11 Solving Equations

General Outcome

- Represent algebraic expressions in multiple ways.

Specific Outcomes

PR3 Demonstrate an understanding of preservation of equality by:
- modelling preservation of equality, concretely, pictorially and symbolically
- applying preservation of equality to solve equations.

PR4 Explain the difference between an expression and an equation.

PR5 Evaluate an expression given the value of the variable(s).

PR6 Model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially and symbolically, where a and b are integers.

PR7 Model and solve problems that can be represented by linear equations of the form:

- $ax + b = c$
- $ax = b$
- $\frac{x}{a} = b, a \neq 0$

concretely, pictorially and symbolically, where a, b and c are whole numbers.

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
11.1	✓ identify constants, numerical coefficients, and variables in expressions and equations
	✓ describe the difference between an expression and an equation
11.2	✓ model problems with equations
	✓ solve equations and record the process
	✓ verify solutions to equations
11.3	✓ model problems with equations
	✓ solve equations and record the process
11.4	✓ model problems with two-step equations
	✓ solve two-step equations and record the process

Assessment *as* Learning	Supported Learning
Use the Before column of **BLM 11–1 Chapter 11 Self-Assessment** to provide students with the big picture for this chapter and to help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• As students complete each section of the chapter or complete the Chapter 11 Review, have them review the related parts on **BLM 11–1 Chapter 11 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Chapter 11 Planning Chart

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter Opener • 20–30 minutes		BLM 11–1 Chapter 11 Self-Assessment BLM 11–2 Solving Equations	• paper • scissors • stapler
11.1 Expressions and Equations • 80–100 minutes	**Essential:** 1–5, 7, 9, Math Link **Typical:** 1–5, 9–12, Math Link **Extension/Enrichment:** 1, 2, 12–15, Math Link	Master 2 Two Stars and One Wish BLM 11–1 Chapter 11 Self-Assessment BLM 11–3 Section 11.1 Extra Practice BLM 11–4 Section 11.1 Math Link	• 2 cups or containers per student pair • at least 15 counters per student pair • transparent counters (optional) • 2-pan balance (optional) • algebra tiles
11.2 Solve One-Step Equations: $x + a = b$ • 80–100 minutes	**Essential:** 1–4, 6, 8, 9, 12, 13, Math Link **Typical:** 2–4, 6, 8, 9, 12–18, Math Link **Extension/Enrichment:** 2, 3, 18–22, Math Link	BLM 11–1 Chapter 11 Self-Assessment BLM 11–5 Section 11.2 Extra Practice BLM 11–6 Section 11.2 Math Link	• 30 loonies (or play money, 1-g masses, or tokens) • cup (foam or lightweight plastic) • 2-pan balance
11.3 Solve One-Step Equations: $ax = b$, $\frac{x}{a} = b$ • 80–100 minutes	**Essential:** 1–4, 6, 8, 10, 11, 13–15, Math Link **Typical:** 1–4, 6, 8, 10, 11, 13–18, 20, Math Link **Extension/Enrichment:** 1–3, 16–21, Math Link	BLM 11–1 Chapter 11 Self-Assessment BLM 11–7 Section 11.3 Extra Practice BLM 11–8 Section 11.3 Math Link	• cups and counters • algebra tiles
11.4 Solve Two-Step Equations: $ax + b = c$ • 80–100 minutes	**Essential:** 1, 2 *or* 3, 4, 6, 7, 9–11, 14, 15, Math Link **Typical:** 1, 2 *or* 3, 4, 6, 7, 9–11, 14–19, Math Link **Extension/Enrichment:** 1, 2 *or* 3, 16–21, Math Link	BLM 11–1 Chapter 11 Self-Assessment BLM 11–9 Section 11.4 Extra Practice BLM 11–10 Section 11.4 Math Link	• cups and counters • algebra tiles • 2-pan balance with masses, including the sizes needed in Example 1 • blocks
Chapter 11 Review • 40–50 minutes	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	BLM 11–1 Chapter 11 Self-Assessment BLM 11–3 Section 11.1 Extra Practice BLM 11–5 Section 11.2 Extra Practice BLM 11–7 Section 11.3 Extra Practice BLM 11–9 Section 11.4 Extra Practice	• cups and counters
Chapter 11 Practice Test • 40–50 minutes	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 2, 6, 7, 9, 10, 12, 13	BLM 11–1 Chapter 11 Self-Assessment BLM 11–11 Chapter 11 Test	• calculator
Chapter 11 Wrap It Up! • 40–50 minutes		Master 1 Project Rubric BLM 11–4 Section 11.1 Math Link BLM 11–6 Section 11.2 Math Link BLM 11–8 Section 11.3 Math Link BLM 11–10 Section 11.4 Math Link BLM 11–12 Chapter 11 Wrap It Up!	
Chapter 11 Math Games • 40–50 minutes			• cups and counters • algebra tiles
Chapter 11 Challenge in Real Life • 60–75 minutes		Master 1 Project Rubric BLM 11–13 Chapter 11 *MathLinks* 7 Student Resource Answers BLM 11–14 Chapter 11 BLM Answers	• shoe box or similar box, wrapping paper, ribbon • flyers and catalogues, phone numbers of local stores, or phone books (optional) • calculator

