Coordinates and Design

10

General Outcomes

- Develop number sense.
- Use patterns to describe the world and solve problems.
- Represent algebraic expressions in multiple ways.

Specific Outcomes

N4 Demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions.
PR1 Demonstrate an understanding of oral or written patterns and their equivalent linear relations.
PR2 Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.
PR4 Explain the difference between an expression and an equation.
PR5 Evaluate an expression given the value of the variable(s).

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
10.1	✓ describe patterns using words, tables, and diagrams
	✓ determine patterns with repeating decimals
10.2	✓ write an expression to represent a pattern
	✓ identify and provide examples of a constant, a numerical coefficient, and a variable in an expression
	✓ write a word statement as an expression
10.3	✓ model an expression
	✓ evaluate an expression
	✓ make a table of values for an expression
10.4	✓ graph a linear relation
	✓ make a graph for a table of values
	✓ describe the relationship shown on a graph
	✓ match linear relations to graphs

Assessment *as* Learning	Supported Learning
Use the Before column of **BLM 10–1 Chapter 10 Self-Assessment** to provide students with the big picture for this chapter and to help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• As students complete each section of the chapter or complete the Chapter 10 Review, have them review the related parts of **BLM 10–1 Chapter 10 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Chapter 10 Planning Chart

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter Opener • 20–30 minutes		BLM 10–1 Chapter 10 Self-Assessment BLM 10–2 Patterns and Expressions	• ruler • stapler • grid paper • scissors • calendar
10.1 Describe Patterns • 80–100 minutes	**Essential:** 1–5, 7, 9, 14, Math Link **Typical:** 1–5, 7, 9, 11–15, Math Link **Extension/Enrichment:** 1–4, 12, 13, 16–18, Math Link	Master 2 Two Stars and One Wish BLM 10–1 Chapter 10 Self-Assessment BLM 10–4 Section 10.1 Extra Practice BLM 10–5 Section 10.1 Math Link BLM 10–14 Wrap It Up! Mystery Pattern	• ruler • scissors • calculator • counters or coloured tiles
10.2 Variables and Expressions • 80–100 minutes	**Essential:** 1–4, 6, 8, 9, 11, 12, Math Link **Typical:** 1–4, 6, 8, 9, 11–13, 15, 16, Math Link **Extension/Enrichment:** 1–4, 13, 14, 17–19, Math Link	BLM 10–1 Chapter 10 Self-Assessment BLM 10–6 Parts of an Expression BLM 10–7 Section 10.2 Extra Practice BLM 10–8 Section 10.2 Math Link BLM 10–14 Wrap It Up! Mystery Pattern	• toothpicks • coloured tiles • calendar
10.3 Evaluate Expressions • 80–100 minutes	**Essential:** 1, 2, 3 *or* 4, 5, 7, 9, 11, 13, Math Link **Typical:** 1, 2, 3 *or* 4, 5, 7, 9, 11, 13, 15, 17, 18, Math Link **Extension/Enrichment:** 1, 2, 3 *or* 4, 16, 19–21, Math Link	BLM 10–1 Chapter 10 Self-Assessment BLM 10–9 Section 10.3 Extra Practice BLM 10–10 Section 10.3 Math Link BLM 10–14 Wrap It Up! Mystery Pattern	• cups or plastic containers • counters • toothpicks • calendar
10.4 Graph Linear Relations • 100–125 minutes	**Essential:** 1 *or* 2, 3, 5, 7–10, Math Link **Typical:** 1 *or* 2, 3, 5, 7–13 , Math Link **Extension/Enrichment:** 1 *or* 2, 12–15, Math Link	Master 8 Centimetre Grid Paper Master 9 0.5 Centimetre Grid Paper BLM 10–1 Chapter 10 Self-Assessment BLM 10–3 Coordinate Grid BLM 10–11 Section 10.4 Extra Practice BLM 10–12 Section 10.4 Math Link BLM 10–14 Wrap It Up! Mystery Pattern	• ruler • grid paper • counters or coloured tiles • stir sticks
Chapter 10 Review • 40–50 minutes	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	BLM 10–1 Chapter 10 Self-Assessment BLM 10–4 Section 10.1 Extra Practice BLM 10–7 Section 10.2 Extra Practice BLM 10–9 Section 10.3 Extra Practice BLM 10–11 Section 10.4 Extra Practice	• unit cubes • cups or plastic containers • counters
Chapter 10 Practice Test • 40–50 minutes	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 2–5, 7, 8	BLM 10–1 Chapter 10 Self-Assessment BLM 10–13 Chapter 10 Test	• grid paper • ruler
Chapter 10 Wrap It Up! • 40–50 minutes		Master 1 Project Rubric BLM 10–5 Section 10.1 Math Link BLM 10–8 Section 10.2 Math Link BLM 10–10 Section 10.3 Math Link BLM 10–12 Section 10.4 Math Link BLM 10–14 Wrap It Up! Mystery Pattern BLM 10–15 Chapter 10 Wrap It Up!	

Chapter 10 Planning Chart (continued)

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter 10 **Math Games** • 60–75 minutes		Master 8 Centimetre Grid Paper BLM 10–16 Puzzling Expressions	
Chapter 10 **Challenge in Real Life** • 60–75 minutes		Master 1 Project Rubric Master 8 Centimetre Grid Paper Master 9 0.5 Centimetre Grid Paper BLM 10–17 Your Life Line Results BLM 10–18 Chapter 10 *MathLinks 7* Student Resource Answers BLM 10–19 Chapter 10 BLM Answers	• clock or watch with second hand, or stopwatch • ruler • coloured pencils or markers

Chapter 10 Assessment Planner

Assessment Options	Type of Assessment	Assessment Tool
Chapter Opener	Assessment *as* Learning (TR page i, 349)	BLM 10–1 Chapter 10 Self-Assessment Chapter 10 Foldable
10.1 Describe Patterns	Assessment *as* Learning (TR pages 353, 354, 356) Assessment *for* Learning (TR pages 352, 353, 355, 357)	Master 2 Two Stars and One Wish Math Learning Log (TR page 356) BLM 10–1 Chapter 10 Self-Assessment
10.2 Variables and Expressions	Assessment *as* Learning (TR pages 359, 362, 364) Assessment *for* Learning (TR pages 360, 361, 362, 364)	Math Learning Log (TR page 364) BLM 10–1 Chapter 10 Self-Assessment
10.3 Evaluate Expressions	Assessment *as* Learning (TR pages 366, 369, 370) Assessment *for* Learning (TR pages 367, 368, 369, 371)	Math Learning Log (TR page 370) BLM 10–1 Chapter 10 Self-Assessment
10.4 Graph Linear Relations	Assessment *as* Learning (TR pages 374, 378, 380) Assessment *for* Learning (TR pages 375, 377, 379, 381)	Math Learning Log (TR page 380) BLM 10–1 Chapter 10 Self-Assessment
Chapter 10 Review	Assessment *for* Learning (TR page 382) Assessment *as* Learning (TR page 383)	Math Learning Log (TR page 383) BLM 10–1 Chapter 10 Self-Assessment
Chapter 10 Practice Test	Assessment *as* Learning (TR page 384) Assessment *of* Learning (TR page 385)	BLM 10–1 Chapter 10 Self-Assessment BLM 10–13 Chapter 10 Test
Chapter 10 Wrap It Up!	Assessment *of* Learning (TR page 384a)	Master 1 Project Rubric
Chapter 10 Math Games	Assessment *for* Learning (TR page 386)	
Chapter 10 Challenge in Real Life	Assessment *for* Learning (TR pages 386a) Assessment *of* Learning (TR pages 386a)	Master 1 Project Rubric

You may wish to use one or more of the following materials to help you assess student readiness for Chapter 10.

Assessment *for* Learning	Supported Learning
Method 1: Have students develop a journal entry to explain what they personally know about patterns and when they might have experienced patterns in real life. **Method 2:** Have students complete **BLM 10–2 Patterns and Expressions** to check their conceptual understanding. Remind students that you are looking for the scope of their knowledge.	• Students who require reinforcement of prerequisite skills may wish to complete the Get Ready materials available in the *MathLinks 7 Workbook* and at the **www.mathlinks7.ca** book site.

Chapter Opener

Suggested Timing
20–30 minutes

Materials
- ruler
- stapler
- grid paper
- scissors
- calendar

Blackline Masters
BLM 10–1 Chapter 10 Self-Assessment

Key Words
pattern
variable
expression
value
constant
numerical coefficient
table of values
linear relation
graph
relationship

What's the Math?

In this chapter, students describe patterns using words, tables, and diagrams. Students learn to use number patterns to change from fractions to numbers with a repeating decimal. They are introduced to using variables and expressions to describe patterns. They also evaluate expressions. Finally, they graph linear relations using tables of values and describe the relationships shown on graphs.

Activity Planning Notes

Begin Chapter 10 by explaining that it is about describing patterns and representing them with expressions. Discuss with students where they have seen or used patterns and expressions in their lives. Try to elicit ideas from all class members. After some discussion, begin to focus on patterns involving time. Direct students' attention to the calendar in the student resource, and bring in a calendar to show to students. Then ask, "What patterns are in the school day? School year? School timetable?"

Math Link

The Math Link provides students with an opportunity to begin thinking about calendars and how number patterns are evident within each month and as the year progresses. Have students discuss the Math Link as a group so that they might share what they discover about the patterns mentioned in the student resource. This Math Link will assist them in working toward the chapter problem wrap-up titled Wrap It Up! on page 385. In the Wrap It Up! students use data from the individual Math Links to identify a mystery pattern. This means that for students to do the Wrap It Up! they need to complete all the Math Links in this chapter. The Math Links could be completed after each section in the chapter or all at the end as part of a project or group assignment.

FOLDABLES™
Study Tool

Have students make the Foldable in the student resource to keep track of the information in the chapter.

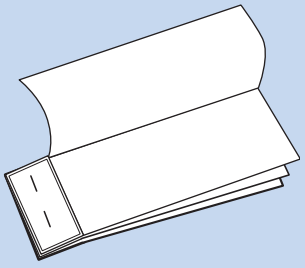
You may wish to have students keep track of Key Words using a design specifically for that purpose. Students can make the following Foldable and write vocabulary terms on the front of each tab. Have them use the space beneath the tab to write definitions and provide examples.

Step 1 Cut a sheet of grid paper horizontally in half. Fold the half sheet in two horizontally.

Step 2 Draw a line 2 cm in from the left side of the folded paper. Cut the top part of the fold along this line.

Step 3 Make one of these folded sheets for each Key Word. Staple the tabs together to make a booklet.

Step 4 Write a Key Word on the front of each tab. Write definitions and give examples underneath the tabs.



Note: You can have students make the complete vocabulary Foldable at the beginning of the chapter. Alternatively, have them add the number of tabs needed for each section as they work through the chapter.

Assessment *as* Learning	Supported Learning
Chapter 10 Foldable As students work on each section in Chapter 10, have them keep track of any problems they are having under the What I Need to Work On tab in their chapter Foldable.	• As students complete each section, have them review the list of items they need to work on, and then have them check off any that have been handled.

Supported Learning

Learning Style

- The creation of the Foldable is a good hands-on activity for concrete, kinesthetic, and visual learners.

Learning Style and Motor

- Students have already made several Foldables like the one in the chapter opener and have made a similar vocabulary Foldable in Chapter 1. You may wish to have them use these as models.

ESL and Language

- Students may benefit from having new vocabulary terms written on study cards and displayed in the classroom on a math word wall.

ESL and Memory

- Students may choose to create their own vocabulary/picture dictionary in their notebook for the Key Words. Matching a picture and/or symbol with each Key Word and its definition helps students consolidate their understanding of the vocabulary.

Meeting the Needs of All Learners

- Discuss patterns used by various communities. For example, Inuit communities have their own divisions of the year, reflecting the seasonal differences experienced in the North. See *The Arctic Sky: Inuit Astronomy, Star Lore, and Legend* by John MacDonald (Royal Ontario Museum, 1998). You might wish to discuss with the class a pattern that is familiar in your community. This could be a quilt pattern displayed in the local museum, a well-known egg-painting pattern, or perhaps a unique local architectural design.

Common Errors

- Students leave all the Math Links to the end of the chapter and rush through them to determine the missing numbers in the Wrap It Up!

R_x Choose the option of having students complete each Math Link as they proceed through the chapter. Alternatively, for the Wrap It Up! have students prepare a report on patterns and expressions based on any two Math Links.

10.1 Describe Patterns

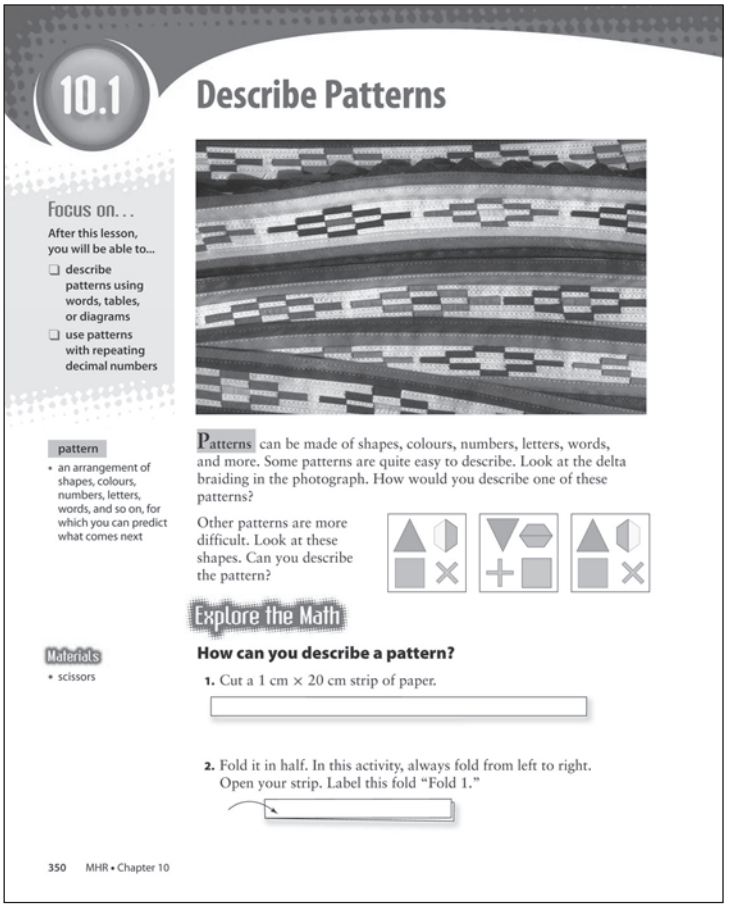
Suggested Timing
80–100 minutes

Materials
- ruler
- scissors
- calculator
- counters or coloured tiles

Blackline Masters
Master 2 Two Stars and One Wish
BLM 10–1 Chapter 10 Self-Assessment
BLM 10–4 Section 10.1 Extra Practice
BLM 10–5 Section 10.1 Math Link
BLM 10–14 Wrap It Up! Mystery Pattern

Mathematical Processes

- [x] Communication
- [x] Connections
- [ ] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [x] Technology
- [x] Visualization



10.1 Describe Patterns

Focus on...
After this lesson, you will be able to...
- describe patterns using words, tables, or diagrams
- use patterns with repeating decimal numbers

pattern
- an arrangement of shapes, colours, numbers, letters, words, and so on, for which you can predict what comes next

Patterns can be made of shapes, colours, numbers, letters, words, and more. Some patterns are quite easy to describe. Look at the delta braiding in the photograph. How would you describe one of these patterns?

Other patterns are more difficult. Look at these shapes. Can you describe the pattern?

Explore the Math

Materials
- scissors

How can you describe a pattern?

1. Cut a 1 cm × 20 cm strip of paper.

2. Fold it in half. In this activity, always fold from left to right. Open your strip. Label this fold "Fold 1."

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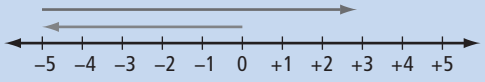
Specific Outcomes

N4 Demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions.

Warm-Up

1. $(-8) + (-5)$ **2.** $(-3) - (-3)$

3. What addition does the diagram represent?



4. a) The results of a class election are shown. Sketch a circle graph to display the data.

b) Explain why you sketched the circle sectors the way you did.

Candidate	Votes
Candidate 1	12
Candidate 2	9
Candidate 3	9
Candidate 4	6

5. $3\frac{3}{4} - 2\frac{1}{2}$

Mental Math

For #6 to #8, is the answer closest to 0, $\frac{1}{2}$, *or* 1? *Explain your choice.*

6. $\frac{4}{5} + \frac{1}{10}$

7. $\frac{3}{4} - \frac{1}{5}$

8. $\frac{1}{3} - \frac{1}{6}$

For #9 and #10, estimate the answer.

9. $1\frac{4}{5} + 2\frac{9}{11}$

10. $1\frac{3}{20} + 5\frac{15}{17}$

3. Fold the strip back as it was in step 2. Fold it in half again. Open your strip. Label each of the new folds "Fold 2."

4. The folds marked D in the diagram point down. Those marked U point up. Was Fold 1 up or down?

5. Fold your strip back as it was in step 3. Predict what the pattern of ups and downs will be after another fold in half.

6. Fold your strip in half again. Open your strip. Was your prediction correct?

7. Label each of the four new folds from step 6 "Fold 3."

8. Copy the table. Finish filling it in to help you find a pattern.

Fold Number	Ups and Downs
1	D
2	U, D
3	

9. Predict what the ups and downs will be after another fold in half.

10. Fold your strip back as it was in step 6. Fold it in half again. Was your prediction correct?

Reflect on Your Findings

11. a) Did you find this pattern difficult to predict? Why or why not?
b) What problem-solving strategies did you use? Which worked best?

10.1 Describe Patterns • MHR 351

Activity Planning Notes

Discuss patterns with the class. Encourage students to work in pairs or small groups to identify patterns they have encountered. Then summarize them with the class.

Explore the Math

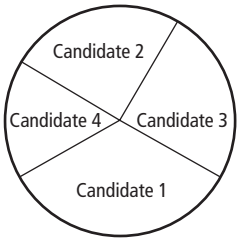
Have students work individually to make a 1 cm × 20 cm strip of paper and then fold it as instructed. This exercise leads to a pattern that most students will not have explored. It is important that students realize that there are many useful patterns outside of the number patterns they normally come across in mathematics.

Answers

Warm-Up

1. −13 **2.** 0 **3.** (−5) + (+8)

4. a)



b) Candidates 2 and 3 each have $\frac{1}{4}$ of the votes. They share half the circle. Candidates 1 and 4 share the other half. Candidate 1 has twice the share of Candidate 4.

5. $1\frac{1}{4}$

6. It is closest to 1 because $\frac{4}{5}$ is close to 1 and you have added something to it.

7. It is closest to $\frac{1}{2}$ because $\frac{3}{4}$ is between $\frac{1}{2}$ and 1 and you take a small amount from $\frac{3}{4}$.

8. It is closest to 0 because you start with less than $\frac{1}{2}$ and take away another fraction.

9. 2 + 3 = 5 **10.** 1 + 6 = 7

Explore the Math

4. down **5.** Answers may vary. For example: U, D, U, D

6. Answers may vary. For example: Yes. The pattern for Fold 3 was U, D, U, D.

8. U, D, U, D

9. Answers may vary. For example: U, D, U, D, U, D, U, D

10. Answers may vary. For example: No. The pattern for Fold 4 was D, U, D, U, D, U, D, U.

11. a) Answers may vary. For example: Yes. It was difficult to determine the pattern based on just three folds.
b) Answers may vary.

Common Errors

- Students may struggle with the pattern in the Explore the Math.

R_x Encourage all students to use the paper-folding model so that they can see what happens after the first three or four folds. Then have students either continue folding or attempt to extend the pattern abstractly.

Answers

Show You Know: Example 1

a) The number of squares is double the figure number, or 2 times the figure number.

b) 10. Double the figure number of 5 is 10.

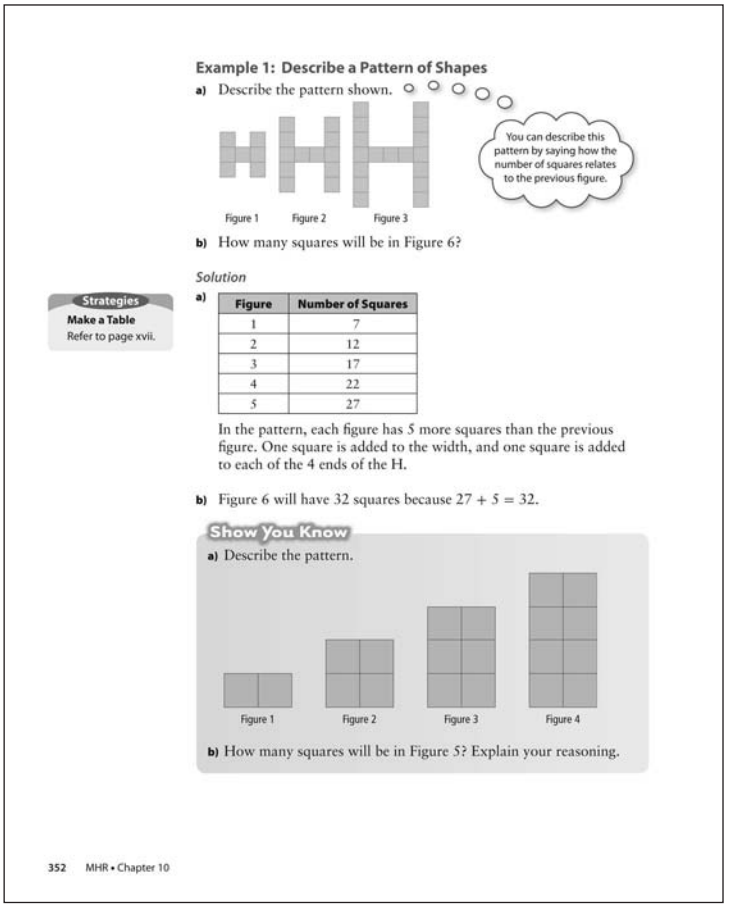
Supported Learning

Learning Style, ESL, and Language

- Have students create a list of words that they can use to describe patterns.
- Allow students to describe the patterns orally with a partner rather than describe them in writing.

Meeting the Needs of All Learners

- Focus on the use of manipulatives for representing patterns and determining the rules for patterns.



Example 1: Describe a Pattern of Shapes

a) Describe the pattern shown.

b) How many squares will be in Figure 6?

Solution

Strategies
Make a Table
Refer to page xvii.

a)

Figure	Number of Squares
1	7
2	12
3	17
4	22
5	27

In the pattern, each figure has 5 more squares than the previous figure. One square is added to the width, and one square is added to each of the 4 ends of the H.

b) Figure 6 will have 32 squares because 27 + 5 = 32.

Show You Know

a) Describe the pattern.

b) How many squares will be in Figure 5? Explain your reasoning.

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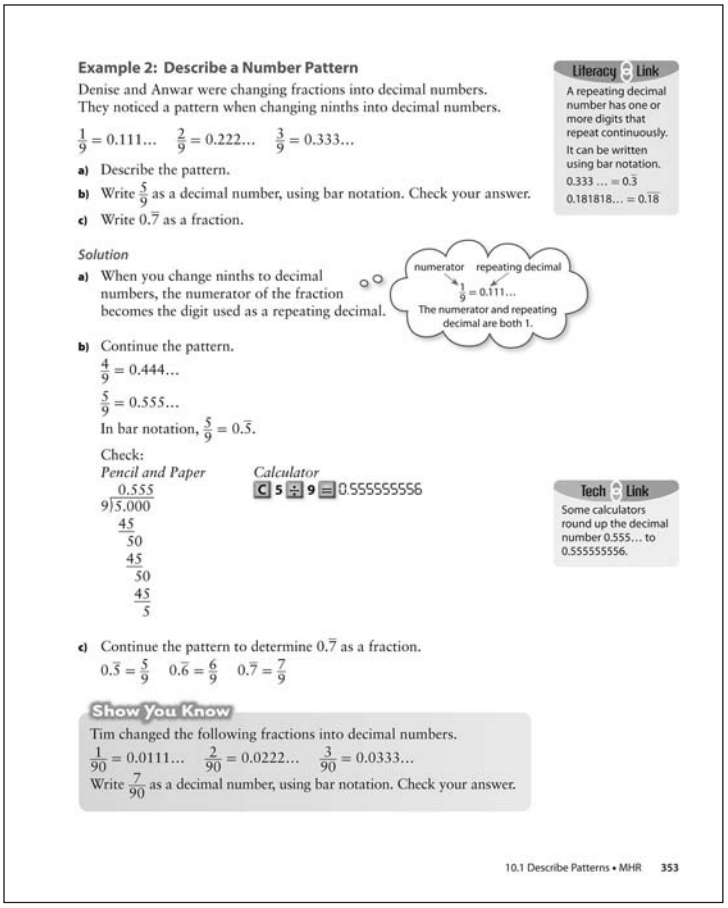
Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss the strategies they used to determine the pattern. During this process, they are generalizing what they have learned during the Explore the Math.	• Suggest that students note strategies their partner used and consider trying them the next time they are predicting a pattern. • Make a list of Strategies for Determining a Pattern and hang it in the classroom for easy reference. Add to the list as the class works through the chapter.

