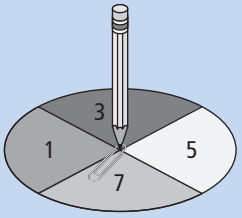
Specific Outcomes

N6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.

Warm-Up

Add #1 to #3 using a number line.

1. (+5) + (+5)
2. (–4) + (+8)
3. (–3) + (–5)
4. This spinner is spun twice. What is the probability that the sum of the numbers is even?



5. Calculate the area of a parallelogram with a base of 1.2 m and a height of 4 m.

Mental Math

6. A circle graph has a 27% sector. Estimate the internal angle of that sector.
7. Sketch the angle you calculated in #6. Explain your thinking.
8. List six multiples for 3 and 6. Then show the least common multiple.
9. List the numbers divisible by 9. How do you know? 45, 79, 81, 2349, 3996, 18 764, 73 944
10. Estimate 563 out of 900 as a percent. Show your thinking.

Answers

Warm-Up

1. (+10) **2.** (+4) **3.** (−8)

4. Students should use a table, tree diagram, or another organizer to show the outcomes. (Sums are in brackets.)
P (even sums) = 100%

	1	3	5	7
1	1, 1 (2)	1, 3 (4)	1, 5 (6)	1, 7 (8)
3	3, 1 (4)	3, 3 (6)	3, 5 (8)	3, 7 (10)
5	5, 1 (6)	5, 3 (8)	5, 5 (10)	5, 7 (12)
7	7, 1 (8)	7, 3 (10)	7, 5 (12)	7, 7 (14)

5. 4.8 m^2

6. 27% is close to 25% or $\frac{1}{4}$. 360° ÷ 4 = 90°

7. Students should sketch a right angle and explain that the angle would be close to the angle in the corner of a book.

8. 3 = 3, 6, 9, 12, 15, 18; 6 = 6, 12, 18, 24, 30, 36.
Least common multiple = 6

9. 45, 81, 2349, 3996, 73 944. The digits of these numbers add up to a number divisible by 9.

10. 50% of 900 = 450
10% of 900 = 90
5% = 45
60% = 450 + 90 = 540. Too low
65% = 450 + 90 + 45 = 585. Too high
Between 60% and 65%.

Explore the Math

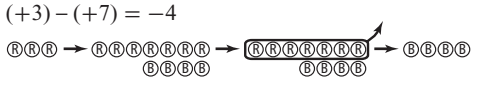
1. a) 5 red chips
b) (+5) − (+2) = +3
c) (+7) − (+4) = +3

2. a) 6 blue chips
b) (−6) − (−4) = −2
c) (−5) − (−2) = −3

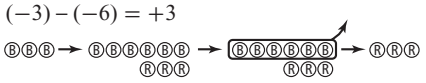
3. a) No. Answers will vary. For example: The second integer is larger than the first integer. The subtraction can be completed by using zero pairs.
b) No. Answers will vary. For example: The second integer is larger than the first integer. The subtraction can be completed by using zero pairs.

4. a) Start with the first integer, add 3 zero pairs, and remove the 5 red chips. What remains is the difference.
(+2) − (+5) = −3
b) Start with the first integer, add 2 zero pairs, and remove the 3 blue chips. What remains is the difference.
(−1) − (−3) = +2

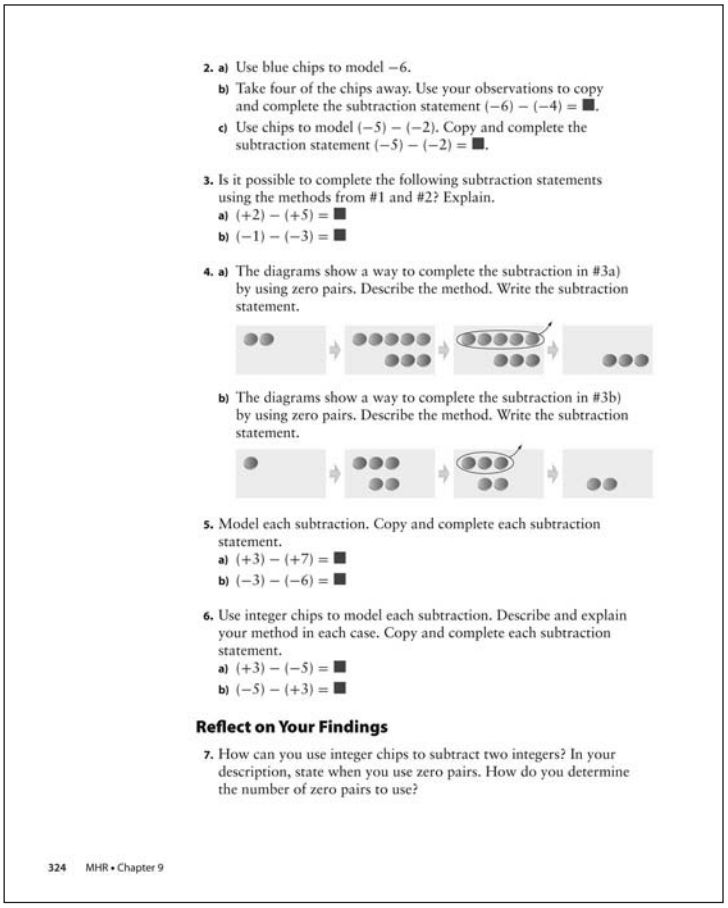
5. a) (+3) − (+7) = −4



b) (−3) − (−6) = +3



6. a) Start with the first integer. Add 5 zero pairs. Subtract the 5 blue chips. The result is 8 red chips.
(+3) − (−5) = +8



2. a) Use blue chips to model −6.
b) Take four of the chips away. Use your observations to copy and complete the subtraction statement (−6) − (−4) = ■.
c) Use chips to model (−5) − (−2). Copy and complete the subtraction statement (−5) − (−2) = ■.

3. Is it possible to complete the following subtraction statements using the methods from #1 and #2? Explain.
a) (+2) − (+5) = ■
b) (−1) − (−3) = ■

4. a) The diagrams show a way to complete the subtraction in #3a) by using zero pairs. Describe the method. Write the subtraction statement.

b) The diagrams show a way to complete the subtraction in #3b) by using zero pairs. Describe the method. Write the subtraction statement.

5. Model each subtraction. Copy and complete each subtraction statement.
a) (+3) − (+7) = ■
b) (−3) − (−6) = ■

6. Use integer chips to model each subtraction. Describe and explain your method in each case. Copy and complete each subtraction statement.
a) (+3) − (−5) = ■
b) (−5) − (+3) = ■

Reflect on Your Findings

7. How can you use integer chips to subtract two integers? In your description, state when you use zero pairs. How do you determine the number of zero pairs to use?

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Activity Planning Notes

Consider having students use **BLM 9–6 Time Zone Map** to help answer the questions. Have students describe the relationship between the time zones and the time differences. They may suggest that time difference is the sum of two integers. Prompt students to consider that the word *difference* implies subtraction.

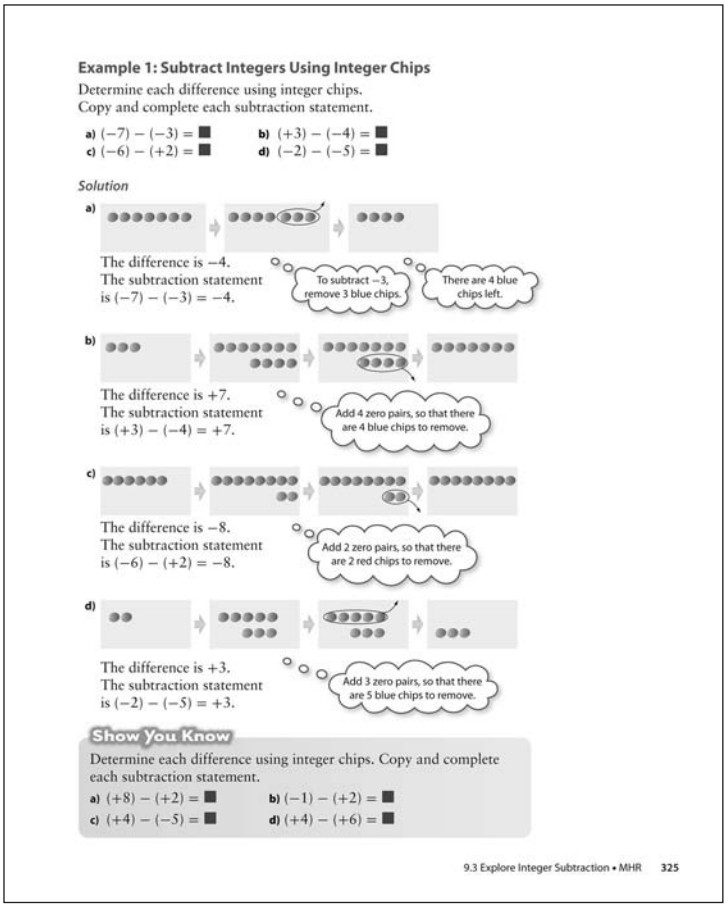
After the investigation, have students revisit the opener and subtract (+1) − (−6) = +7 to show that Rome is 7 h ahead of Winnipeg, or subtract (−6) − (+1) = −7 to show that Winnipeg is 7 h behind Rome.

Explore the Math

Students develop a model for subtracting integers.

Method 1: Have students work in pairs. Give each pair a set of integer chips (at least eight of each colour), or other manipulatives.

Method 2: Have students draw and colour circles to represent integers. Have them cross out or cover up any chips that need to be removed.



Example 1: Subtract Integers Using Integer Chips

Determine each difference using integer chips.
Copy and complete each subtraction statement.

a) $(-7) - (-3) =$ ■ b) $(+3) - (-4) =$ ■
c) $(-6) - (+2) =$ ■ d) $(-2) - (-5) =$ ■

Solution

a)
The difference is −4.
The subtraction statement is $(-7) - (-3) = -4$.
To subtract −3, remove 3 blue chips.
There are 4 blue chips left.

b)
The difference is +7.
The subtraction statement is $(+3) - (-4) = +7$.
Add 4 zero pairs, so that there are 4 blue chips to remove.

c)
The difference is −8.
The subtraction statement is $(-6) - (+2) = -8$.
Add 2 zero pairs, so that there are 2 red chips to remove.

d)
The difference is +3.
The subtraction statement is $(-2) - (-5) = +3$.
Add 3 zero pairs, so that there are 5 blue chips to remove.

Show You Know

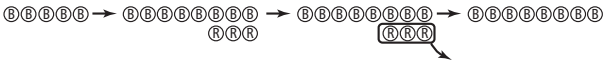
Determine each difference using integer chips. Copy and complete each subtraction statement.

a) $(+8) - (+2) =$ ■ b) $(-1) - (+2) =$ ■
c) $(+4) - (-5) =$ ■ d) $(+4) - (+6) =$ ■

9.3 Explore Integer Subtraction • MHR 325

Answers

b) Start with the first integer. Add 3 zero pairs. Subtract 3 red chips. The result is 8 blue chips. $(-5) - (+3) = -8$



7. Answers will vary.

Show You Know: Example 1

a) $(+8) - (+2) = +6$ **b)** $(-1) - (+2) = -3$
c) $(+4) - (-5) = +9$ **d)** $(+4) - (+6) = -2$

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students describe how to use integer chips to subtract two integers. Check student responses to #7 for ability to generalize what they learned during the activity.	• You may wish to have students demonstrate subtracting using integer chips. Have them record how to tell which problems need zero pairs. For example, if they have one negative integer chip, they can't take three negative integer chips away from it. • Students may add more zero pairs than necessary. This is not a problem as it will not affect the answer.

Method 3: Demonstrate subtracting integers by using transparent chips on an overhead projector. Have students draw and colour circles for each example you show, and write the corresponding subtraction statement.

Example 1 illustrates subtracting integers using integer chips. The example uses a strategy (i.e., Model It).

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• Have students talk through their thinking with their partner. • You may wish to provide additional questions very close to those in the Show You Know for students who would benefit from them: **a)** (–5) – (+3) (–8. Add 3 zero pairs, so that there are 3 red chips to remove.) **b)** (–1) – (+5) (–6. Add 5 zero pairs, so that there are 5 red chips to remove.) **c)** (–6) – (–2) (–4. Add 4 zero pairs, so that there are 2 blue chips to remove.) **d)** (+2) – (–5) (+7. Add 5 zero pairs, so that there are 5 blue chips to remove.)

Supported Learning

Learning Style

- **BLM 9–6 Time Zone Map** shows the location of other cities. You may wish to use an overhead of the map and provide extra oral practice by having students walk across the time zones to answer questions you pose.

ESL and Language

- English language learners may have difficulty with the terms *time zones* and *below street level.* Use the Did You Know? on page 323 to help explain time zones.

Meeting the Needs of All Learners

- Some students may have difficulty processing the examples and need coaching. Pay particular attention to the concept of adding zero pairs.

Common Errors

- Some students may ignore the signs of the integers they are subtracting.

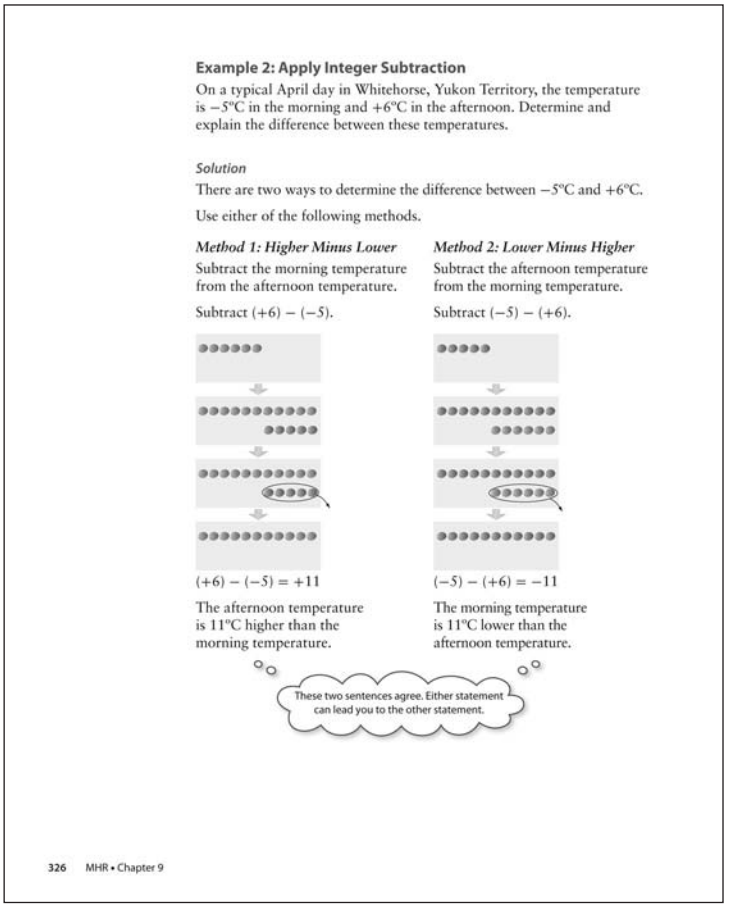
R_x Reinforce the use of two colours of manipulatives to represent positive and negative integers.

- Some students may not grasp that the order of subtraction of two different integers affects the value of the difference (unlike in addition, where the order of the terms does not matter).

R_x Reinforce the significance of using the two colours of manipulatives to represent positive and negative integers. Have students practise modelling the subtraction of two different integers in both orders, so that they see the two results represented by equal numbers of integer chips of different colours. In other words, the two differences are opposite integers.

- Some students may have difficulty deciding how many zero pairs to add when modelling a subtraction.

R_x Stress that a zero pair represents zero. Adding any number of zero pairs does not change the value that a model represents. Students may need to use trial and error at first to find the minimum number of zero pairs to add. (You might mention that there is no maximum number. If students add too many zero pairs, they can just remove them without affecting the value that a model represents. It is most efficient to add the minimum number of zero pairs to avoid removing excess zero pairs.) As students become better at adding zero pairs, encourage them to develop strategies for deciding the minimum number of zero pairs to add.



Example 2: Apply Integer Subtraction

On a typical April day in Whitehorse, Yukon Territory, the temperature is −5°C in the morning and +6°C in the afternoon. Determine and explain the difference between these temperatures.

Solution

There are two ways to determine the difference between −5°C and +6°C.

Use either of the following methods.

Method 1: Higher Minus Lower

Subtract the morning temperature from the afternoon temperature.

Subtract (+6) − (−5).

(+6) − (−5) = +11

The afternoon temperature is 11°C higher than the morning temperature.

Method 2: Lower Minus Higher

Subtract the afternoon temperature from the morning temperature.

Subtract (−5) − (+6).