Chapter 11 Assessment Planner

Assessment Options	Type of Assessment	Assessment Tool
Chapter Opener	Assessment *as* Learning (TR pages i, 389)	BLM 11–1 Chapter 11 Self-Assessment Chapter 11 Foldable
11.1 Expressions and Equations	Assessment *as* Learning (TR pages 392, 393, 394) Assessment *for* Learning (TR pages 392, 393, 394)	Master 2 Two Stars and One Wish Math Learning Log (TR page 394) BLM 11–1 Chapter 11 Self-Assessment
11.2 Solve One-Step Equations: $x + a = b$	Assessment *as* Learning (TR pages 397, 399, 400) Assessment *for* Learning (TR pages 397, 398, 399, 401)	Math Learning Log (TR page 400) BLM 11–1 Chapter 11 Self-Assessment
11.3 Solve One-Step Equations: $ax = b$, $\frac{x}{a} = b$	Assessment *as* Learning (TR pages 403, 406, 407) Assessment *for* Learning (TR pages 404, 405, 407)	Math Learning Log (TR page 407) BLM 11–1 Chapter 11 Self-Assessment
11.4 Solve Two-Step Equations: $ax + b = c$	Assessment *as* Learning (TR pages 410, 412, 413) Assessment *for* Learning (TR pages 410, 411, 412, 413)	Math Learning Log (TR page 413) BLM 11–1 Chapter 11 Self-Assessment
Chapter 11 Review	Assessment *for* Learning (TR page 414) Assessment *as* Learning (TR page 415)	Math Learning Log (TR page 415) BLM 11–1 Chapter 11 Self-Assessment
Chapter 11 Practice Test	Assessment *as* Learning (TR page 416) Assessment *of* Learning (TR page 417)	BLM 11–1 Chapter 11 Self-Assessment BLM 11–11 Chapter 11 Test
Chapter 11 Wrap It Up!	Assessment *of* Learning (TR pages 416a)	Master 1 Project Rubric
Chapter 11 Math Games	Assessment *for* Learning (TR page 418)	
Chapter 11 Challenge in Real Life	Assessment *for* Learning (TR page 418a) Assessment *of* Learning (TR page 418a)	Master 1 Project Rubric

You may wish to use one or more of the following materials to help you assess student readiness for Chapter 11.

Assessment *for* Learning	Supported Learning
Method 1: Have students develop a journal to explain what they personally know about equations and number patterns, and how they use them in their lives. **Method 2:** Have students complete **BLM 11–2 Solving Equations** to check their conceptual understanding. Remind students that you are looking for the scope of their knowledge.	• Students who require reinforcement of prerequisite skills may wish to complete the Get Ready materials available in the *MathLinks 7 Workbook* and at the **www.mathlinks7.ca** book site.

Chapter Opener

Suggested Timing
20–30 minutes

Materials
- paper
- scissors
- stapler

Blackline Masters
BLM 11–1 Chapter 11 Self-Assessment

Key Words
equation
opposite operation

What's the Math?

In this chapter, students work with algebraic expressions and equations. They start by identifying the differences between algebraic expressions and equations and learn related terms. Students develop skills in solving different types of linear algebraic equations and use these skills to solve problems.

Activity Planning Notes

As a class, read the information about coded messages. Have students work on their own or with a partner to identify different ways that codes can be used. As a class activity, you might post a message using a simple code and have students decode it. You could code the alphabet by substituting a number for each letter, in order (e.g., $a = 1$). Have students identify ways to break the code using an algebraic expression. Ask students how codes are similar to solving the unknown in an equation.

Explain that Chapter 11 is about solving problems using equations. Encourage students to share where they have seen or used equations in their lives. Try to elicit ideas from all class members.

Assessment *as* Learning	Supported Learning
Chapter 11 Foldable As students work on each section in Chapter 11, have them keep track of any problems they are having under the What I Need to Work On tab in their chapter Foldable.	• As students complete each section, have them review the list of items they need to work on, and then have them check off any that have been handled.

Math Link

Initiate a class discussion about secret codes that students may have created and used with friends. Brainstorm different ways to create a code. Students might suggest math symbols and numbers they have used to substitute for letters.

You may wish to read the Wrap It Up! for this chapter problem, which is on page 417. The Math Links in this chapter lead directly to the Wrap It Up! All students should do all of these Math Links.

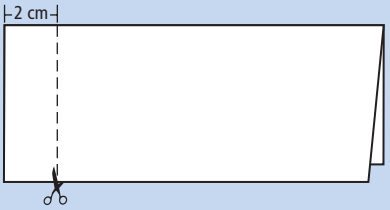
FOLDABLES™
Study Tool

Have students make the Foldable in the student resource to keep track of the information in the chapter. Filling in the What I Need to Work On tab as they progress through the chapter will assist them in identifying and solving any difficulties with concepts, skills, and processes.

You may prefer to have students keep track of Key Words using a design specifically for that purpose. Since many of the terms used in Chapter 10 are necessary for Chapter 11, you may wish to have students use the same Foldable design and staple the new pages to the end of the Chapter 10 Key Words Foldable.

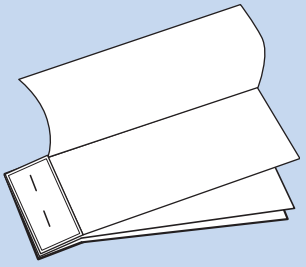
Step 1 Cut a sheet of grid paper horizontally in half. Fold the half sheet in two horizontally.

Step 2 Draw a line 2 cm in from the left side of the folded paper. Cut the top part of the fold along this line.



Step 3 Make one of these folded sheets for each Key Word. Staple the tabs together to make a booklet.

Step 4 Write a Key Word on the front of each tab. Write definitions and give examples underneath the tabs.



Remind students to take notes about key ideas, examples, Key Words, and what they need to work on under the appropriate tab of the chapter Foldable.

Supported Learning

Learning Style and Motor

- Create the Foldable ahead of time to use as a model.

ESL and Language

- Consider displaying Key Words on a math word wall. Students may also create their own vocabulary/ picture dictionary.

Common Errors

- Some students may think that codes result only from substituting numbers and symbols for letters.

R_x Provide some examples of codes that result from different mathematical operations, such as using an expression (e.g., $2x + 1$).