For Example 1, at this point students should be asked to describe the patterns using words, not expressions.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• You may wish to have students, especially concrete and kinesthetic learners, use tiles to construct the first five figures in the Show You Know before they attempt to answer the questions. Make sure that they get Figure 5 correct. • Ask students who are having trouble describing the pattern to say how the number of squares relates to the figure number and to think about what operation they use on the figure number to determine the number of squares.



Example 2: Describe a Number Pattern

Denise and Anwar were changing fractions into decimal numbers. They noticed a pattern when changing ninths into decimal numbers.

$\frac{1}{9} = 0.111\ldots \quad \frac{2}{9} = 0.222\ldots \quad \frac{3}{9} = 0.333\ldots$

a) Describe the pattern.

b) Write $\frac{5}{9}$ as a decimal number, using bar notation. Check your answer.

c) Write $0.\overline{7}$ as a fraction.

Literacy Link

A repeating decimal number has one or more digits that repeat continuously.

It can be written using bar notation.

$0.333\ldots = 0.\overline{3}$

$0.181818\ldots = 0.\overline{18}$

Solution

a) When you change ninths to decimal numbers, the numerator of the fraction becomes the digit used as a repeating decimal.

numerator repeating decimal

$\frac{1}{9} = 0.111\ldots$

The numerator and repeating decimal are both 1.

b) Continue the pattern.

$\frac{4}{9} = 0.444\ldots$

$\frac{5}{9} = 0.555\ldots$

In bar notation, $\frac{5}{9} = 0.\overline{5}$.

Check:

Pencil and Paper

$9\overline{)5.000}$ = 0.555

45, 50, 45, 50, 45, 5

Calculator

C 5 ÷ 9 = 0.555555556

Tech Link

Some calculators round up the decimal number 0.555… to 0.555555556.

c) Continue the pattern to determine $0.\overline{7}$ as a fraction.

$0.\overline{5} = \frac{5}{9} \quad 0.\overline{6} = \frac{6}{9} \quad 0.\overline{7} = \frac{7}{9}$

Show You Know

Tim changed the following fractions into decimal numbers.

$\frac{1}{90} = 0.0111\ldots \quad \frac{2}{90} = 0.0222\ldots \quad \frac{3}{90} = 0.0333\ldots$

Write $\frac{7}{90}$ as a decimal number, using bar notation. Check your answer.

10.1 Describe Patterns • MHR 353

Answers

Show You Know: Example 2

$0.0\overline{7}$

Supported Learning

Gifted and Enrichment

- Have students explore what happens to the pattern for ninths and ninetieths at every eighth term, for example, $\frac{9}{9} = 1$ and $\frac{9}{90} = 0.1$. All students will explore this further in Apply #12 on page 356.

In Example 2, students are introduced to bar notation, which is a useful shortcut. Ensure that they make a habit of expressing repeating decimals using bar notation.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• Students tend to have more difficulty changing from repeating decimals to fractions than vice versa, so provide additional practice with this process, as necessary. • You may wish to give students who will benefit from it an additional repeating decimal pattern to describe: $\frac{1}{11} = 0.\overline{09}$, $\frac{2}{11} = 0.\overline{18}$, $\frac{3}{11} = 0.\overline{27}\ldots$. Allow students to use a calculator. Coach them through describing how you get from $\frac{1}{11} = 0.\overline{09}$ to $\frac{2}{11} = 0.\overline{18}$, and then have them describe on their own how you get from $\frac{2}{11} = 0.\overline{18}$ to $\frac{3}{11} = 0.\overline{27}$, and what comes next in the pattern.

Answers

Communicate the Ideas

1. a)–d) Answers will vary.

2. a) Answers may vary. For example:

Figure 1 Figure 2 Figure 3 Figure 4

b) Answers will vary.

3. Yes. To get the next number in the pattern, subtract 0.3.

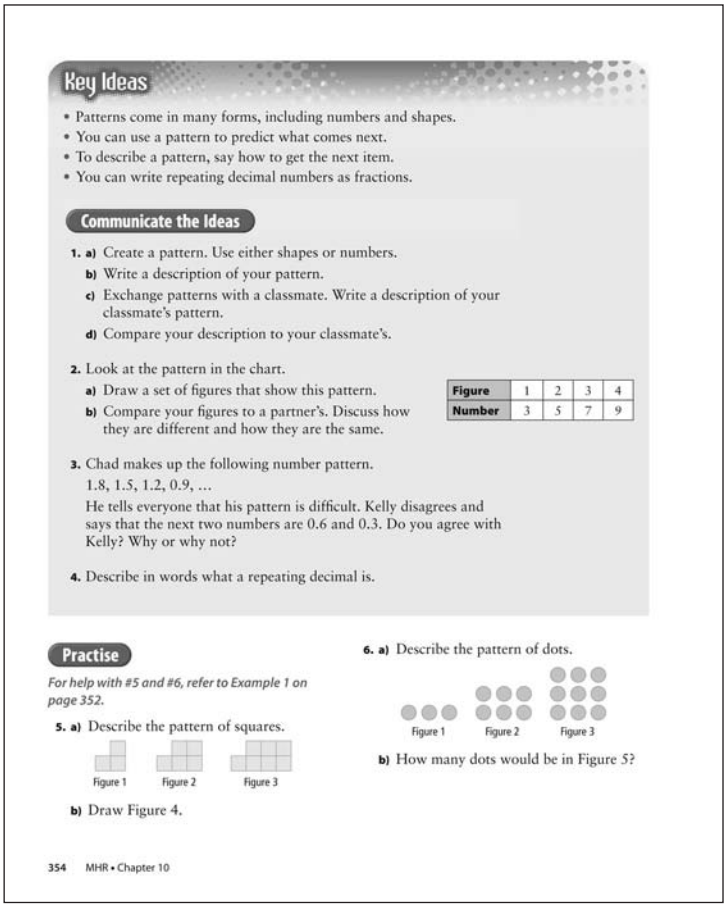
4. Answers may vary. For example: A repeating decimal has a number or a series of numbers after the decimal point that forms a repeating pattern.

Supported Learning

ESL and Language

- Give students sentence stems to help them describe different patterns. For example:
  - –The pattern increases (has more) by …
  - –The number is decreasing (has less) by …
  - –The number is divided in …
  - –The number is doubled …

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students complete #1 in pairs and #2 to #4 individually.	• Allow students to create the patterns in #1 and #2 using concrete materials, such as counters or tiles. • Due to the open-ended nature of #2, some students may need guidance to understand that there is more than one answer. • In #3, it may be helpful to ask students specifically what math operation is needed to move from the first number to the second number, the second number to the third number, etc. • Consider handing out **Master 2 Two Stars and One Wish** to have students critique a classmate's work in #1. Work with students to develop criteria for judging their classmate's answers. For example, criteria might include – described what the first item in the pattern was – described how to get the next item in the pattern – used mathematical words – described in a logical and easy-to-follow way



Key Ideas

- Patterns come in many forms, including numbers and shapes.
- You can use a pattern to predict what comes next.
- To describe a pattern, say how to get the next item.
- You can write repeating decimal numbers as fractions.

Communicate the Ideas

1. a) Create a pattern. Use either shapes or numbers.
   b) Write a description of your pattern.
   c) Exchange patterns with a classmate. Write a description of your classmate's pattern.
   d) Compare your description to your classmate's.
2. Look at the pattern in the chart.
   a) Draw a set of figures that show this pattern.
   b) Compare your figures to a partner's. Discuss how they are different and how they are the same.

Figure	1	2	3	4
Number	3	5	7	9

3. Chad makes up the following number pattern.
   1.8, 1.5, 1.2, 0.9, …
   He tells everyone that his pattern is difficult. Kelly disagrees and says that the next two numbers are 0.6 and 0.3. Do you agree with Kelly? Why or why not?
4. Describe in words what a repeating decimal is.

Practise

For help with #5 and #6, refer to Example 1 on page 352.

5. a) Describe the pattern of squares.

Figure 1 Figure 2 Figure 3

b) Draw Figure 4.

6. a) Describe the pattern of dots.

Figure 1 Figure 2 Figure 3

b) How many dots would be in Figure 5?

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Key Ideas

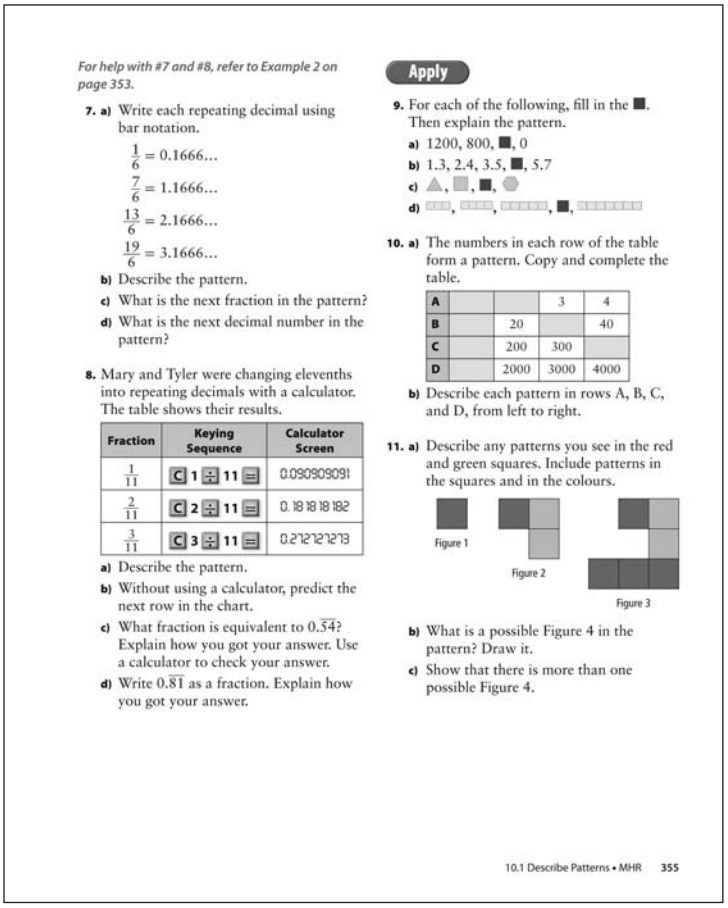
Have students prepare their own list of Key Ideas to put in their chapter Foldable, including examples.

It is important for students to begin describing patterns in terms of the first figure or term, since this will become necessary in subsequent sections of this chapter when they use expressions to describe patterns.

Communicate the Ideas

These questions allow students to demonstrate their understanding of describing patterns and how this understanding can be applied to writing or drawing a pattern. Students also have an opportunity to explain their understanding of repeating decimals.

Before students begin #2, discuss how this question is the same as what they encountered in Example 1, but reversed. Example 1 began with a diagram and asked students to describe the pattern. This question provides a table that shows the pattern and asks students to create the diagrams.



For help with #7 and #8, refer to Example 2 on page 353.

7. a) Write each repeating decimal using bar notation.

$\frac{1}{6} = 0.1666\ldots$

$\frac{7}{6} = 1.1666\ldots$

$\frac{13}{6} = 2.1666\ldots$

$\frac{19}{6} = 3.1666\ldots$

b) Describe the pattern.

c) What is the next fraction in the pattern?

d) What is the next decimal number in the pattern?

8. Mary and Tyler were changing elevenths into repeating decimals with a calculator. The table shows their results.

Fraction	Keying Sequence	Calculator Screen
$\frac{1}{11}$	C 1 ÷ 11 =	0.090909091
$\frac{2}{11}$	C 2 ÷ 11 =	0.181818182
$\frac{3}{11}$	C 3 ÷ 11 =	0.272727273

a) Describe the pattern.

b) Without using a calculator, predict the next row in the chart.

c) What fraction is equivalent to $0.\overline{54}$? Explain how you got your answer. Use a calculator to check your answer.

d) Write $0.\overline{81}$ as a fraction. Explain how you got your answer.

Apply

9. For each of the following, fill in the ■. Then explain the pattern.

a) 1200, 800, ■, 0

b) 1.3, 2.4, 3.5, ■, 5.7

c) △, □, ■, ⬟

d) ▭, ▭▭, ▭▭▭, ■, ▭▭▭▭▭▭▭

10. a) The numbers in each row of the table form a pattern. Copy and complete the table.

A			3	4
B		20		40
C		200	300	
D		2000	3000	4000

b) Describe each pattern in rows A, B, C, and D, from left to right.

11. a) Describe any patterns you see in the red and green squares. Include patterns in the squares and in the colours.

Figure 1 Figure 2 Figure 3

b) What is a possible Figure 4 in the pattern? Draw it.

c) Show that there is more than one possible Figure 4.

10.1 Describe Patterns • MHR 355

Common Errors

- Students have difficulty changing repeating decimals to fractions.

R_x The curriculum expects students to express repeating decimals as fractions. At this level, students need to know a limited number of these decimals, such as thirds, sixths, ninths, and elevenths. You might post a list of these fractions and their corresponding repeating decimals. Encourage students to memorize the most commonly used conversions, such as $\frac{1}{3} = 0.\overline{3}$ and $\frac{2}{3} = 0.\overline{6}$.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–5, 7, 9, 14, Math Link
Typical	1–5, 7, 9, 11–15, Math Link
Extension/Enrichment	1–4, 12, 13, 16–18, Math Link

Practise

The Practise questions give a good indication of whether students understand simple geometric patterns as well as number patterns dealing with repeating decimals.

Assessment *for* Learning	Supported Learning
Practise and Apply Have students do #5, #7, and #9. Students who have no problems with these questions can go on to the rest of the Apply questions.	• Students who have problems with #5, #7, and #9 will need additional coaching. • Students having difficulty with #5 will need to refer back to Example 1. Coach them through #6a), and then have them complete b) on their own. • Students having difficulty with #7 will need to refer back to Example 2. Coach them through #8a), and then have them complete the rest of the question on their own. • Ask students having difficulty with #9 to explain what strategies they are using to describe patterns. Clarify any misunderstandings. Coach students through row A in #10a), and then have them complete b) on their own. • Check back with them several times to make sure that they understand the concepts.

Common Errors

- Students may extend patterns incorrectly.

R_x Reinforce that each item of a pattern is generated by the previous item.

Supported Learning

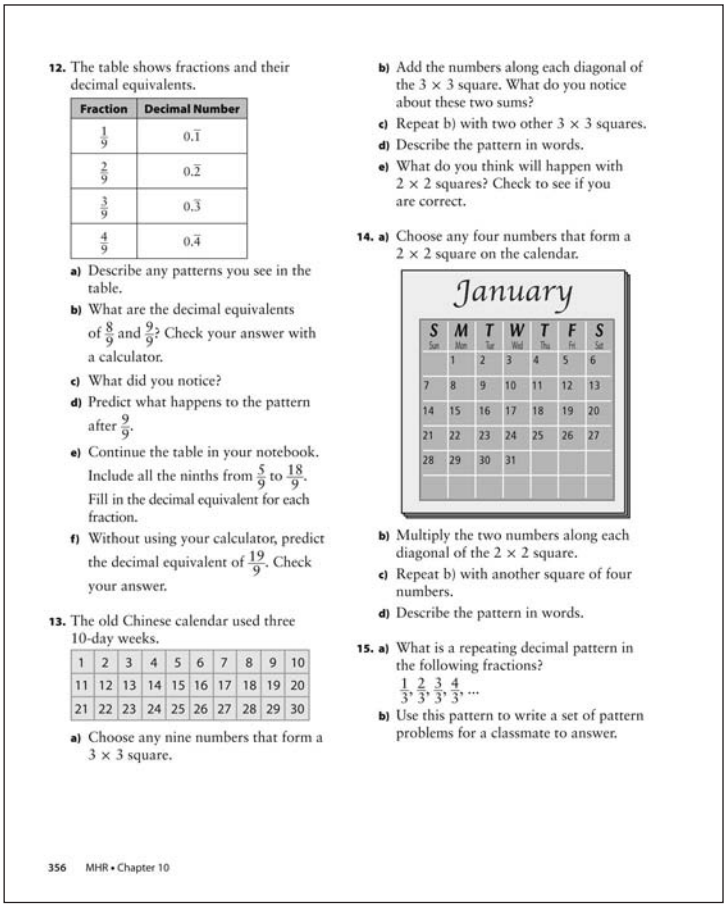
ESL and Language

- For #13 and #14, give students a visual of what is meant by a 3 × 3 square or a 2 × 2 square on the calendar.

Learning Style and Memory

- You may wish to provide **BLM 10–4 Section 10.1 Extra Practice** to students who require more practice.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • Which patterns used in this section do you most commonly see or use in your life? When might you want to use those patterns?	• Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Work with students to develop a plan for dealing with the areas in which they are having difficulty. • Depending on students' learning style, have them provide oral or written answers. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • Have students review the part related to Section 10.1 in **BLM 10–1 Chapter 10 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.



12. The table shows fractions and their decimal equivalents.

Fraction	Decimal Number
$\frac{1}{9}$	$0.\overline{1}$
$\frac{2}{9}$	$0.\overline{2}$
$\frac{3}{9}$	$0.\overline{3}$
$\frac{4}{9}$	$0.\overline{4}$

a) Describe any patterns you see in the table.
b) What are the decimal equivalents of $\frac{8}{9}$ and $\frac{9}{9}$? Check your answer with a calculator.
c) What did you notice?
d) Predict what happens to the pattern after $\frac{9}{9}$.
e) Continue the table in your notebook. Include all the ninths from $\frac{5}{9}$ to $\frac{18}{9}$. Fill in the decimal equivalent for each fraction.
f) Without using your calculator, predict the decimal equivalent of $\frac{19}{9}$. Check your answer.

13. The old Chinese calendar used three 10-day weeks.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

a) Choose any nine numbers that form a 3 × 3 square.
b) Add the numbers along each diagonal of the 3 × 3 square. What do you notice about these two sums?
c) Repeat b) with two other 3 × 3 squares.
d) Describe the pattern in words.
e) What do you think will happen with 2 × 2 squares? Check to see if you are correct.

14. a) Choose any four numbers that form a 2 × 2 square on the calendar.

January

S Sun	M Mon	T Tue	W Wed	T Thu	F Fri	S Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

b) Multiply the two numbers along each diagonal of the 2 × 2 square.
c) Repeat b) with another square of four numbers.
d) Describe the pattern in words.

15. a) What is a repeating decimal pattern in the following fractions?
$\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \ldots$
b) Use this pattern to write a set of pattern problems for a classmate to answer.

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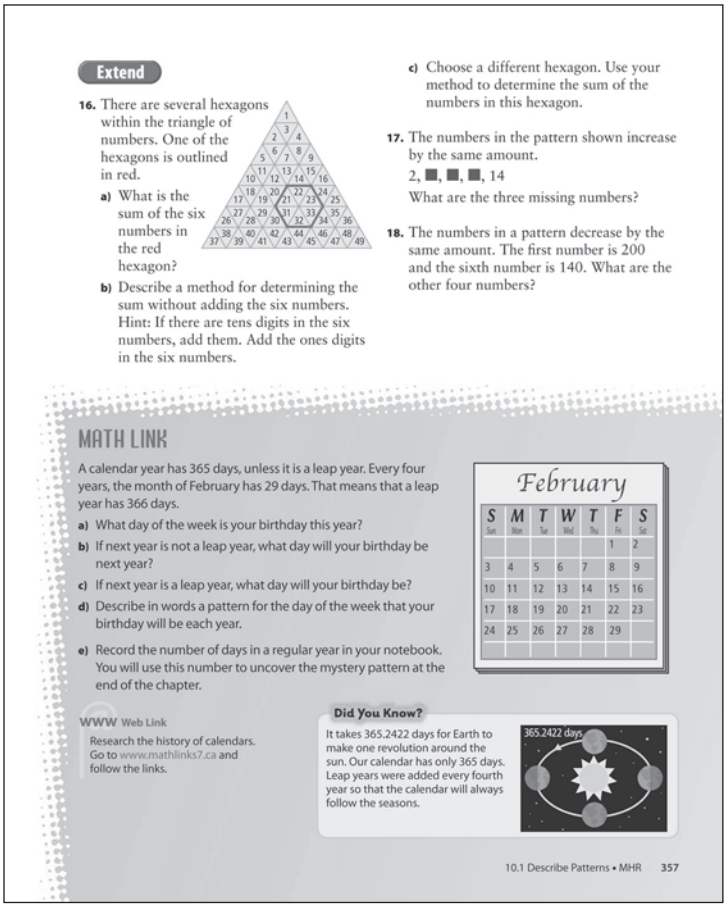
Apply and Extend

For #11 on page 355, encourage students to create multiple possible patterns. The intention of this question is for students to be more divergent in their thought processes.

Questions 13 and 14 contain variations on patterns in calendars. These patterns should help students as they complete the Math Links in this chapter.

If students require assistance to determine the method in #16b), read the provided hint and ask, "How do these sums help you to determine the sum of the numbers in the red hexagon?"

Students are not expected to solve #17 using a formula; they can use Guess and Check. Another strategy is to ask, "What is the number midway between 2 and 14?" The answer of 8 goes in the middle blank. Then ask, "What is the number midway between 2 and 8?" The answer of 5 goes in the first blank. Complete the third blank in the same way.



Extend

16. There are several hexagons within the triangle of numbers. One of the hexagons is outlined in red.
   a) What is the sum of the six numbers in the red hexagon?
   b) Describe a method for determining the sum without adding the six numbers. Hint: If there are tens digits in the six numbers, add them. Add the ones digits in the six numbers.
   c) Choose a different hexagon. Use your method to determine the sum of the numbers in this hexagon.

17. The numbers in the pattern shown increase by the same amount.
   2, ■, ■, ■, 14
   What are the three missing numbers?

18. The numbers in a pattern decrease by the same amount. The first number is 200 and the sixth number is 140. What are the other four numbers?

MATH LINK

A calendar year has 365 days, unless it is a leap year. Every four years, the month of February has 29 days. That means that a leap year has 366 days.

a) What day of the week is your birthday this year?
b) If next year is not a leap year, what day will your birthday be next year?
c) If next year is a leap year, what day will your birthday be?
d) Describe in words a pattern for the day of the week that your birthday will be each year.
e) Record the number of days in a regular year in your notebook. You will use this number to uncover the mystery pattern at the end of the chapter.

February

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	

WWW Web Link
Research the history of calendars. Go to www.mathlinks7.ca and follow the links.

Did You Know?
It takes 365.2422 days for Earth to make one revolution around the sun. Our calendar has only 365 days. Leap years were added every fourth year so that the calendar will always follow the seasons.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 357 gives students an opportunity to describe patterns in the calendar year. It is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 385.	• If students have difficulty with these questions, use calendars for more than one year so that students can confirm their predictions. • Have students who are struggling look up a specific holiday for three consecutive non-leap years. Ask, "On what day of the week does the holiday fall in year 1? Year 2? Year 3?" • Observe and clarify what patterns students describe. • Allow students to share and talk with a partner. • Students who are having difficulty getting started could use **BLM 10–5 Section 10.1 Math Link**, which provides scaffolding for this activity.