(−5) − (+6) = −11

The morning temperature is 11°C lower than the afternoon temperature.

These two sentences agree. Either statement can lead you to the other statement.

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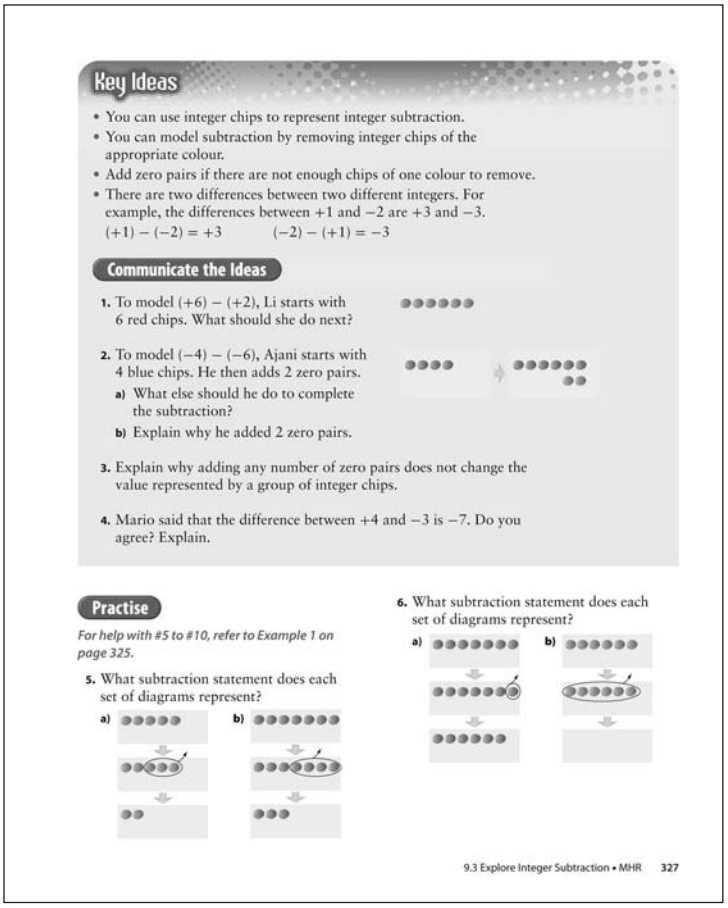
Example 2 provides two methods for subtracting integers: Method 1: Higher Minus Lower and Method 2: Lower Minus Higher. The two methods show that two integers can be subtracted in either order in a word problem unless the problem specifies an order, such as when an incomplete subtraction statement is provided that shows the order of subtraction. The emphasis in Example 2 is on explaining the meaning of the two differences and showing that the meanings are consistent with each other.

Make sure to stress both interpretations of the difference when the order in which the subtraction is performed is not important, that is, when the wording of the question does not specify the order. Help students understand that the two interpretations in Example 2 are consistent with each other.

You might ask students which method they would use to find how many degrees the afternoon temperature is above the morning temperature. Many students may say Method 1 but Method 2 is also acceptable. The summary statement from Method 2 readily leads to the summary statement from Method 1. Encourage students to interpret the integer answers to word problems and make sure that the final statement in the solution is consistent with the wording in the problem.



Key Ideas

- You can use integer chips to represent integer subtraction.
- You can model subtraction by removing integer chips of the appropriate colour.
- Add zero pairs if there are not enough chips of one colour to remove.
- There are two differences between two different integers. For example, the differences between +1 and −2 are +3 and −3.
  (+1) − (−2) = +3 (−2) − (+1) = −3

Communicate the Ideas

1. To model (+6) − (+2), Li starts with 6 red chips. What should she do next?
2. To model (−4) − (−6), Ajani starts with 4 blue chips. He then adds 2 zero pairs.
   a) What else should he do to complete the subtraction?
   b) Explain why he added 2 zero pairs.
3. Explain why adding any number of zero pairs does not change the value represented by a group of integer chips.
4. Mario said that the difference between +4 and −3 is −7. Do you agree? Explain.

Practise

For help with #5 to #10, refer to Example 1 on page 325.

5. What subtraction statement does each set of diagrams represent?
   a) b)
6. What subtraction statement does each set of diagrams represent?
   a) b)

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Answers

Communicate the Ideas

1. Add two zero pairs.

2. **a)** Remove six blue chips.

b) Answers will vary. For example: To complete the statement he needs to add two zero pairs so he would have six blue chips to remove.

3. Answers will vary. For example: Zero does not add value to any number.

4. Answers will vary. For example: Yes. One of the two differences is (−3) − (+4) = −7.
Start with 3 blue chips. Add 4 zero pairs. Remove 4 red chips. There are 7 blue chips left.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–5, 7, 9, 11, 13–15, 18
Typical	1–5, 7, 9, 11, 13–18
Extension/Enrichment	1–4, 19–21

Key Ideas

This section summarizes how to use integer chips to represent integer subtraction. Stress that there are two differences between two different integers. Have students explain the meanings of the two differences shown in Key Ideas. (The difference of +3 shows that +1 is 3 more than −2. The difference of −3 shows that −2 is 3 less than +1. The meanings of the two differences are consistent with each other.)

Communicate the Ideas

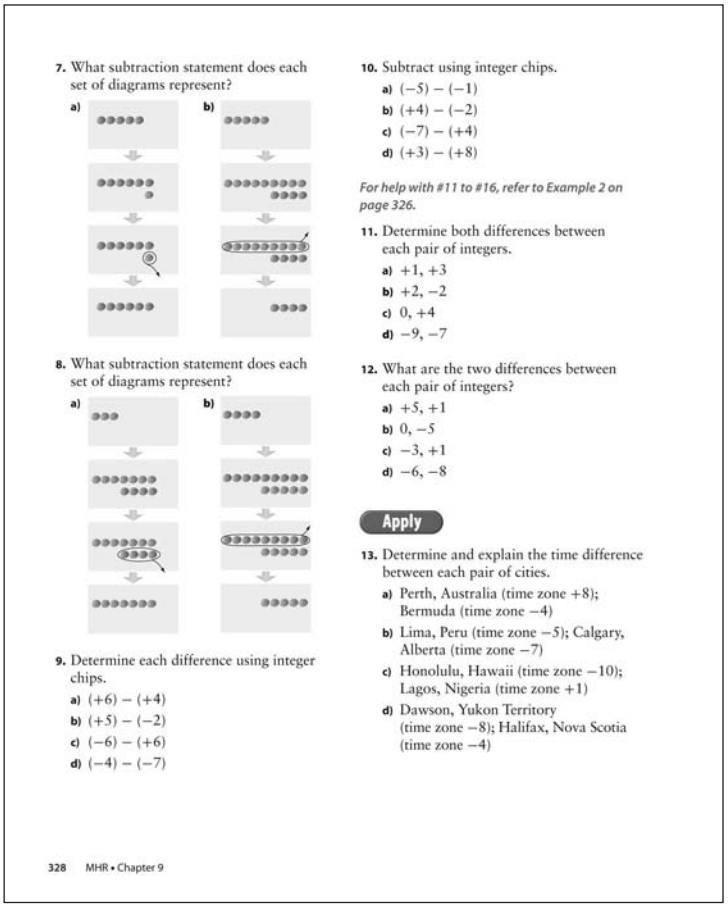
These questions allow students to apply their understanding of integer subtraction using integer chips. In #1 and #2, students explain how to use integer chips for subtraction. In #3, students check their understanding of zero pairs. In #4, students review a solution to a problem.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Students should answer #1 to #4, as they are all key questions. Make sure that students understand the concepts. Consider allowing them to use integer chips to help support their answers.	• Consider having students work in a group to complete and then discuss the answers. • In #4, encourage students to discuss their answer with neighbouring students and listen to each other's explanations. • In #4, students should find that there are two differences between +4 and −3. You might ask them to explain the meanings of the two differences and to check that they are consistent with each other.

Supported Learning

Learning Style and Memory

- Provide **BLM 9–7 Section 9.3 Extra Practice** to students who require more practice.



7. What subtraction statement does each set of diagrams represent?

a) b)

8. What subtraction statement does each set of diagrams represent?

a) b)

9. Determine each difference using integer chips.
   a) (+6) − (+4)
   b) (+5) − (−2)
   c) (−6) − (+6)
   d) (−4) − (−7)

10. Subtract using integer chips.
   a) (−5) − (−1)
   b) (+4) − (−2)
   c) (−7) − (+4)
   d) (+3) − (+8)

For help with #11 to #16, refer to Example 2 on page 326.

11. Determine both differences between each pair of integers.
   a) +1, +3
   b) +2, −2
   c) 0, +4
   d) −9, −7

12. What are the two differences between each pair of integers?
   a) +5, +1
   b) 0, −5
   c) −3, +1
   d) −6, −8

Apply

13. Determine and explain the time difference between each pair of cities.
   a) Perth, Australia (time zone +8); Bermuda (time zone −4)
   b) Lima, Peru (time zone −5); Calgary, Alberta (time zone −7)
   c) Honolulu, Hawaii (time zone −10); Lagos, Nigeria (time zone +1)
   d) Dawson, Yukon Territory (time zone −8); Halifax, Nova Scotia (time zone −4)

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WWW Web Link

For online activities that provide games that use subtraction of integers, go to **www.mathlinks7.ca** and follow the links.

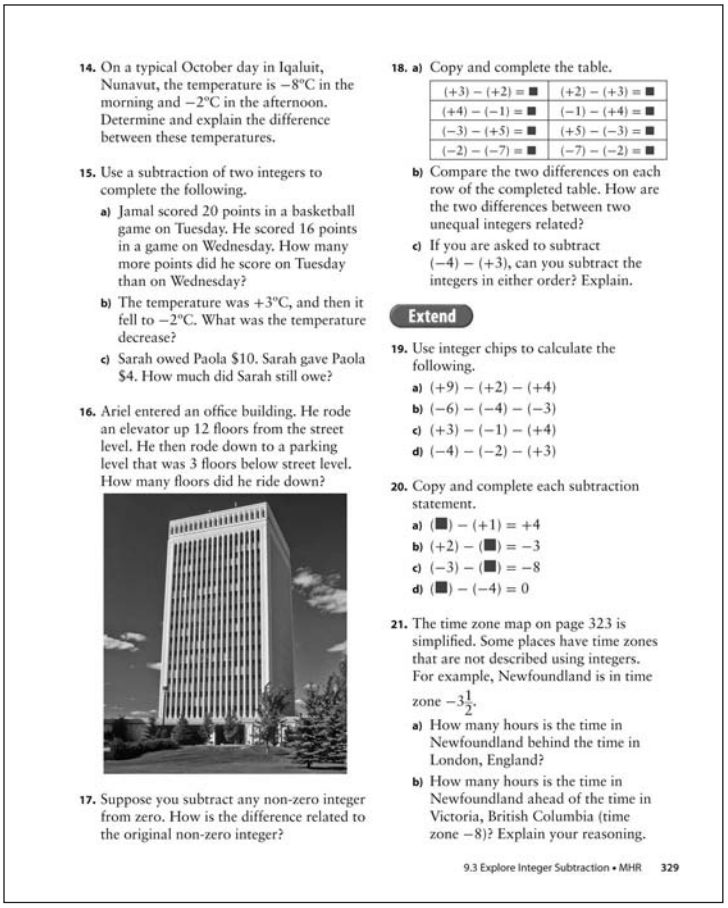
Practise

Assessment *for* Learning	Supported Learning
Practise Have students do #5, #7, #9, and #11. Students who have no problems with these questions can go on to the Apply questions. Students will need integer chips.	• Students who have problems with #5, #7, #9, and #11 will need additional coaching. Have students explain their thinking on these questions; clarify any misunderstandings. Coach students through #6a), #8a), #10a), and #12a), and then have them complete the remaining parts of those questions on their own. Have students refer back to examples in the student resource. Check back with them several times to make sure that they understand the concepts. • In #5 to #10, there is only one correct answer for each part (i.e., the order of subtraction is specified). However, in #11 and #12, students must find both differences between two different integers. Ask students who have difficulty with #11 and #12 to explain the meanings of the two differences and to check that they are consistent with each other.

Apply and Extend

In #13 to #16, students work with real-world applications. Consider having students use **BLM 9–6 Time Zone Map** to help answer #13 and #21. In #21, students apply their understanding of subtraction of integers by working with fractions.

In #13 and #14, the order of the subtraction is not specified, and students can determine and explain either difference. You might have students share answers and check that their explanations are consistent with each other.



14. On a typical October day in Iqaluit, Nunavut, the temperature is −8°C in the morning and −2°C in the afternoon. Determine and explain the difference between these temperatures.

15. Use a subtraction of two integers to complete the following.
a) Jamal scored 20 points in a basketball game on Tuesday. He scored 16 points in a game on Wednesday. How many more points did he score on Tuesday than on Wednesday?
b) The temperature was +3°C, and then it fell to −2°C. What was the temperature decrease?
c) Sarah owed Paola $10. Sarah gave Paola $4. How much did Sarah still owe?

16. Ariel entered an office building. He rode an elevator up 12 floors from the street level. He then rode down to a parking level that was 3 floors below street level. How many floors did he ride down?

17. Suppose you subtract any non-zero integer from zero. How is the difference related to the original non-zero integer?

18. a) Copy and complete the table.

(+3) − (+2) = ■	(+2) − (+3) = ■
(+4) − (−1) = ■	(−1) − (+4) = ■
(−3) − (+5) = ■	(+5) − (−3) = ■
(−2) − (−7) = ■	(−7) − (−2) = ■

b) Compare the two differences on each row of the completed table. How are the two differences between two unequal integers related?
c) If you are asked to subtract (−4) − (+3), can you subtract the integers in either order? Explain.

Extend

19. Use integer chips to calculate the following.
a) (+9) − (+2) − (+4)
b) (−6) − (−4) − (−3)
c) (+3) − (−1) − (+4)
d) (−4) − (−2) − (+3)

20. Copy and complete each subtraction statement.
a) (■) − (+1) = +4
b) (+2) − (■) = −3
c) (−3) − (■) = −8
d) (■) − (−4) = 0

21. The time zone map on page 323 is simplified. Some places have time zones that are not described using integers. For example, Newfoundland is in time zone $-3\frac{1}{2}$.
a) How many hours is the time in Newfoundland behind the time in London, England?
b) How many hours is the time in Newfoundland ahead of the time in Victoria, British Columbia (time zone −8)? Explain your reasoning.

9.3 Explore Integer Subtraction • MHR 329

Assessment *as* Learning	Supported Learning
Math Learning Log Have students complete the following statements: • The part I understand the best about subtracting integers is ... • What I find most confusing about subtracting integers is ...	• Have students refer back to the What I Need to Work On tab of their chapter Foldable to answer the second question. • Concrete and kinesthetic learners may need to use integer chips and number lines to help them explain what is clear and not so clear. • You may wish to have students review the part related to Section 9.3 in **BLM 9–1 Chapter 9 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

In #15 and #16, there is only one possible answer. Students may, however, use either difference to get the answer. For example, in #15a), determining a difference of −4 is not incorrect as long as the value is interpreted correctly. It means that Jamal scored four fewer points on Wednesday than on Tuesday, and hence that he scored four more points Tuesday than on Wednesday. However, you might point out that the question "How many more points …?" implies a positive difference.

If students incorrectly believe that the order of integers in subtraction does not affect the difference, have them refer to Example 2 in Section 9.3 and then complete #18. You might use #18c) to discuss order of integers in subtraction. The difference (−4) − (+3) has only one value (−7) because the order in which the integers are subtracted is specified. Some students may reason correctly that they could calculate (+3) − (−4) to get the integer +7, because they know from part b) that the required answer to part c) will be the opposite of this integer. If this reasoning is used, you might take the opportunity to discuss efficiency.

Since students have no prior experience with negative fractions, you might assign #21 with discretion. Students might solve the problem by plotting points on a number line and counting the distance between them. Or, they might interpolate. For example, if time zone −3 is 5 hours ahead of time zone −8, and time zone −4 is 4 hours ahead of time zone −8, then time zone $-3\frac{1}{2}$ (halfway between −3 and −4) is $4\frac{1}{2}$ hours ahead of time zone −8 (because $4\frac{1}{2}$ is halfway between 5 and 4).