11.1 Expressions and Equations

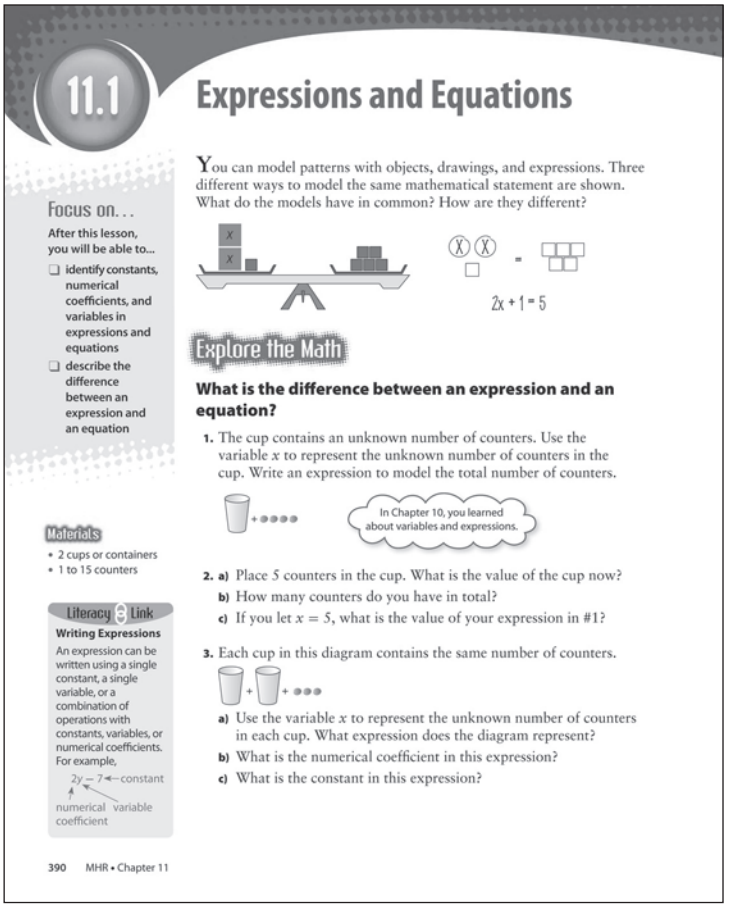
Suggested Timing
80–100 minutes

Materials
- 2 cups or containers per student pair
- at least 15 counters per student pair
- transparent counters (optional)
- 2-pan balance (optional)
- algebra tiles

Blackline Masters
Master 2 Two Stars and One Wish
BLM 11–1 Chapter 11 Self-Assessment
BLM 11–3 Section 11.1 Extra Practice
BLM 11–4 Section 11.1 Math Links

Mathematical Processes

- [x] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



11.1 Expressions and Equations

Focus on...
After this lesson, you will be able to...
- identify constants, numerical coefficients, and variables in expressions and equations
- describe the difference between an expression and an equation

You can model patterns with objects, drawings, and expressions. Three different ways to model the same mathematical statement are shown. What do the models have in common? How are they different?

2x + 1 = 5

Explore the Math

What is the difference between an expression and an equation?

1. The cup contains an unknown number of counters. Use the variable x to represent the unknown number of counters in the cup. Write an expression to model the total number of counters.

In Chapter 10, you learned about variables and expressions.

Materials
- 2 cups or containers
- 1 to 15 counters

2. a) Place 5 counters in the cup. What is the value of the cup now?
b) How many counters do you have in total?
c) If you let $x = 5$, what is the value of your expression in #1?

3. Each cup in this diagram contains the same number of counters.
a) Use the variable x to represent the unknown number of counters in each cup. What expression does the diagram represent?
b) What is the numerical coefficient in this expression?
c) What is the constant in this expression?

Literacy Link
Writing Expressions
An expression can be written using a single constant, a single variable, or a combination of operations with constants, variables, or numerical coefficients. For example,
2y − 7 ← constant
numerical coefficient, variable

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Specific Outcomes

PR4 Explain the difference between an expression and equation.

Warm-Up

Use the information in the table to answer #1 and #2.

x	1	2	3	4
y	0	1	2	3

1. Draw a graph using the ordered pairs in the table of values.
2. Describe the pattern on your graph.
3. $(-1) - (+6)$
4. Draw a circle with a diameter of 7 cm.
5. $1\frac{1}{6} + 2\frac{1}{2}$

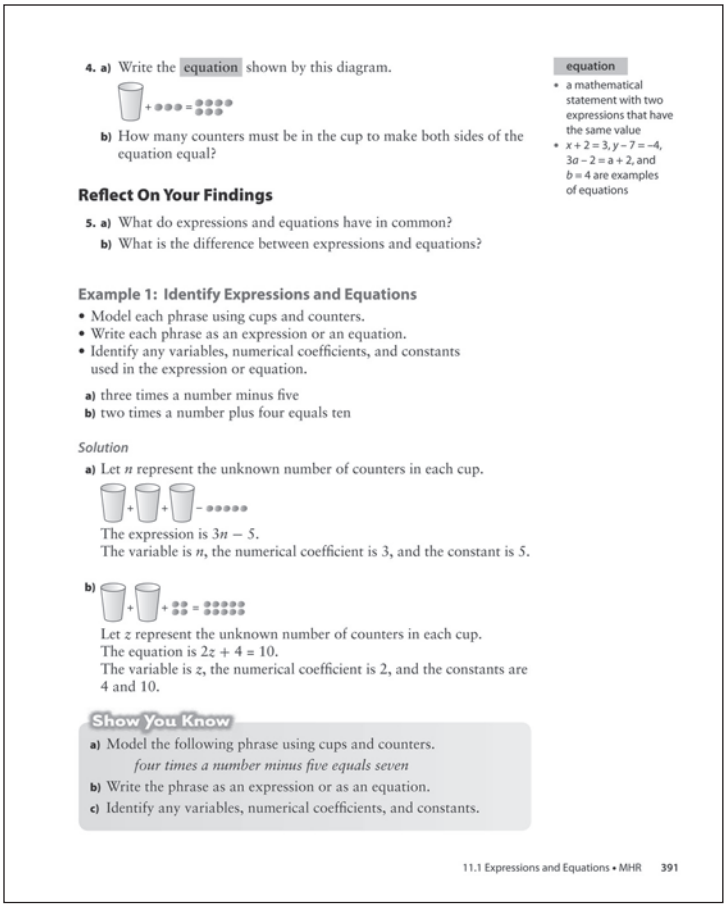
Mental Math

For #6 to #8, decide whether the answer is closest to 0, $\frac{1}{2}$, or 1. Explain your choice.