Answers

Math Link

a) Answers may vary. For example: Tuesday

b) Answers may vary. For example: Wednesday

c) Answers may vary. For example: Thursday

d) Every year for three years, the day of the week for my birthday will be one day later than the previous year. Every fourth year, it will be two days later than the previous year.

e) 365

Supported Learning

Gifted and Enrichment

- Have students describe other calendar patterns such as
  - the number of days in each year (365 days for three years, then 366 days the following year, then 365 days for three years, etc.)
  - the number of days in February (28 for three years, then 29 for one year, then 28 for three years, etc.)

WWW Web Link

For students who would benefit from referring to calendars as they work on the Math Link, go to **www.mathlinks7.ca** and follow the links.

MATH LINK

The Math Link introduces students to calendar patterns in a way that relates to their own lives. Remind students that in part e) they need to record the number of days in a regular calendar year to help them uncover the mystery pattern in the Wrap It Up! at the end of the chapter. You may wish to give them **BLM 10–14 Wrap It Up! Mystery Pattern**, on which they can record this number.

10.2 Variables and Expressions

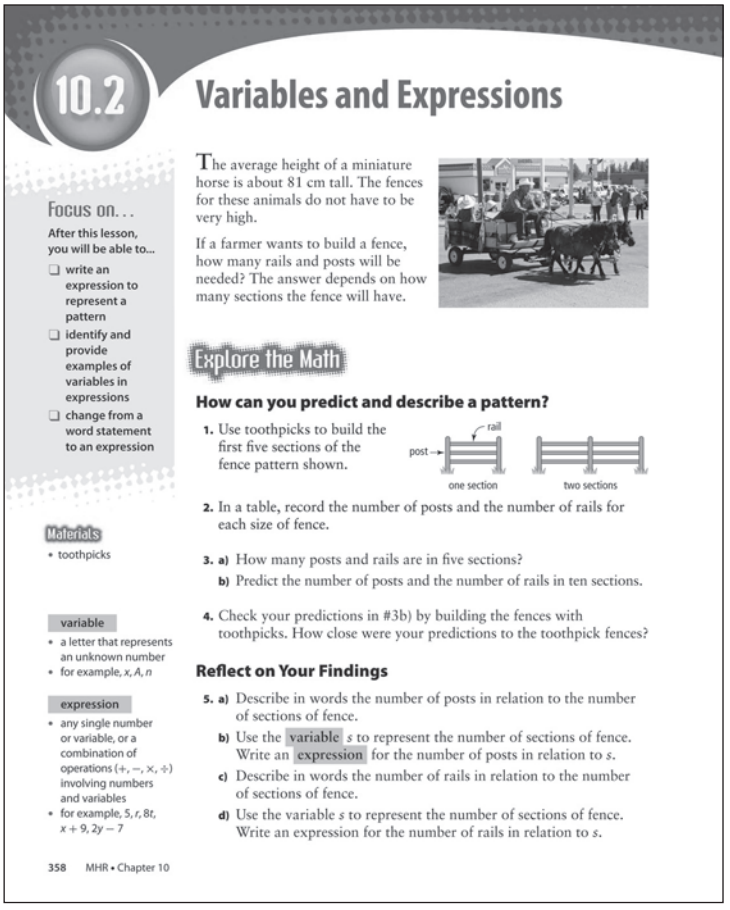
Suggested Timing
80–100 minutes

Materials
- toothpicks
- coloured tiles
- calendar

Blackline Masters
BLM 10–1 Chapter 10 Self-Assessment
BLM 10–6 Parts of an Expression
BLM 10–7 Section 10.2 Extra Practice
BLM 10–8 Section 10.2 Math Link
BLM 10–14 Wrap It Up! Mystery Pattern

Mathematical Processes

- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



10.2 Variables and Expressions

Focus on...
After this lesson, you will be able to...
- write an expression to represent a pattern
- identify and provide examples of variables in expressions
- change from a word statement to an expression

The average height of a miniature horse is about 81 cm tall. The fences for these animals do not have to be very high.

If a farmer wants to build a fence, how many rails and posts will be needed? The answer depends on how many sections the fence will have.

Explore the Math

How can you predict and describe a pattern?

1. Use toothpicks to build the first five sections of the fence pattern shown.

post, rail
one section
two sections

2. In a table, record the number of posts and the number of rails for each size of fence.

3. a) How many posts and rails are in five sections?
b) Predict the number of posts and the number of rails in ten sections.

4. Check your predictions in #3b) by building the fences with toothpicks. How close were your predictions to the toothpick fences?

Reflect on Your Findings

5. a) Describe in words the number of posts in relation to the number of sections of fence.
b) Use the variable *s* to represent the number of sections of fence. Write an expression for the number of posts in relation to *s*.
c) Describe in words the number of rails in relation to the number of sections of fence.
d) Use the variable *s* to represent the number of sections of fence. Write an expression for the number of rails in relation to *s*.

Materials
- toothpicks

variable
- a letter that represents an unknown number
- for example, *x*, *A*, *n*

expression
- any single number or variable, or a combination of operations (+, −, ×, ÷) involving numbers and variables
- for example, 5, *r*, 8*t*, $x + 9$, $2y - 7$

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Specific Outcomes

PR1 Demonstrate an understanding of oral or written patterns and their equivalent linear relations.
PR4 Explain the difference between an expression and an equation.

Warm-Up

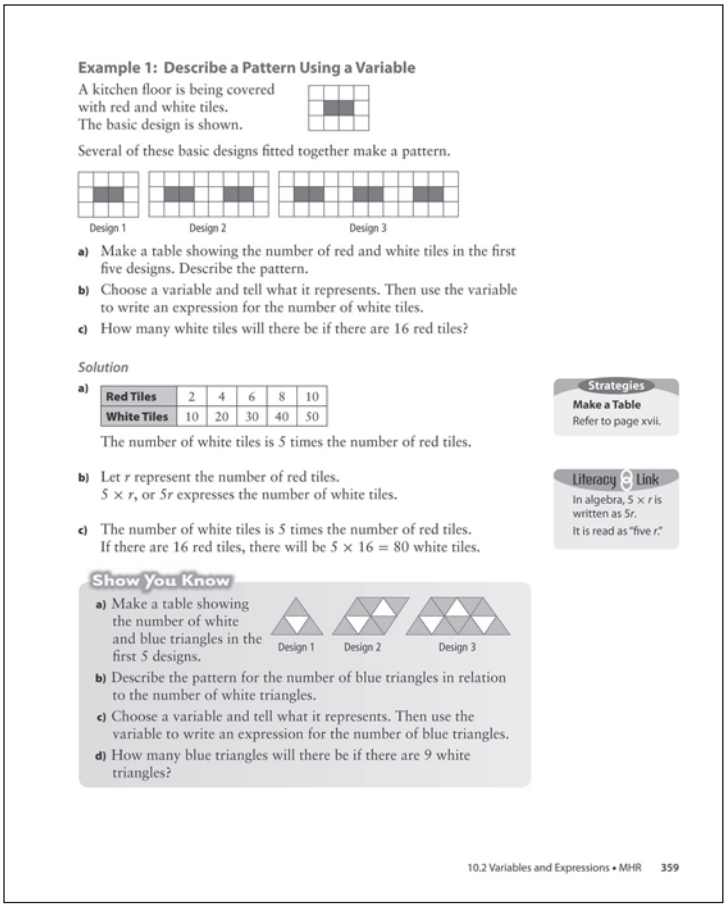
For #1 to #3, use the following numbers.

$\frac{1}{11} = 0.0909\ldots$ $\frac{2}{11} = 0.1818\ldots$ $\frac{3}{11} = 0.2727\ldots$

1. Describe the pattern.
2. Predict the next term in the pattern.
3. Show each decimal number using bar notation.
4. Determine the next three numbers in the pattern: 2, 8, 14, ____, ____, ____.
5. $\frac{23}{24} - \frac{11}{24}$

Mental Math

6. The diameter of a circle is 4 m. What is the radius?
7. The radius of a circle is 8.5 cm. What is the diameter?
8. The diameter of a circle is 3 cm. Estimate the circumference.
9. The radius of a circle is 12 m. Estimate the circumference.
10. You want to show 26% of a circle graph. Estimate the size of the interior angle.



Example 1: Describe a Pattern Using a Variable

A kitchen floor is being covered with red and white tiles. The basic design is shown.

Several of these basic designs fitted together make a pattern.

Design 1 Design 2 Design 3

a) Make a table showing the number of red and white tiles in the first five designs. Describe the pattern.

b) Choose a variable and tell what it represents. Then use the variable to write an expression for the number of white tiles.

c) How many white tiles will there be if there are 16 red tiles?

Solution

a)

Red Tiles	2	4	6	8	10
White Tiles	10	20	30	40	50

The number of white tiles is 5 times the number of red tiles.

Strategies
Make a Table
Refer to page xvii.

b) Let r represent the number of red tiles.
$5 \times r$, or $5r$ expresses the number of white tiles.

Literacy Link
In algebra, $5 \times r$ is written as $5r$.
It is read as "five r."

c) The number of white tiles is 5 times the number of red tiles.
If there are 16 red tiles, there will be $5 \times 16 = 80$ white tiles.

Show You Know

a) Make a table showing the number of white and blue triangles in the first 5 designs.

Design 1 Design 2 Design 3

b) Describe the pattern for the number of blue triangles in relation to the number of white triangles.

c) Choose a variable and tell what it represents. Then use the variable to write an expression for the number of blue triangles.

d) How many blue triangles will there be if there are 9 white triangles?

10.2 Variables and Expressions • MHR 359

Activity Planning Notes

Have a brief class discussion about miniature horses, encouraging students to relate any personal experiences with these animals. Look for pictures of miniature horses on the Internet.

Explore the Math

All students should try this Explore the Math since it introduces the terms *variable* and *expression* through a relatively easy pattern involving a post-and-rail fence. Patterns are developed separately for posts and for rails, using different sizes of fence.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students describe the two patterns using words and expressions. During this process, they are generalizing what they have learned during the Explore the Math.	• Consider pre-teaching the Key Words for this section. If you teach Language Arts in additional to Math, include the terms in your spelling list for the week. Have students write definitions and examples for each term in their chapter Foldable or vocabulary Foldable.

Answers

Warm-Up

1. The repeating decimals are multiples of 9, increasing in value.

2. $\frac{4}{11} = 0.3636\ldots$

3. $0.\overline{09}$, $0.\overline{18}$, $0.\overline{27}$

4. 20, 26, 32

5. $\frac{12}{24} = \frac{1}{2}$

6. 2 m

7. 17 cm

8. $3 \times 3 = 9$ cm

9. $12 \times 2 \times 3 = 72$ m

10. 26% is close to 25%, or $\frac{1}{4}$. $360° \div 4 = 90°$

Explore the Math

2.

Number of Sections	1	2	3	4	5
Number of Posts	2	3	4	5	6
Number of Rails	3	6	9	12	15

3. a) posts: 6; rails: 15

b) Answers may vary. For example: posts: 11; rails: 30

4. Answers may vary.

5. a) Answers may vary. For example: The number of posts is one more than the number of sections.

b) $s + 1$

c) Answers may vary. For example: There are 3 times as many rails as there are sections.

d) $s \times 3$, or $3s$

Show You Know: Example 1

a)

Number of White Triangles	1	2	3	4	5
Number of Blue Triangles	3	6	9	12	15

b) There are 3 times as many blue triangles as there are white triangles.

c) w = white triangles; $3w$

d) 27

Supported Learning

Learning Style

- Kinesthetic and concrete learners will benefit from the hands-on nature of the Explore the Math.

Answers

Show You Know: Example 2

a) $s - 2$

b) $4c$

Supported Learning

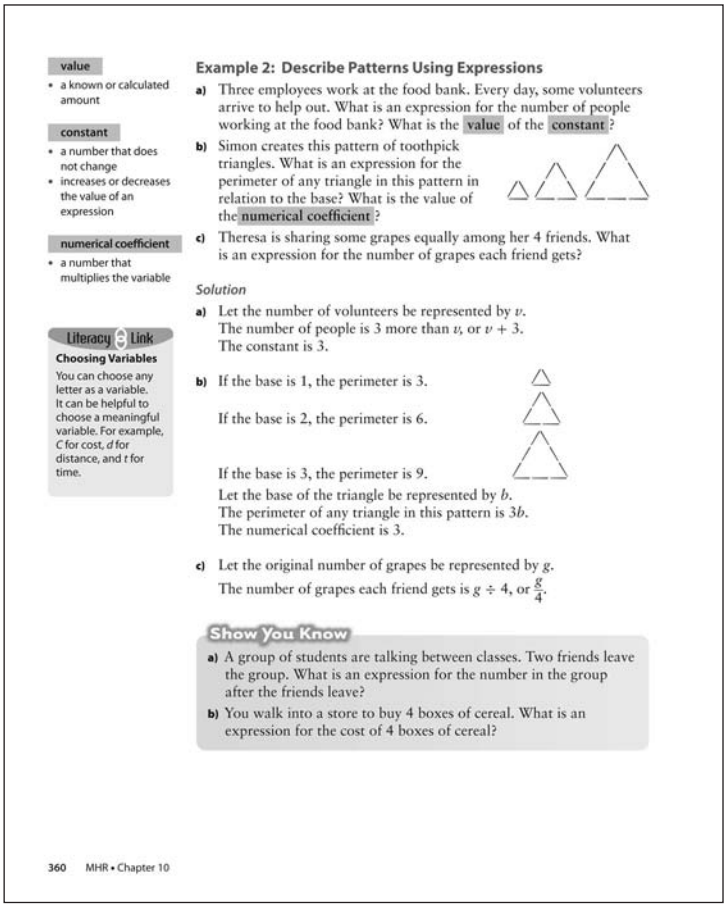
Learning Style

- Allow students the choice of drawing or modelling the various patterns in this section.

Common Errors

- Some students may be confused by the expression $3b$ in Example 2.

R_x Tell them that $3b$ means $3 \times b$. Explain that when no operation is shown between the numerical coefficient (3) and the variable (b), it is understood to be multiplication.



value
- a known or calculated amount

constant
- a number that does not change
- increases or decreases the value of an expression

numerical coefficient
- a number that multiplies the variable

Literacy Link

Choosing Variables

You can choose any letter as a variable. It can be helpful to choose a meaningful variable. For example, C for cost, d for distance, and t for time.

Example 2: Describe Patterns Using Expressions

a) Three employees work at the food bank. Every day, some volunteers arrive to help out. What is an expression for the number of people working at the food bank? What is the value of the constant?

b) Simon creates this pattern of toothpick triangles. What is an expression for the perimeter of any triangle in this pattern in relation to the base? What is the value of the numerical coefficient?

c) Theresa is sharing some grapes equally among her 4 friends. What is an expression for the number of grapes each friend gets?

Solution

a) Let the number of volunteers be represented by v.
The number of people is 3 more than v, or $v + 3$.
The constant is 3.

b) If the base is 1, the perimeter is 3.
If the base is 2, the perimeter is 6.
If the base is 3, the perimeter is 9.
Let the base of the triangle be represented by b.
The perimeter of any triangle in this pattern is $3b$.
The numerical coefficient is 3.

c) Let the original number of grapes be represented by g.
The number of grapes each friend gets is $g \div 4$, or $\frac{g}{4}$.

Show You Know

a) A group of students are talking between classes. Two friends leave the group. What is an expression for the number in the group after the friends leave?

b) You walk into a store to buy 4 boxes of cereal. What is an expression for the cost of 4 boxes of cereal?

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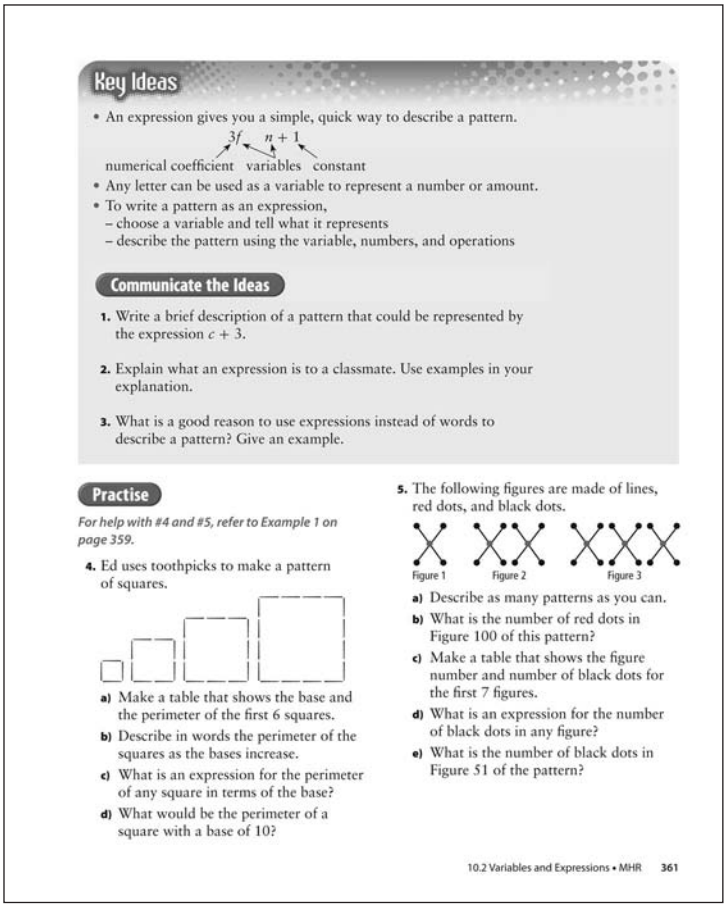
Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know on page 359 related to Example 1.	• Have concrete and kinesthetic learners students use cards to practise identifying the parts of an expression and forming expressions. Provide a collection of cards for each pair of students with the following symbols on them: math operations ($+, -, \times, \div, =$), variables (x, a, b, y), and two sets of numbers from 1 to 9. Label the top of each operation card "Operation," and label the variable cards "Variable." Label one set of numbers "Constant," and the second set "Numerical Coefficient." You may wish to use **BLM 10–6 Parts of an Expression**, which provides templates for these cards.

For Example 1, encourage students to build the pattern using two colours of tiles. Have students count the number of red tiles and the number of white tiles in each of the first three designs and predict the number of red and white tiles in Design 4. Each time students use a variable they should explicitly identify what it represents. For example, "Let w represent the number of white triangles," or "$3w$, where w is the number of white triangles."

In Example 2, students translate from words to an expression, which is an important skill in describing patterns. Brainstorm "operations words" with students to help them to understand what operation to perform. Record them in a chart like this:

Adding	Subtracting	Multiplying	Dividing
altogether, combine, count, increase, more, plus, sum, tally, together, total	decrease, deduct, difference, fewer, less, minus, take away	double, triple (and so on), product, times	distribute, equal parts, half, third (and so on), quotient, share, split

Make sure students understand that not all of these words tell conclusively what operation to use, but they offer possible avenues to explore.



Key Ideas

- An expression gives you a simple, quick way to describe a pattern.

$3f$ $n + 1$

numerical coefficient variables constant

- Any letter can be used as a variable to represent a number or amount.
- To write a pattern as an expression,
  - choose a variable and tell what it represents
  - describe the pattern using the variable, numbers, and operations

Communicate the Ideas

1. Write a brief description of a pattern that could be represented by the expression $c + 3$.
2. Explain what an expression is to a classmate. Use examples in your explanation.
3. What is a good reason to use expressions instead of words to describe a pattern? Give an example.

Practise

For help with #4 and #5, refer to Example 1 on page 359.

4. Ed uses toothpicks to make a pattern of squares.

a) Make a table that shows the base and the perimeter of the first 6 squares.
b) Describe in words the perimeter of the squares as the bases increase.
c) What is an expression for the perimeter of any square in terms of the base?
d) What would be the perimeter of a square with a base of 10?

5. The following figures are made of lines, red dots, and black dots.

Figure 1 Figure 2 Figure 3

a) Describe as many patterns as you can.
b) What is the number of red dots in Figure 100 of this pattern?
c) Make a table that shows the figure number and number of black dots for the first 7 figures.
d) What is an expression for the number of black dots in any figure?
e) What is the number of black dots in Figure 51 of the pattern?

10.2 Variables and Expressions • MHR 361

Answers

Communicate the Ideas

1. Answers may vary. For example: Carl is three years older than Jane.
2. Answers may vary. For example: It is any single number or variable, or a combination of operations $(+, -, \times, \div)$ involving numbers and variables; e.g., 5, r, $8t$, $x + 9$, $2y - 7$.
3. Answers may vary. For example: Using an expression is a quicker way to describe a pattern than using words. For example, the candies were shared by three friends, or $\frac{c}{3}$.

Supported Learning

ESL

- Be sure to explicitly teach the words in the chart of "operations words" on the opposite page. Many English language learners will be unfamiliar with the various ways to talk about mathematical operations.

ESL and Language

- For Communicate the Ideas #1 and #2, have students work with a classmate who has a clear understanding of the terminology.

Example 2 provides three scenarios that can be translated into expressions. Again, students should define each variable: "Let x represent …" or "where x represents … ."

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know on page 360 related to Example 2.	• Allow kinesthetic and concrete learners to use toothpicks and other materials to model the patterns. • Read the Literacy Link on page 360 with students and encourage them to use variables that relate to the words in the question; for example, g for grapes.

Common Errors

- Students may neglect to say what their variable represents.

R$_x$ Reinforce that a variable can represent anything; hence, the need to state what the variable stands for. You may wish to allow students to identify the variable in an abbreviated way: "b = boys."

Key Ideas

Students should prepare their own list of Key Ideas to put in their chapter Foldable, including examples.

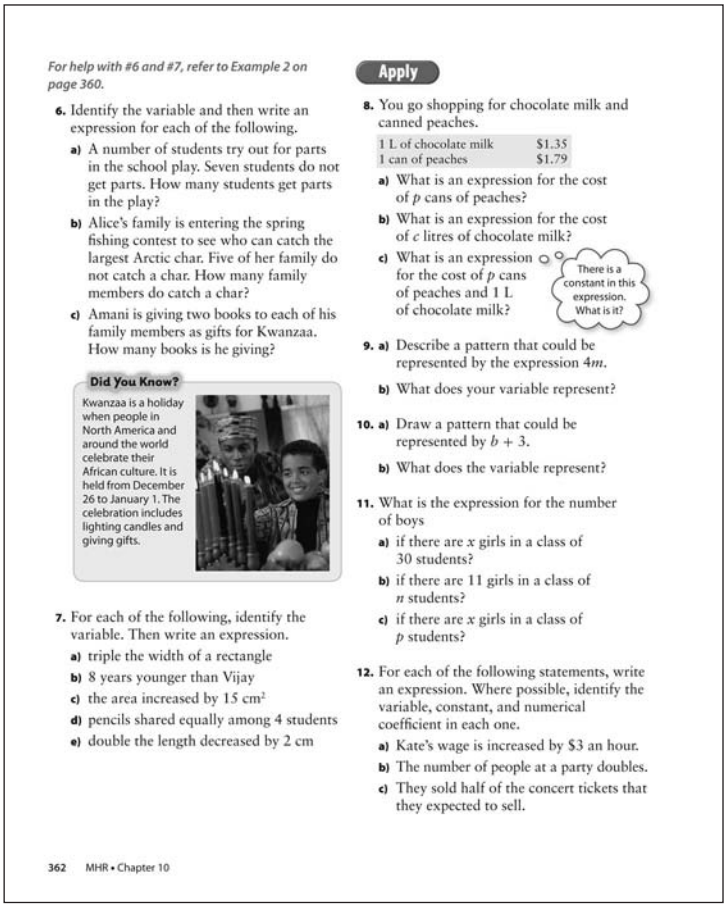
Communicate the Ideas

The Communicate the Ideas allows students to explain their understanding of how patterns can be represented using mathematical expressions. Students have an opportunity to determine a real-world application for a given expression, explain what an expression is to a classmate, as well as describe the benefits of using expressions instead of words.

All three questions are important for student understanding of both patterns and expressions. Note that you may wish to use the term *algebraic expression* for #2 to clarify what sort of expression is being referred to.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students complete #1 individually, and #2 with a classmate. Discuss #3 as a class before students answer the question on their own	• After students have completed #1, have a couple of volunteers explain the process they used to create their pattern and why they used it. Students might then share their idea in #1 with a partner to confirm that their application is clear and logical. • As a class, discuss #3. Invite students to come up with examples to give them further practice in using expressions to describe word patterns. Then have students write individual answers to this question.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8, 9, 11, 12, Math Link
Typical	1–4, 6, 8, 9, 11–13, 15, 16, Math Link
Extension/Enrichment	1–4, 13, 14, 17–19, Math Link



For help with #6 and #7, refer to Example 2 on page 360.