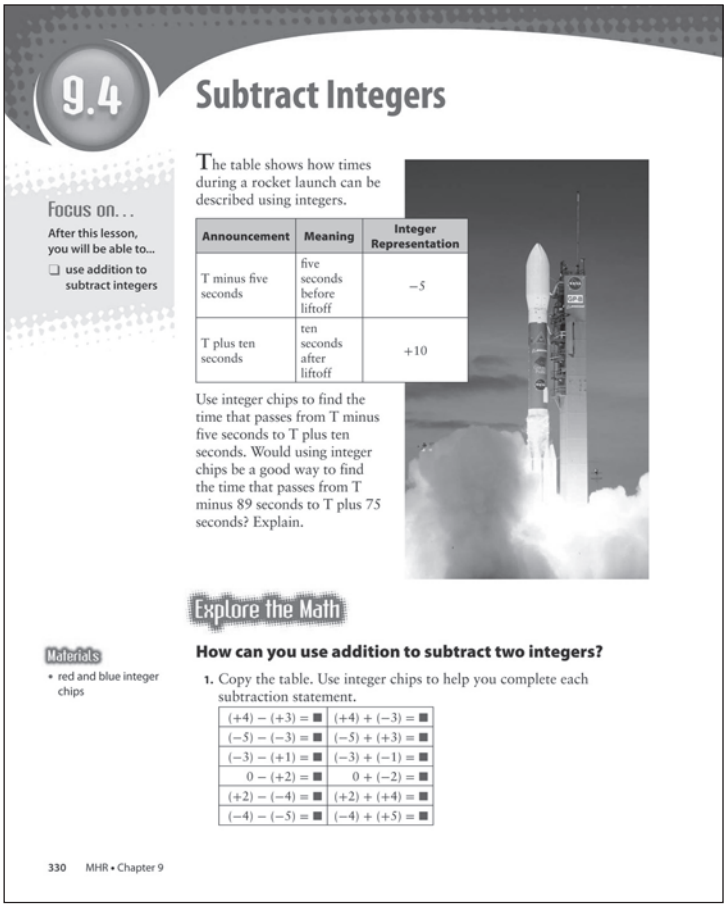
9.4 Subtract Integers

Suggested Timing
40–50 minutes

Materials
- red and blue integer chips
- transparent chips (optional)

Blackline Masters
Master 3 Integer Number Lines
Master 4 Vertical and Horizontal Number Lines
BLM 9–1 Chapter 9 Self-Assessment
BLM 9–8 Section 9.4 Explore the Math
BLM 9–9 Section 9.4 Extra Practice
BLM 9–10 Section 9.4 Math Link

Mathematical Processes
- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



9.4 Subtract Integers

Focus on...
After this lesson, you will be able to...
❑ use addition to subtract integers

The table shows how times during a rocket launch can be described using integers.

Announcement	Meaning	Integer Representation
T minus five seconds	five seconds before liftoff	−5
T plus ten seconds	ten seconds after liftoff	+10

Use integer chips to find the time that passes from T minus five seconds to T plus ten seconds. Would using integer chips be a good way to find the time that passes from T minus 89 seconds to T plus 75 seconds? Explain.

Explore the Math

Materials
- red and blue integer chips

How can you use addition to subtract two integers?

1. Copy the table. Use integer chips to help you complete each subtraction statement.

(+4) − (+3) = ■	(+4) + (−3) = ■
(−5) − (−3) = ■	(−5) + (+3) = ■
(−3) − (+1) = ■	(−3) + (−1) = ■
0 − (+2) = ■	0 + (−2) = ■
(+2) − (−4) = ■	(+2) + (+4) = ■
(−4) − (−5) = ■	(−4) + (+5) = ■

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Specific Outcomes

N6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.

Warm-Up

Use integer chips for #1 to #3.

1. (+7) − (+5)

2. (+3) − (−6)

3. (−1) − (−6)

Use a number line for #4 and #5.

4. (+2) + (+6)

5. (−3) + (−6)

Mental Math

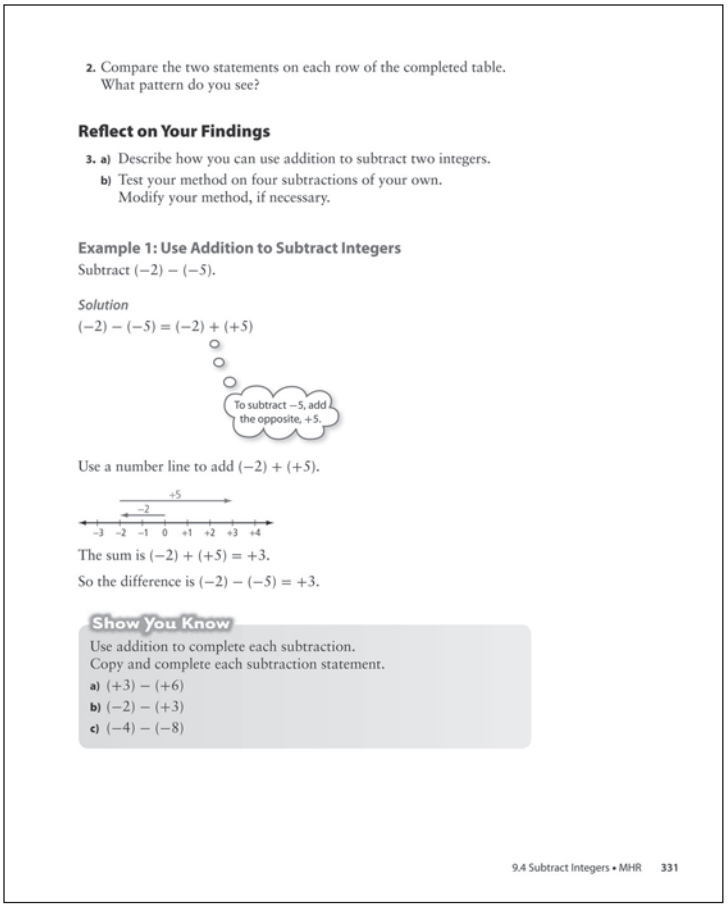
6. A circle graph has a 12% sector. Estimate the internal angle of that sector.

7. List the numbers divisible by 6. How do you know?
45, 79, 81, 2349, 3996, 18 764, 73 944

8. Calculate 30% of $12.

9. Calculate 45% of 132. Explain your thinking.

10. Estimate 23 out of 96 as a percent.



2. Compare the two statements on each row of the completed table. What pattern do you see?

Reflect on Your Findings

3. a) Describe how you can use addition to subtract two integers.
   b) Test your method on four subtractions of your own. Modify your method, if necessary.

Example 1: Use Addition to Subtract Integers

Subtract $(-2) - (-5)$.

Solution

$(-2) - (-5) = (-2) + (+5)$

To subtract −5, add the opposite, +5.

Use a number line to add $(-2) + (+5)$.

+5, −2

−3 −2 −1 0 +1 +2 +3 +4

The sum is $(-2) + (+5) = +3$.

So the difference is $(-2) - (-5) = +3$.

Show You Know

Use addition to complete each subtraction. Copy and complete each subtraction statement.

a) $(+3) - (+6)$
b) $(-2) - (+3)$
c) $(-4) - (-8)$

9.4 Subtract Integers • MHR 331

Activity Planning Notes

For the first question in the opener, students should find that 15 seconds pass from T minus 5 seconds to T plus 10 seconds.

For the second question, it is possible to use chips. However, using large numbers of integer chips is unwieldy.

Explore the Math

In this investigation, students develop another method for subtracting integers that is useful for problems in which the values of the integers are large.

Method 1: Have students complete the table in #1 individually, and then discuss their results in pairs or small groups. Give each student a set of integer chips (at least six of each colour). In advance, prepare an overhead of the table and provide students with the table on **BLM 9–8 Section 9.4 Explore the Math**. Make sure that the table is accurate before students look for a pattern.

Answers

Warm-Up

1. $(+2)$ **2.** $(+9)$ **3.** $(+5)$ **4.** $(+8)$

5. (-9) **6.** 12% is close to 10% or $\frac{1}{10}$; $360° \div 10 = 36°$

7. 3996, 73 944. These numbers are divisible by both 2 and 3.

8. 10% of \$12 = \$1.20. $30\% = 3 \times \$1.20 = \$3 + \$0.60 = \$3.60$

9. 10% of 132 = 13.2
5% of 132 = 13.2 ÷ 2 = 6.6
$45\% = 13.2 + 13.2 + 13.2 + 13.2 + 6.6 = 40 + 12 + 0.8 + 6.6 = 58 + 0.8 + 0.6 = 59.4$
Various methods are possible. Encourage students to try to put numbers that are easy to add together.

10. 50% of 96 = 48
25% of 96 = 24
The answer is a little lower than 25%.

Explore the Math

1.

$(+4) - (+3) = +1$	$(+4) + (-3) = +1$
$(-5) - (-3) = -2$	$(-5) + (+3) = -2$
$(-3) - (+1) = -4$	$(-3) + (-1) = -4$
$0 - (+2) = -2$	$0 + (-2) = -2$
$(+2) - (-4) = +6$	$(+2) + (+4) = +6$
$(-4) - (-5) = +1$	$(-4) + (+5) = +1$

2. Answers will vary. For example: The opposite integer is added each time, but the answer is the same.

3. **a)** Answers may vary. For example: Add the opposite integer.
b) Answers will vary.

Supported Learning

ESL and Language

- English language learners may have difficulty with terms such as *rocket, launch, elevation, above sea level, melting, boiling,* and *dominated hockey*.

Motor

- Have students use **BLM 9–8 Section 9.4 Explore the Math**, which provides a copy of the table on page 330.

Common Errors

- Some students may have difficulty in rewriting a subtraction as an addition.

R_x Remind students that they can continue to subtract using integer chips without rewriting a subtraction as an addition. As students gain experience, encourage them to consider other pairs of statements like those in the table in #1 on page 330. Stress that rewriting a subtraction as an addition involves changing both the operation symbol and the sign of the second integer in the expression.

Answers

Show You Know: Example 1

a) $(+3) - (+6) = (+3) + (-6) = -3$

b) $(-2) - (+3) = (-2) + (-3) = -5$

c) $(-4) - (-8) = (-4) + (+8) = +4$

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss their answers to #3.	• Encourage students to generalize what they have learned to help them develop a method that they understand. • Make sure that students test their methods on each type of subtraction shown in the table on page 330. They need to realize that some methods will work for some types of subtraction, but not for others.

Supported Learning

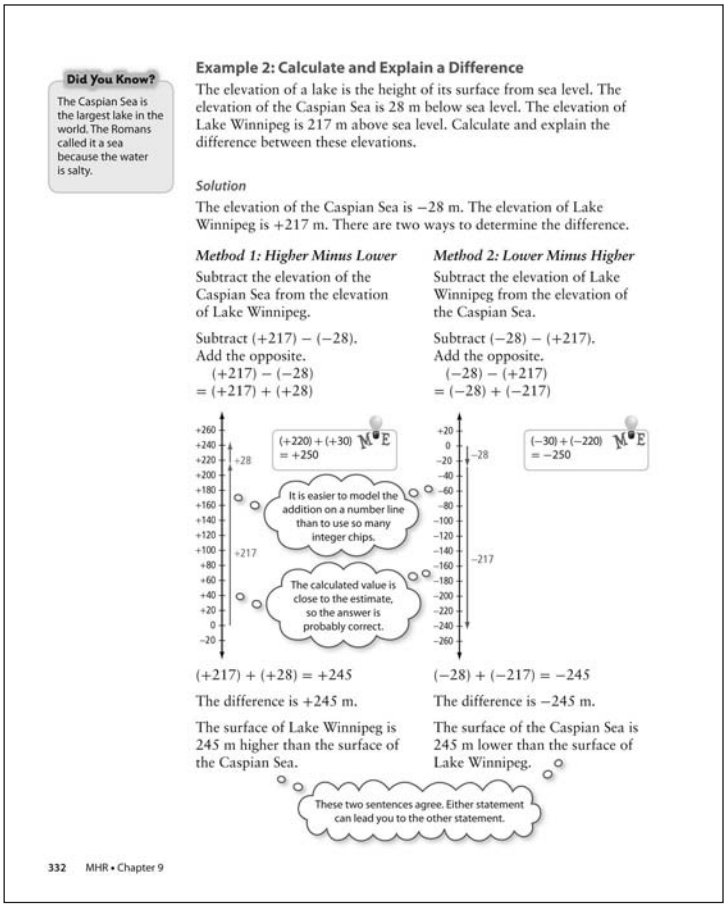
Learning Style and Memory

- Have students represent the subtraction in Example 2 on a vertical number line such as **Master 4 Vertical and Horizontal Number Lines**.

Meeting the Needs of All Learners

- Model solutions using integer chips and number lines.
- For Example 2, use drawings to show the relationship between the elevation of the lake and the sea and sea level.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1 on page 331.	• Have students talk through their thinking with a partner. • You may wish to provide additional questions very close to those in the Show You Know for students who would benefit from them: **a)** $(+4) - (+7)$ (–3. To subtract +7, add the opposite (–7). $(+4) + (-7) = (-3)$) **b)** $(-6) - (+8)$ (–14. To subtract +8, add the opposite (–8). $(-6) + (-8) = (-14)$) **c)** $(-9) - (-3)$ (–6. To subtract –3, add the opposite (+3). $(-9) + (+3) = (-6)$) Coach students through a). Make sure that they understand the procedure, then have them try b) and c) on their own.



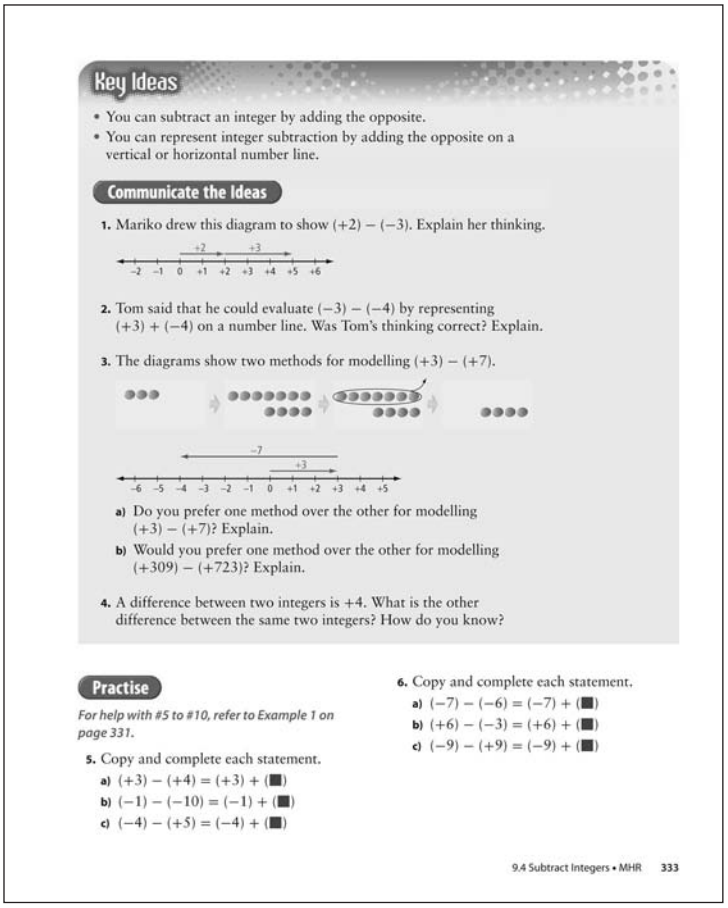
Method 2: Demonstrate the subtractions and additions using transparent chips on an overhead projector. Have students record the results on their copy of the table. Then have students look for a pattern individually and share their findings.

Example 1 illustrates subtracting integers by adding the opposite using a number line.

Example 2 provides two methods for subtracting integers: Method 1: Higher Minus Lower and Method 2: Lower Minus Higher. The two methods show that two integers can be subtracted in either order in a word problem unless the problem specifies an order, such as when an incomplete subtraction statement is provided that shows the order of subtraction. The emphasis in Example 2 is on explaining the meaning of the two differences and showing that the meanings are consistent with each other.

Stress the importance of estimation in problems in which the values are large.

Encourage students to interpret the integer answers to word problems and make sure that the final statement is consistent with the wording in the problem.



Key Ideas

- You can subtract an integer by adding the opposite.
- You can represent integer subtraction by adding the opposite on a vertical or horizontal number line.

Communicate the Ideas

1. Mariko drew this diagram to show (+2) − (−3). Explain her thinking.

2. Tom said that he could evaluate (−3) − (−4) by representing (+3) + (−4) on a number line. Was Tom's thinking correct? Explain.

3. The diagrams show two methods for modelling (+3) − (+7).

a) Do you prefer one method over the other for modelling (+3) − (+7)? Explain.

b) Would you prefer one method over the other for modelling (+309) − (+723)? Explain.

4. A difference between two integers is +4. What is the other difference between the same two integers? How do you know?

Practise

For help with #5 to #10, refer to Example 1 on page 331.