6. $\frac{1}{100} + \frac{1}{50}$
7. $\frac{3}{4} + \frac{2}{8}$
8. $\frac{5}{8} - \frac{1}{20}$

For #9 and #10, estimate the answer.

9. $2\frac{1}{3} + 3\frac{4}{5}$
10. $5\frac{9}{10} - 4\frac{1}{7}$



4. a) Write the equation shown by this diagram.

b) How many counters must be in the cup to make both sides of the equation equal?

equation
- a mathematical statement with two expressions that have the same value
- $x + 2 = 3$, $y - 7 = -4$, $3a - 2 = a + 2$, and $b = 4$ are examples of equations

Reflect On Your Findings

5. a) What do expressions and equations have in common?
b) What is the difference between expressions and equations?

Example 1: Identify Expressions and Equations

- Model each phrase using cups and counters.
- Write each phrase as an expression or an equation.
- Identify any variables, numerical coefficients, and constants used in the expression or equation.

a) three times a number minus five
b) two times a number plus four equals ten

Solution

a) Let n represent the unknown number of counters in each cup.

The expression is $3n - 5$.
The variable is n, the numerical coefficient is 3, and the constant is 5.

b)

Let z represent the unknown number of counters in each cup.
The equation is $2z + 4 = 10$.
The variable is z, the numerical coefficient is 2, and the constants are 4 and 10.

Show You Know

a) Model the following phrase using cups and counters.
four times a number minus five equals seven
b) Write the phrase as an expression or as an equation.
c) Identify any variables, numerical coefficients, and constants.

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Activity Planning Notes

As a class, discuss the three ways that the mathematical statement $2x + 1 = 5$ is modelled. Discuss what the models have in common (e.g., variables, coefficients, constants) that allow them to represent the same idea. You may need to review these terms. Prompt students to use these terms to identify the similarities and differences (e.g., use of objects, numbers, and symbols; use of equal signs) among the models. Consider making a class chart of the similarities and differences.

Explore the Math

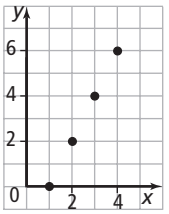
In this investigation, students determine the differences between algebraic expressions and equations. Focus on terms such as *constant*, *coefficient*, *variable*, *expression*, and *equation*.

Method 1: Have students work with a partner and use cups and counters to complete and then discuss their work. Check that students understand the differences between an expression and an equation.

Method 2: Demonstrate the expressions and equations by using cups and transparent counters on an overhead projector. For #4, consider modelling the equation using a two-pan balance. Have students use words and drawings to record each example you show.

Answers

Warm-Up

1. 

2. Answers will vary. For example:
   - Every time you move right 1, you move up 2.
   - $y = 2x - 2$
3. -7
4. Check that students use a radius of 3.5 cm.
5. $1\frac{1}{6} + 2\frac{3}{6} = 3\frac{4}{6} = 3\frac{2}{3}$
6. The answer is close to 0 because both of these fractions are very small.
7. The answer is close to 1 because $\frac{3}{4}$ is bigger than $\frac{1}{2}$ and then you have added to it.
8. The answer is close to $\frac{1}{2}$ because $\frac{5}{8}$ is close to that and you have subtracted a small amount.
9. $2 + 4 = 6$
10. $6 - 4 = 2$

Explore the Math

1. $x + 4$
2. **a)** 5 **b)** 9 **c)** $5 + 4 = 9$
3. **a)** $2x + 3$ **b)** 2 **c)** 3
4. **a)** $x + 3 = 7$ **b)** 4
5. **a)** Answers may vary. Expressions and equations have variables, and/or constants, and/or numerical coefficients, and/or mathematical operations.
   b) Answers may vary. Expressions do not have an equal sign and cannot be solved until the value of the variable is given. Equations have equal signs and can be solved using mathematical operations to find the value of the variable.

Answers

Show You Know: Example 1

a) 

b) $4x - 5 = 7$

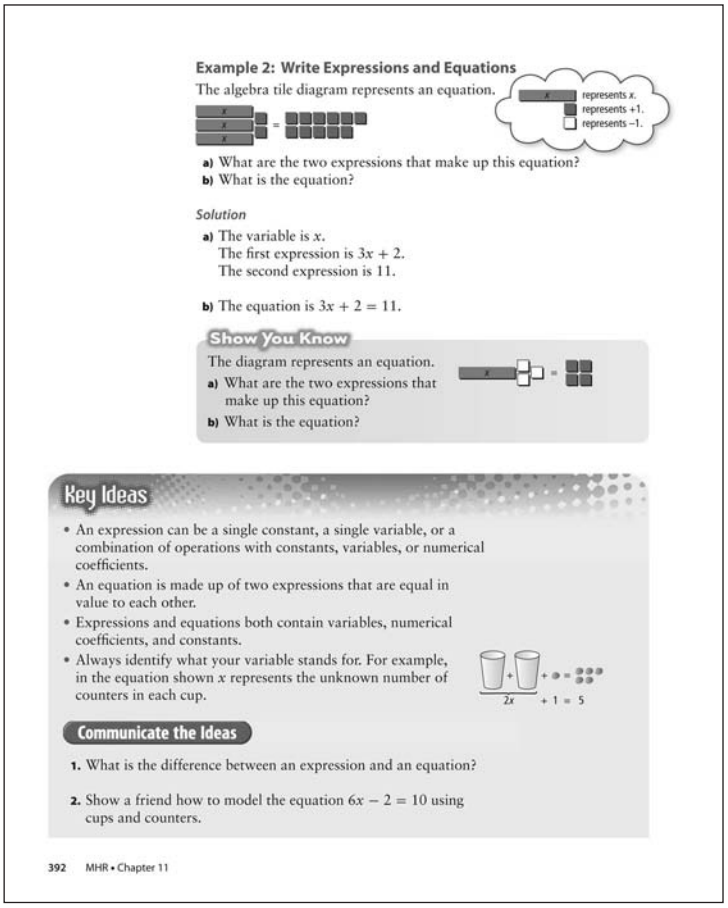
c) variable: x; numerical coefficient: 4; constants: 5, 7

Show You Know: Example 2

a) $x - 3, 4$ **b)** $x - 3 = 4$

Communicate the Ideas

1. An expression can be a single constant, a single variable, or a combination of operations with constants, variables, or numerical coefficients. An equation is made up of two expressions that are equal in value to each other. (Both expressions and equations contain variables, numerical coefficients, and constants.)
2. 