6. Identify the variable and then write an expression for each of the following.

a) A number of students try out for parts in the school play. Seven students do not get parts. How many students get parts in the play?

b) Alice's family is entering the spring fishing contest to see who can catch the largest Arctic char. Five of her family do not catch a char. How many family members do catch a char?

c) Amani is giving two books to each of his family members as gifts for Kwanzaa. How many books is he giving?

Did You Know?

Kwanzaa is a holiday when people in North America and around the world celebrate their African culture. It is held from December 26 to January 1. The celebration includes lighting candles and giving gifts.

7. For each of the following, identify the variable. Then write an expression.

a) triple the width of a rectangle

b) 8 years younger than Vijay

c) the area increased by 15 cm^2

d) pencils shared equally among 4 students

e) double the length decreased by 2 cm

Apply

8. You go shopping for chocolate milk and canned peaches.

1 L of chocolate milk	$1.35
1 can of peaches	$1.79

a) What is an expression for the cost of p cans of peaches?

b) What is an expression for the cost of c litres of chocolate milk?

c) What is an expression for the cost of p cans of peaches and 1 L of chocolate milk?

There is a constant in this expression. What is it?

9. a) Describe a pattern that could be represented by the expression $4m$.

b) What does your variable represent?

10. a) Draw a pattern that could be represented by $b + 3$.

b) What does the variable represent?

11. What is the expression for the number of boys

a) if there are x girls in a class of 30 students?

b) if there are 11 girls in a class of n students?

c) if there are x girls in a class of p students?

12. For each of the following statements, write an expression. Where possible, identify the variable, constant, and numerical coefficient in each one.

a) Kate's wage is increased by $3 an hour.

b) The number of people at a party doubles.

c) They sold half of the concert tickets that they expected to sell.

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Practise

Assessment *for* Learning	Supported Learning
Practise Have students do #4 and #6. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #4 and #6 will need additional coaching. Have these students explain their strategies for describing patterns using expressions. Clarify any misunderstandings. • Students having difficulty with #4 will need to refer back to Example 1. Coach them through #5a), and then have them complete the rest of the question on their own. • Students having difficulty with #6 will need to refer back to Example 2. Coach them through #7a), and then have them complete the rest of the question on their own. • Check back with them several times to make sure that they understand the concepts.

13. Salma gets $7 per hour to baby-sit. She gets a bonus if she has to baby-sit past 10 p.m. The expression $7h + 3$ represents what Salma was paid last night. She baby-sat from 5:30 p.m. to 10:30 p.m.

a) What is the variable in the expression? Explain what it represents.

b) What does 3 represent?

c) How much did she earn last night?

14. A grocer orders w white loaves and b brown loaves of bread every day for d days.

Describe in words what each if the following expressions represents.

a) $w + b$

b) wd

15. Large bags of dog food hold 25 kg each. Small bags hold 10 kg each.

What is the expression for the total mass in kilograms if

a) the number of large bags of dog food is x?

b) the number of small bags of dog food is y?

c) there are x large bags plus y small bags of dog food?

16. Where possible, identify the variable, numerical coefficient, and constant in each expression.

a) $7a - 8$

b) $100 - 3h$

c) $8q$

d) $\frac{n}{3} - 5$

17. Study the red H on the hundreds chart.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

a) What patterns do you notice in the numbers in the red H?

b) What is the sum of the numbers in the red H?

c) Describe a method for determining the sum without adding all the numbers. Hint: Look at the middle number, 24. Then count how many numbers are in the H.

d) Make another H in the chart that is the same size and shape as the first one. Use your method to determine the sum of the numbers in your H. Check your answer.

e) Find an expression for the sum of the numbers in an H in the hundreds chart. Make your variable represent the middle number in the H.

Apply and Extend

These questions provide a variety of contexts in which to use variables and expressions. It is important that students take the time necessary to read each question carefully prior to attempting an answer.

In #11, students who are struggling might be encouraged to replace the variable with a number. Then, have them tell what mathematical operation they could use to find the answer. Once they figure this out they can use the same operation with the variable.

Note that #17 is similar to the calendar problem in the Math Link on page 364: students describe a pattern involving several numbers in terms of only one of those numbers. If students have trouble describing a method in part c), read the provided hint and then ask, "How do these numbers help you to determine the sum of the numbers in the red H?"

Some students will find #19 more challenging, depending on their previous exposure to similar patterns.

Supported Learning

Learning Style and Memory

- You may wish to provide **BLM 10–7 Section 10.2 Extra Practice** to students who require more practice.

Answers

Math Link

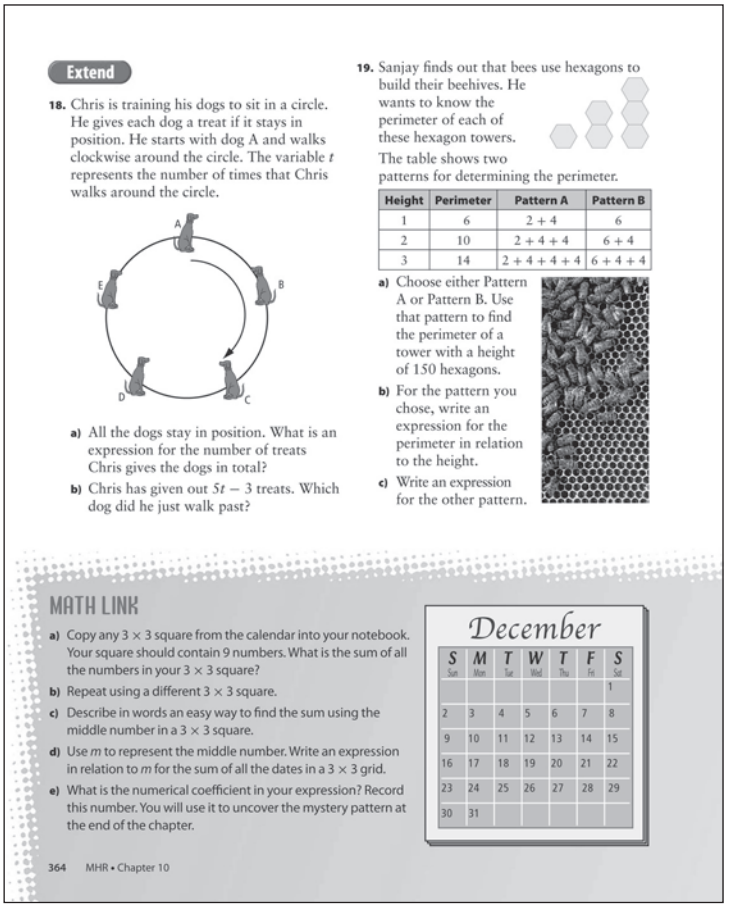
a) Answers may vary.

b) Answers may vary.

c) Multiply the middle number by 9.

d) $9m$

e) 9



Extend

18. Chris is training his dogs to sit in a circle. He gives each dog a treat if it stays in position. He starts with dog A and walks clockwise around the circle. The variable t represents the number of times that Chris walks around the circle.

a) All the dogs stay in position. What is an expression for the number of treats Chris gives the dogs in total?

b) Chris has given out $5t - 3$ treats. Which dog did he just walk past?

19. Sanjay finds out that bees use hexagons to build their beehives. He wants to know the perimeter of each of these hexagon towers. The table shows two patterns for determining the perimeter.

Height	Perimeter	Pattern A	Pattern B
1	6	2 + 4	6
2	10	2 + 4 + 4	6 + 4
3	14	2 + 4 + 4 + 4	6 + 4 + 4

a) Choose either Pattern A or Pattern B. Use that pattern to find the perimeter of a tower with a height of 150 hexagons.

b) For the pattern you chose, write an expression for the perimeter in relation to the height.

c) Write an expression for the other pattern.

MATH LINK

a) Copy any 3 × 3 square from the calendar into your notebook. Your square should contain 9 numbers. What is the sum of all the numbers in your 3 × 3 square?

b) Repeat using a different 3 × 3 square.

c) Describe in words an easy way to find the sum using the middle number in a 3 × 3 square.

d) Use m to represent the middle number. Write an expression in relation to m for the sum of all the dates in a 3 × 3 grid.

e) What is the numerical coefficient in your expression? Record this number. You will use it to uncover the mystery pattern at the end of the chapter.

December

S Sun	M Mon	T Tue	W Wed	T Thu	F Fri	S Sat
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

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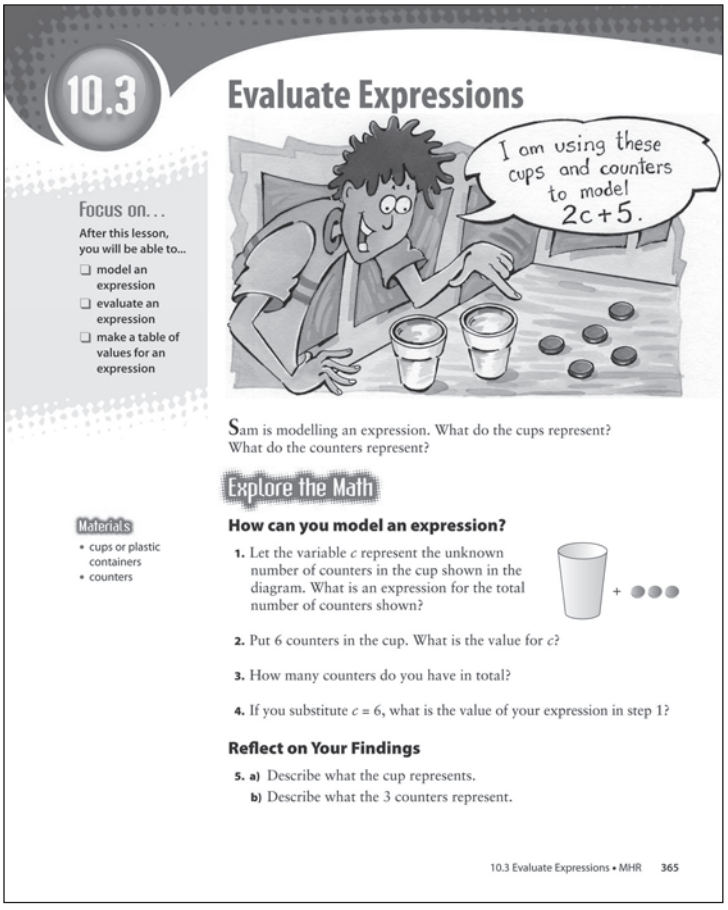
Assessment *as* Learning	Supported Learning
Math Learning Log Have students reflect on two or three items they have improved on and how they think they have improved. Have students also pick one area in which they feel they did not learn as much as they should or could have. Have them reflect on how they plan to improve in this area of learning.	• Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Work with students to develop a plan for dealing with the areas in which they are having difficulty. • Depending on students' learning style, have them provide oral or written answers. • Keep a record of student reflections in their learning portfolio. You may wish to have students return to these reflections at the end of the chapter. • Have students review the part related to Section 10.2 in **BLM 10–1Chapter 10 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Math Link

This Math Link continues the theme of exploring the patterns found in a calendar. Remind students that in part e) they need to record the value of the numerical coefficient to help uncover the mystery pattern in the Wrap It Up! at the end of the chapter. You may wish to have them record this number on **BLM 10–14 Wrap It Up! Mystery Pattern**.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 364 allows students to write an expression for a calendar pattern. It is intended to help students work toward the chapter problem wrap-uptitled Wrap It Up! on page 385.	• To ensure success, make sure students take time to read each part of the question and work through the Math Link step by step. • Encourage students to discuss whether it makes a difference which 3 × 3 square is chosen. Ask them to explain their reasoning. • Students who are having difficulty getting started could use **BLM 10–8 Section 10.2 Math Link**, which provides scaffolding for this activity.

10.3 Evaluate Expressions



10.3

Evaluate Expressions

Focus on...

After this lesson, you will be able to...

- model an expression
- evaluate an expression
- make a table of values for an expression

Sam is modelling an expression. What do the cups represent? What do the counters represent?

Explore the Math

Materials

- cups or plastic containers
- counters

How can you model an expression?

1. Let the variable c represent the unknown number of counters in the cup shown in the diagram. What is an expression for the total number of counters shown?

2. Put 6 counters in the cup. What is the value for c?
3. How many counters do you have in total?
4. If you substitute $c = 6$, what is the value of your expression in step 1?

Reflect on Your Findings

5. a) Describe what the cup represents.
   b) Describe what the 3 counters represent.

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Suggested Timing
80–100 minutes

Materials
- cups or plastic containers
- counters
- toothpicks
- calendar

Blackline Masters
BLM 10–1 Chapter 10 Self-Assessment
BLM 10–9 Section 10.3 Extra Practice
BLM 10–10 Section 10.3 Math Link
BLM 10–14 Wrap It Up! Mystery Pattern

Mathematical Processes

- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization

Specific Outcomes

PR5 Evaluate an expression given the value of the variable(s).

Warm-Up

Use the diagram below to answer #1 to #4.

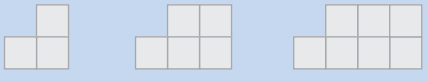


Figure 1 Figure 2 Figure 3

1. Make a table that shows the figure number and the number of boxes.
2. Describe in words the number of boxes as the figure number increases.
3. What is the expression for the number of boxes in any figure?
4. How many squares will be in Figure 10?
5. In Canada, what is the probability that Christmas will fall on December 25? Explain.

Mental Math

For #6 and #7, estimate the answer.

6. $5\frac{7}{8} - 2\frac{1}{4}$
7. $1\frac{4}{5} + 3\frac{9}{10}$
8. Show $\frac{1}{4}$ as a decimal and as a percent.
9. Show 35 out of 70 as a percent.
10. A circle has a radius of 5 m. Estimate the area.

Answers

Warm-Up

1. Look for a table such as the following. Encourage students to identify a number pattern.

Figure Number	Number of Boxes
1	$3 = 1 + 2$
2	$5 = 1 + 2 + 2$
3	$7 = 1 + 2 + 2 + 2$

2. The boxes increase by 2 every time the figure number increases. The number of boxes in each figure is 1 plus 2 times the figure number.

3. $1 + 2n$, where n is the number of boxes

4. $1 + 2n = 1 + 2(10) = 21$

5. 100%. Christmas is traditionally celebrated on that date.

6. $6 - 2 = 4$ **7.** $2 + 4 = 6$ **8.** 0.25, 25% **9.** 50%

10. $3 \times 5 \times 5 = 75\ m^2$

Explore the Math

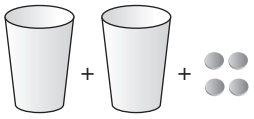
1. $c + 3$ **2.** $c = 6$ **3.** 9 **4.** 9

5. a) The cup represents the variable, or the unknown number of counters.

b) The three counters represent the constant, or the known number of counters.

Show You Know: Example 1

a)

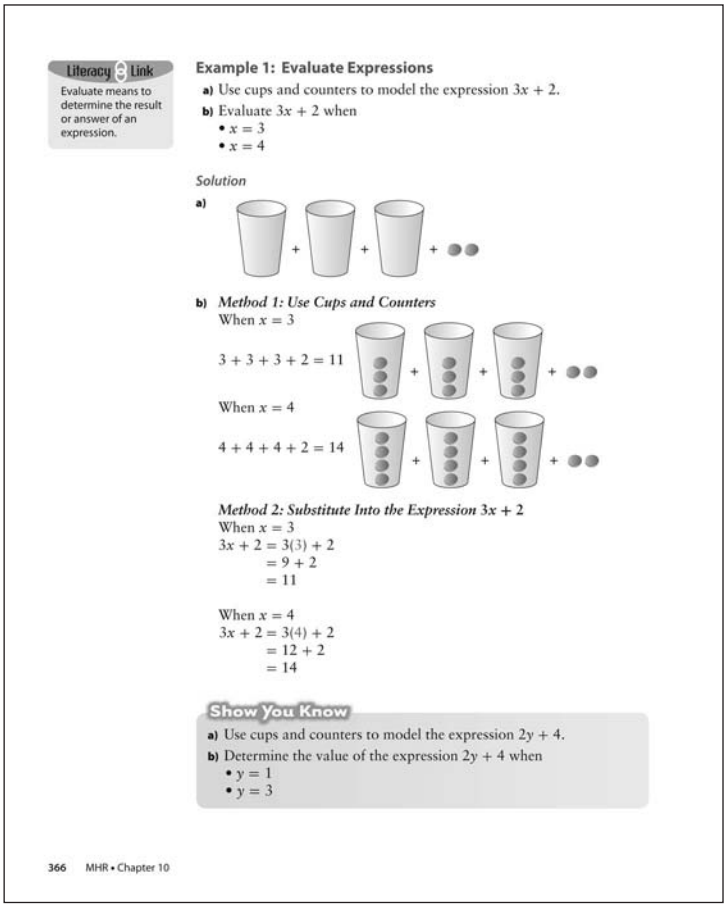


b) 6, 10

Supported Learning

Learning Style

- Have students use real cups and counters to solve the expressions.



Literacy Link
Evaluate means to determine the result or answer of an expression.

Example 1: Evaluate Expressions

a) Use cups and counters to model the expression $3x + 2$.

b) Evaluate $3x + 2$ when
- $x = 3$
- $x = 4$

Solution

a)

b) *Method 1: Use Cups and Counters*

When $x = 3$

$3 + 3 + 3 + 2 = 11$

When $x = 4$

$4 + 4 + 4 + 2 = 14$

Method 2: Substitute Into the Expression $3x + 2$

When $x = 3$

$3x + 2 = 3(3) + 2$
$= 9 + 2$
$= 11$

When $x = 4$

$3x + 2 = 3(4) + 2$
$= 12 + 2$
$= 14$

Show You Know

a) Use cups and counters to model the expression $2y + 4$.

b) Determine the value of the expression $2y + 4$ when
- $y = 1$
- $y = 3$

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Activity Planning Notes

In this section, each student should have an opportunity to physically handle cups and counters. Use an oral exercise to get started; for example, ask students to model $x + 2$, $2x + 1$, etc. Reverse the situation by showing a model and asking students to provide a matching expression.

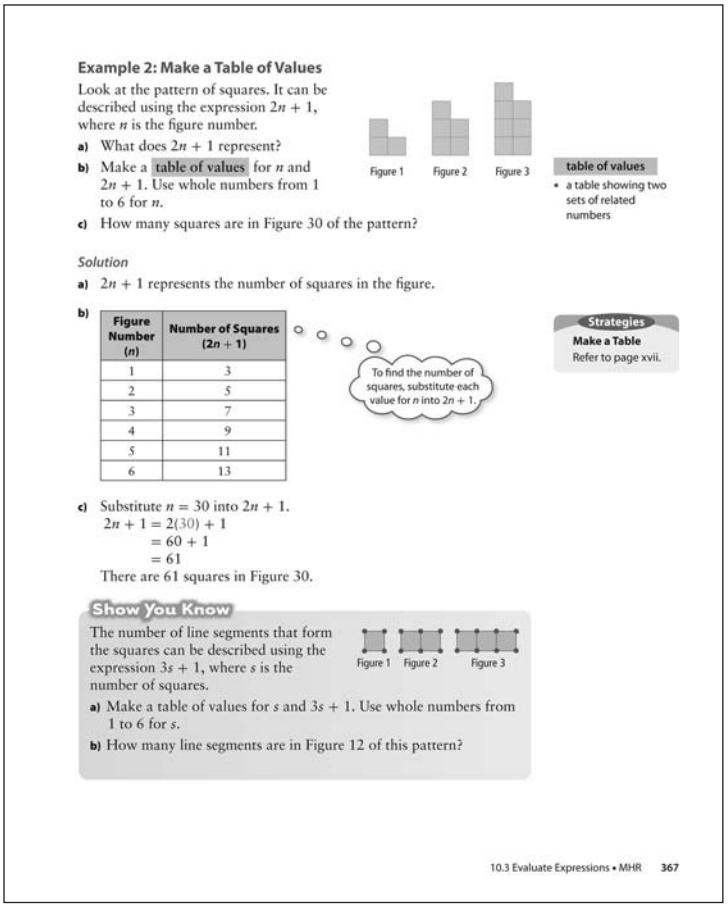
You may choose to begin with the Explore the Math or with Example 1.

Explore the Math

Have students work through the Explore the Math in small groups. Summarize to ensure that all students understand the modelling process.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Pay attention as students describe what the cups and counters represent. During this process, they are generalizing what they have learned during the Explore the Math.	• Ensure that students conceptually understand how the model shows an expression with variables and constants. Repeatedly reinforce the terminology and symbols that correspond to the model. This practice allows students to connect concrete, pictorial, and symbolic learning.

For Example 1, encourage students to use the concrete materials, not just draw a picture of them. Be sure that all students understand the cups/counters model before introducing substitution as the second method of evaluating expressions.



Example 2: Make a Table of Values

Look at the pattern of squares. It can be described using the expression $2n + 1$, where n is the figure number.

a) What does $2n + 1$ represent?

b) Make a table of values for n and $2n + 1$. Use whole numbers from 1 to 6 for n.

c) How many squares are in Figure 30 of the pattern?

Figure 1 Figure 2 Figure 3

table of values
- a table showing two sets of related numbers

Solution

a) $2n + 1$ represents the number of squares in the figure.

b)

Figure Number (n)	Number of Squares ($2n + 1$)
1	3
2	5
3	7
4	9
5	11
6	13

To find the number of squares, substitute each value for n into $2n + 1$.

Strategies
Make a Table
Refer to page xvii.

c) Substitute $n = 30$ into $2n + 1$.

$2n + 1 = 2(30) + 1$
$= 60 + 1$
$= 61$

There are 61 squares in Figure 30.

Show You Know

The number of line segments that form the squares can be described using the expression $3s + 1$, where s is the number of squares.

Figure 1 Figure 2 Figure 3

a) Make a table of values for s and $3s + 1$. Use whole numbers from 1 to 6 for s.

b) How many line segments are in Figure 12 of this pattern?

10.3 Evaluate Expressions • MHR 367

Answers

Show You Know: Example 2

a)

Number of Squares	Number of Line Segments ($3s + 1$)
1	4
2	7
3	10
4	13
5	16
6	19

b) 37

Supported Learning

Motor

- Students who have motor challenges may find it difficult to draw a table. Ensure that they use a ruler and are given adequate time to complete the task. Alternatively, consider allowing students to use a computer to create the table.

Meeting the Needs of All Learners

- Use visuals, concrete experiences, diagrams, charts, and pictures to assist in the learning of these concepts.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know on page 366 related to Example 1.	• In part b), encourage students to use one method for $y = 1$ and another method for $y = 3$. That way, you can check whether students understand both methods. You may wish to allow concrete and kinesthetic learners to use cups and counters to complete both parts a) and b). • Provide additional questions to do with evaluating expressions for students who will benefit from them: **a)** $a + 2$, when $a = 2$ and when $a = 5$ (Begin with an expression that only involves one operation. Have students physically put the number of counters in the cup and then count the total.) **b)** $3b + 5$, when $b = 1$ and when $b = 3$ (Move on to an expression with two operations.) Coach students through one additional question, and then have them try the second one on their own. • You may wish to have students do the Math Games on page 386. This activity provides a fun way to practise evaluating expressions.

Common Errors

- Students may have difficulty relating geometric patterns to patterns shown in a table of values.

R_x Refer students to Example 2. Make sure they recognize that Figure 1, Figure 2, Figure 3, etc. in the pattern correspond to the figure number in the table, and that the number of squares in each figure corresponds to the number of squares in the table.

For Example 2, check that students understand that both the variable and the expression have a specific meaning in questions of this type. Reinforce that n represents the figure number and $2n + 1$ represents the number of squares.