5. Copy and complete each statement.
   a) (+3) − (+4) = (+3) + (■)
   b) (−1) − (−10) = (−1) + (■)
   c) (−4) − (+5) = (−4) + (■)

6. Copy and complete each statement.
   a) (−7) − (−6) = (−7) + (■)
   b) (+6) − (−3) = (+6) + (■)
   c) (−9) − (+9) = (−9) + (■)

9.4 Subtract Integers • MHR 333

Key Ideas

This section summarizes how to subtract an integer by adding the opposite and how to represent integer subtraction on a number line. Emphasize that *the opposite* is the opposite of the integer that is being subtracted. For example, to subtract −2 from −3, add +2 to –3, but not +3 to −2.

Communicate the Ideas

These questions allow students to model and explain how diagrams or subtraction statements demonstrate integer subtraction. In #1, students interpret a number line. In #2, students reinforce their understanding of adding the opposite in integer subtraction. In #3, students review the two methods of modelling subtraction, and in #4, they reinforce their understanding about the two differences between two integers.

Answers

Communicate the Ideas

1. She added the opposite of −3, that is, (+2) + (+3).

2. No. Answers will vary. For example: Tom changed the subtraction to addition without changing the sign of the integer.

3. a) Answers will vary. **b)** Answers will vary.

4. –4. Answers will vary. For example: When the integers are reversed, the difference is the opposite of the other difference.

Supported Learning

Learning Style and Gifted and Enrichment

- You might show students how to use a calculator to do some of questions. Emphasize that it is not necessary to rewrite a subtraction as an addition in order to use a calculator. The following key sequence is for a TI-30Xa calculator: [C] 2 [+/−] [−] 5 [+/−] [=]

  Students with other calculators may need to explore a way to enter negative integers on their calculators, and may need assistance. Clarify that for most calculators the integer sign key and the subtraction key are not the same. Encourage students to write out the correct key sequence for their own calculator and to compare with other students. When students can successfully complete Example 1 using their calculators, you might ask them to verify the numerical answers in Example 2.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–5, 7, 9, 11, 13, 15, Math Link
Typical	1–5, 7, 9, 11, 13, 15–22, Math Link
Extension/Enrichment	1–4, 16, 17, 21–24

Assessment *as* Learning	Supported Learning
Communicate the Ideas Most students should do all of the questions. Check that their responses show that they can generalize what they have learned during this section.	• Have students work in small groups and discuss their answers. This may be especially helpful for #3, where students' preferences and the reasons for them will differ. In #3a), more students may favour the use of integer chips because the concept of adding the opposite is new to them. However, #3b) illustrates a limitation of the concrete model when integers include numerals with large values. • Check each student's answer to #1. Make sure they understand the concept of using number • In #2, allow students to draw a diagram to help them explain their answer.

Supported Learning

Learning Style, ESL, Language, and Memory

- Some students may have difficulty processing the steps in word problems and need some coaching.

Learning Style and Memory

- Provide **BLM 9–9 Section 9.4 Extra Practice** to students who require more practice.

Common Errors

- Some students may change the sign of both the minuend and the subtrahend.

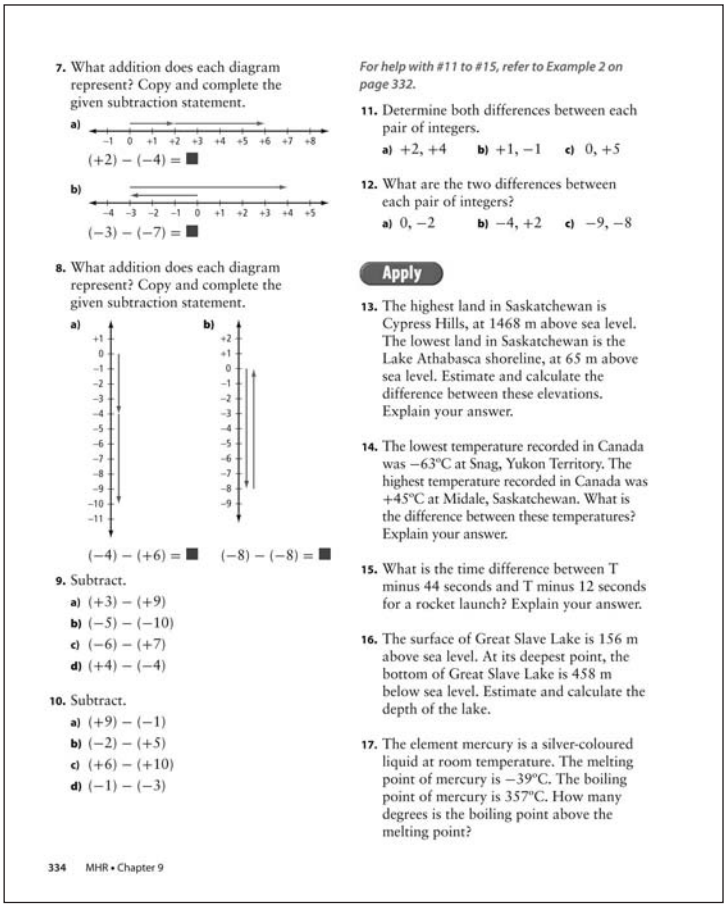
R_x Stress that rewriting a subtraction as an addition involves changing both the operation symbol and the sign of the second integer in the expression.

- Some students may have difficulty with word problems.

R_x Suggest that students read each problem carefully, and then list what they know. Have students estimate the answer.

- Some students may try to move too quickly to the symbolic method for subtracting integers and make mistakes as a result.

R_x Encourage students to continue using concrete or semi-concrete methods until they have a clear understanding of how to subtract integers.



7. What addition does each diagram represent? Copy and complete the given subtraction statement.

a) $(+2) - (-4) = \blacksquare$

b) $(-3) - (-7) = \blacksquare$

8. What addition does each diagram represent? Copy and complete the given subtraction statement.

a) $(-4) - (+6) = \blacksquare$ b) $(-8) - (-8) = \blacksquare$

9. Subtract.
a) $(+3) - (+9)$
b) $(-5) - (-10)$
c) $(-6) - (+7)$
d) $(+4) - (-4)$

10. Subtract.
a) $(+9) - (-1)$
b) $(-2) - (+5)$
c) $(+6) - (+10)$
d) $(-1) - (-3)$

For help with #11 to #15, refer to Example 2 on page 332.

11. Determine both differences between each pair of integers.
a) $+2, +4$ b) $+1, -1$ c) $0, +5$

12. What are the two differences between each pair of integers?
a) $0, -2$ b) $-4, +2$ c) $-9, -8$

Apply

13. The highest land in Saskatchewan is Cypress Hills, at 1468 m above sea level. The lowest land in Saskatchewan is the Lake Athabasca shoreline, at 65 m above sea level. Estimate and calculate the difference between these elevations. Explain your answer.

14. The lowest temperature recorded in Canada was −63°C at Snag, Yukon Territory. The highest temperature recorded in Canada was +45°C at Midale, Saskatchewan. What is the difference between these temperatures? Explain your answer.

15. What is the time difference between T minus 44 seconds and T minus 12 seconds for a rocket launch? Explain your answer.

16. The surface of Great Slave Lake is 156 m above sea level. At its deepest point, the bottom of Great Slave Lake is 458 m below sea level. Estimate and calculate the depth of the lake.

17. The element mercury is a silver-coloured liquid at room temperature. The melting point of mercury is −39°C. The boiling point of mercury is 357°C. How many degrees is the boiling point above the melting point?

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Practise

In #5 to #8, guide students to subtract integers by adding the opposite on a number line.

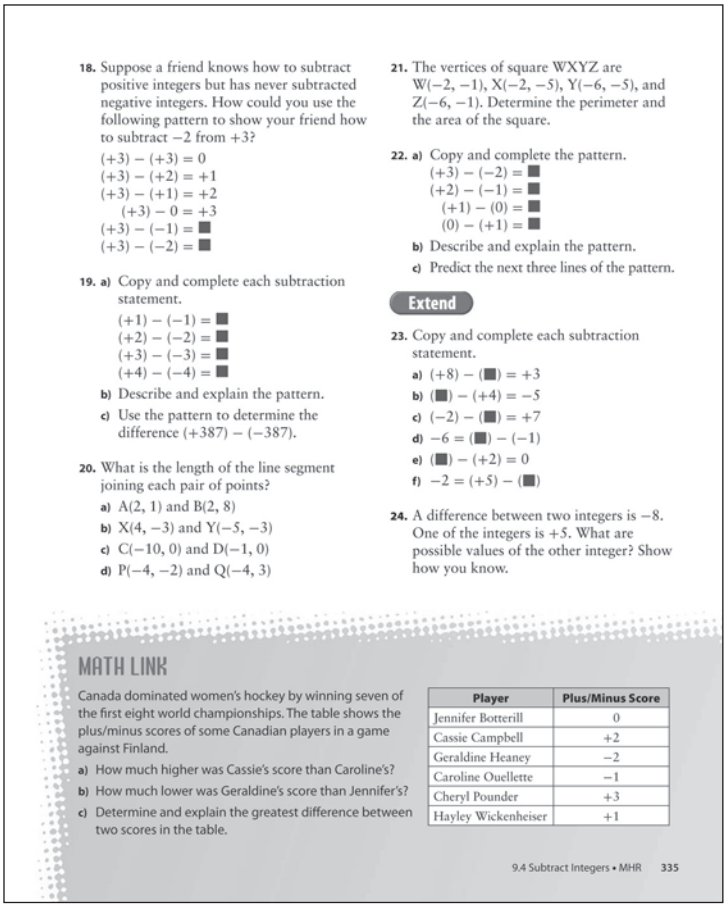
In #5 to #10, there is only one correct answer for each part (i.e., the order of subtraction is specified). However, in #11 and #12, students must find both differences between two different integers. If you wish, ask students to explain the meanings of the two differences and to check that they are consistent with each other.

In #9 and #10, allow students to choose a method. Some students may be more comfortable using integer chips; others may not need to use manipulatives or diagrams.

Assessment *for* Learning	Supported Learning
Practise and Apply Have students do #5, #7, #9, #11, and #13. Students who have no problems with these questions can go on to the rest of the Apply questions.	• Students who have problems with #5, #7, #9, #11, and #13 will need additional coaching. Have students explain their thinking on these questions; clarify any misunderstandings. Coach students through #6a), #8a), #10a), and #12a), and then have them complete the remaining parts of each question on their own. Have students refer back to examples in the student resource. Check back with them several times to make sure that they understand the concepts. • Once students feel confident of their skills, have them try #14.

Web Link

For an online game that allows students to solve addition and subtraction problems using a number line, go to **www.mathlinks7.ca** and follow the links.



18. Suppose a friend knows how to subtract positive integers but has never subtracted negative integers. How could you use the following pattern to show your friend how to subtract −2 from +3?
(+3) − (+3) = 0
(+3) − (+2) = +1
(+3) − (+1) = +2
(+3) − 0 = +3
(+3) − (−1) = ■
(+3) − (−2) = ■

19. a) Copy and complete each subtraction statement.
(+1) − (−1) = ■
(+2) − (−2) = ■
(+3) − (−3) = ■
(+4) − (−4) = ■
b) Describe and explain the pattern.
c) Use the pattern to determine the difference (+387) − (−387).

20. What is the length of the line segment joining each pair of points?
a) A(2, 1) and B(2, 8)
b) X(4, −3) and Y(−5, −3)
c) C(−10, 0) and D(−1, 0)
d) P(−4, −2) and Q(−4, 3)

21. The vertices of square WXYZ are W(−2, −1), X(−2, −5), Y(−6, −5), and Z(−6, −1). Determine the perimeter and the area of the square.

22. a) Copy and complete the pattern.
(+3) − (−2) = ■
(+2) − (−1) = ■
(+1) − (0) = ■
(0) − (+1) = ■
b) Describe and explain the pattern.
c) Predict the next three lines of the pattern.

Extend

23. Copy and complete each subtraction statement.
a) (+8) − (■) = +3
b) (■) − (+4) = −5
c) (−2) − (■) = +7
d) −6 = (■) − (−1)
e) (■) − (+2) = 0
f) −2 = (+5) − (■)

24. A difference between two integers is −8. One of the integers is +5. What are possible values of the other integer? Show how you know.

MATH LINK

Canada dominated women's hockey by winning seven of the first eight world championships. The table shows the plus/minus scores of some Canadian players in a game against Finland.
a) How much higher was Cassie's score than Caroline's?
b) How much lower was Geraldine's score than Jennifer's?
c) Determine and explain the greatest difference between two scores in the table.

Player	Plus/Minus Score
Jennifer Botterill	0
Cassie Campbell	+2
Geraldine Heaney	−2
Caroline Ouellette	−1
Cheryl Pounder	+3
Hayley Wickenheiser	+1

9.4 Subtract Integers • MHR 335

Apply and Extend

In #13 to #15, the order of the subtraction is not specified, and students can determine and explain either difference.

Questions 16 and 17 are worded in such a way that only one answer is possible in each case. However, students may use either difference to get the answer. For example, in #16, a numerical difference of +614 or −614 is acceptable, as long as either value can be correctly interpreted. Deciding that the surface of the lake is 614 m above the bottom, or that the bottom of the lake is 614 m below the surface, will lead to a depth of 614 m.

In #13 and #16, students estimate and calculate a difference. Have students use their estimation skills in #14, #15, and #17 as well.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 335 helps students work toward the chapter problem wrap-up titled Wrap It Up! on page 345.	• Have students write the subtraction statements as they solve the problems. As students work, observe and have them self-observe for how well they solve each one. • Students who are having difficulty getting started could use **BLM 9–10 Section 9.4 Math Link**.

Answers

Math Link

a) 3 units higher

b) 2 units lower

c) 5 units difference between Geraldine and Cheryl;
(+3) − (−2) = (+3) + (+2) = +5

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • What methods do you know for subtracting two integers? • What do you like and dislike about each method?	• Encourage concrete and kinesthetic learners to use integer chips and number lines to help them answer the questions. • Depending on students' learning style, have them provide verbal or written answers. • You may wish to have students review the part related to Section 9.4 in **BLM 9–1 Chapter 9 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

MATH LINK

This Math Link provides students another opportunity to practise using plus/minus scores. Doing this Math Link will help them do the chapter problem titled Wrap It Up!

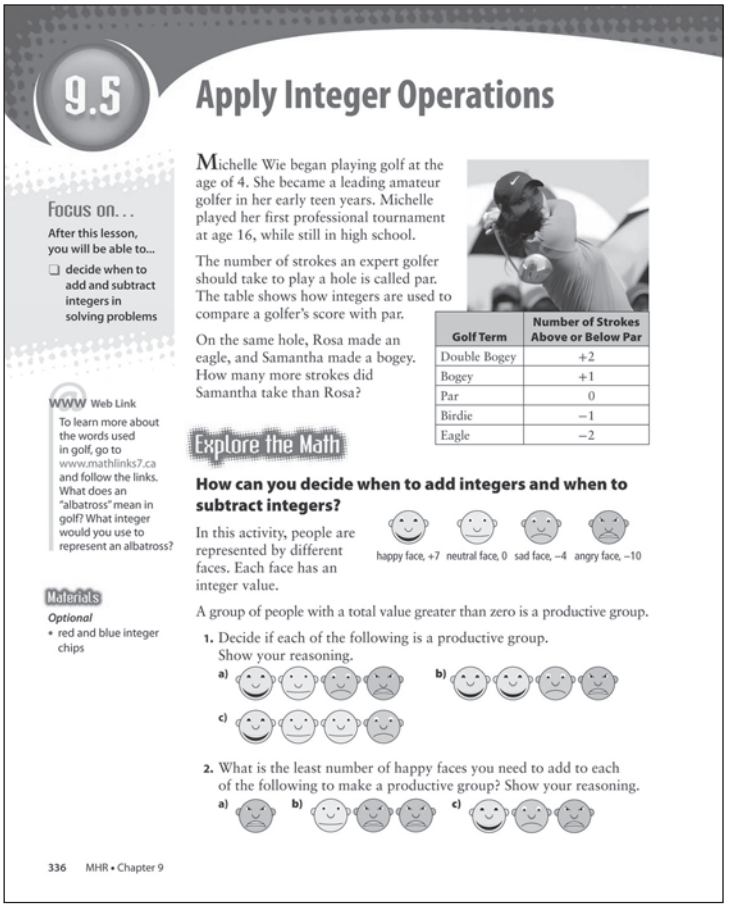
9.5 Apply Integer Operations

Suggested Timing
60–75 minutes

Materials
- red and blue integer chips (optional)
- scissors

Blackline Masters
Master 3 Integer Number Lines
Master 4 Vertical and Horizontal Number Lines
BLM 9–1 Chapter 9 Self-Assessment
BLM 9–6 Time Zone Map
BLM 9–11 Faces
BLM 9–12 Section 9.5 Extra Practice
BLM 9–13 Section 9.5 Science Link

Mathematical Processes
- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



9.5 Apply Integer Operations

Focus on...
After this lesson, you will be able to...
- decide when to add and subtract integers in solving problems

Michelle Wie began playing golf at the age of 4. She became a leading amateur golfer in her early teen years. Michelle played her first professional tournament at age 16, while still in high school.