Example 2: Write Expressions and Equations

The algebra tile diagram represents an equation.

represents x.
represents +1.
represents −1.

a) What are the two expressions that make up this equation?
b) What is the equation?

Solution

a) The variable is x.
The first expression is $3x + 2$.
The second expression is 11.

b) The equation is $3x + 2 = 11$.

Show You Know

The diagram represents an equation.
a) What are the two expressions that make up this equation?
b) What is the equation?

Key Ideas

- An expression can be a single constant, a single variable, or a combination of operations with constants, variables, or numerical coefficients.
- An equation is made up of two expressions that are equal in value to each other.
- Expressions and equations both contain variables, numerical coefficients, and constants.
- Always identify what your variable stands for. For example, in the equation shown x represents the unknown number of counters in each cup.

$2x + 1 = 5$

Communicate the Ideas

1. What is the difference between an expression and an equation?
2. Show a friend how to model the equation $6x - 2 = 10$ using cups and counters.

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Supported Learning

Meeting the Need of All Learners

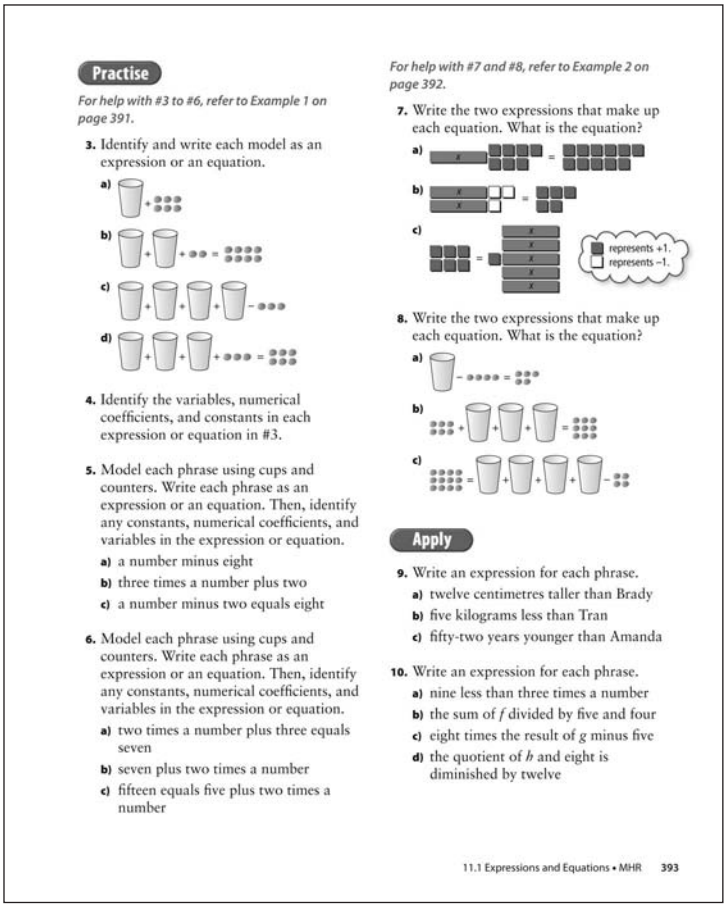
- Allow students to continue to use cups and counters as necessary throughout the chapter.
- Have English language learners use an organizer such as Frayer's model to show the differences between an algebraic expression and an equation. Frayer's model divides a page into four equal parts for each term. Students use one part for each of the following: definition (e.g., *expression*); characteristics (e.g., can be a single constant, a single variable, or a combination of operations with constants, variables, and/or numerical coefficients); example (e.g., x, $3x + 2$); and an example of what it is not (e.g., $3x + 2 = 11$). Have students repeat the process for the term *equation*.
- Point out the Literacy Link on page 390 and have students add an illustrated example of *expression* to their Foldable.
- Consider using a balance and manipulatives to show the different models of equality.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students describe the differences and similarities between expressions and equations.	• Check student responses to #5 for comprehension. • Encourage students to use examples to explain the similarities and differences.

In Example 1, students reinforce their understanding of expressions and equations and identify variables, numerical coefficients, and constants.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1 on page 391.	• Have students continue to use cups and counters to help identify variables, numerical coefficients, and constants. • You may wish to provide additional questions to students who would benefit from them: **a)** Model three times a number plus four equals ten. (3 cups + 4 counters = 10 counters) **b)** Write the phrase as an expression or equation. ($3x + 4 = 10$) **c)** Identify each part of the expression or equation. (variable: x; numerical coefficient: 3; constants: 4, 10)

Example 2 illustrates writing expressions and equations. Make sure that students understand the use of positive and negative algebra tiles to represent the quantities in the equation.



Practise

For help with #3 to #6, refer to Example 1 on page 391.

3. Identify and write each model as an expression or an equation.

a)

b)

c)

d)

4. Identify the variables, numerical coefficients, and constants in each expression or equation in #3.

5. Model each phrase using cups and counters. Write each phrase as an expression or an equation. Then, identify any constants, numerical coefficients, and variables in the expression or equation.

a) a number minus eight

b) three times a number plus two

c) a number minus two equals eight

6. Model each phrase using cups and counters. Write each phrase as an expression or an equation. Then, identify any constants, numerical coefficients, and variables in the expression or equation.

a) two times a number plus three equals seven

b) seven plus two times a number

c) fifteen equals five plus two times a number

For help with #7 and #8, refer to Example 2 on page 392.

7. Write the two expressions that make up each equation. What is the equation?

a)

b)

c)

represents +1.
represents −1.