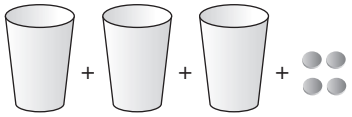
Ensure that all students understand how to make a table of values, since this is a critical skill in mathematics. Tables of values can be arranged vertically, as in part b) of the solution, or horizontally, as shown here:

Figure Number (n)	1	2	3	4	5	6
Number of Squares ($2n + 1$)	3	5	7	9	11	13

Answers

Communicate the Ideas

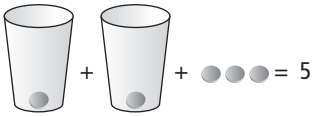
1.



a) The cups represent three of the same variable or unknown number.

b) The counters represent the constant of 4.

2.

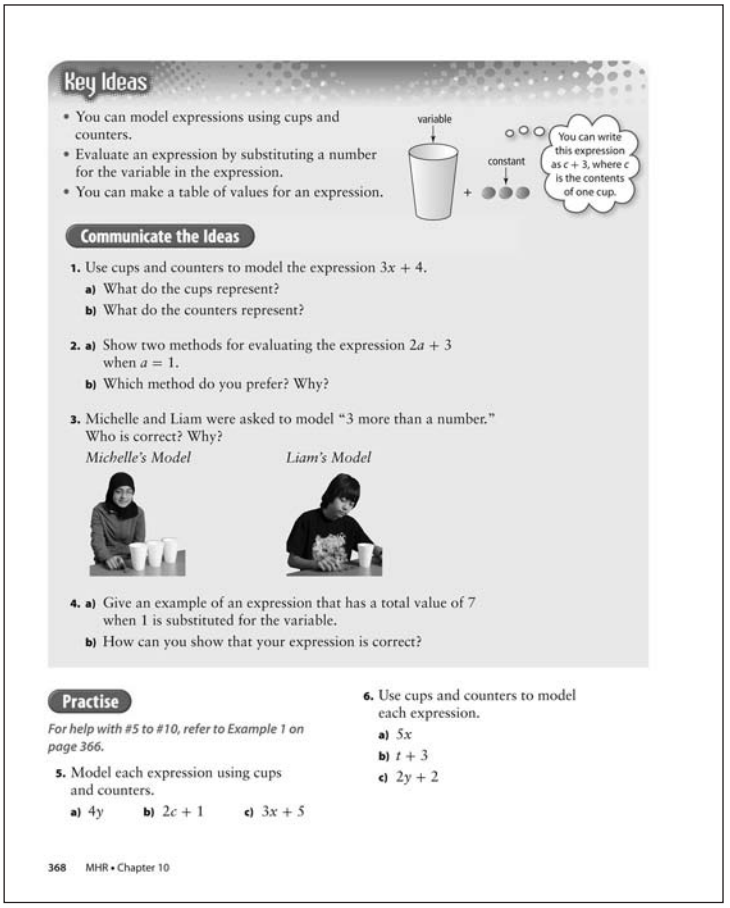


$2a + 3 = 2(1) + 3$ when $a = 1$
$= 2 + 3$
$= 5$

3. Liam is correct because 3 is the constant and n is the variable or unknown number, which makes the expression $n + 3$.

4. a) Answers may vary. For example: $2x + 5$

b) Use substitution: $2(1) + 5 = 7$.



Key Ideas

- You can model expressions using cups and counters.
- Evaluate an expression by substituting a number for the variable in the expression.
- You can make a table of values for an expression.

variable
constant
You can write this expression as $c + 3$, where c is the contents of one cup.

Communicate the Ideas

1. Use cups and counters to model the expression $3x + 4$.
a) What do the cups represent?
b) What do the counters represent?

2. a) Show two methods for evaluating the expression $2a + 3$ when $a = 1$.
b) Which method do you prefer? Why?

3. Michelle and Liam were asked to model "3 more than a number." Who is correct? Why?
Michelle's Model *Liam's Model*

4. a) Give an example of an expression that has a total value of 7 when 1 is substituted for the variable.
b) How can you show that your expression is correct?

Practise

For help with #5 to #10, refer to Example 1 on page 366.

5. Model each expression using cups and counters.
a) $4y$ **b)** $2c + 1$ **c)** $3x + 5$

6. Use cups and counters to model each expression.
a) $5x$
b) $t + 3$
c) $2y + 2$

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Supported Learning

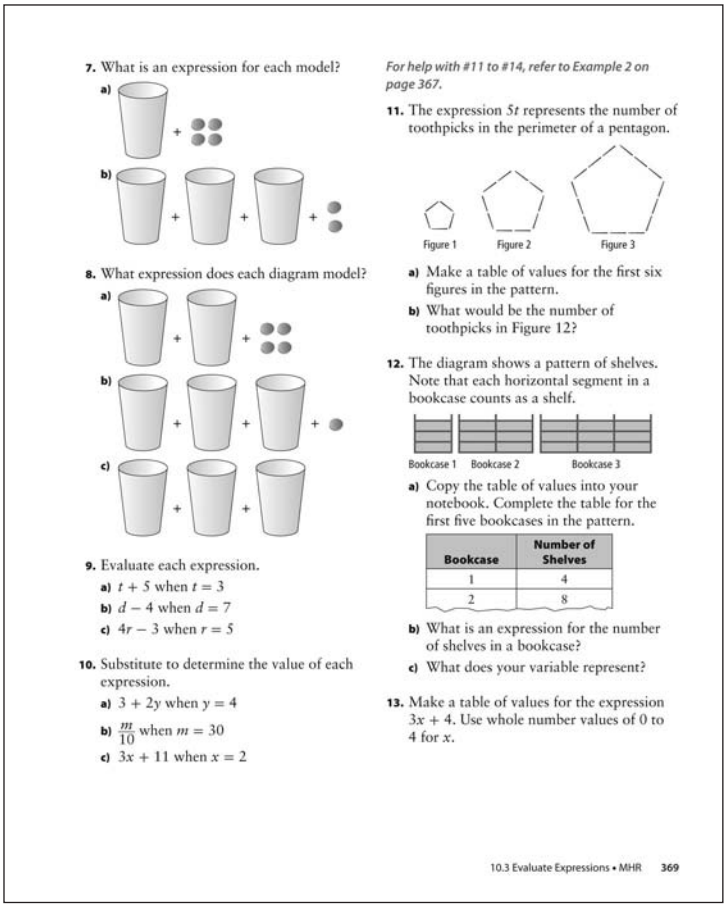
ESL and Language

- Assist students in understanding the meaning of the following words: *represent*, *substitute*, *expression*, and *constant*.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know on page 367 related to Example 2.	• Make sure students can identify what s and $3s + 1$ represent. • You may wish to provide visual learners with **Master 8 Centimetre Grid Paper** so that they can draw Figures 1 to 6 before developing the table. They might also draw Figures 7 to 12 to check their answer to b). • Provide an additional question to students who will benefit from it: Look at #6a) on page 354. The pattern of green dots can be described using the expression $3f$, where f is the figure number. **a)** Make a table of values for f and $3f$. Use whole numbers from 1 to 7 for f. **b)** How many dots are in Figure 20 of this pattern? Coach students through this additional question.

Key Ideas

Have students prepare their own list of Key Ideas and put these in their chapter Foldable. It is quite acceptable to include more or fewer concepts than what are present in the student resource. The intention of this section is to show students a number of different ways to visualize and evaluate expressions. Students should be encouraged to use strategies that match their learning style.



7. What is an expression for each model?

a)

b)

8. What expression does each diagram model?

a)

b)

c)

9. Evaluate each expression.

a) $t + 5$ when $t = 3$

b) $d - 4$ when $d = 7$

c) $4r - 3$ when $r = 5$

10. Substitute to determine the value of each expression.

a) $3 + 2y$ when $y = 4$

b) $\frac{m}{10}$ when $m = 30$

c) $3x + 11$ when $x = 2$

For help with #11 to #14, refer to Example 2 on page 367.

11. The expression $5t$ represents the number of toothpicks in the perimeter of a pentagon.

Figure 1 Figure 2 Figure 3

a) Make a table of values for the first six figures in the pattern.

b) What would be the number of toothpicks in Figure 12?

12. The diagram shows a pattern of shelves. Note that each horizontal segment in a bookcase counts as a shelf.

Bookcase 1 Bookcase 2 Bookcase 3

a) Copy the table of values into your notebook. Complete the table for the first five bookcases in the pattern.

Bookcase	Number of Shelves
1	4
2	8

b) What is an expression for the number of shelves in a bookcase?

c) What does your variable represent?

13. Make a table of values for the expression $3x + 4$. Use whole number values of 0 to 4 for x.

10.3 Evaluate Expressions • MHR 369

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students complete #1 and #2. Give them a choice of completing #3 or #4. Giving students a choice assists with their mathematical development and encourages a sense of responsibility in their learning.	• In #2, students are given the opportunity to consider which strategy is best suited to their own learning style. • In #3, students view two completed solutions and decide which is correct and why. As you circulate, ask students to explain the "why" to you.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 2, 3 *or* 4, 5, 7, 9, 11, 13, Math Link
Typical	1, 2, 3 *or* 4, 5, 7, 9, 11, 13, 15, 17, 18, Math Link
Extension/Enrichment	1, 2, 3 *or* 4, 16, 19–21, Math Link

Communicate the Ideas

The Communicate the Ideas allows students to explain and justify their understanding of how to evaluate an expression using diagrams and substitution.

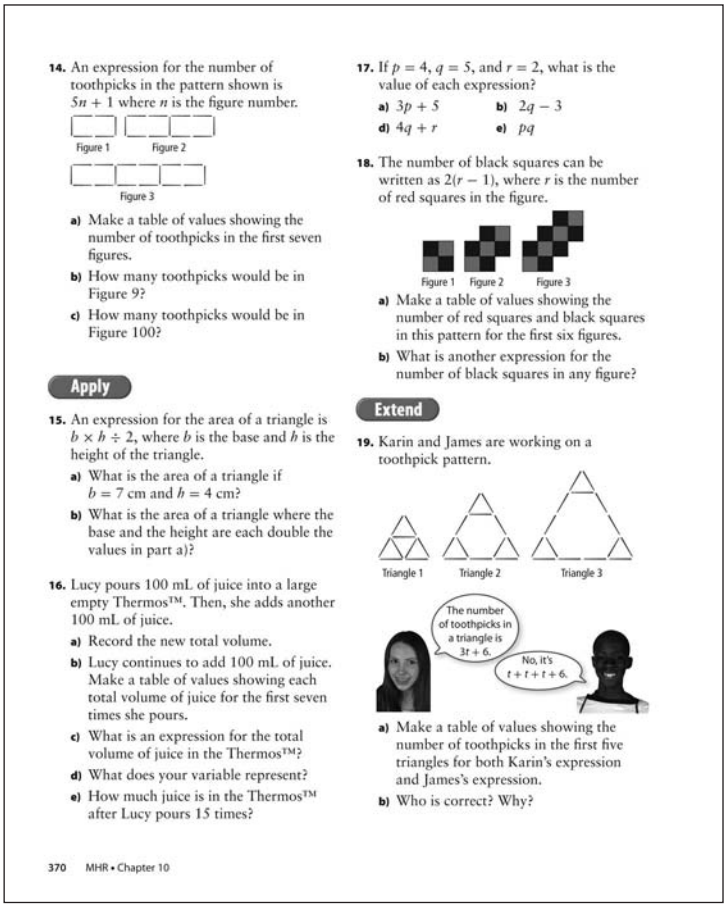
Practise

Assessment *for* Learning	Supported Learning
Practise Have students do #5, #7, #9, #11, and #13. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #5, #7, #9, #11, and #13 will need additional coaching. • For students having difficulty with #5 and #7, have them explain their understanding of how models represent expressions. Clarify any misunderstandings. Coach them through #6a) and #8a), and then have them complete b) and c) of each question on their own. • For students having difficulty with #9, have them explain their understanding of how to evaluate expressions. Clarify any misunderstandings. Coach them through #10a), and then have them complete b) and c) on their own. • For students having difficulty with #11 and #13, have them explain their understanding of how to make a table of values for an expression. Clarify any misunderstandings. Coach them through #12, and then have them complete #14 on their own. • Check back with them several times to make sure that they understand the concepts.

Supported Learning

Learning Style and Memory

- You may wish to provide **BLM 10–9 Section 10.3 Extra Practice** to students who require more practice.



14. An expression for the number of toothpicks in the pattern shown is $5n + 1$ where n is the figure number.

Figure 1 Figure 2

Figure 3

a) Make a table of values showing the number of toothpicks in the first seven figures.

b) How many toothpicks would be in Figure 9?

c) How many toothpicks would be in Figure 100?

Apply

15. An expression for the area of a triangle is $b \times h \div 2$, where b is the base and h is the height of the triangle.

a) What is the area of a triangle if $b = 7$ cm and $h = 4$ cm?

b) What is the area of a triangle where the base and the height are each double the values in part a)?

16. Lucy pours 100 mL of juice into a large empty Thermos™. Then, she adds another 100 mL of juice.

a) Record the new total volume.

b) Lucy continues to add 100 mL of juice. Make a table of values showing each total volume of juice for the first seven times she pours.

c) What is an expression for the total volume of juice in the Thermos™?

d) What does your variable represent?

e) How much juice is in the Thermos™ after Lucy pours 15 times?

17. If $p = 4$, $q = 5$, and $r = 2$, what is the value of each expression?

a) $3p + 5$ b) $2q - 3$

d) $4q + r$ e) pq

18. The number of black squares can be written as $2(r - 1)$, where r is the number of red squares in the figure.

Figure 1 Figure 2 Figure 3

a) Make a table of values showing the number of red squares and black squares in this pattern for the first six figures.

b) What is another expression for the number of black squares in any figure?

Extend

19. Karin and James are working on a toothpick pattern.

Triangle 1 Triangle 2 Triangle 3

The number of toothpicks in a triangle is $3t + 6$.

No, it's $t + t + t + 6$.

a) Make a table of values showing the number of toothpicks in the first five triangles for both Karin's expression and James's expression.

b) Who is correct? Why?

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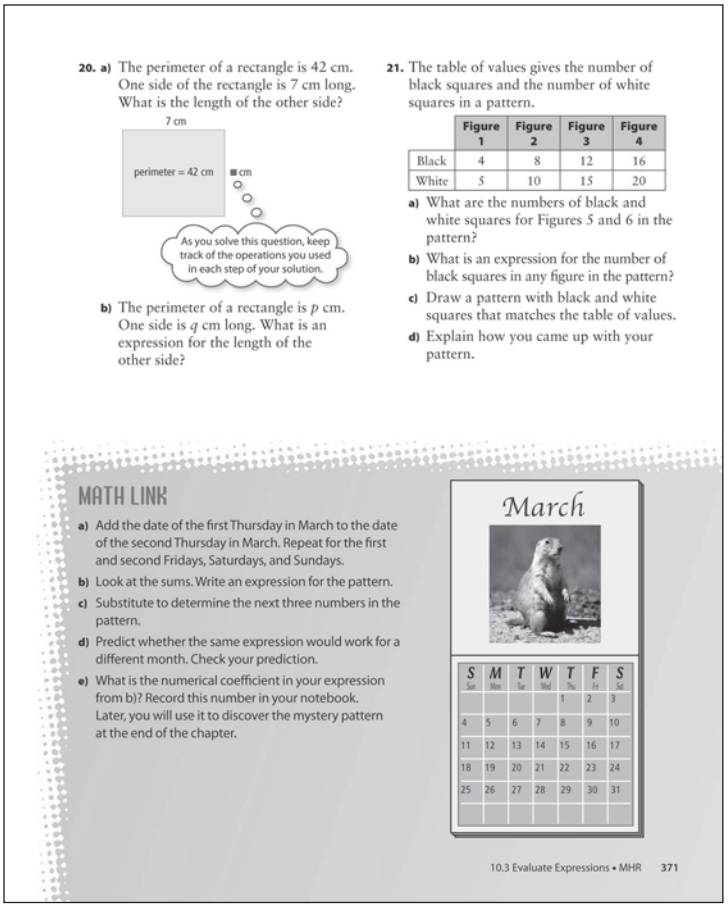
Apply and Extend

You may find #18 particularly insightful for students since it shows them that, for some patterns, there are multiple ways to write an expression.

In #20, students solve part a) using numbers and then move to the more abstract by solving part b) using only letters. Direct students' attention to the thought bubble, which hints that they will need to apply the operations they used in part a) to their expression in part b).

Assessment *as* Learning	Supported Learning
Math Learning Log Have students reflect on two or three items they have improved on and how they think they have improved. Students might also write about one area in which they feel they did not learn as much as they should or could have. Have them reflect on how they plan to improve this area of learning.	• Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Work with students to develop a plan for dealing with the areas in which they are having difficulty. • Depending on students' learning style, have them provide oral or written answers. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • Have students review the part related to Section 10.3 in **BLM 10–1 Chapter 10 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.



20. a) The perimeter of a rectangle is 42 cm. One side of the rectangle is 7 cm long. What is the length of the other side?

7 cm

perimeter = 42 cm

■ cm

As you solve this question, keep track of the operations you used in each step of your solution.

b) The perimeter of a rectangle is p cm. One side is q cm long. What is an expression for the length of the other side?

21. The table of values gives the number of black squares and the number of white squares in a pattern.

	Figure 1	Figure 2	Figure 3	Figure 4
Black	4	8	12	16
White	5	10	15	20

a) What are the numbers of black and white squares for Figures 5 and 6 in the pattern?

b) What is an expression for the number of black squares in any figure in the pattern?

c) Draw a pattern with black and white squares that matches the table of values.

d) Explain how you came up with your pattern.

MATH LINK

a) Add the date of the first Thursday in March to the date of the second Thursday in March. Repeat for the first and second Fridays, Saturdays, and Sundays.

b) Look at the sums. Write an expression for the pattern.

c) Substitute to determine the next three numbers in the pattern.

d) Predict whether the same expression would work for a different month. Check your prediction.

e) What is the numerical coefficient in your expression from b)? Record this number in your notebook. Later, you will use it to discover the mystery pattern at the end of the chapter.

Answers

Math Link

a) 9, 11, 13, 15

b) $2t + 7$

c) $2(5) + 7 = 17$; $2(6) + 7 = 19$; $2(7) + 7 = 21$

d) Answers may vary.

e) 2

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 371 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 385. It provides students with an opportunity to apply their understanding of how patterns of sums on a calendar would be written as an expression.	• If students are not sure how to determine an expression for the pattern of sums, suggest that they try the Guess and Check strategy. • Explain to students that the expression they must determine in part b) should generate the sum of the two dates based on the term number ($2t + 7$ where t is the term number). For example, when term number 1 is substituted into the expression $2t + 7$, the result is 9 (which is the sum of the dates of the first and second Thursdays: $1 + 8 = 9$). • For part d), students may choose one of two ways to test their expression: – starting with the first Thursday in a different month ($2t + 7$ will not work if the first day of the month is not Thursday) – starting with the first day of a different month ($2t + 7$ will always work) • Note that some students may determine the expression in part b) to be $s + 2$, where s is the previous sum. In this case, for part e) students will need to record the constant instead of the numerical coefficient. • You may wish to provide to students **BLM 10–10 Section 10.3 Math Link**, which provides scaffolding for this activity.

MATH LINK

With the Math Link, students continue to explore patterns in a calendar. Students are asked to write an expression for a calendar pattern. Then they predict if the expression could be generalized to another month of the calendar. Remind students to record in their notebook the numerical coefficient in part e), or have them record it on **BLM 10–14 Wrap It Up! Mystery Pattern**.

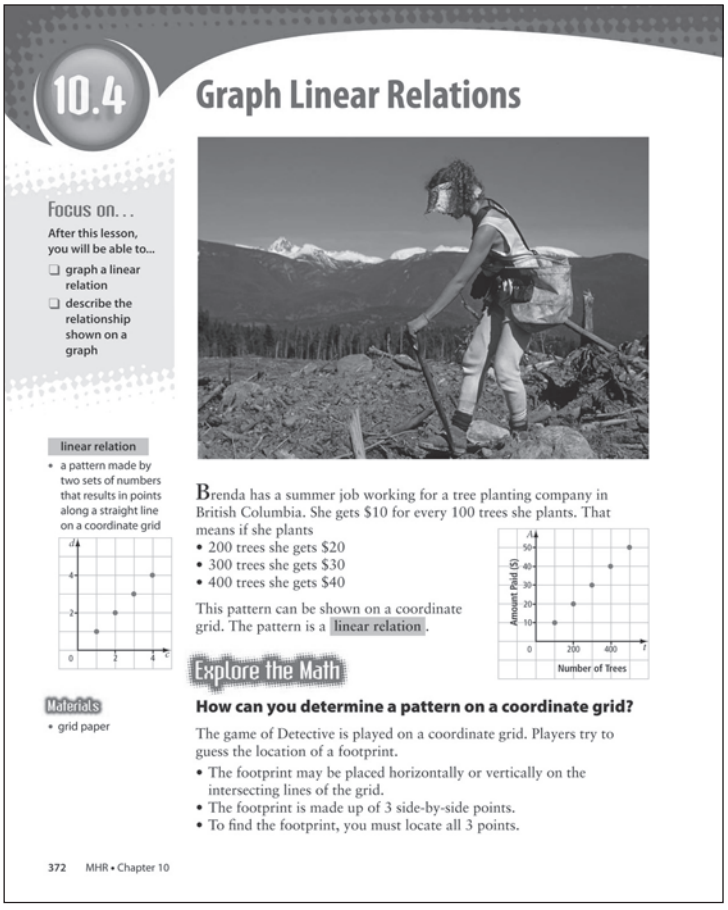
10.4 Graph Linear Relations

Suggested Timing
100–125 minutes

Materials
- ruler
- grid paper
- counters or coloured tiles
- stir sticks

Blackline Masters
Master 8 Centimetre Grid Paper
Master 9 0.5 Centimetre Grid Paper
BLM 10–1 Chapter 10 Self-Assessment
BLM 10–3 Coordinate Grid
BLM 10–11 Section 10.4 Extra Practice
BLM 10–12 Section 10.4 Math Link
BLM 10–14 Wrap It Up! Mystery Pattern

Mathematical Processes
- ☑ Communication
- ☑ Connections
- ☑ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☐ Technology
- ☑ Visualization


10.4 Graph Linear Relations

Focus on...
After this lesson, you will be able to...
- graph a linear relation
- describe the relationship shown on a graph

linear relation
- a pattern made by two sets of numbers that results in points along a straight line on a coordinate grid

Brenda has a summer job working for a tree planting company in British Columbia. She gets $10 for every 100 trees she plants. That means if she plants
- 200 trees she gets $20
- 300 trees she gets $30
- 400 trees she gets $40

This pattern can be shown on a coordinate grid. The pattern is a linear relation.

Amount Paid ($)
Number of Trees

Explore the Math

Materials
- grid paper

How can you determine a pattern on a coordinate grid?

The game of Detective is played on a coordinate grid. Players try to guess the location of a footprint.
- The footprint may be placed horizontally or vertically on the intersecting lines of the grid.
- The footprint is made up of 3 side-by-side points.
- To find the footprint, you must locate all 3 points.

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Specific Outcomes

PR2 Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.

Warm-Up

1. Model the expression $3u + 6$.

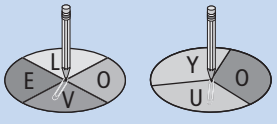
2. Evaluate the expression in #1 when $u = 5$.

3. Determine the value of $\frac{t}{5}$ when $t = 25$.

Use the spinners to answer #4 and #5.

4. Organize the sample space.

5. What is the probability of spinning two vowels? *Y* is not a vowel for this question.



Mental Math

Use the following list for #6 to #8. Explain your reasoning.
200, 205, 210, 215, 220, 225, 230, 235, 240, 245, 250

6. List the numbers divisible by 6.

7. List the numbers divisible by 5.

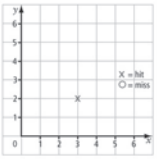
8. List the numbers divisible by 4.

9. Estimate 49% of 634.

10. Estimate 125 out of 380 as a percent.

1. Draw a coordinate grid. Label the x-axis and y-axis from 0 to 6.

2. You make a guess of (3, 2). You hit the footprint! Mark (3, 2) with X. What two lines could the footprint be along?



3. Your next guess is (3, 3). It's a miss. Mark (3, 3) with ○. Could the footprint be along the vertical line through (3, 2)? Why?

4. Next, you try (2, 2). It's a hit! Mark (2, 2) with X. What are the possible coordinates for the last point of the footprint?