The number of strokes an expert golfer should take to play a hole is called par. The table shows how integers are used to compare a golfer's score with par.

Golf Term	Number of Strokes Above or Below Par
Double Bogey	+2
Bogey	+1
Par	0
Birdie	−1
Eagle	−2

On the same hole, Rosa made an eagle, and Samantha made a bogey. How many more strokes did Samantha take than Rosa?

WWW Web Link
To learn more about the words used in golf, go to www.mathlinks7.ca and follow the links. What does an "albatross" mean in golf? What integer would you use to represent an albatross?

Explore the Math

How can you decide when to add integers and when to subtract integers?

In this activity, people are represented by different faces. Each face has an integer value.

happy face, +7 neutral face, 0 sad face, −4 angry face, −10

Materials
Optional
- red and blue integer chips

A group of people with a total value greater than zero is a productive group.

1. Decide if each of the following is a productive group. Show your reasoning.

a) b) c)

2. What is the least number of happy faces you need to add to each of the following to make a productive group? Show your reasoning.

a) b) c)

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Specific Outcomes

N6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.

Warm-Up

Subtract for #1 to #3.

1. (+9) – (–1)

2. (–3) – (+7)

3. (+5) – (+8)

Determine both differences between each pair of integers for #4 and #5.

4. +2, +6

5. –3, –6

Mental Math

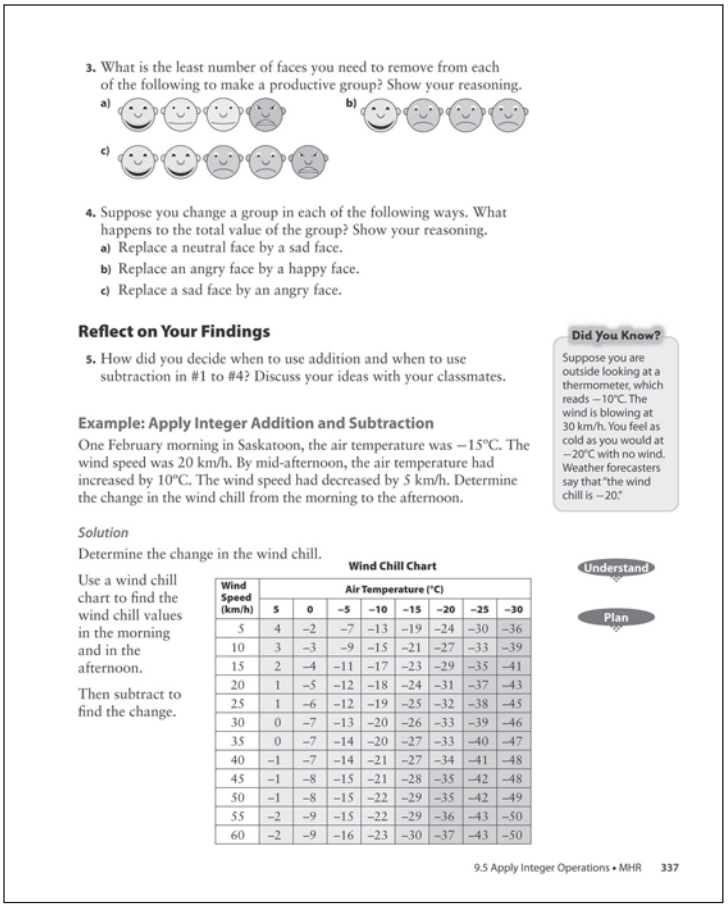
6. 52% of grade 7 students have read Harry Potter. Estimate the internal angle you would need to show this percent in a circle graph.

7. Sketch the internal angle for #6. Explain the angle you chose.

8. List the numbers divisible by 8. How do you know?
45, 56, 78, 80, 342, 760, 944, 992

9. Calculate 95% of $36 in two ways.

10. Estimate 135 out of 400 as a percent.



3. What is the least number of faces you need to remove from each of the following to make a productive group? Show your reasoning.

a) b) c)

4. Suppose you change a group in each of the following ways. What happens to the total value of the group? Show your reasoning.
   a) Replace a neutral face by a sad face.
   b) Replace an angry face by a happy face.
   c) Replace a sad face by an angry face.

Reflect on Your Findings

5. How did you decide when to use addition and when to use subtraction in #1 to #4? Discuss your ideas with your classmates.

Did You Know?

Suppose you are outside looking at a thermometer, which reads −10°C. The wind is blowing at 30 km/h. You feel as cold as you would at −20°C with no wind. Weather forecasters say that "the wind chill is −20."

Example: Apply Integer Addition and Subtraction

One February morning in Saskatoon, the air temperature was −15°C. The wind speed was 20 km/h. By mid-afternoon, the air temperature had increased by 10°C. The wind speed had decreased by 5 km/h. Determine the change in the wind chill from the morning to the afternoon.

Solution

Determine the change in the wind chill.

Understand

Use a wind chill chart to find the wind chill values in the morning and in the afternoon.

Then subtract to find the change.

Plan

Wind Chill Chart

Wind Speed (km/h)	Air Temperature (°C)							
	5	0	−5	−10	−15	−20	−25	−30
5	4	−2	−7	−13	−19	−24	−30	−36
10	3	−3	−9	−15	−21	−27	−33	−39
15	2	−4	−11	−17	−23	−29	−35	−41
20	1	−5	−12	−18	−24	−31	−37	−43
25	1	−6	−12	−19	−25	−32	−38	−45
30	0	−7	−13	−20	−26	−33	−39	−46
35	0	−7	−14	−20	−27	−33	−40	−47
40	−1	−7	−14	−21	−27	−34	−41	−48
45	−1	−8	−15	−21	−28	−35	−42	−48
50	−1	−8	−15	−22	−29	−35	−42	−49
55	−2	−9	−15	−22	−29	−36	−43	−50
60	−2	−9	−16	−23	−30	−37	−43	−50

9.5 Apply Integer Operations • MHR 337

Answers

Warm-Up

1. (+10) **2.** (−10) **3.** (−3)

4. (+2) − (+6) = −4, (+6) − (+2) = 4

5. (−3) − (−6) = +3, (−6) − (−3) = −3

6. 52% is close to 50% or $\frac{1}{2}$. 360° ÷ 2 = 180°

7. Students should draw a straight line because that is 180°.

8. 56, 80, 760, 944, 992. These numbers are divisible by 2 three times.

9. 100% of \$36 = \$36
10% of \$36 = \$3.60
5% of \$36 = \$1.80
95% = 100% − 5% = \$36 − \$1.80 = \$36 − \$2 + \$0.20 = \$34.20
95% = 9 × 10% + 5% = 9 × \$3.60 + \$1.80 = \$27 + \$5.40 + \$1.80 = \$33 + \$1.20 = \$34.20

Supported Learning

ESL and Language

- English language learners may have difficulty with terms such as *golf, leading amateur golfer, teen years, professional tournament, strokes, expert golfer, bogey, productive, par,* and *defeated.*

Meeting the Needs of All Learners

- Allow students to use integer chips and number lines to help them model problems.

Activity Planning Notes

Consider asking students who are familiar with golf to explain the terms used in golf. Most students will probably use subtraction to show that Samantha took three strokes more than Rosa. Some students may use addition. If they do, ask them to explain the addition they would carry out. It is possible that they use addition to add the opposite.

Explore the Math

In this investigation students decide when to add or subtract integers.

Method 1: Have students work in small groups. Provide students who need them with integer chips. Have students cut out and use the faces from **BLM 9–11 Faces** to help them. Encourage them to use mental arithmetic to answer the questions. As a class, discuss #5 to compare results and students' use of operations.

Method 2: In advance, make an overhead of **BLM 9–11 Faces** and cut out the faces. Demonstrate how to answer similar questions by placing the transparent faces on an overhead projector. When students are comfortable with the process, have them proceed with the activity as in Method 1.

Web Link

Have students use the Web Link on page 336 to find out that an albatross is a score of three strokes below par on a hole. If it were part of the table in the opener, an albatross would be represented as −3.

Answers

10. 50% of 400 = 200
25% of 400 = 100
10% of 400 = 40
5% of 400 = 20
35% = 100 + 40 = 140. A little high
30% = 120. Too low
Between 30% and 35%, but closer to 35%.

Explore the Math

1. a) Not productive; (+7) + (0) + (−4) + (−10) = −7
b) Not productive; (+7) + (+7) + (−4) + (−10) = 0
c) Productive; (+7) + (0) + (0) + (−4) = +3

2. a) 2 happy faces
b) 3 happy faces
c) 2 happy faces

3. a) 1 angry face
b) 2 sad faces
c) 1 angry face

4. a) Answers will vary. For example: Productivity will decrease.
b) Answers will vary. For example: Productivity will increase.
c) Answers will vary. For example: Productivity will decrease.

5. Answers will vary.

Supported Learning

Learning Style, ESL, Language, and Memory

- Clarify the meaning of "the wind chill in the afternoon was 13 higher than the wind chill in the morning." Some students may be confused by the word *higher* if it is interpreted as *colder*.

Learning Style and Motor

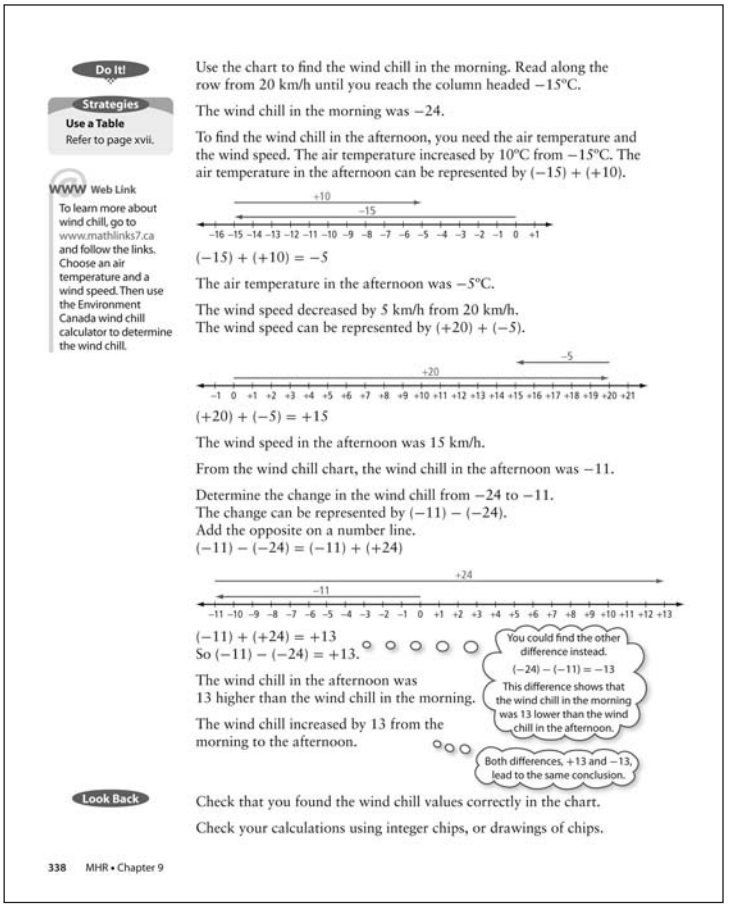
- Provide students with a copy of **Master 4 Vertical and Horizontal Number Lines**.

Meeting the Needs of All Learners

- Draw students' attention to the Did You Know? on page 337. Discuss how a wind above 50 km/h will begin to produce blizzard conditions. At these wind speeds, frostbite to exposed skin can occur in a matter of minutes. Forecasters often give the number of minutes to frostbite as part of the weather forecast.

Gifted and Enrichment

- Encourage students to consult the Environment Canada web site via the link in the Web Link on page 338 to find out more about wind chill.



Do It!

Strategies
Use a Table
Refer to page xvii.

WWW Web Link
To learn more about wind chill, go to www.mathlinks7.ca and follow the links. Choose an air temperature and a wind speed. Then use the Environment Canada wind chill calculator to determine the wind chill.

Use the chart to find the wind chill in the morning. Read along the row from 20 km/h until you reach the column headed −15°C.

The wind chill in the morning was −24.

To find the wind chill in the afternoon, you need the air temperature and the wind speed. The air temperature increased by 10°C from −15°C. The air temperature in the afternoon can be represented by (−15) + (+10).

(−15) + (+10) = −5

The air temperature in the afternoon was −5°C.

The wind speed decreased by 5 km/h from 20 km/h.
The wind speed can be represented by (+20) + (−5).

(+20) + (−5) = +15

The wind speed in the afternoon was 15 km/h.

From the wind chill chart, the wind chill in the afternoon was −11.

Determine the change in the wind chill from −24 to −11.
The change can be represented by (−11) − (−24).
Add the opposite on a number line.
(−11) − (−24) = (−11) + (+24)

(−11) + (+24) = +13
So (−11) − (−24) = +13.

The wind chill in the afternoon was 13 higher than the wind chill in the morning.

The wind chill increased by 13 from the morning to the afternoon.

You could find the other difference instead.
(−24) − (−11) = −13
This difference shows that the wind chill in the morning was 13 lower than the wind chill in the afternoon.

Both differences, +13 and −13, lead to the same conclusion.

Look Back

Check that you found the wind chill values correctly in the chart.

Check your calculations using integer chips, or drawings of chips.

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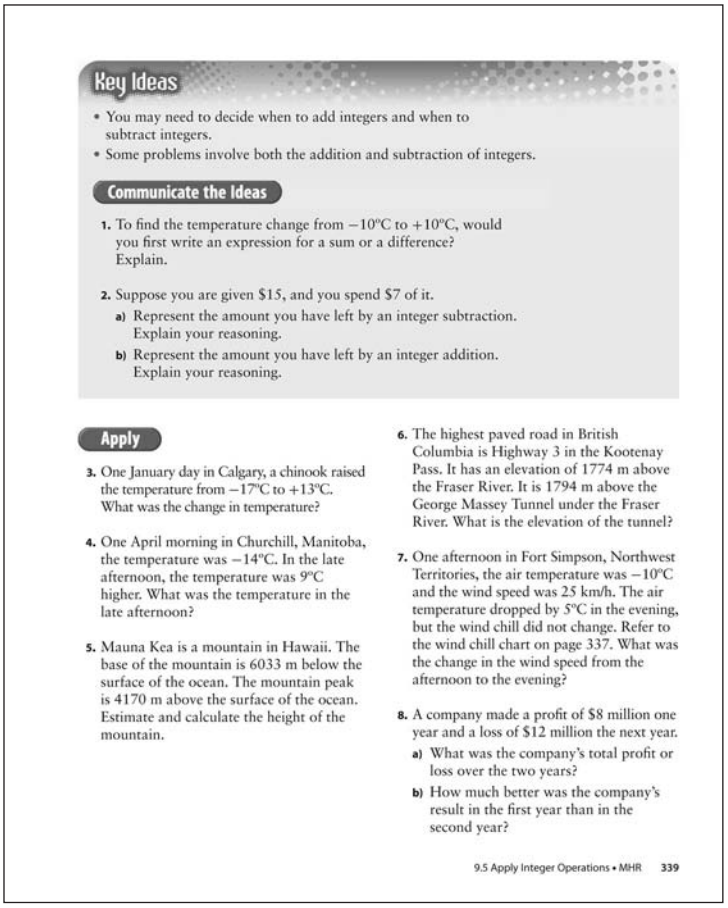
Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss what they did during the Explore the Math and read their explanations in #5.	• Ask students who are having difficulty with this question to use the class responses as springboards to similar ones of their own. • You might extend #5 by asking students to suggest a way to modify the activity. Have them develop two questions about the modified game and ask a classmate to answer them.

The example illustrates how to use the addition and subtraction of integers to solve an extended problem. The context should be familiar, so students should be aware that when there is a significant wind, the wind chill is lower than the air temperature.

Reinforce the idea that the Example uses a familiar problem solving strategy (i.e., Use a Table). You may wish to refer students to other examples of tables used in problem solving by pointing out the Strategies box on page 338.

Point out that the stages of the problem solving model are indicated beside the solution. Within the Do It! stage, indicate the two distinct parts: determining the wind chill in the morning and determining the wind chill in the afternoon. Both of these values are needed to determine the change in the wind chill from the morning to the afternoon. Reinforce the idea that the change can be found by subtracting the integers in either order. The solution shows the determination of a difference of +13 but the thought bubble indicates that a difference of −13 would also be acceptable, as long as students are able to interpret this integer in relation to the wording of the original problem.