8. Write the two expressions that make up each equation. What is the equation?

a)

b)

c)

Apply

9. Write an expression for each phrase.

a) twelve centimetres taller than Brady

b) five kilograms less than Tran

c) fifty-two years younger than Amanda

10. Write an expression for each phrase.

a) nine less than three times a number

b) the sum of f divided by five and four

c) eight times the result of g minus five

d) the quotient of h and eight is diminished by twelve

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Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2 on page 392.	• Have students use algebra tiles to model the equation. • You may wish to provide additional questions to students who would benefit from them: **a)** Use algebra tiles to model $2x - 3 = 5$. (Look for 2 green tiles and 3 white tiles on one side, with 5 red tiles on the other.) **b)** What are the two expressions that make up this equation? ($2x - 3$ and 5) **c)** What is the equation? ($2x - 3 = 5$)

Key Ideas

This section summarizes expressions and equations and how to write an equation. Stress that x represents an unknown number.

Communicate the Ideas

In #1, students describe the difference between expressions and equations. In #2, students model an equation using cups and counters.

Practise

In #4, observe that students correctly identify the variables, coefficients, and constants.

Common Errors

- Some students may confuse variables, coefficients, and constants.

R_x Encourage students to use manipulatives such as cups and counters to represent variables, coefficients, and constants.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students do #1 and #2. Encourage students to discuss their answer with neighbouring students and to listen to each other's explanations.	• These are key questions; make sure that students understand the concepts. • For #1, consider having students use a Venn diagram. You may decide to do this as a whole class activity. • Use **Master 2 Two Stars and One Wish** to have students critique other students' writing pieces. Work as a group to decide on assessment criteria. This master allows them to write two things they like about a piece and one thing they would like to improve.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–5, 7, 9, Math Link
Typical	1–5, 9–12, Math Link
Extension/Enrichment	1, 2, 12–15, Math Link

Assessment *for* Learning	Supported Learning
Practise Have students do #3 to #5, and #7. Students who have no problems with these questions can go on to the Apply questions.	• Provide students with cups, counters, and algebra tiles. • Students who have problems with #3 to #5 and #7 will need additional coaching. Have students explain their thinking on these questions and then clarify any misunderstandings. Have students correct #3 to #5 and then do #6 on their own. Coach students through #8a), and then have them complete the remaining parts of the question on their own. Check several times to make sure that they understand the concepts.

Answers

Math Link

a) Answers will vary. **b)** Answers will vary.

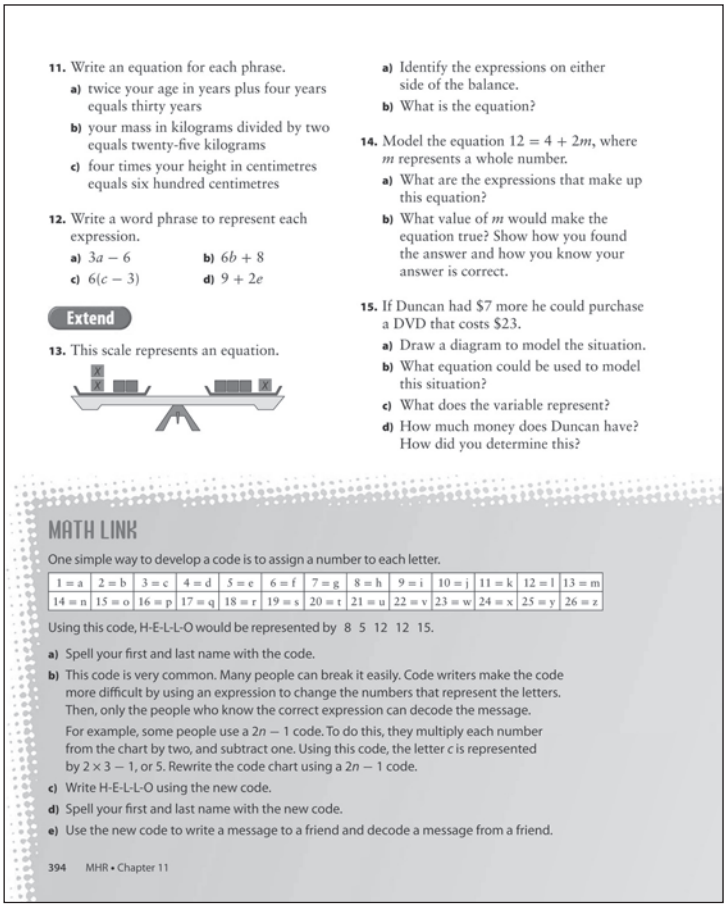
c) 15, 9, 23, 23, 29 **d)** Answers will vary.

e) Answers will vary.

Supported Learning

Meeting the Needs of All Learners

- Have students make a table of the four basic operations and list words or phrases that translate into each operation (e.g., +: *add, sum, plus, more, increase by*).
- Some students may need extra reinforcement to model and write expressions and equations. Consider having partners take turns to explain the models and written expressions to each other.
- Provide **BLM 11–3 Section 11.1 Extra Practice** to students who require more practice.
- Encourage students who need them to continue to use concrete materials for modelling phrases.



11. Write an equation for each phrase.
- **a)** twice your age in years plus four years equals thirty years
- **b)** your mass in kilograms divided by two equals twenty-five kilograms
- **c)** four times your height in centimetres equals six hundred centimetres

12. Write a word phrase to represent each expression.
- **a)** $3a - 6$
- **b)** $6b + 8$
- **c)** $6(c - 3)$
- **d)** $9 + 2e$

Extend

13. This scale represents an equation.

- **a)** Identify the expressions on either side of the balance.
- **b)** What is the equation?

14. Model the equation $12 = 4 + 2m$, where m represents a whole number.
- **a)** What are the expressions that make up this equation?
- **b)** What value of m would make the equation true? Show how you found the answer and how you know your answer is correct.

15. If Duncan had $7 more he could purchase a DVD that costs $23.
- **a)** Draw a diagram to model the situation.
- **b)** What equation could be used to model this situation?
- **c)** What does the variable represent?
- **d)** How much money does Duncan have? How did you determine this?