Reflect on Your Findings

5. a) Imagine you hit a footprint on your first try. How many possible locations are there for the rest of the points on the footprint?
   b) Why is it necessary to play Detective on a coordinate grid instead of on a plain piece of paper?
   c) Describe strategies for locating a footprint.

Activity Planning Notes

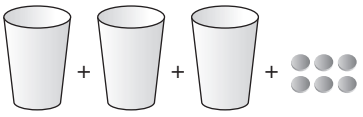
Read the opening paragraph with students. Lead a brief discussion on tree planting. Ask students: "Do you know anyone who has been tree planting? How much did that person get paid? How much is Brenda getting paid?"

Explore the Math

This activity is similar to the game Battleship™. The intent is for students to analyse possible locations for missing points. This exercise acts as an introduction to identifying patterns on a coordinate grid. You may wish to provide students with **Master 8 Centimetre Grid Paper** or **Master 9 0.5 Centimetre Grid Paper** for this activity.

Answers

Warm-Up

1. 

2. $3(5) + 6 = 21$

3. $\frac{25}{5} = 5$

4. Students should make a table, tree diagram, or other organizer. For example:

	Y	O	U
L	L, Y	L, O	L, U
O	O, Y	O, O	O, U
V	V, Y	V, O	V, U
E	E, Y	E, O	E, U

5. These outcomes include (O, O), (O, U), (E, O), (E, U).

   $P(\text{2 vowels}) = \frac{4}{12} = \frac{1}{3}$

6. 210, 240. The numbers are divisible by 2 and 3.

7. 200, 205, 210, 215, 220, 225, 230, 235, 240, 245, 250. The last digit is either 0 or 5.

8. 200, 220, 240. The number formed by the last two digits is divisible by 2 at least twice.

9. 49% is about 50%. $50\% = \frac{1}{2}$. $634 \div 2 = 317$

10. 50% of 380 =190
    25% of 380 = 95
    10% of 380 = 38
    5% of 380 = 19
    35% of 380 = 95 + 38 = 133 Too high
    30% of 380 = 38 × 3 = 114 Too low
    The answer is between 30% and 35%.

Explore the Math

2. $x = 3$; $y = 2$

3. Yes. Answers may vary. For example: The footprint could lie along points (3, 2), (3, 1), and (3, 0).

4. (1, 2) or (4, 2)

5. **a)** 8
   b) Answers may vary. For example: It is easier to describe a location.
   c) Answers may vary. For example: Once you hit the first point on the footprint, try left, right, up, and down from that point. Once you hit the second point, try to find the point that completes the straight line.

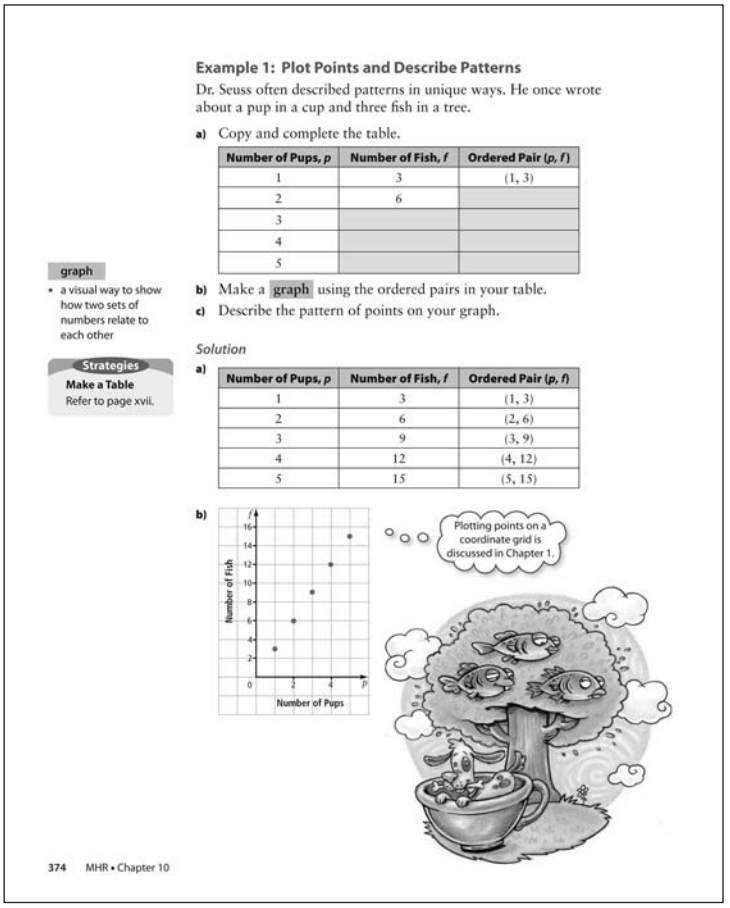
Supported Learning

ESL

- As students work on the Explore the Math, ensure that English language learners understand the meaning of the word *footprint*.
- For Example 1, ensure that English language learners understand the meaning of the word *pup*.

Motor

- Provide **BLM 10–3 Coordinate Grid** rather than having them draw the coordinate grid by hand. Students will be able to focus on their math skills without being distracted by the task of drawing.



Example 1: Plot Points and Describe Patterns

Dr. Seuss often described patterns in unique ways. He once wrote about a pup in a cup and three fish in a tree.

a) Copy and complete the table.

Number of Pups, p	Number of Fish, f	Ordered Pair (p, f)
1	3	(1, 3)
2	6	
3		
4		
5		

graph
- a visual way to show how two sets of numbers relate to each other

b) Make a graph using the ordered pairs in your table.

c) Describe the pattern of points on your graph.

Solution

a)

Number of Pups, p	Number of Fish, f	Ordered Pair (p, f)
1	3	(1, 3)
2	6	(2, 6)
3	9	(3, 9)
4	12	(4, 12)
5	15	(5, 15)

Strategies
Make a Table
Refer to page xvii.

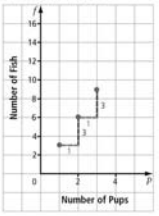
b)

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss strategies for locating points on a coordinate grid. During this process, they are generalizing what they have learned during the Explore the Math.	• Discuss with students the pattern on the coordinate grid that is made by the footprint. Ask, "Is it a straight line? Is it a linear relation? How do you know? Is there any way to place the footprint on the grid so that it is not a linear relation?" Assist students in understanding that any straight line on the grid is a linear relation. • If students need additional practice with using a coordinate grid, you may wish to have them play the game called Going Fishing on page 40 of the student resource.

In Example 1, students use two important skills in mathematics: drawing a graph on a coordinate grid and describing the pattern of points on a graph. The description of the pattern of points can be presented using words, horizontal and vertical distances between points, and an algebraic relationship. All three methods for describing the points on a graph are acceptable, and students should have some practice with each before settling on a preferred method.

c) *Method 1: Describe Using Words*
The pattern of points forms a straight line, starting at (1, 3).
The f-coordinate is 3 times the p-coordinate.

Method 2: Describe Using Horizontal and Vertical Distances
The points increase by 1 unit to the right, then 3 units up, starting at (1, 3).



Method 3: Describe Using a Relationship
The number of pups is p. The number of fish is f.
The coordinates of the points are (p, f).
The relationship between p and f is linear and can be expressed as $(p, 3p)$ or $f = 3p$.

relationship
- a pattern formed by two sets of numbers

Show You Know

Imagine that for every pup in a cup, there are four fish at the store.

a) In your notebook, copy and complete the table.

Number of Pups, p	Number of Fish, f	Ordered Pair (p, f)
1	4	
2	8	
3	12	
4		
5		

b) Graph the ordered pairs.

c) Describe the pattern of points on your graph. Use words, horizontal and vertical distances, and a relationship.

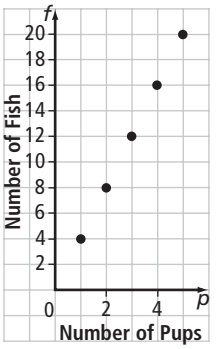
Answers

Show You Know: Example 1

a)

Number of Pups, p	Number of Fish, f	Ordered Pair (p, f)
1	4	(1, 4)
2	8	(2, 8)
3	12	(3, 12)
4	16	(4, 16)
5	20	(5, 20)

b)



c) Answers may vary.
For example:

Words: The pattern of points forms a straight line, starting at (1, 4). The f coordinate is 4 times the p coordinate.

Horizontal and vertical distances on a graph: The points increase by 1 unit right, then 4 units up, starting at (1, 4).

Relationship: The coordinates of the points are (p, f). The relationship between p and f is linear. It can be written as $(p, 4p)$ or $f = 4p$.

Assessment *for* Learning

Example 1
Have students do the Show You Know related to Example 1.

Supported Learning

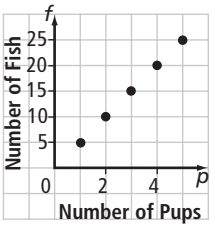
- Some students may need to review their work in Chapter 1 on plotting points on a coordinate grid.
- The introduction of a second variable may be challenging for some students. It is important that students can identify the expression and realize that the additional variable represents the solution to the expression.
- Provide students who will benefit from it with additional practice. You may wish to use a simpler statement: Imagine that for every pup there are five fish.

a) Complete a table of values for the pattern.

Number of Pups, p	Number of Fish, f	Ordered Pair (p, f)
1	5	(1, 5)
2	10	(2, 10)
3	15	(3, 15)
4	20	(4, 20)
5	25	(5, 25)

b) Graph the ordered pairs.

c) Describe the pattern of points on the graph. (Answers may vary. For example: For every point right, the graph goes up 5 points. The pattern forms a straight line. $f = 5p$)



Coach students through the first two rows of the table of values, and then have them complete the same steps as in Example 1 on their own.

Supported Learning

ESL and Language

- For the Show You Know for Examples 1 and 2, allow English language learners and students with language difficulties to describe the graph showing horizontal and vertical distances on a graph or by writing a relationship, instead of using words.

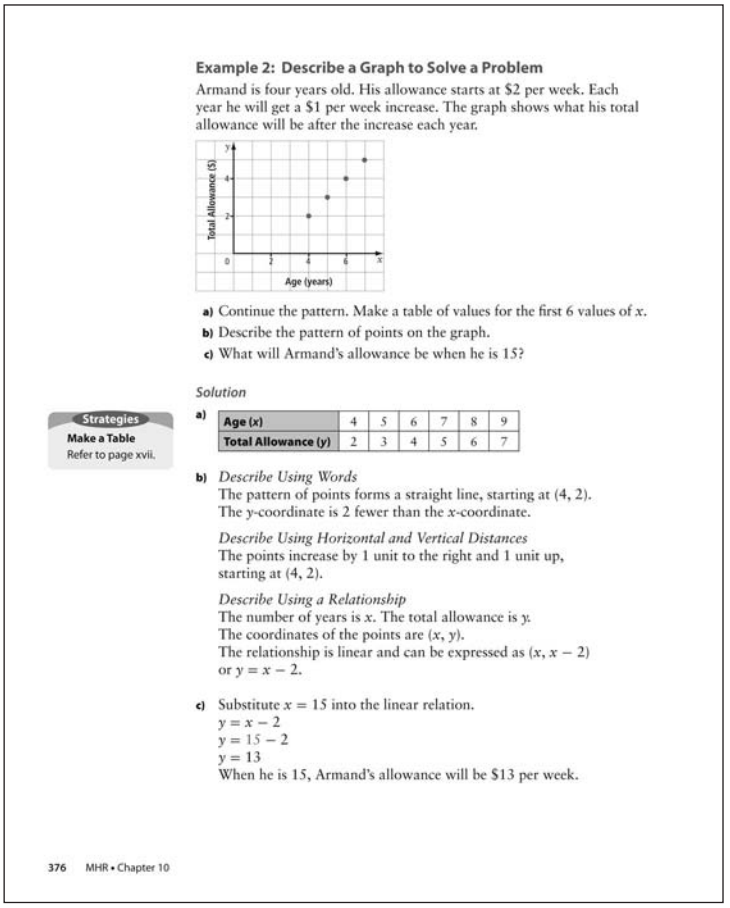
Supported Learning

ESL

- For Example 2, ensure that English language learners understand the meaning of the word *allowance*.

Meeting the Needs of All Learners

- Relate the material to experiences, events, and routines in students' lives. For example, the Example 2 Show You Know could be the number of beads in a container as someone is making a necklace.



Example 2: Describe a Graph to Solve a Problem

Armand is four years old. His allowance starts at $2 per week. Each year he will get a $1 per week increase. The graph shows what his total allowance will be after the increase each year.

a) Continue the pattern. Make a table of values for the first 6 values of x.
b) Describe the pattern of points on the graph.
c) What will Armand's allowance be when he is 15?

Solution

Strategies
Make a Table
Refer to page xvii.

a)

Age (x)	4	5	6	7	8	9
Total Allowance (y)	2	3	4	5	6	7

b) *Describe Using Words*
The pattern of points forms a straight line, starting at (4, 2).
The y-coordinate is 2 fewer than the x-coordinate.

Describe Using Horizontal and Vertical Distances
The points increase by 1 unit to the right and 1 unit up, starting at (4, 2).

Describe Using a Relationship
The number of years is x. The total allowance is y.
The coordinates of the points are (x, y).
The relationship is linear and can be expressed as $(x, x - 2)$ or $y = x - 2$.

c) Substitute $x = 15$ into the linear relation.
$y = x - 2$
$y = 15 - 2$
$y = 13$
When he is 15, Armand's allowance will be $13 per week.

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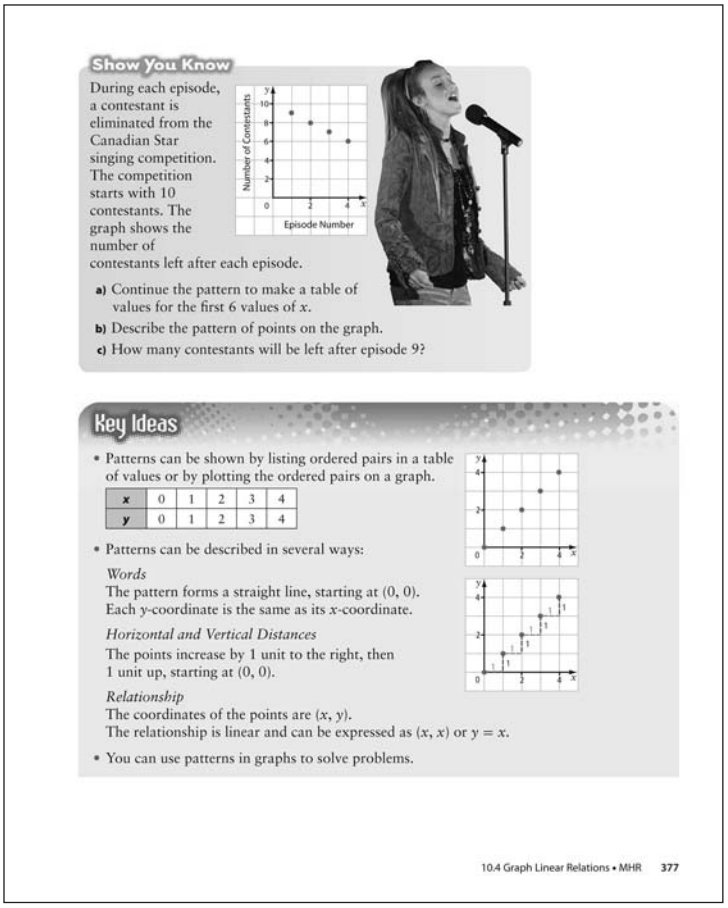
Example 2 introduces an everyday problem application that can be solved using a graph on a coordinate grid. The questions are the same as in Example 1, except students must solve the word problem by determining Armand's allowance when he is 15 years old.

Key Ideas

Students should prepare their own summary of section 10.4, using some of the thoughts in the Key Ideas on page 377 and other concepts they feel are important. Make sure students do not simply copy what is in the student resource. Real learning occurs when students put concepts in their own words and make them part of their understanding. Have students place their summary in their chapter Foldable.



Show You Know

During each episode, a contestant is eliminated from the Canadian Star singing competition. The competition starts with 10 contestants. The graph shows the number of contestants left after each episode.

a) Continue the pattern to make a table of values for the first 6 values of x.

b) Describe the pattern of points on the graph.

c) How many contestants will be left after episode 9?

Key Ideas

- Patterns can be shown by listing ordered pairs in a table of values or by plotting the ordered pairs on a graph.

x	0	1	2	3	4
y	0	1	2	3	4

- Patterns can be described in several ways:

Words
The pattern forms a straight line, starting at (0, 0). Each y-coordinate is the same as its x-coordinate.

Horizontal and Vertical Distances
The points increase by 1 unit to the right, then 1 unit up, starting at (0, 0).

Relationship
The coordinates of the points are (x, y).
The relationship is linear and can be expressed as (x, x) or $y = x$.

- You can use patterns in graphs to solve problems.

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Answers

Show You Know: Example 2

a)

Episode Number, x	Number of Contestants, y
1	9
2	8
3	7
4	6
6	5
6	4

b) Answers may vary. For example:

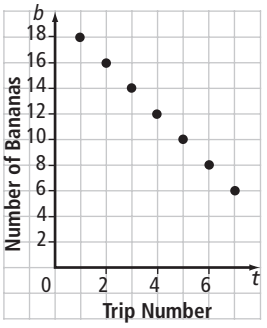
Words: The pattern of points forms a straight line, starting at (1, 9). To get the y-coordinate, subtract the x-coordinate from 10.

Horizontal and vertical distances on a graph: The points decrease by 1 unit right, and then 1 unit down, starting at (1, 9).

Relationship: The episode number is x. The number of contestants is y. The coordinates of the points are (x, y). The relationship between x and y is linear. It can be written as $(x, 10 - x)$ or $y = 10 - x$.

c) 1

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• The Show You Know shows a graph with a pattern of decreasing values. Make sure students understand that a straight line on a graph represents a linear relation, no matter the orientation of the line. • This relationship is in the form of 10 – x, with the constant at the beginning of the expression. Some students may need addition guidance in describing this less familiar type of relationship. • Provide students who will benefit from it with additional practice using descending values: There are 20 bananas in the fruit bowl. Raoul takes 2 each time he comes into the kitchen. The graph shows the number of bananas left after each trip.



a) Make a table of values for the first seven values of b, where t represents the trip number and b represents the number of bananas left in the bowl.

t	1	2	3	4	5	6	7
b	18	16	14	12	10	8	6

b) Describe the pattern of points on the graph. (Answers may vary. For example: The points are in a straight line starting at (1, 18). The b value goes down 2 for every increase by 1 of the t value.)

c) How many bananas will be left after 8 trips? (4)

Supported Learning

ESL and Language

- For the Show You Know related to Example 2, students may need guidance in understanding the meaning of the words *episode* and *contestant*.

Answers

Communicate the Ideas

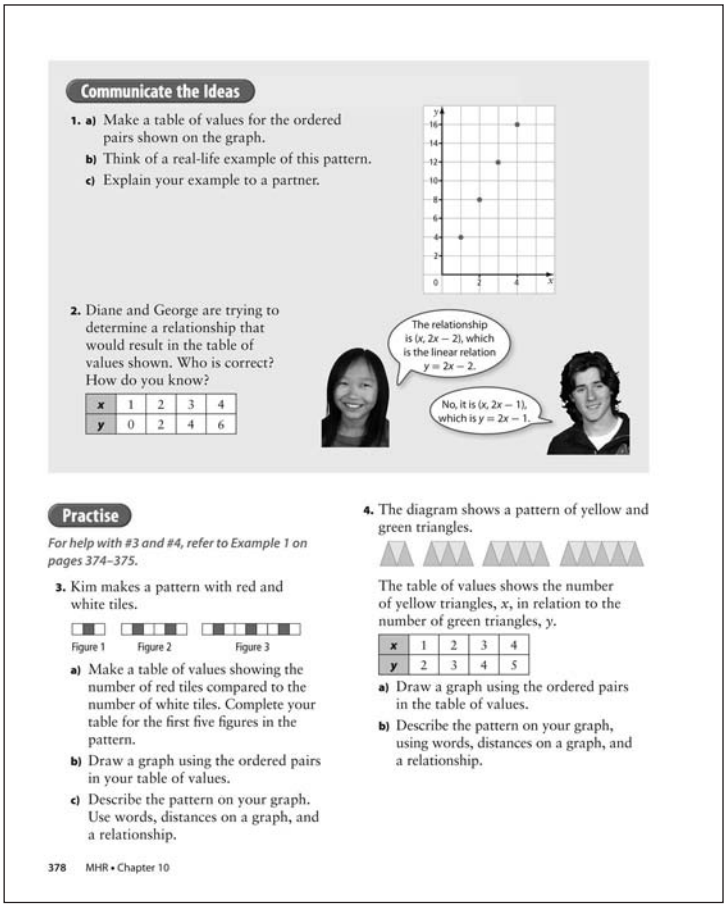
1. a)

x	y
1	4
2	8
3	2
4	16

b) Answers may vary. For example: The total cost of comic books if they cost $4 each

2. Diane is correct. Answers may vary. For example: When you substitute the values of x into each relationship, the y values from Diane's relationship match the table of values, and the y values from George's relationship do not match the table of values.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1 *or* 2, 3, 5, 7–10, Math Link
Typical	1 *or* 2, 3, 5, 7–13, Math Link
Extension/Enrichment	1 *or* 2, 12–15, Math Link



Communicate the Ideas

1. a) Make a table of values for the ordered pairs shown on the graph.
b) Think of a real-life example of this pattern.
c) Explain your example to a partner.

2. Diane and George are trying to determine a relationship that would result in the table of values shown. Who is correct? How do you know?

x	1	2	3	4
y	0	2	4	6

The relationship is $(x, 2x - 2)$, which is the linear relation $y = 2x - 2$.

No, it is $(x, 2x - 1)$, which is $y = 2x - 1$.

Practise

For help with #3 and #4, refer to Example 1 on pages 374–375.

3. Kim makes a pattern with red and white tiles.

Figure 1 Figure 2 Figure 3

a) Make a table of values showing the number of red tiles compared to the number of white tiles. Complete your table for the first five figures in the pattern.
b) Draw a graph using the ordered pairs in your table of values.
c) Describe the pattern on your graph. Use words, distances on a graph, and a relationship.

4. The diagram shows a pattern of yellow and green triangles.

The table of values shows the number of yellow triangles, x, in relation to the number of green triangles, y.

x	1	2	3	4
y	2	3	4	5

a) Draw a graph using the ordered pairs in the table of values.
b) Describe the pattern on your graph, using words, distances on a graph, and a relationship.

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Supported Learning

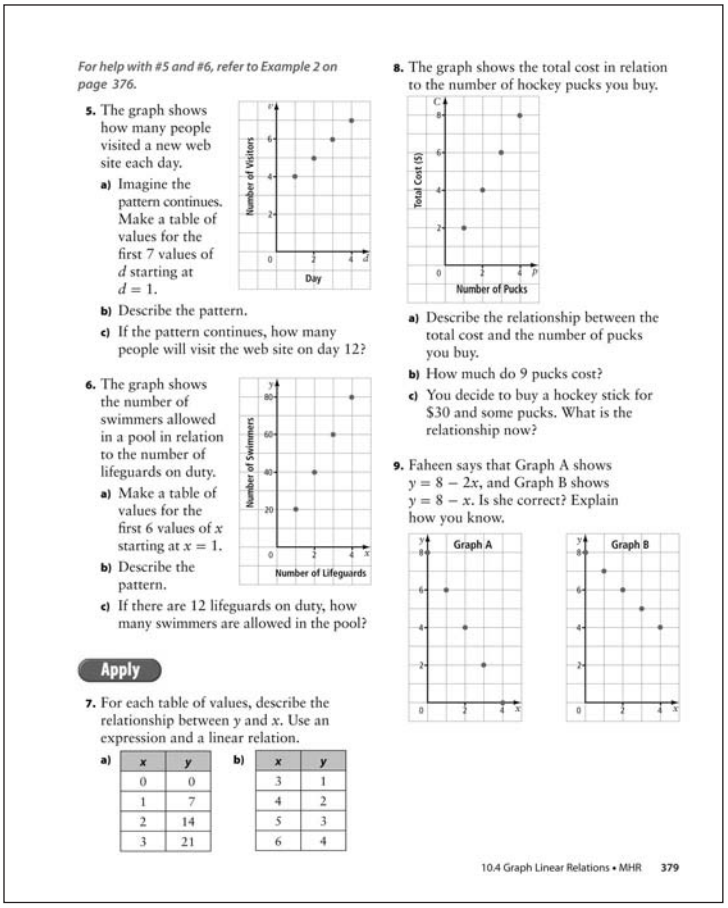
Learning Style

- Allow concrete and kinesthetic learners to model #3 and #4 of the Practise with counters or coloured tiles.
- You may wish to allow students to use a method of their choice for describing the patterns on the graphs in #3c) and #4b), instead of having them use all three methods.