Key Ideas

- You may need to decide when to add integers and when to subtract integers.
- Some problems involve both the addition and subtraction of integers.

Communicate the Ideas

1. To find the temperature change from −10°C to +10°C, would you first write an expression for a sum or a difference? Explain.

2. Suppose you are given $15, and you spend $7 of it.
   a) Represent the amount you have left by an integer subtraction. Explain your reasoning.
   b) Represent the amount you have left by an integer addition. Explain your reasoning.

Apply

3. One January day in Calgary, a chinook raised the temperature from −17°C to +13°C. What was the change in temperature?

4. One April morning in Churchill, Manitoba, the temperature was −14°C. In the late afternoon, the temperature was 9°C higher. What was the temperature in the late afternoon?

5. Mauna Kea is a mountain in Hawaii. The base of the mountain is 6033 m below the surface of the ocean. The mountain peak is 4170 m above the surface of the ocean. Estimate and calculate the height of the mountain.

6. The highest paved road in British Columbia is Highway 3 in the Kootenay Pass. It has an elevation of 1774 m above the Fraser River. It is 1794 m above the George Massey Tunnel under the Fraser River. What is the elevation of the tunnel?

7. One afternoon in Fort Simpson, Northwest Territories, the air temperature was −10°C and the wind speed was 25 km/h. The air temperature dropped by 5°C in the evening, but the wind chill did not change. Refer to the wind chill chart on page 337. What was the change in the wind speed from the afternoon to the evening?

8. A company made a profit of $8 million one year and a loss of $12 million the next year.
   a) What was the company's total profit or loss over the two years?
   b) How much better was the company's result in the first year than in the second year?

9.5 Apply Integer Operations • MHR 339

Key Ideas

As a class, read the information about using both addition and subtraction to solve problems.

Communicate the Ideas

These questions allow students to apply their understanding of using integer addition and subtraction to solve problems.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Most students should complete both questions.	• Encourage students to share their answers with neighbouring students and listen to each other's explanations. • Check each student's answer to #1 and #2. Make sure that they understand the concept of using number lines. In #2, allow students to draw a diagram to help them explain their answer. • Parts a) and b) of #2 are intended to reinforce the idea that subtracting +7 and adding −7 are two equivalent ways of representing the $7 spent.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–5, Science Link
Typical	1–5, 9–15, Science Link
Extension/Enrichment	1, 2, 13, 15–18

Answers

Communicate the Ideas

1. Answers may vary. For example: Write an expression for a difference, and then change the expression to a sum.
   $(+10) - (-10) = (+10) + (+10) = +20$

2. **a)** $(+15) - (+7) = +8$
   b) $(+15) + (-7) = +8$

Supported Learning

Learning Style

- Encourage students to use integer chips and number lines to help model problems in the Apply section (where appropriate).

ESL and Language

- Some students may have difficulty knowing when to add and when to subtract. Before having them do any problems, read through the questions with students and identify the words that suggest adding or subtracting. You may wish to post a list of these on a word wall with the relevant sign posted beside them. Add to the list as students do other questions.

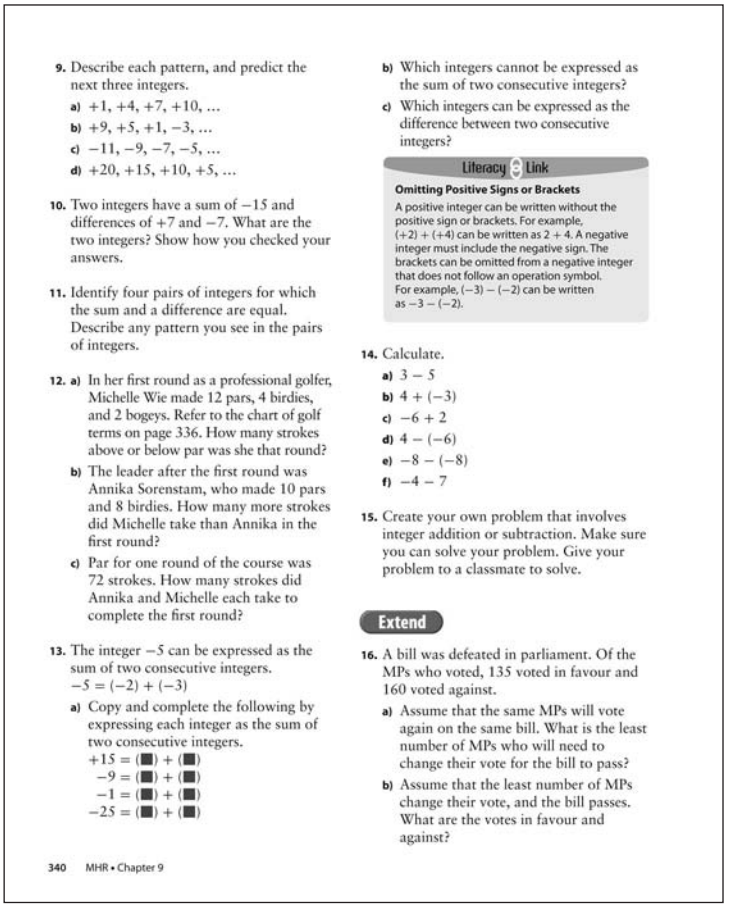
Motor

- Provide students who need to model problems with **Master 3 Integer Number Lines** or **Master 4 Vertical and Horizontal Number Lines**.

Common Errors

- Some students may have difficulty deciding when to add and when to subtract.

R_x Encourage students to try many problems and learn to identify the verbal cues in the wording of questions, such as those that require the total or sum (for addition), and the difference or change (for subtraction). Also, encourage students to check that the answer to a problem makes sense in relation to the given data.



9. Describe each pattern, and predict the next three integers.
   a) +1, +4, +7, +10, ...
   b) +9, +5, +1, −3, ...
   c) −11, −9, −7, −5, ...
   d) +20, +15, +10, +5, ...

10. Two integers have a sum of −15 and differences of +7 and −7. What are the two integers? Show how you checked your answers.

11. Identify four pairs of integers for which the sum and a difference are equal. Describe any pattern you see in the pairs of integers.

12. a) In her first round as a professional golfer, Michelle Wie made 12 pars, 4 birdies, and 2 bogeys. Refer to the chart of golf terms on page 336. How many strokes above or below par was she that round?
    b) The leader after the first round was Annika Sorenstam, who made 10 pars and 8 birdies. How many more strokes did Michelle take than Annika in the first round?
    c) Par for one round of the course was 72 strokes. How many strokes did Annika and Michelle each take to complete the first round?

13. The integer −5 can be expressed as the sum of two consecutive integers.
    −5 = (−2) + (−3)
    a) Copy and complete the following by expressing each integer as the sum of two consecutive integers.
    +15 = (■) + (■)
    −9 = (■) + (■)
    −1 = (■) + (■)
    −25 = (■) + (■)
    b) Which integers cannot be expressed as the sum of two consecutive integers?
    c) Which integers can be expressed as the difference between two consecutive integers?

Literacy Link

Omitting Positive Signs or Brackets

A positive integer can be written without the positive sign or brackets. For example, (+2) + (+4) can be written as 2 + 4. A negative integer must include the negative sign. The brackets can be omitted from a negative integer that does not follow an operation symbol. For example, (−3) − (−2) can be written as −3 − (−2).

14. Calculate.
    a) 3 − 5
    b) 4 + (−3)
    c) −6 + 2
    d) 4 − (−6)
    e) −8 − (−8)
    f) −4 − 7

15. Create your own problem that involves integer addition or subtraction. Make sure you can solve your problem. Give your problem to a classmate to solve.

Extend

16. A bill was defeated in parliament. Of the MPs who voted, 135 voted in favour and 160 voted against.
    a) Assume that the same MPs will vote again on the same bill. What is the least number of MPs who will need to change their vote for the bill to pass?
    b) Assume that the least number of MPs change their vote, and the bill passes. What are the votes in favour and against?

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Supported Learning

Learning Style and Memory

- Provide **BLM 9–13 Section 9.5 Extra Practice** to students who require more practice.

ESL and Language

- For #5, consider using a diagram to show what is meant by the base and peak of a mountain. For #6, use a diagram to show the relationship between the highway and the tunnel and river.
- For #16, have students vote on an issue. After the vote, indicate what was defeated. Have English language learners add the word *defeated* to their dictionary.

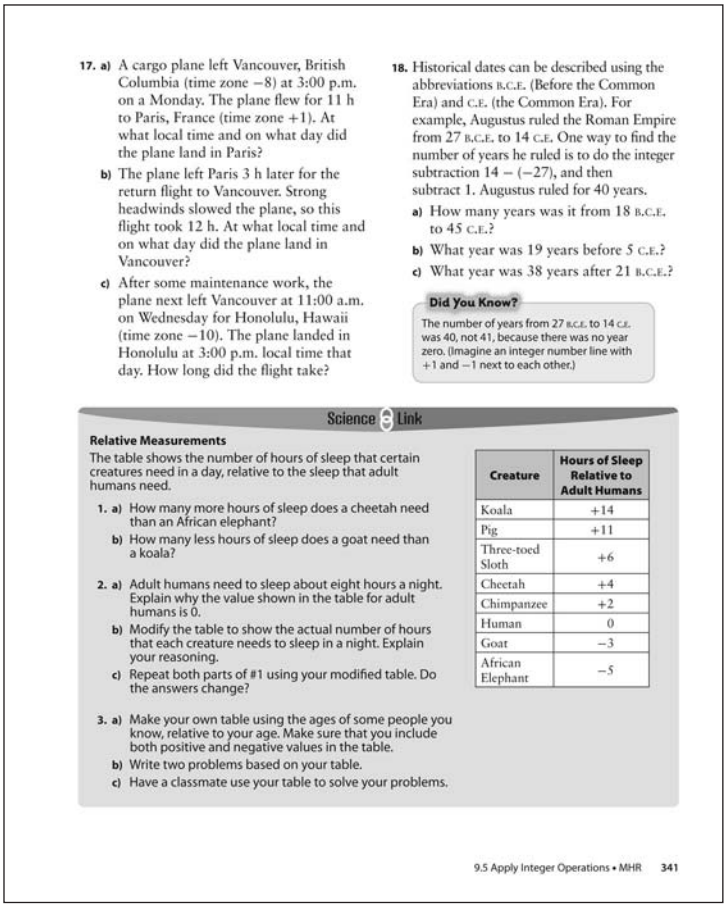
Apply

In #3 to #8 and #12, students need to choose between addition and subtraction to solve problems in real-world contexts. Before students complete #13, read and discuss the Literacy Link on page 340.

Assessment *for* Learning	Supported Learning
Apply Have students do #3 to #5. Students who have no problems with these questions can continue with the Apply questions.	• Students who have problems with #3 to #5 will need additional coaching. Have students explain their thinking on these questions; clarify any misunderstandings. Have students refer back to examples in the student resource and coach them through fixing their errors. Check back with them several times to make sure that they understand the concepts. • Once students appear to understand what is expected, have them do #6 to #8.

Extend

Consider having students use **BLM 9–6 Time Zone Map** to help them with #17. Question 18 includes a method for determining the number of years Augustus ruled. Make sure that students refer to the accompanying Did You Know? for an explanation of the method.



17. a) A cargo plane left Vancouver, British Columbia (time zone −8) at 3:00 p.m. on a Monday. The plane flew for 11 h to Paris, France (time zone +1). At what local time and on what day did the plane land in Paris?

b) The plane left Paris 3 h later for the return flight to Vancouver. Strong headwinds slowed the plane, so this flight took 12 h. At what local time and on what day did the plane land in Vancouver?

c) After some maintenance work, the plane next left Vancouver at 11:00 a.m. on Wednesday for Honolulu, Hawaii (time zone −10). The plane landed in Honolulu at 3:00 p.m. local time that day. How long did the flight take?

18. Historical dates can be described using the abbreviations B.C.E. (Before the Common Era) and C.E. (the Common Era). For example, Augustus ruled the Roman Empire from 27 B.C.E. to 14 C.E. One way to find the number of years he ruled is to do the integer subtraction 14 − (−27), and then subtract 1. Augustus ruled for 40 years.

a) How many years was it from 18 B.C.E. to 45 C.E.?

b) What year was 19 years before 5 C.E.?

c) What year was 38 years after 21 B.C.E.?

Did You Know?

The number of years from 27 B.C.E. to 14 C.E. was 40, not 41, because there was no year zero. (Imagine an integer number line with +1 and −1 next to each other.)

Science Link

Relative Measurements

The table shows the number of hours of sleep that certain creatures need in a day, relative to the sleep that adult humans need.

Creature	Hours of Sleep Relative to Adult Humans
Koala	+14
Pig	+11
Three-toed Sloth	+6
Cheetah	+4
Chimpanzee	+2
Human	0
Goat	−3
African Elephant	−5

1. a) How many more hours of sleep does a cheetah need than an African elephant?
   b) How many less hours of sleep does a goat need than a koala?

2. a) Adult humans need to sleep about eight hours a night. Explain why the value shown in the table for adult humans is 0.
   b) Modify the table to show the actual number of hours that each creature needs to sleep in a night. Explain your reasoning.
   c) Repeat both parts of #1 using your modified table. Do the answers change?

3. a) Make your own table using the ages of some people you know, relative to your age. Make sure that you include both positive and negative values in the table.
   b) Write two problems based on your table.
   c) Have a classmate use your table to solve your problems.

Answers

Science Link

1. a) $(+4) - (-5) = 9$. A cheetah needs 9 h more sleep than an African elephant.

b) $(-3) - (+14) = -17$. A goat needs 17 h less sleep than a koala.

2. a) Answers will vary. For example: The value for humans is the point of reference for all the other values in the chart.

b)

Creature	Hours of Sleep Needed
Koala	22
Pig	19
Three-toed Sloth	15
Cheetah	12
Chimpanzee	10
Human	8
Goat	5
African Elephant	3

c) The answers do not change. Now the numbers are relative to 8.

3. a) Answers will vary.

b) Answers will vary.

c) Answers will vary.

The Science Link on page 341 is an optional activity that provides students with the opportunity to apply integer operations to relative measurements. Students analyse the amount of sleep required for various animals relative to adult humans. Students work with a relative scale in order to see that the choice of the zero on the scale affects individual measurements, but not the difference between two measurements. Students also create and apply a table of relative values of their own. Due to the high interest level of this activity, you may wish to have all students do it.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • What terms in a word problem suggest that you need to add? • What terms in a word problem suggest that you need to subtract? • What do you find most interesting about word problems? • What do you find most difficult?	• Depending on students' learning style, have them provide verbal or written answers. • You may wish to have students review the part related to Section 9.5 in **BLM 9–1 Chapter 9 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Supported Learning

Learning Style

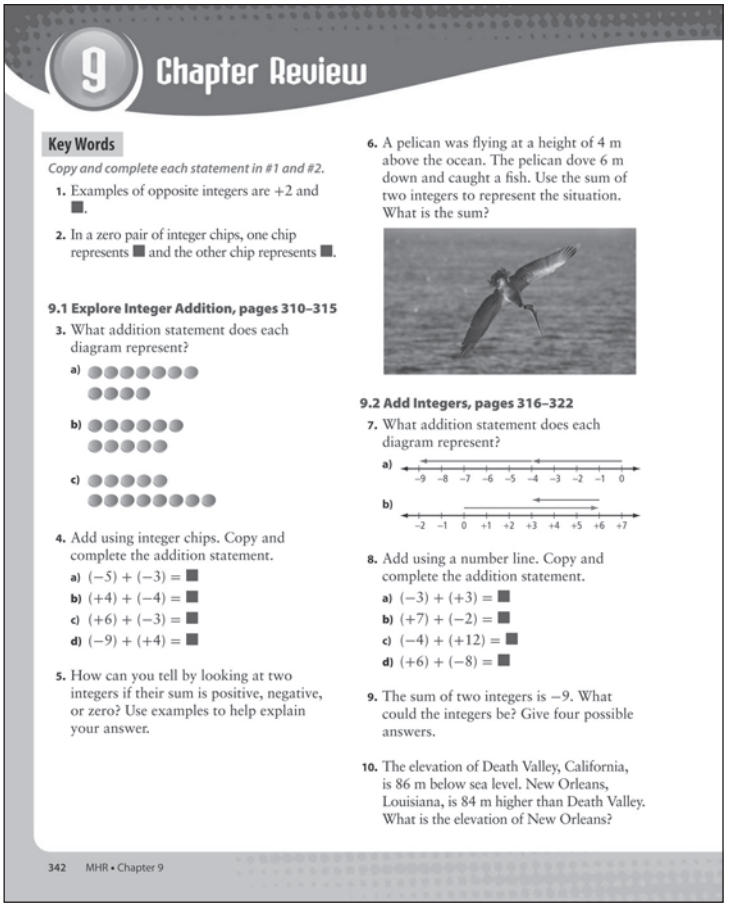
- **BLM 9–13 Section 9.5 Science Link** provides scaffolding for the Science Link activity for those who need it.