MATH LINK

One simple way to develop a code is to assign a number to each letter.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g	8 = h	9 = i	10 = j	11 = k	12 = l	13 = m
14 = n	15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u	22 = v	23 = w	24 = x	25 = y	26 = z

Using this code, H-E-L-L-O would be represented by 8 5 12 12 15.

- **a)** Spell your first and last name with the code.
- **b)** This code is very common. Many people can break it easily. Code writers make the code more difficult by using an expression to change the numbers that represent the letters. Then, only the people who know the correct expression can decode the message.

  For example, some people use a $2n - 1$ code. To do this, they multiply each number from the chart by two, and subtract one. Using this code, the letter c is represented by $2 \times 3 - 1$, or 5. Rewrite the code chart using a $2n - 1$ code.
- **c)** Write H-E-L-L-O using the new code.
- **d)** Spell your first and last name with the new code.
- **e)** Use the new code to write a message to a friend and decode a message from a friend.

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Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • What do you know about expressions and equations? • What do you find difficult about expressions and equations?	• Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • You may wish to have students review the part related to Section 11.1 in **BLM 11–1 Chapter 11 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Apply and Extend

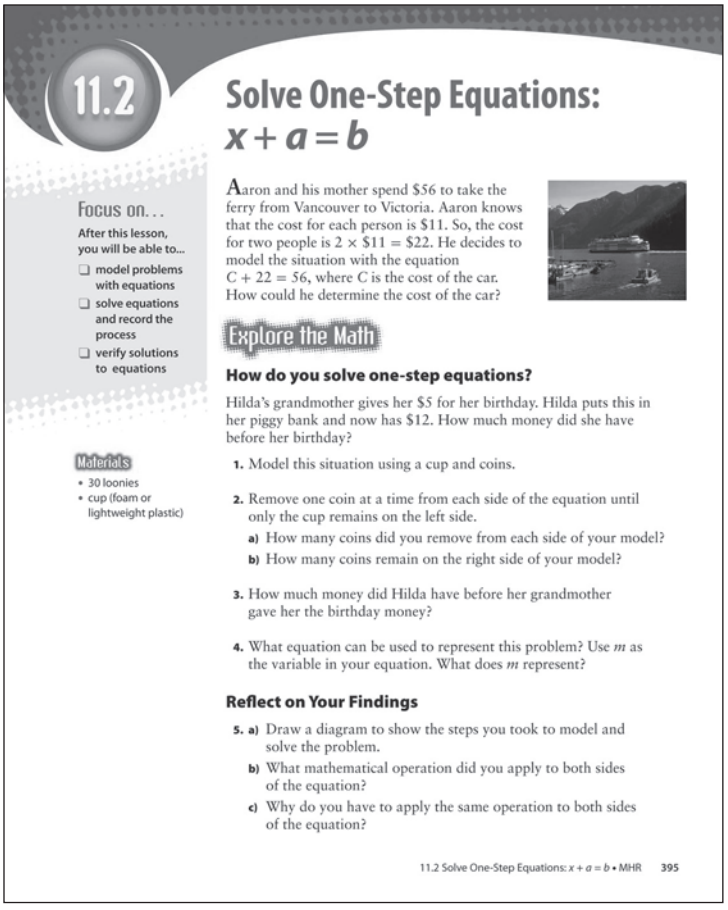
In #9, students convert phrases to expressions. Have students highlight key words (e.g., *taller, less than*) and discuss how these key words translate into mathematical operations (e.g., 12 cm taller = + 12). Students will use this information in #10 and #11.

Math Link

This Math Link provides students with an opportunity to write a code and decode messages using an expression.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 394 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 417.	• It is recommended that all students do this Math Link. • Students who are having difficulty getting started could use **BLM 11–4 Section 11.1 Math Link**, which provides scaffolding for this activity. • Invite students to check each other's accuracy.

11.2 Solve One-Step Equations: $x + a = b$



11.2

Solve One-Step Equations: $x + a = b$

Focus on...

After this lesson, you will be able to...

- model problems with equations
- solve equations and record the process
- verify solutions to equations

Aaron and his mother spend $56 to take the ferry from Vancouver to Victoria. Aaron knows that the cost for each person is $11. So, the cost for two people is 2 × $11 = $22. He decides to model the situation with the equation $C + 22 = 56$, where C is the cost of the car. How could he determine the cost of the car?

Explore the Math

How do you solve one-step equations?

Hilda's grandmother gives her $5 for her birthday. Hilda puts this in her piggy bank and now has $12. How much money did she have before her birthday?

Materials
- 30 loonies
- cup (foam or lightweight plastic)

1. Model this situation using a cup and coins.
2. Remove one coin at a time from each side of the equation until only the cup remains on the left side.
   a) How many coins did you remove from each side of your model?
   b) How many coins remain on the right side of your model?
3. How much money did Hilda have before her grandmother gave her the birthday money?
4. What equation can be used to represent this problem? Use m as the variable in your equation. What does m represent?

Reflect on Your Findings

5. a) Draw a diagram to show the steps you took to model and solve the problem.
   b) What mathematical operation did you apply to both sides of the equation?
   c) Why do you have to apply the same operation to both sides of the equation?

11.2 Solve One-Step Equations: $x + a = b$ • MHR 395

Suggested Timing
80–100 minutes

Materials
- 30 loonies (or play money, 1-g masses, or tokens)
- cup (foam or lightweight plastic)
- 2-pan balance

Blackline Masters
BLM 11–1 Chapter 11 Self-Assessment
BLM 11–5 Section 11.2 Extra Practice
BLM 11–6 Section 11.2 Math Link

Mathematical Processes

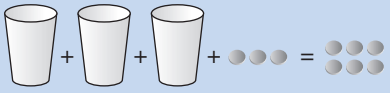
- ☑ Communication
- ☑ Connections
- ☑ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☐ Technology
- ☑ Visualization

Specific Outcomes

PR6 Model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially and symbolically, where a and b are integers.

Warm-Up

Use this diagram for #1 and #2.