Communicate the Ideas

The Communicate the Ideas allows students to explain how patterns can be shown using words, tables, graphs, and linear relations. Either question is useful to increase understanding. You may wish to allow students to choose between #1 and #2.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students complete #1 or #2.	• Encourage students to write in their chapter Foldable an example of how words, tables, graphs, and linear relations can be used to represent patterns. Have them share their examples with a partner to confirm that their selections are accurate.



For help with #5 and #6, refer to Example 2 on page 376.

5. The graph shows how many people visited a new web site each day.

a) Imagine the pattern continues. Make a table of values for the first 7 values of d starting at $d = 1$.

b) Describe the pattern.

c) If the pattern continues, how many people will visit the web site on day 12?

6. The graph shows the number of swimmers allowed in a pool in relation to the number of lifeguards on duty.

a) Make a table of values for the first 6 values of x starting at $x = 1$.

b) Describe the pattern.

c) If there are 12 lifeguards on duty, how many swimmers are allowed in the pool?

Apply

7. For each table of values, describe the relationship between y and x. Use an expression and a linear relation.

a)

x	y
0	0
1	7
2	14
3	21

b)

x	y
3	1
4	2
5	3
6	4

8. The graph shows the total cost in relation to the number of hockey pucks you buy.

a) Describe the relationship between the total cost and the number of pucks you buy.

b) How much do 9 pucks cost?

c) You decide to buy a hockey stick for $30 and some pucks. What is the relationship now?

9. Faheen says that Graph A shows $y = 8 - 2x$, and Graph B shows $y = 8 - x$. Is she correct? Explain how you know.

Supported Learning

ESL

- For #6, ensure that English language learners understand the meaning of the word *lifeguard*.

Common Errors

- Students may confuse x-coordinates and y-coordinates.

R_x Reinforce that (x, y) is an ordered pair where x is the horizontal coordinate and y is the vertical coordinate. Note how, for example, the ordered pair (3, 1) is different from (1, 3).

Practise

Assessment *for* Learning	Supported Learning
Practise and Apply Have students do #3 and #5. Students who have no problems with these questions can go on to #9 and then to the rest of the Apply questions.	• Students who have problems with #3 will need additional coaching with Example 1. Have these students explain how they draw graphs using a table of values and describe a pattern on a graph. Clarify any misunderstandings. Coach students through #4. • Students who have problems with #5 will need additional coaching with Example 2. Have these students explain how they make a table of vales from a graph, describe a pattern, and solve a problem using a graph. Clarify any misunderstandings. Coach students through #6. • Once students understand the work in #3 to #5, have them do #9. If they are having difficulty, coach them through the question and then assign #11, which is similar. • Check back with them several times to make sure that they understand the concepts in all of these questions.

Supported Learning

Learning Style

- You may wish to provide stir sticks to students who would benefit from using a model for #15 on page 381.

Learning Style and Memory

- You may wish to provide **BLM 10–11 Section 10.4 Extra Practice** to students who require more practice.

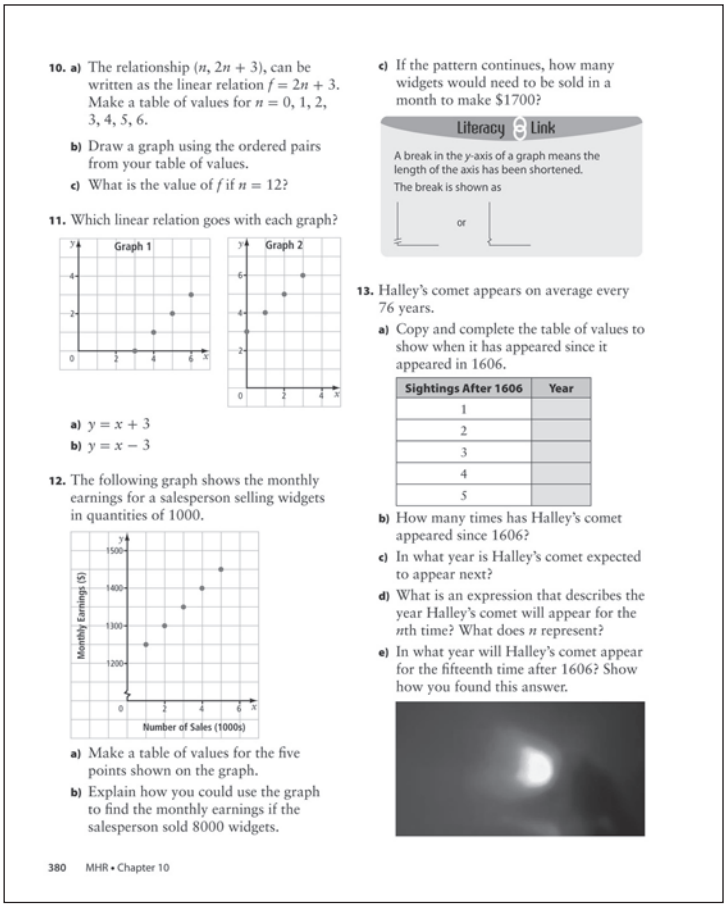
Common Errors

- Students may confuse the two variables in a linear relation.

R$_x$ Encourage students to maintain a consistent convention for tables. For example, they could choose to present their tables in a vertical format where the "generating" or *x*-variable is in the left column and the "generated" or *y*-variable is in the right column.

- Students may have difficulty knowing how to express a relationship.

R$_x$ Allow students to use an ordered pair or an equation to express the relationship (note, however, that the term *equation* is not introduced until Chapter 11). For example, $(x, 2x + 1)$ and $y = 2x + 1$ both represent the same relationship, and either format is acceptable.



10. a) The relationship $(n, 2n + 3)$, can be written as the linear relation $f = 2n + 3$. Make a table of values for $n = 0, 1, 2, 3, 4, 5, 6$.

b) Draw a graph using the ordered pairs from your table of values.

c) What is the value of f if $n = 12$?

11. Which linear relation goes with each graph?

Graph 1

Graph 2

a) $y = x + 3$

b) $y = x - 3$

12. The following graph shows the monthly earnings for a salesperson selling widgets in quantities of 1000.

Monthly Earnings ($)

Number of Sales (1000s)

a) Make a table of values for the five points shown on the graph.

b) Explain how you could use the graph to find the monthly earnings if the salesperson sold 8000 widgets.

c) If the pattern continues, how many widgets would need to be sold in a month to make $1700?

Literacy Link

A break in the *y*-axis of a graph means the length of the axis has been shortened. The break is shown as

or

13. Halley's comet appears on average every 76 years.

a) Copy and complete the table of values to show when it has appeared since it appeared in 1606.

Sightings After 1606	Year
1	
2	
3	
4	
5	

b) How many times has Halley's comet appeared since 1606?

c) In what year is Halley's comet expected to appear next?

d) What is an expression that describes the year Halley's comet will appear for the *n*th time? What does *n* represent?

e) In what year will Halley's comet appear for the fifteenth time after 1606? Show how you found this answer.

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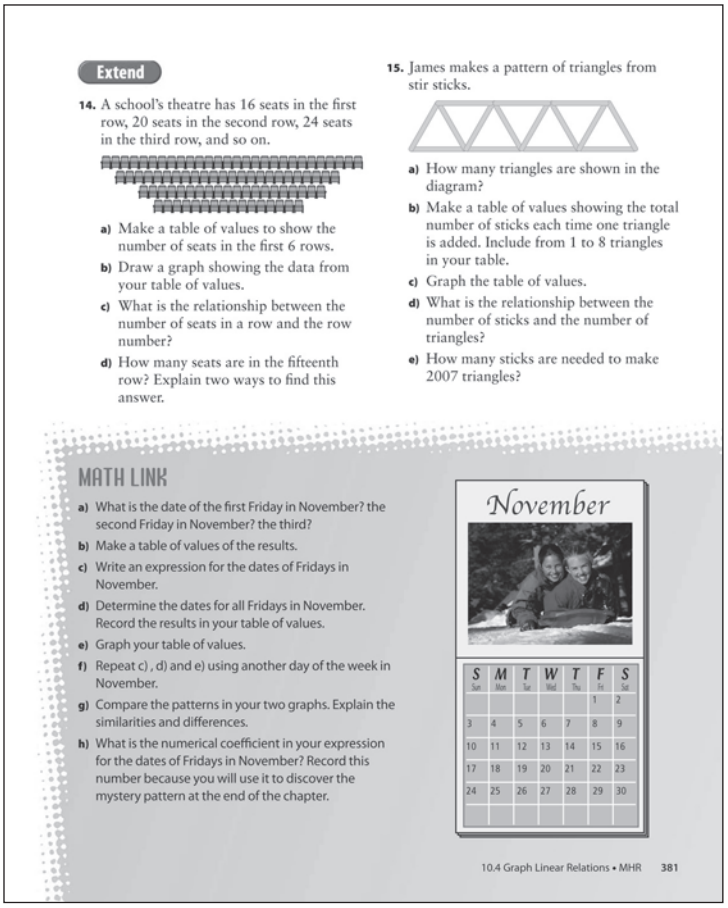
Apply and Extend

Students who are weaker readers may demonstrate their understanding of the concepts and skills from section 10.4 without an excessive amount of reading by doing #7, #9, #10, and #11. However, most students should experience questions built around a context, such as #8, #12, and #13.

The Extend questions on page 381 are contextually based and do not provide the same degree of leading as the Apply questions do.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • Why is it helpful to understand more than one way to describe the points on a graph? Explain with examples.	• Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Work with students to develop a plan for dealing with the areas in which they are having difficulty. • Depending on students' learning style, have them provide oral or written answers. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • Have students review the part related to section 10.4 in **BLM 10–1 Chapter 10 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.



Extend

14. A school's theatre has 16 seats in the first row, 20 seats in the second row, 24 seats in the third row, and so on.

a) Make a table of values to show the number of seats in the first 6 rows.
b) Draw a graph showing the data from your table of values.
c) What is the relationship between the number of seats in a row and the row number?
d) How many seats are in the fifteenth row? Explain two ways to find this answer.

15. James makes a pattern of triangles from stir sticks.

a) How many triangles are shown in the diagram?
b) Make a table of values showing the total number of sticks each time one triangle is added. Include from 1 to 8 triangles in your table.
c) Graph the table of values.
d) What is the relationship between the number of sticks and the number of triangles?
e) How many sticks are needed to make 2007 triangles?

MATH LINK

a) What is the date of the first Friday in November? the second Friday in November? the third?
b) Make a table of values of the results.
c) Write an expression for the dates of Fridays in November.
d) Determine the dates for all Fridays in November. Record the results in your table of values.
e) Graph your table of values.
f) Repeat c), d) and e) using another day of the week in November.
g) Compare the patterns in your two graphs. Explain the similarities and differences.
h) What is the numerical coefficient in your expression for the dates of Fridays in November? Record this number because you will use it to discover the mystery pattern at the end of the chapter.

November

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

10.4 Graph Linear Relations • MHR 381

MATH LINK

The Math Link continues the study of patterns in a calendar. Suggest to students who are not sure how to determine the expression to try the Guess and Check strategy. Remind students to record in their notebook the numerical coefficient in part h), or have them write it on **BLM 10–14 Wrap It Up! Mystery Pattern**.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 381 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 385. It gives students an opportunity to explain calendar patterns using a table of values, an expression, and a graph.	• Students who are having difficulty getting started could use **BLM 10–12 Section 10.4 Math Link**, which provides scaffolding for this activity. • Observe students, making sure their tables of value, expressions, and graphs are correct. Note the similarities and differences they identify when comparing the two graphs.

Answers

Math Link

a) 1, 8, 15

b)

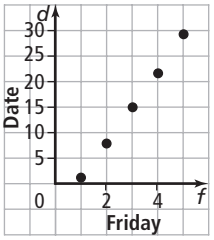
Friday, *f*	Date, *d*
1	1
2	8
4	15

c) $7f - 6$

d)

Friday, *f*	Date, *d*
1	1
2	8
3	15
4	22
5	29

e)



f) Answers may vary. For example:

Wednesday, *f*	Date, *d*
1	6
2	13
3	20
4	27

$7w - 1$

g) Similarities: Both graphs are linear. The points on both graphs increase by 1 unit right, then 7 units up. Differences: The graphs start at different points.

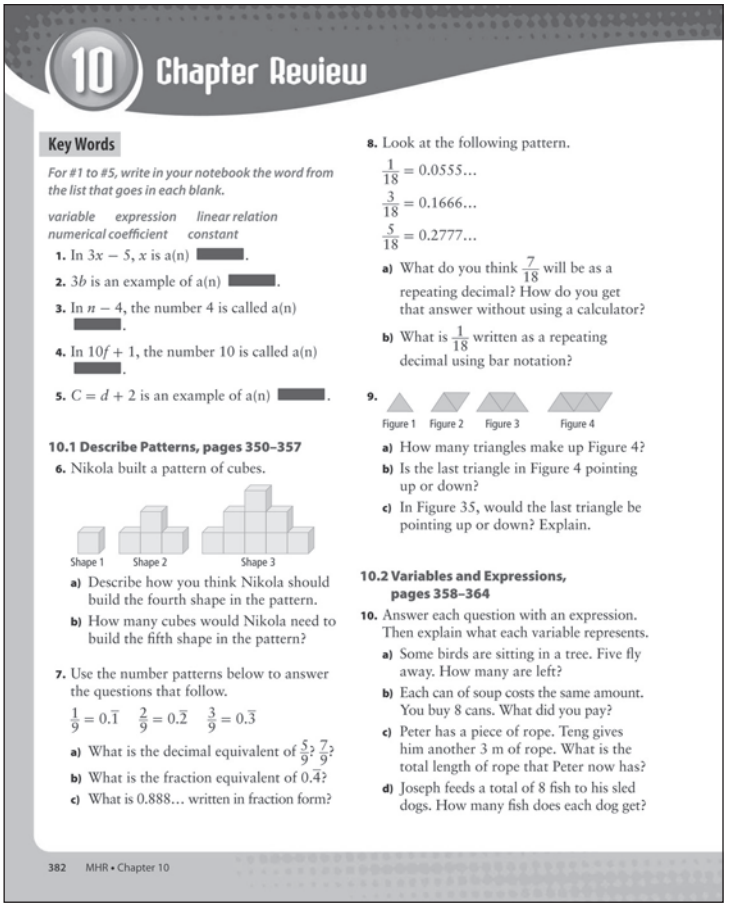
h) 7

10 Chapter Review

Suggested Timing
40–50 minutes

Materials
- unit cubes
- cups or plastic containers
- counters

Blackline Masters
BLM 10–1 Chapter 10 Self-Assessment
BLM 10–4 Section 10.1 Extra Practice
BLM 10–7 Section 10.2 Extra Practice
BLM 10–9 Section 10.3 Extra Practice
BLM 10–11 Section 10.4 Extra Practice



10 Chapter Review

Key Words

For #1 to #5, write in your notebook the word from the list that goes in each blank.

variable expression linear relation numerical coefficient constant

1. In $3x - 5$, x is a(n) ▬.
2. $3b$ is an example of a(n) ▬.
3. In $n - 4$, the number 4 is called a(n) ▬.
4. In $10f + 1$, the number 10 is called a(n) ▬.
5. $C = d + 2$ is an example of a(n) ▬.

10.1 Describe Patterns, pages 350–357

6. Nikola built a pattern of cubes.

Shape 1 Shape 2 Shape 3

a) Describe how you think Nikola should build the fourth shape in the pattern.
b) How many cubes would Nikola need to build the fifth shape in the pattern?

7. Use the number patterns below to answer the questions that follow.

$\frac{1}{9} = 0.\overline{1}$ $\frac{2}{9} = 0.\overline{2}$ $\frac{3}{9} = 0.\overline{3}$

a) What is the decimal equivalent of $\frac{5}{9}$? $\frac{7}{9}$?
b) What is the fraction equivalent of $0.\overline{4}$?
c) What is 0.888... written in fraction form?

8. Look at the following pattern.

$\frac{1}{18} = 0.0555...$
$\frac{3}{18} = 0.1666...$
$\frac{5}{18} = 0.2777...$

a) What do you think $\frac{7}{18}$ will be as a repeating decimal? How do you get that answer without using a calculator?
b) What is $\frac{1}{18}$ written as a repeating decimal using bar notation?

9.

Figure 1 Figure 2 Figure 3 Figure 4

a) How many triangles make up Figure 4?
b) Is the last triangle in Figure 4 pointing up or down?
c) In Figure 35, would the last triangle be pointing up or down? Explain.

10.2 Variables and Expressions, pages 358–364

10. Answer each question with an expression. Then explain what each variable represents.

a) Some birds are sitting in a tree. Five fly away. How many are left?
b) Each can of soup costs the same amount. You buy 8 cans. What did you pay?
c) Peter has a piece of rope. Teng gives him another 3 m of rope. What is the total length of rope that Peter now has?
d) Joseph feeds a total of 8 fish to his sled dogs. How many fish does each dog get?

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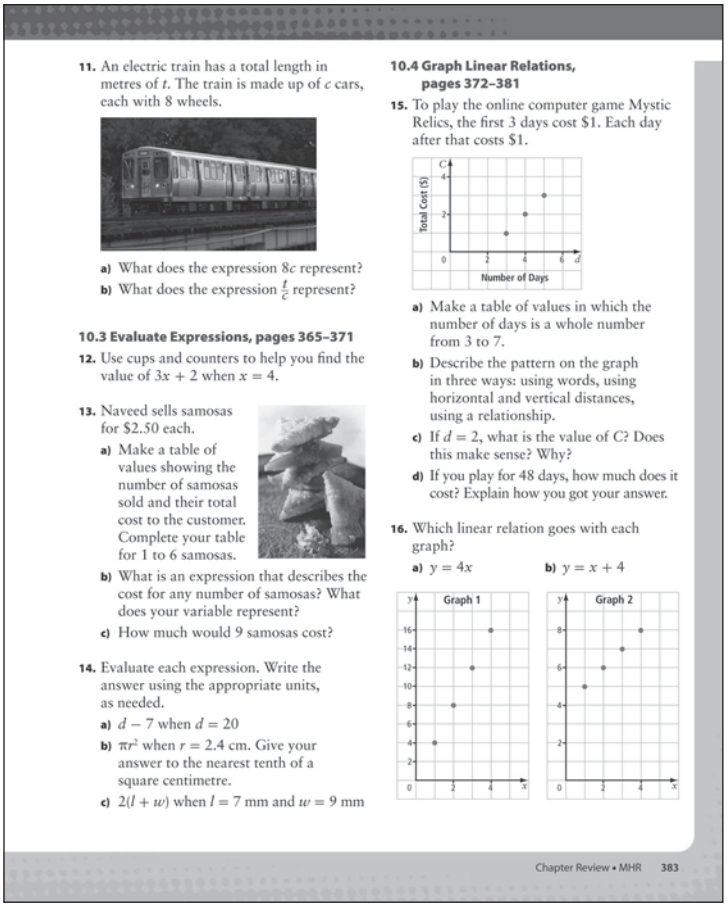
Activity Planning Notes

Allow students to work independently or in pairs on the Chapter 10 Review. Provide an opportunity for students to discuss any questions, consider alternative methods of solving, and ask about questions they found difficult.

Tell students to write the numbers 6 to 26 in two columns in their notebooks. Have them look at the question related to the number in their student resource. Students can then circle the questions using the colours that they used on **BLM 10–1 Chapter 10 Self-Assessment** to identify which ones they need help with or do not yet understand.

Assessment *for* Learning	Supported Learning
Chapter 10 Review The Chapter 10 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Tell students to check the contents of the What I Need to Work On tab of their chapter Foldable. Have students do at least one question related to any concept, skill, or process that has been giving them trouble. • Once they have completed the Chapter 10 Review, encourage students to list in their chapter Foldable any questions they found difficult. • Have students revisit any section they are having difficulty with prior to working on the Chapter 10 Practice Test.



11. An electric train has a total length in metres of t. The train is made up of c cars, each with 8 wheels.

a) What does the expression $8c$ represent?

b) What does the expression $\frac{t}{c}$ represent?

10.3 Evaluate Expressions, pages 365–371

12. Use cups and counters to help you find the value of $3x + 2$ when $x = 4$.

13. Naveed sells samosas for $2.50 each.

a) Make a table of values showing the number of samosas sold and their total cost to the customer. Complete your table for 1 to 6 samosas.

b) What is an expression that describes the cost for any number of samosas? What does your variable represent?

c) How much would 9 samosas cost?

14. Evaluate each expression. Write the answer using the appropriate units, as needed.

a) $d - 7$ when $d = 20$

b) πr^2 when $r = 2.4$ cm. Give your answer to the nearest tenth of a square centimetre.

c) $2(l + w)$ when $l = 7$ mm and $w = 9$ mm

10.4 Graph Linear Relations, pages 372–381

15. To play the online computer game Mystic Relics, the first 3 days cost $1. Each day after that costs $1.

a) Make a table of values in which the number of days is a whole number from 3 to 7.

b) Describe the pattern on the graph in three ways: using words, using horizontal and vertical distances, using a relationship.

c) If $d = 2$, what is the value of C? Does this make sense? Why?

d) If you play for 48 days, how much does it cost? Explain how you got your answer.

16. Which linear relation goes with each graph?

a) $y = 4x$ b) $y = x + 4$

Supported Learning

Learning Style

- Allow students to complete the Chapter 10 Review using any combination of oral description, diagrams, and numerical answers.
- Kinesthetic and concrete learners might benefit from using unit cubes to build the pattern in #6.
- Provide students with cups or plastic containers and counters to use for #12.

Learning Style and Memory

- Students who require more practice on a particular topic may refer to **BLM 10–4 Section 10.1 Extra Practice**, **BLM 10–7 Section 10.2 Extra Practice**, **BLM 10–9 Section 10.3 Extra Practice**, and **BLM 10–11 Section 10.4 Extra Practice**.

ESL and Language

- Encourage students to practise the vocabulary terms using flash cards. Have students work in pairs to quiz each other.

Gifted and Enrichment

- Some students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge, go to **www.mathlinks7.ca** and follow the links.

Assessment *as* Learning	Supported Learning
Math Learning Log Once students have completed the Chapter 10 Review, have them reflect on their progress and write a journal entry that completes the following statements: – I am comfortable with following parts of the chapter … – The method I prefer for representing a pattern is … – I am having difficulty with … – Here's how I plan to address the areas I am having difficult with …	• Have students use the What I Need to Work On tab of their chapter Foldable to provide information about what they continue to have problems with and what problems they had that have now been resolved. • You may wish to have students refer to **BLM 10–1 Chapter 10 Self-Assessment** when they report on what they are comfortable with and what they are having difficulty with.