9 Chapter Review

Suggested Timing
40–50 minutes

Materials
- red and blue integer chips

Blackline Masters
Master 3 Integer Number Lines
Master 4 Vertical and Horizontal Number Lines
BLM 9–1 Chapter 9 Self-Assessment
BLM 9–3 Section 9.1 Extra Practice
BLM 9–4 Section 9.2 Extra Practice
BLM 9–6 Time Zone Map
BLM 9–7 Section 9.3 Extra Practice
BLM 9–9 Section 9.4 Extra Practice
BLM 9–12 Section 9.5 Extra Practice



9 Chapter Review

Key Words

Copy and complete each statement in #1 and #2.

1. Examples of opposite integers are +2 and ■.
2. In a zero pair of integer chips, one chip represents ■ and the other chip represents ■.

9.1 Explore Integer Addition, pages 310–315

3. What addition statement does each diagram represent?

a) ●●●●●●● ●●●●

b) ●●●●●● ●●●●●

c) ●●●●● ●●●●●●●●

4. Add using integer chips. Copy and complete the addition statement.
   a) (−5) + (−3) = ■
   b) (+4) + (−4) = ■
   c) (+6) + (−3) = ■
   d) (−9) + (+4) = ■
5. How can you tell by looking at two integers if their sum is positive, negative, or zero? Use examples to help explain your answer.
6. A pelican was flying at a height of 4 m above the ocean. The pelican dove 6 m down and caught a fish. Use the sum of two integers to represent the situation. What is the sum?

9.2 Add Integers, pages 316–322

7. What addition statement does each diagram represent?

a)

b)

8. Add using a number line. Copy and complete the addition statement.
   a) (−3) + (+3) = ■
   b) (+7) + (−2) = ■
   c) (−4) + (+12) = ■
   d) (+6) + (−8) = ■
9. The sum of two integers is −9. What could the integers be? Give four possible answers.
10. The elevation of Death Valley, California, is 86 m below sea level. New Orleans, Louisiana, is 84 m higher than Death Valley. What is the elevation of New Orleans?

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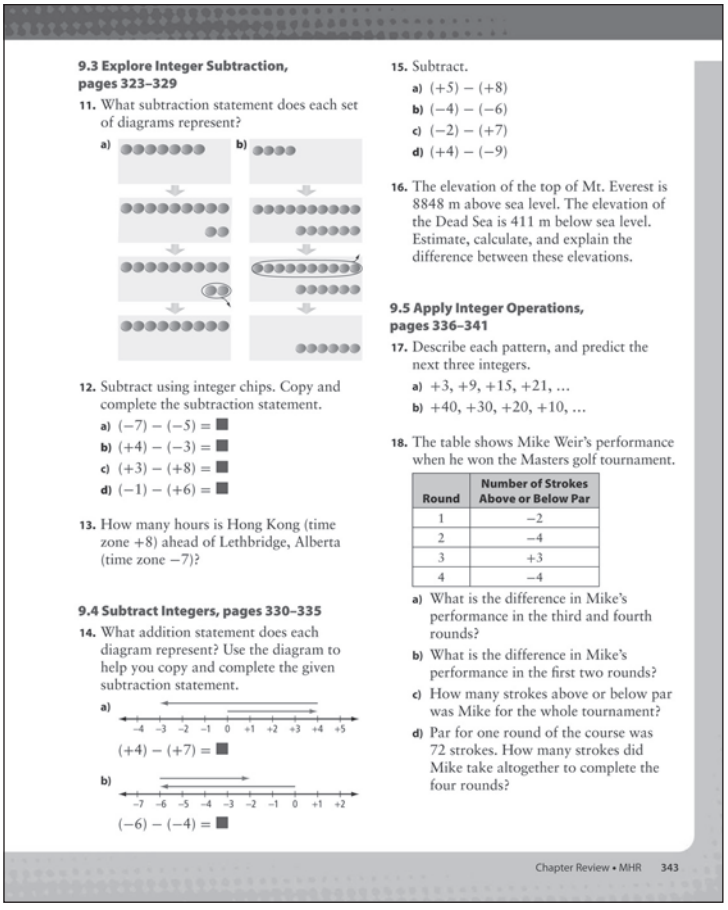
Activity Planning Notes

Have students work independently to complete the review questions. If students encounter difficulties, they could discuss strategies with other students. Encourage them to refer to the information in their chapter Foldable and then to the specific section in the student resource or their Foldable.

Students will need integer chips in order to complete #4.

Assessment *for* Learning	Supported Learning
Chapter 9 Review The chapter review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Tell students to check the contents of the What I Need to Work On tab of their chapter Foldable. Have students do at least one question related to any concept, skill, or process that has been giving them trouble. • Have students revisit any section they are having difficulty with prior to working on the Chapter 9 Practice Test.



9.3 Explore Integer Subtraction, pages 323–329

11. What subtraction statement does each set of diagrams represent?

a)

b)

12. Subtract using integer chips. Copy and complete the subtraction statement.

a) $(-7) - (-5) =$ ■

b) $(+4) - (-3) =$ ■

c) $(+3) - (+8) =$ ■

d) $(-1) - (+6) =$ ■

13. How many hours is Hong Kong (time zone +8) ahead of Lethbridge, Alberta (time zone −7)?

9.4 Subtract Integers, pages 330–335

14. What addition statement does each diagram represent? Use the diagram to help you copy and complete the given subtraction statement.

a)

$(+4) - (+7) =$ ■

b)

$(-6) - (-4) =$ ■

15. Subtract.

a) $(+5) - (+8)$

b) $(-4) - (-6)$

c) $(-2) - (+7)$

d) $(+4) - (-9)$

16. The elevation of the top of Mt. Everest is 8848 m above sea level. The elevation of the Dead Sea is 411 m below sea level. Estimate, calculate, and explain the difference between these elevations.

9.5 Apply Integer Operations, pages 336–341

17. Describe each pattern, and predict the next three integers.

a) +3, +9, +15, +21, ...

b) +40, +30, +20, +10, ...

18. The table shows Mike Weir's performance when he won the Masters golf tournament.

Round	Number of Strokes Above or Below Par
1	−2
2	−4
3	+3
4	−4

a) What is the difference in Mike's performance in the third and fourth rounds?

b) What is the difference in Mike's performance in the first two rounds?

c) How many strokes above or below par was Mike for the whole tournament?

d) Par for one round of the course was 72 strokes. How many strokes did Mike take altogether to complete the four rounds?

Chapter Review • MHR 343

Supported Learning

Learning Style and Memory

- Have students use **BLM 9–6 Time Zone Map** to help them answer #13.
- Refer students to the blackline master that provides additional reinforcement for each section. See **BLM 9–3 Section 9.1 Extra Practice**, **BLM 9–4 Section 9.2 Extra Practice**, **BLM 9–7 Section 9.3 Extra Practice**, **BLM 9–9 Section 9.4 Extra Practice**, and **BLM 9–12 Section 9.5 Extra Practice**.

Learning Style

- Allow students to complete the Chapter 9 Review using any combination of oral answers, written answers, and diagrams.
- Allow students to use integer chips and number lines to complete the Chapter 9 Review.

ESL, Language, and Memory

- Encourage students to use their chapter Foldable during the Chapter 9 Review and to add any notes into the pertinent sections.

Motor

- Provide students with a copy of **Master 3 Integer Number Lines** or **Master 4 Vertical and Horizontal Number Lines** to model problems.

Gifted and Enrichment

- Students may already be familiar with the skills handled in this review. To provide extra questions, go to **www.mathlinks7.ca** and follow the links.

Assessment *as* Learning	Supported Learning
Math Learning Log Once students have completed the chapter review, have them reflect on their progress and complete a journal entry for each statement: – I am comfortable with the following parts of the chapter … – I prefer to add and subtract integers using ... – I am having difficulty with … – Here's how I plan to address the areas I am having difficulty with …	• Have students refer back to the What I Need to Work On tab of their chapter Foldable and answer these questions from the contents of that tab. • You may wish to have students refer to **BLM 9–1 Chapter 9 Self-Assessment** when they report on what they are comfortable with, what they continue to have difficulty with, and what they plan to do about it.

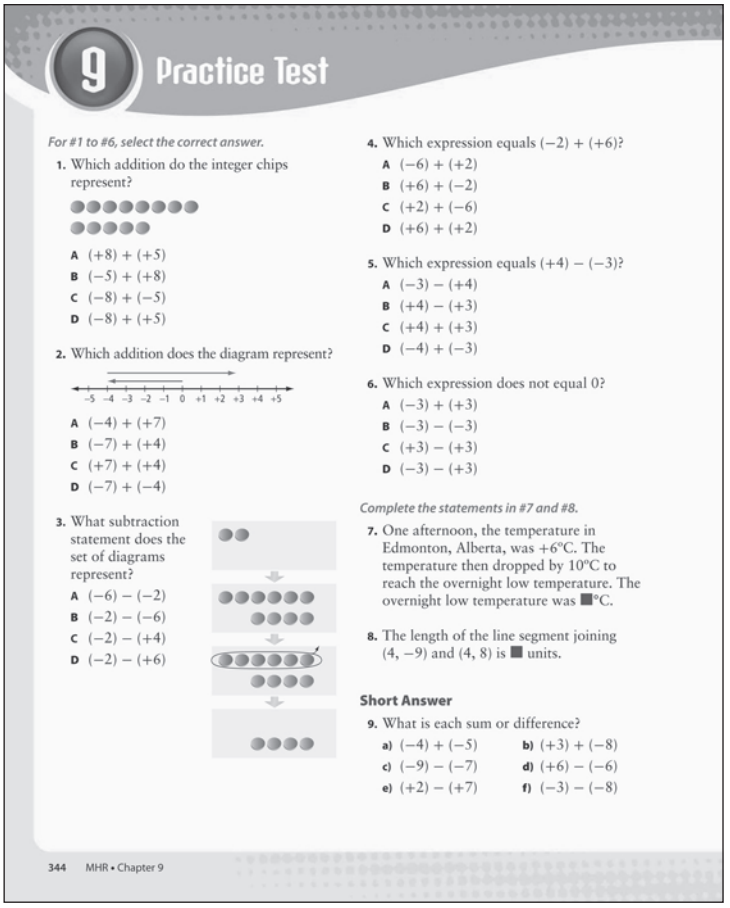
9 Practice Test

Suggested Timing
40–50 minutes

Materials
- red and blue integer chips

Blackline Masters
BLM 9–1 Chapter 9 Self-Assessment
BLM 9–14 Chapter 9 Test

Assessment *as* Learning	Supported Learning
Chapter 9 Self-Assessment Have students review their earlier responses on **BLM 9–1 Chapter 9 Self-Assessment**.	• Have students use their responses on the practice test and work they completed earlier in the chapter to complete the After column of this self-assessment. Before the Chapter 9 Test, coach them in the areas in which they are having problems.



9 Practice Test

For #1 to #6, select the correct answer.

1. Which addition do the integer chips represent?

A (+8) + (+5)
B (−5) + (+8)
C (−8) + (−5)
D (−8) + (+5)

2. Which addition does the diagram represent?

A (−4) + (+7)
B (−7) + (+4)
C (+7) + (+4)
D (−7) + (−4)

3. What subtraction statement does the set of diagrams represent?

A (−6) − (−2)
B (−2) − (−6)
C (−2) − (+4)
D (−2) − (+6)

4. Which expression equals (−2) + (+6)?

A (−6) + (+2)
B (+6) + (−2)
C (+2) + (−6)
D (+6) + (+2)

5. Which expression equals (+4) − (−3)?

A (−3) − (+4)
B (+4) − (+3)
C (+4) + (+3)
D (−4) + (−3)

6. Which expression does not equal 0?

A (−3) + (+3)
B (−3) − (−3)
C (+3) − (+3)
D (−3) − (+3)

Complete the statements in #7 and #8.

7. One afternoon, the temperature in Edmonton, Alberta, was +6°C. The temperature then dropped by 10°C to reach the overnight low temperature. The overnight low temperature was ■°C.

8. The length of the line segment joining (4, −9) and (4, 8) is ■ units.

Short Answer

9. What is each sum or difference?

a) (−4) + (−5)
b) (+3) + (−8)
c) (−9) − (−7)
d) (+6) − (−6)
e) (+2) − (+7)
f) (−3) − (−8)

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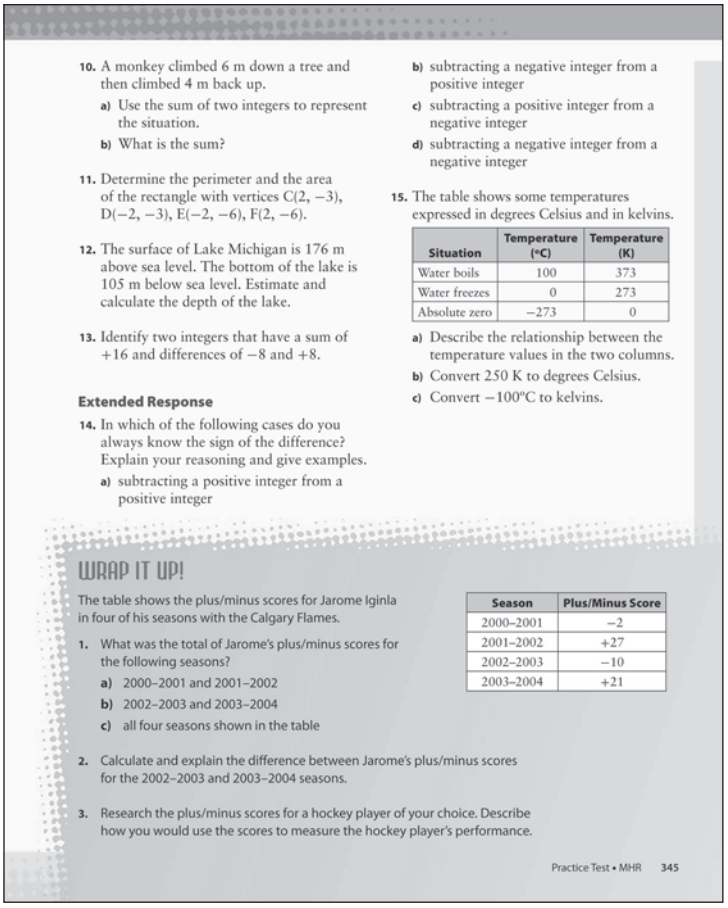
Study Guide

Question(s)	Section(s)	Refer to	I can …
1, 2, 10	9.1 9.2	Example 1 Example 1	✓ add integers using integer chips ✓ add integers using a number line
3	9.3	Example 1	✓ subtract integers using integer chips
4, 5, 6, 9, 15	9.1 9.2 9.3 9.4	Example 1 Example 1 Example 1 Example 2	✓ add integers using integer chips ✓ add integers using a number line ✓ subtract integers using integer chips ✓ subtract integers using a number line
7, 8, 11, 12, 13	9.3 9.4	Example 2 Example 2	✓ subtract integers using integer chips ✓ use addition to subtract integers
14	9.5	Example	✓ decide when to add and subtract integers in problem solving



10. A monkey climbed 6 m down a tree and then climbed 4 m back up.
    a) Use the sum of two integers to represent the situation.
    b) What is the sum?

11. Determine the perimeter and the area of the rectangle with vertices C(2, −3), D(−2, −3), E(−2, −6), F(2, −6).

12. The surface of Lake Michigan is 176 m above sea level. The bottom of the lake is 105 m below sea level. Estimate and calculate the depth of the lake.

13. Identify two integers that have a sum of +16 and differences of −8 and +8.

Extended Response

14. In which of the following cases do you always know the sign of the difference? Explain your reasoning and give examples.
    a) subtracting a positive integer from a positive integer
    b) subtracting a negative integer from a positive integer
    c) subtracting a positive integer from a negative integer
    d) subtracting a negative integer from a negative integer

15. The table shows some temperatures expressed in degrees Celsius and in kelvins.

Situation	Temperature (°C)	Temperature (K)
Water boils	100	373
Water freezes	0	273
Absolute zero	−273	0

    a) Describe the relationship between the temperature values in the two columns.
    b) Convert 250 K to degrees Celsius.
    c) Convert −100°C to kelvins.

WRAP IT UP!

The table shows the plus/minus scores for Jarome Iginla in four of his seasons with the Calgary Flames.

Season	Plus/Minus Score
2000–2001	−2
2001–2002	+27
2002–2003	−10
2003–2004	+21

1. What was the total of Jarome's plus/minus scores for the following seasons?
   a) 2000–2001 and 2001–2002
   b) 2002–2003 and 2003–2004
   c) all four seasons shown in the table

2. Calculate and explain the difference between Jarome's plus/minus scores for the 2002–2003 and 2003–2004 seasons.

3. Research the plus/minus scores for a hockey player of your choice. Describe how you would use the scores to measure the hockey player's performance.

Practice Test • MHR 345

Supported Learning

Learning Style

- Allow students to use integer chips and number lines to complete the practice test.

ESL, Language, and Memory

- Consider allowing students to use their chapter Foldable during the practice test.

Activity Planning Notes

This practice test can be assigned as an in-class or take-home assignment. These are the minimum questions that will meet the related curriculum outcomes: #1–#7, #9, #10, and #12.

Answers to the Chapter 9 Practice Test are provided on **BLM 9–17 Chapter 9 *MathLinks 7* Student Resource Answers**.

Note: In #14, the temperature unit on the Kelvin scale is the kelvin (K). The plural is kelvins. There is no such unit as the "degree Kelvin (°K)," though this is a common misuse.

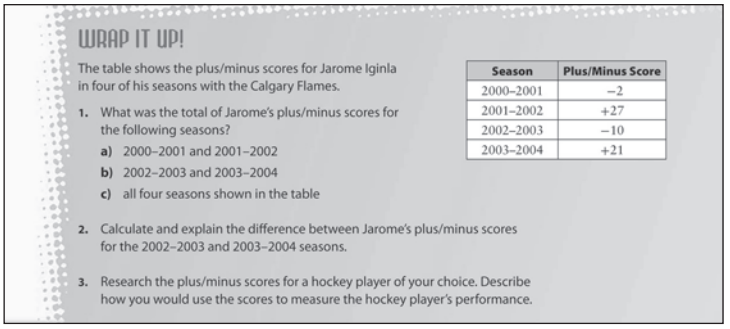
Assessment *of* Learning	Supported Learning
Chapter 9 Test After students complete the practice test, you may wish to use **BLM 9–14 Chapter 9 Test** as a summative assessment.	• Consider allowing students to use their chapter Foldable. • Consider using the Math Games on page 346 or the Challenge in Real Life on page 347 to assess the knowledge and skills of students who have difficulty with tests.

Wrap It Up!

Suggested Timing
40–50 minutes (allow more time if students are conducting their own research)

Materials
- research materials or a computer with Internet access

Blackline Masters
Master 1 Project Rubric
BLM 9–5 Section 9.2 Math Link
BLM 9–10 Section 9.4 Math Link
BLM 9–15 Chapter 9 Wrap It Up!



WRAP IT UP!

The table shows the plus/minus scores for Jarome Iginla in four of his seasons with the Calgary Flames.

Season	Plus/Minus Score
2000–2001	−2
2001–2002	+27
2002–2003	−10
2003–2004	+21

1. What was the total of Jarome's plus/minus scores for the following seasons?
   a) 2000–2001 and 2001–2002
   b) 2002–2003 and 2003–2004
   c) all four seasons shown in the table
2. Calculate and explain the difference between Jarome's plus/minus scores for the 2002–2003 and 2003–2004 seasons.
3. Research the plus/minus scores for a hockey player of your choice. Describe how you would use the scores to measure the hockey player's performance.

Specific Outcomes

N6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.

Web Link

For updated plus/minus ratings of NHL players, go to **www.mathlinks7.ca** and follow the links.

Answers

Wrap It Up!

1. a) +25
   b) +11
   c) +36
2. Look for one of the following answers. Students are not expected to give both differences and their respective explanations.
   - There was a difference of +31. Jarome's score was 31 higher in 2003–2004 than in 2002–2003.
   - There was a difference of −31. Jarome's score was 31 lower in 2002–2003 than in 2003–2004.
3. Answers will vary.

Activity Planning Notes

This chapter problem can be scaled up or down to meet the needs of your class. Students who are interested in hockey may enjoy researching plus/minus scores of their favourite hockey player and making comparisons to Jarome Iginla.

Method 1: Consider allowing students to choose and then research a player's plus/minus scores independently. They may find season-to-season data or even more detail, since it is possible to find a player's plus/minus score for every game in a season.

Method 2: To minimize the time spent on this Wrap It Up!, provide students with plus/minus scores for a number of players and let them choose from among those players.

Assessment *of* Learning	Supported Learning
Wrap It Up! The Wrap It Up! allows students to apply their understanding of integers to plus/minus scores for hockey. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this Wrap It Up! Page 345a provides notes on how to use the rubric for this Wrap It Up!	• You may wish to have students review the work they have completed in the Math Links in Sections 9.2 and 9.4 before they begin. • If students have not completed the Math Links earlier in the chapter, you may wish to provide them with **BLM 9–5 Section 9.2 Math Link** and **BLM 9–10 Section 9.4 Math Link**. • You may wish to have students use **BLM 9–15 Chapter 9 Wrap It Up!**, which provides scaffolding for the chapter problem wrap-up.

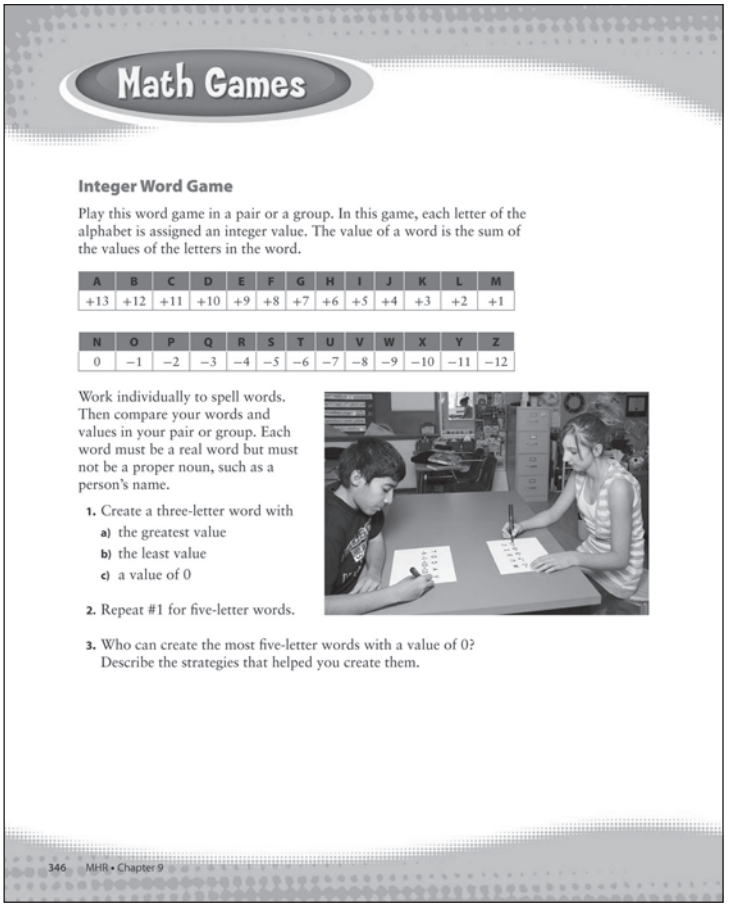
The chart below shows **Master 1 Project Rubric** for tasks such as that in the Wrap It Up! and provides notes that specify how to identify the level of specific answers.

Score/Level	Holistic Descriptor	Specific Question Notes
5 (Standard of Excellence)	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution
4 (Above Acceptable)	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete solution with a calculation error in the scores *or* • provides a weak explanation in part 2 or 3
3 (Meets Acceptable)	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides a complete and correct solution to any two parts of the question (1, 2 or 1, 3 or 2, 3)
2 (Below Acceptable)	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a complete and correct solution to any one part of the question (1 or 2 or 3)
1 (Beginning)	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• makes a start to any one part of the question

Math Games

Suggested Timing
40–50 minutes

Blackline Masters
Master 3 Integer Number Lines



Math Games

Integer Word Game

Play this word game in a pair or a group. In this game, each letter of the alphabet is assigned an integer value. The value of a word is the sum of the values of the letters in the word.

A	B	C	D	E	F	G	H	I	J	K	L	M
+13	+12	+11	+10	+9	+8	+7	+6	+5	+4	+3	+2	+1

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−11	−12

Work individually to spell words. Then compare your words and values in your pair or group. Each word must be a real word but must not be a proper noun, such as a person's name.

1. Create a three-letter word with
   a) the greatest value
   b) the least value
   c) a value of 0
2. Repeat #1 for five-letter words.
3. Who can create the most five-letter words with a value of 0? Describe the strategies that helped you create them.

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Specific Outcomes

N6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.

Supported Learning

Learning Style and Memory

- Provide students who have difficulty adding integers with integer chips or number lines. You may wish to provide **Master 3 Integer Number Lines**.

ESL and Language

- This game requires students to know a wide variety of words. Partner English language learners with each other to make the game fair. Consider giving a point advantage to English language learners.

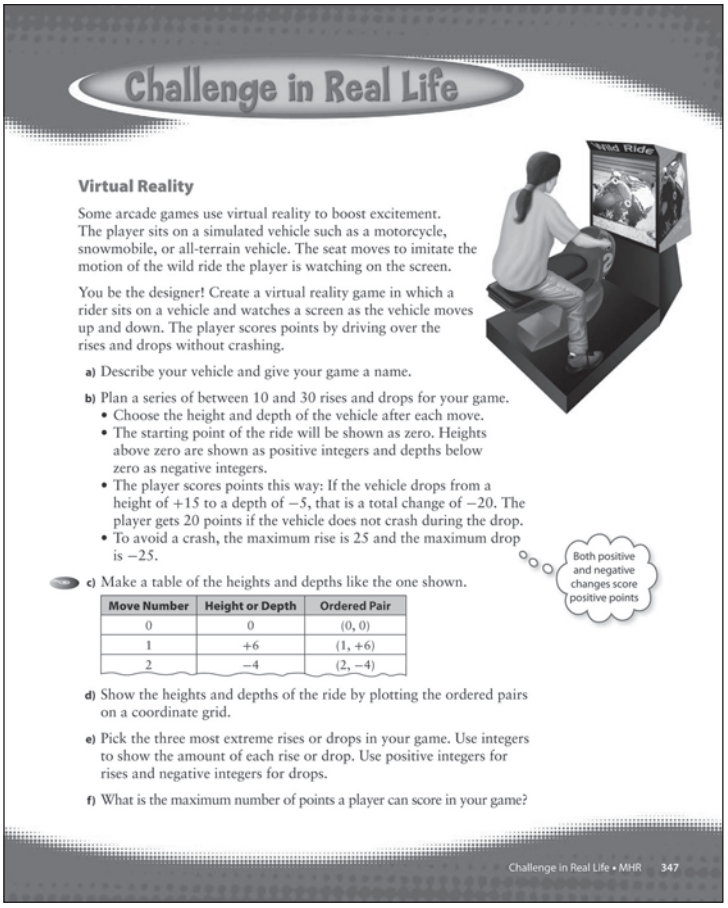
Activity Planning Notes

Give this game some context by talking about codes and ciphers that have been used to send secret messages throughout history. Secret forms of communication are used by businesses to protect their products and information. Armies and spies hide coded orders in messages.

This Integer Word Game uses a common number code. As they play, students will work backward by creating a word and then calculating its value based on the given number code. Have students play with a partner or in a small group.

Assessment *for* Learning	Supported Learning
Integer Word Game Monitor students to check for accuracy and efficiency as they add integers.	• Check that students are able to add integers correctly. • Have students record and check each other's sums to make sure they are correct. • Have students compare the strategies they used to create three-letter words of the greatest value, least value, and zero value.

Challenge in Real Life


Challenge in Real Life

Virtual Reality

Some arcade games use virtual reality to boost excitement. The player sits on a simulated vehicle such as a motorcycle, snowmobile, or all-terrain vehicle. The seat moves to imitate the motion of the wild ride the player is watching on the screen.

You be the designer! Create a virtual reality game in which a rider sits on a vehicle and watches a screen as the vehicle moves up and down. The player scores points by driving over the rises and drops without crashing.

a) Describe your vehicle and give your game a name.

b) Plan a series of between 10 and 30 rises and drops for your game.
- Choose the height and depth of the vehicle after each move.
- The starting point of the ride will be shown as zero. Heights above zero are shown as positive integers and depths below zero as negative integers.
- The player scores points this way: If the vehicle drops from a height of +15 to a depth of −5, that is a total change of −20. The player gets 20 points if the vehicle does not crash during the drop.
- To avoid a crash, the maximum rise is 25 and the maximum drop is −25.

c) Make a table of the heights and depths like the one shown.

Move Number	Height or Depth	Ordered Pair
0	0	(0, 0)
1	+6	(1, +6)
2	−4	(2, −4)

Both positive and negative changes score positive points

d) Show the heights and depths of the ride by plotting the ordered pairs on a coordinate grid.

e) Pick the three most extreme rises or drops in your game. Use integers to show the amount of each rise or drop. Use positive integers for rises and negative integers for drops.

f) What is the maximum number of points a player can score in your game?

Challenge in Real Life • MHR 347

Suggested Timing
60–75 minutes

Materials
- ruler

Blackline Masters
Master 1 Project Rubric
Master 4 Vertical and Horizontal Number Lines
Master 9 0.5 Centimetre Grid Paper
BLM 9–16 Coordinate Grid

Mathematical Processes
- ☑ Communication
- ☑ Connections
- ☐ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☐ Technology
- ☑ Visualization

Specific Outcomes

N6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.

SS4 Identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs.

Activity Planning Notes

You may wish to use the following steps to introduce and complete this Challenge in Real Life:

1. Read through Virtual Reality as a class. Have students share their experiences with various virtual reality games. Invite them to tell how the games work and how the seat moves during the game. If students have played only video games, have them describe games that simulate the movement of a vehicle.
2. Discuss with students what features a virtual reality game could include, what makes these games exciting, and what safety concerns there might be (e.g., players may be required to wear a seat belt). You may wish to set up criteria with students for the games they will design. As a class, decide what makes a ride most enjoyable. For example, would up-down maximum drops be more enjoyable than slowly increasing the drops to a giant finale?

Supported Learning

Gifted and Enrichment
- Have students include going up or down a summit in their game so that, for example, the player begins at a height of 100 and then must go down to zero, or vice versa.
- Students could be challenged to research how actual virtual reality is being developed and some of the math involved.

Supported Learning

Learning Style and Motor

- Instead of plotting points on a coordinate grid, consider having students use arrows on a vertical number line to show the rises and drops using **Master 4 Vertical and Horizontal Number Lines**.

Learning Style

- Encourage concrete and kinesthetic learners to use an object that represents a vehicle and physically move it on a coordinate grid to plan their game.

ESL and Language

- For the first part of the activity, allow students to draw or cut out a picture instead of describing their game in words.

Motor

- Provide students with a copy of **BLM 9–16 Coordinate Grid** before beginning their graph.

3. Discuss how the rises and drops in a game might be shown on a coordinate grid. You may wish to have on hand a model grid for an existing game to show students what the grid might look like. Have students work individually to design their game and make their grid. Provide each student with a copy of **Master 9 0.5 Centimetre Grid Paper**.
4. Clarify that the task is to
   - name and describe a game
   - plan a series of rises and drops for the game, including 10 to 30 movements with a maximum rise or drop of 25
   - make a table of the rises and drops
   - plot the rises and drops as ordered pairs on a coordinate grid
   - use integers to describe the amount of the three most extreme rises and drops
   - determine the maximum number of points a player can score
5. Review the **Master 1 Project Rubric** with students so that they will know what is expected.

Note: If you set up criteria for the most "fun" game before the activity, the class could perform a follow-up evaluation of all the completed rides.

This challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment *for* Learning	Supported Learning
Virtual Reality Discuss the challenge with the class. Have students work together to develop a response, and then provide separate designs.	• Review with students how to plot points on a coordinate grid.

Assessment *of* Learning	Supported Learning
Virtual Reality Discuss the challenge with the class. Have students work together to develop a response, and then provide separate designs.	• Use **Master 1 Project Rubric** to assist you in assessing student work. Page 347a provides notes on how to use this rubric for this challenge. • To view student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as that in the Challenge in Real Life and provides notes that specify how to identify the level of specific answers.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a correct response to all parts of the question with possibly minor errors that do not affect the final conclusion
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete response to parts a) through d) *or* • provides a complete response to parts a) through d), and f)
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides a correct response to parts a), b), and c) *or* • provides a correct response to parts a), b), and d) *or* • partially completes parts a) through d)
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a correct response to parts a) and b) with minimal communication
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides a correct response to part a) *or* • provides a diagram that could lead to a correct response