10 Practice Test

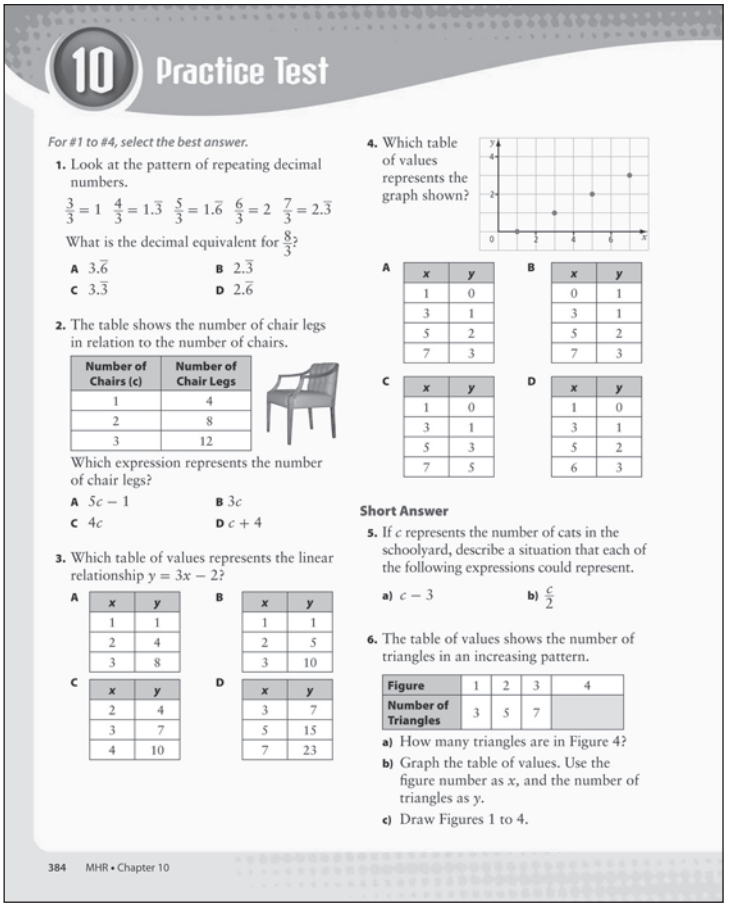
Suggested Timing
40–50 minutes

Materials
- grid paper
- ruler

Blackline Masters
BLM 10–1 Chapter 10 Self-Assessment

BLM 10–13 Chapter 10 Test

Assessment *as* Learning	Supported Learning
Chapter 10 Self-Assessment Have students review their earlier responses on **BLM 10–1 Chapter 10 Self-Assessment**.	• Have students use their responses on the Chapter 10 Practice Test and work they completed earlier in the chapter to complete the After column of this self-assessment. • Before students do the Chapter 10 Test, coach them in the areas in which they are having problems.



10 Practice Test

For #1 to #4, select the best answer.

1. Look at the pattern of repeating decimal numbers.

$\frac{3}{3} = 1 \quad \frac{4}{3} = 1.\overline{3} \quad \frac{5}{3} = 1.\overline{6} \quad \frac{6}{3} = 2 \quad \frac{7}{3} = 2.\overline{3}$

What is the decimal equivalent for $\frac{8}{3}$?

A $3.\overline{6}$ B $2.\overline{3}$
C $3.\overline{3}$ D $2.\overline{6}$

2. The table shows the number of chair legs in relation to the number of chairs.

Number of Chairs (c)	Number of Chair Legs
1	4
2	8
3	12

Which expression represents the number of chair legs?

A $5c - 1$ B $3c$
C $4c$ D $c + 4$

3. Which table of values represents the linear relationship $y = 3x - 2$?

A

x	y
1	1
2	4
3	8

B

x	y
1	1
2	5
3	10

C

x	y
2	4
3	7
4	10

D

x	y
3	7
5	15
7	23

4. Which table of values represents the graph shown?

A

x	y
1	0
3	1
5	2
7	3

B

x	y
0	1
3	1
5	2
7	3

C

x	y
1	0
3	1
5	3
7	5

D

x	y
1	0
3	1
5	2
6	3

Short Answer

5. If c represents the number of cats in the schoolyard, describe a situation that each of the following expressions could represent.

a) $c - 3$ b) $\frac{c}{2}$

6. The table of values shows the number of triangles in an increasing pattern.

Figure	1	2	3	4
Number of Triangles	3	5	7	

a) How many triangles are in Figure 4?

b) Graph the table of values. Use the figure number as x, and the number of triangles as y.

c) Draw Figures 1 to 4.

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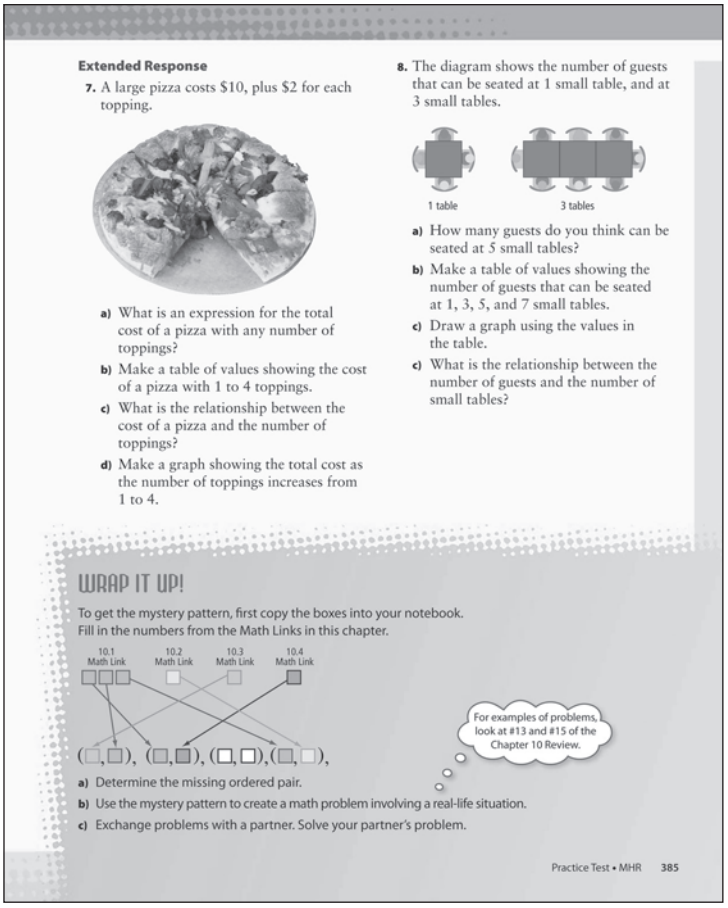
Study Guide

Question(s)	Section(s)	Refer to	I can …
1	10.1	Example 2	✓ determine patterns with repeating decimals
2, 7, 8	10.3	Explore the Math Examples 1, 2	✓ evaluate an expression
2, 3, 7, 8	10.2	Example 1	✓ write an expression to represent a pattern
4	10.4	Example 1	✓ match linear relations to graphs ✓ make a graph for a table of values
5	10.2	Example 2	✓ write a word statement as an expression
6, 7	10.4	Example 1	✓ graph a linear relation
7, 8	10.4	Examples 1, 2	✓ make a table of values for an expression ✓ make a graph for a table of values ✓ describe the relationship shown on a graph



Extended Response

7. A large pizza costs $10, plus $2 for each topping.

a) What is an expression for the total cost of a pizza with any number of toppings?

b) Make a table of values showing the cost of a pizza with 1 to 4 toppings.

c) What is the relationship between the cost of a pizza and the number of toppings?

d) Make a graph showing the total cost as the number of toppings increases from 1 to 4.

8. The diagram shows the number of guests that can be seated at 1 small table, and at 3 small tables.

1 table 3 tables

a) How many guests do you think can be seated at 5 small tables?

b) Make a table of values showing the number of guests that can be seated at 1, 3, 5, and 7 small tables.

c) Draw a graph using the values in the table.

d) What is the relationship between the number of guests and the number of small tables?

WRAP IT UP!

To get the mystery pattern, first copy the boxes into your notebook. Fill in the numbers from the Math Links in this chapter.

10.1 Math Link 10.2 Math Link 10.3 Math Link 10.4 Math Link

(□,□), (□,■), (□,□),(□,),

For examples of problems, look at #13 and #15 of the Chapter 10 Review.

a) Determine the missing ordered pair.

b) Use the mystery pattern to create a math problem involving a real-life situation.

c) Exchange problems with a partner. Solve your partner's problem.

Practice Test • MHR 385

Supported Learning

Learning Style

- Allow students to complete the Chapter 10 Practice Test using any combination of verbal description, diagrams, and numerical answers.

Activity Planning Notes

Have students start the practice test the same way they started the chapter review, by writing the question numbers in their notebooks and circling each number according to the colours they used for **BLM 10–1 Chapter 10 Self-Assessment**.

This practice test can be assigned as an in-class or take-home assignment. These are the minimum questions that will meet the related curriculum outcomes: #2–#5, #7, and #8.

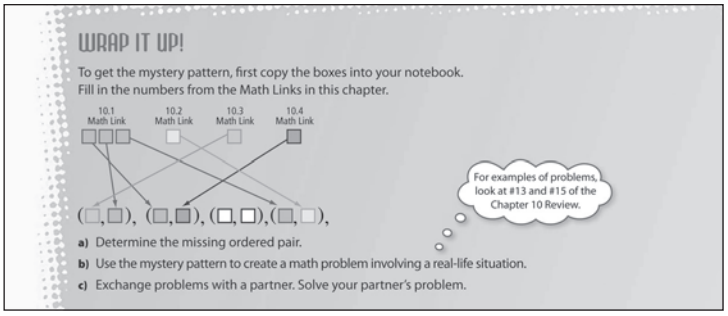
Answers to the Chapter 10 Practice Test are provided on **BLM 10-18 Chapter 10 *MathLinks 7* Student Resource Answers**.

Assessment *of* Learning	Supported Learning
Chapter 10 Test After students complete the practice test, you may wish to use **BLM 10–13 Chapter 10 Test** as a summative assessment.	• Consider allowing students to use their chapter Foldable as they complete the test. • You may wish to use the Math Games on page 386 or the Challenge in Real Life on page 387 to assess the knowledge and skills of students who have difficulty with tests.

Wrap It Up!

Suggested Timing
40–50 minutes

Blackline Masters
Master 1 Project Rubric
BLM 10–5 Section 10.1 Math Link
BLM 10–8 Section 10.2 Math Link
BLM 10–10 Section 10.3 Math Link
BLM 10–12 Section 10.4 Math Link
BLM 10–14 Wrap It Up! Mystery Pattern
BLM 10–15 Chapter 10 Wrap It Up!



Specific Outcomes

PR1 Demonstrate an understanding of oral or written patterns and their equivalent linear relations.
PR2 Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.
PR5 Evaluate an expression given the value of the variable(s).

Answers

Wrap It Up!

a) (2, 6), (3, 7), (4, 8), (5, 9)

b), c) Answers will vary.

Common Errors

- Students may not have collected all the correct numbers to write in the mystery pattern boxes.

R_x Encourage students to complete each Math Link as they reach the end of each section of Chapter 10. That way, you can check to see which students are having difficulty and help them at that time. This should make it possible for all students to have the correct numbers to use in the Wrap It Up! Alternatively, have students compare their numbers with a partner's at the end of the chapter.

Activity Planning Notes

This chapter problem can be modified to meet the needs of the class.

Method 1: It could include only answers to part a) and part b).

Method 2: It could involve a written explanation for two of the Math Links in Chapter 10.

Assessment *of* Learning	Supported Learning
Wrap It Up! The Wrap It Up! allows students to apply their knowledge of patterns and expressions to creating a math problem of their own. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work. Page 385a provides notes on how to use the rubric for this Wrap It Up!	• The numbers students obtained for the Math Links in sections 10.1, 10.2, 10.3, and 10.4 will provide the missing values for the mystery pattern boxes. • If students have been using **BLM 10–14 Wrap It Up! Mystery Pattern** throughout the Math Links in this chapter, they can refer to these now to determine the missing ordered pair and solve the mystery pattern. If not, you may wish to give this master to students to fill in. • It is recommended that students do all Math Links leading to this Wrap It Up! If they have not, you may wish to have them do these now. **BLM 10–5 Section 10.1 Math Link**, **BLM 10–8 Section 10.2 Math Link**, **BLM 10–10 Section 10.3 Math Link**, and **BLM 10–12 Section 10.4 Math Link** will assist students with the earlier Math Links. • Some students might benefit from using **BLM 10–15 Chapter 10 Wrap It Up!**, which provides scaffolding for the chapter problem wrap-up. • Observe how students use what they have learned in the chapter and how they follow the examples to create a math problem.

The chart below shows the Master 1 Project Rubric for tasks such as the Wrap It Up!, and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete solution with poor communication *or* • provides a complete solution with a minor error in the real-world application *or* • provides three correct ordered pairs and a correct real-world application
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides four correct ordered pairs, but no real-world application *or* • provides three correct ordered pairs and an attempt at the real-world application
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides two correct ordered pairs, and there may be an attempt at the real-world application
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides one correct ordered pair

Math Games

Suggested Timing
60–75 minutes

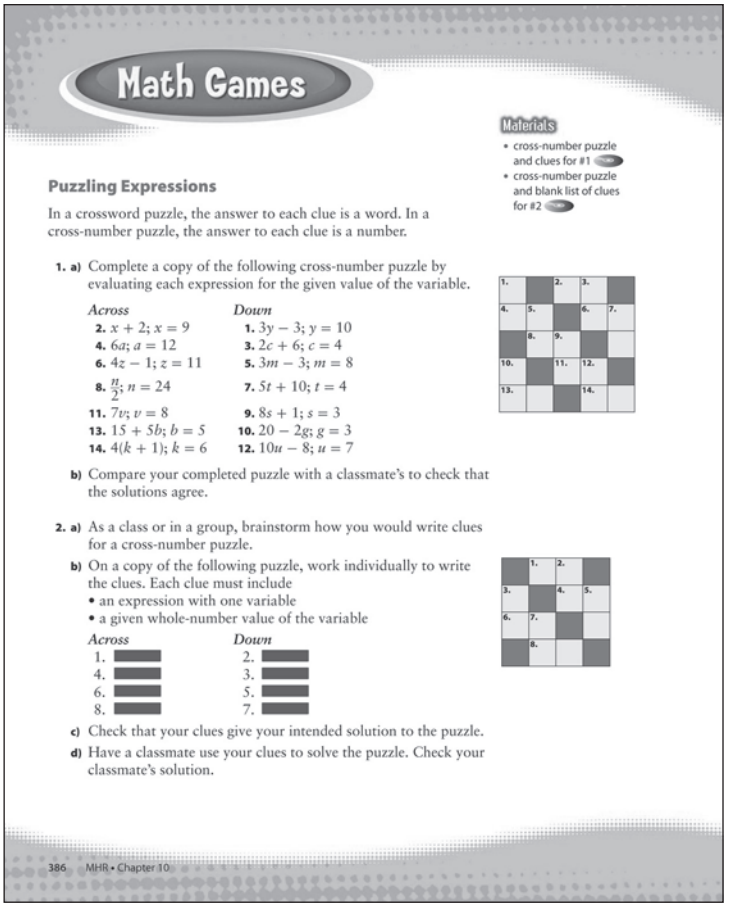
Blackline Masters
Master 8 Centimetre Grid Paper
BLM 10–16 Puzzling Expressions

Supported Learning

Gifted and Enrichment

- Have students make up their own puzzle blank on **Master 8 Centimetre Grid Paper**. You may need to provide some guidelines, such as the minimum and maximum number of clues to use.

Assessment *for* Learning	Supported Learning
Puzzling Expressions Ask students to describe the steps in evaluating an expression.	• Review the rules for the order of operations, since students need to know them as they evaluate expressions. • When students are substituting values for variables, encourage them to write each step in the solution. This practice will help them to analyse their errors.



Math Games

Materials
- cross-number puzzle and clues for #1
- cross-number puzzle and blank list of clues for #2

Puzzling Expressions

In a crossword puzzle, the answer to each clue is a word. In a cross-number puzzle, the answer to each clue is a number.

1. a) Complete a copy of the following cross-number puzzle by evaluating each expression for the given value of the variable.

Across
2. $x + 2; x = 9$
4. $6a; a = 12$
6. $4z - 1; z = 11$
8. $\frac{n}{2}; n = 24$
11. $7v; v = 8$
13. $15 + 5b; b = 5$
14. $4(k + 1); k = 6$

Down
1. $3y - 3; y = 10$
3. $2c + 6; c = 4$
5. $3m - 3; m = 8$
7. $5t + 10; t = 4$
9. $8s + 1; s = 3$
10. $20 - 2g; g = 3$
12. $10u - 8; u = 7$

b) Compare your completed puzzle with a classmate's to check that the solutions agree.

2. a) As a class or in a group, brainstorm how you would write clues for a cross-number puzzle.

b) On a copy of the following puzzle, work individually to write the clues. Each clue must include
- an expression with one variable
- a given whole-number value of the variable

Across
1. ■
4. ■
6. ■
8. ■

Down
2. ■
3. ■
5. ■
7. ■

c) Check that your clues give your intended solution to the puzzle.

d) Have a classmate use your clues to solve the puzzle. Check your classmate's solution.

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Answers

Math Games

1. a)

1. 2	■	2. 1	3. 1	■
4. 7	5. 2	■	6. 4	7. 3
■	8. 1	9. 2	■	0
10. 1	■	11. 5	12. 6	■
13. 4	0	■	14. 2	8

Specific Outcomes

PR5 Evaluate an expression given the value of the variable(s).

Activity Planning Notes

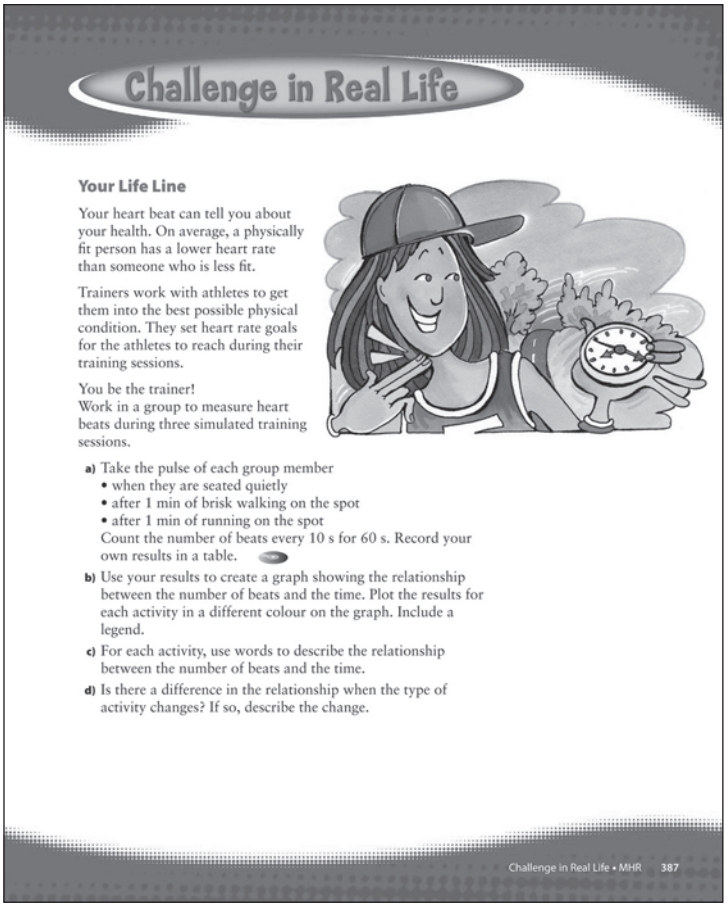
A cross-number puzzle is much like a crossword puzzle, except that each blank cell is filled with a digit from zero to nine. No answer to a clue may start with a leading zero. Have students use the first page of **BLM 10–16 Puzzling Expressions** to answer #1. Encourage them to show their work.

During the brainstorm for #2a), show students how to work backward to write clues. For example, they might decide to put 24 for #1 across:

Starting with $24 = 20 + 4$ could lead to the clue $2x + 4$, $x = 10$.
Starting with $24 = 30 - 6$ could lead to the clue $3z - 6$, $z = 10$.
Starting with $24 = 3 \times 8$ could lead to the clue $3d$, $d = 8$, or $3(t + 3)$, $t = 5$.

Give to students the second page of **BLM 10–16 Puzzling Expressions** to answer #2. Have students develop their own clue for #1 Across that results in the answer 24. Direct them to consider how this choice would affect the answer for #2 Down. They should understand that the answer will have to start with 4, so the clue for #2 Down will need to have an answer from 40 to 49.

Challenge in Real Life



Challenge in Real Life

Your Life Line

Your heart beat can tell you about your health. On average, a physically fit person has a lower heart rate than someone who is less fit.

Trainers work with athletes to get them into the best possible physical condition. They set heart rate goals for the athletes to reach during their training sessions.

You be the trainer!
Work in a group to measure heart beats during three simulated training sessions.

a) Take the pulse of each group member
- when they are seated quietly
- after 1 min of brisk walking on the spot
- after 1 min of running on the spot

Count the number of beats every 10 s for 60 s. Record your own results in a table.

b) Use your results to create a graph showing the relationship between the number of beats and the time. Plot the results for each activity in a different colour on the graph. Include a legend.

c) For each activity, use words to describe the relationship between the number of beats and the time.

d) Is there a difference in the relationship when the type of activity changes? If so, describe the change.

Challenge in Real Life • MHR 387

Suggested Timing
60–75 minutes

Materials
- clock or watch with second hand, or stopwatch
- ruler
- coloured pencils or markers

Blackline Masters
Master 1 Project Rubric
Master 8 Centimetre Grid Paper
Master 9 0.5 Centimetre Grid Paper
BLM 10–17 Your Life Line Results

Mathematical Processes

- [x] Communication
- [x] Connections
- [ ] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [x] Technology
- [x] Visualization

Specific Outcomes

PR2 Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.

Activity Planning Notes

You may wish to use the following steps to introduce and complete this challenge:

1. Read through Your Life Line as a class. Ask students what they predict will happen to their heart rate when they are sitting, walking, and running.
2. Students have likely learned how to take their pulse, either in PE or Science class. Invite a student to explain how it is done. Direct students' attention to the illustration in the student resource, which shows someone taking a neck pulse. Point out that it is best to use their first two fingers.

Supported Learning

ESL and Language

- You may need to assist students with challenging vocabulary, e.g., *pulse, brisk*.

Motor

- Students with mobility problems might be asked to record or watch the clock only.

Gifted and Enrichment

- Have students create a linear expression based on the results for their sitting pulse rate.
- Ask students to research why heart rates differ depending on a person's physical fitness.
- Encourage students to research and report on why heart rates change during exercise, along with what happens during recovery after exercise. They could also research what other factors influence heart rate (e.g., age).

3. Divide students into groups of three. Give students these instructions:
   - Student 1 does the activity and counts the heart beats aloud. Instruct students to start counting immediately after they stop walking or running. Clarify that they will have to restart counting after each 10-s interval.
   - Student 2 watches the clock and calls out the time every 10 s.
   - Student 3 records the data.

   Tell students to rotate roles so that each student takes on a different role for each activity.
4. Provide students with **Master 9 0.5 Centimetre Grid Paper**. Have them discuss as a group how to graph time vs. number of beats.
5. Clarify that the task is to
   - take the pulse of each group member when seated, after brisk walking, and after running
   - organize their own results in a table
   - graph their own results, showing the relationship between the number of beats and the time
   - for each set of results, describe the relationship between the number of beats and the time
   - determine if there is a difference in the relationship when the type of activity changes and describe any difference
6. Review **Master 1 Project Rubric** with students so that they will know what is expected.

This challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment *for* Learning	Supported Learning
Your Life Line Discuss the challenge with the class. Have students work in groups of three to complete part a) and work individually to answer parts b) to d).	• You may wish to provide to students **BLM 10–17 Your Life Line Results**, which is a table for organizing their results. • Give students with weaker motor skills the option of drawing their graph on **Master 8 Centimetre Grid Paper**. • For parts c) and d), students with language difficulties might tell or tape-record their descriptions. • For enrichment, ask students to describe their graph if they went through an intense physical training program. • For a second challenge, complete with teaching notes and student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

Assessment *of* Learning	Supported Learning
Your Life Line Discuss the challenge with the class. Have students work in groups of three to complete part a) and work individually to answer parts b) to d).	• Use **Master 1 Project Rubric** to assist you in assessing student work. Page 387a provides notes on how to use this rubric for this challenge. • To view exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows **Master 1 Project Rubric** for tasks such as the Challenge in Real Life and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution with more than one reasonable suggestion given for part d)
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides complete responses to parts a), b), and c), and an incorrect response to part d) *or* • provides complete responses to parts a), b), and d), with an incorrect or omitted response to part c)
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides correct responses to parts a) and b) *or* • provides a correct response to part b) and legend may or may not be present
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a correct response to part a) with a table that is clearly labelled and data clearly recorded
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides a correct start to part a) with data recorded but not organized in a table *or* • provides a correct response to part b) based on an incorrect part a)