1. Write the model as an equation.
2. Identify the variables, numerical coefficients, and constants in the equation for #1.
3. Show $\frac{2}{3}$ as a decimal using bar notation.
4. Show in lowest terms.
   a) $\frac{8}{10}$ b) $\frac{14}{16}$
5. You flip a coin 10 times. You get 7 heads and 3 tails. Does this show experimental probability or theoretical probability? Explain.

Mental Math

6. The radius of a circle is 4.5 cm. What is the diameter?
7. The diameter of a circle is 9.2 cm. What is the radius?
8. The radius of a circle is 2 m. Estimate the area.
9. The diameter of a circle is 6 m. Estimate the circumference.
10. You want to show 49% on a circle graph. Estimate the size of the interior angle.

Answers

Warm-Up

1. $3c + 3 = 6$ or $c + c + c + 3 = 6$

2. variable: c; numerical coefficient: 3 before the c; constants: 3, 6

3. $0.\overline{6}$

4. a) $\frac{4}{5}$ **b)** $\frac{7}{8}$

5. Experimental probability. According to theoretical probability, you would most likely get 5 heads and 5 tails. This is different.

6. 9 cm

7. 4.6 cm

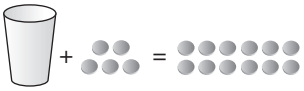
8. $3 \times 2 \times 2 = 12\ m^2$

9. 18 m

10. 49% is almost 50%. $50\% = \frac{1}{2}$. $360° \div 2 = 180°$

Explore the Math

1.

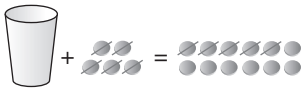


2. a) 5 **b)** 7

3. $7

4. $m + 5 = 12$; m represents the amount of money Hilda had before her birthday.

5. a)



$$m + 5 = 12$$
$$m + 5 - 5 = 12 - 5$$
$$m = 7$$

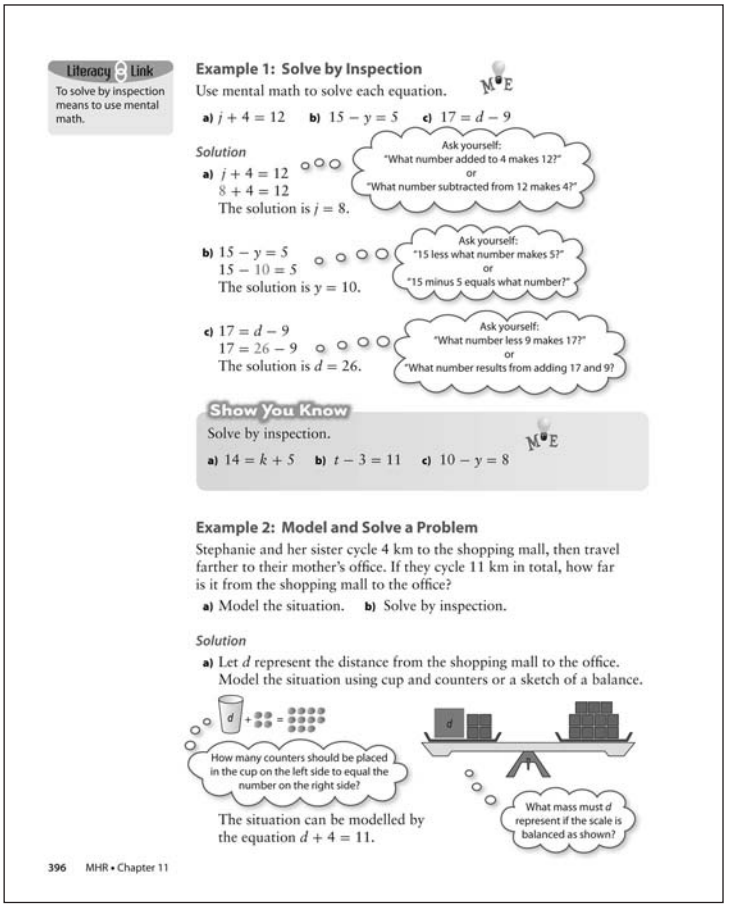
b) Subtract.

c) Answers will vary. For example: The same operation has to be applied to both sides of the equation to balance the equation.

Common Errors

- Some students may have difficulty extracting information from a problem situation and converting it into an equation.

R_x Review key words that help signify operations, such as *before/after, more/less,* and *decrease/increase.*



Literacy Link
To solve by inspection means to use mental math.

Example 1: Solve by Inspection
Use mental math to solve each equation.
a) $j + 4 = 12$ **b)** $15 - y = 5$ **c)** $17 = d - 9$

Solution
a) $j + 4 = 12$
$8 + 4 = 12$
The solution is $j = 8$.
Ask yourself: "What number added to 4 makes 12?" or "What number subtracted from 12 makes 4?"

b) $15 - y = 5$
$15 - 10 = 5$
The solution is $y = 10$.
Ask yourself: "15 less what number makes 5?" or "15 minus 5 equals what number?"

c) $17 = d - 9$
$17 = 26 - 9$
The solution is $d = 26$.
Ask yourself: "What number less 9 makes 17?" or "What number results from adding 17 and 9?"

Show You Know
Solve by inspection.
a) $14 = k + 5$ **b)** $t - 3 = 11$ **c)** $10 - y = 8$

Example 2: Model and Solve a Problem
Stephanie and her sister cycle 4 km to the shopping mall, then travel farther to their mother's office. If they cycle 11 km in total, how far is it from the shopping mall to the office?
a) Model the situation. **b)** Solve by inspection.

Solution
a) Let d represent the distance from the shopping mall to the office. Model the situation using cup and counters or a sketch of a balance.
How many counters should be placed in the cup on the left side to equal the number on the right side?
What mass must d represent if the scale is balanced as shown?
The situation can be modelled by the equation $d + 4 = 11$.

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Activity Planning Notes

Review the problem to ensure that students understand how the variable and constants of the equation are derived from the given information. Explain how using a modelling process helps solve problems.

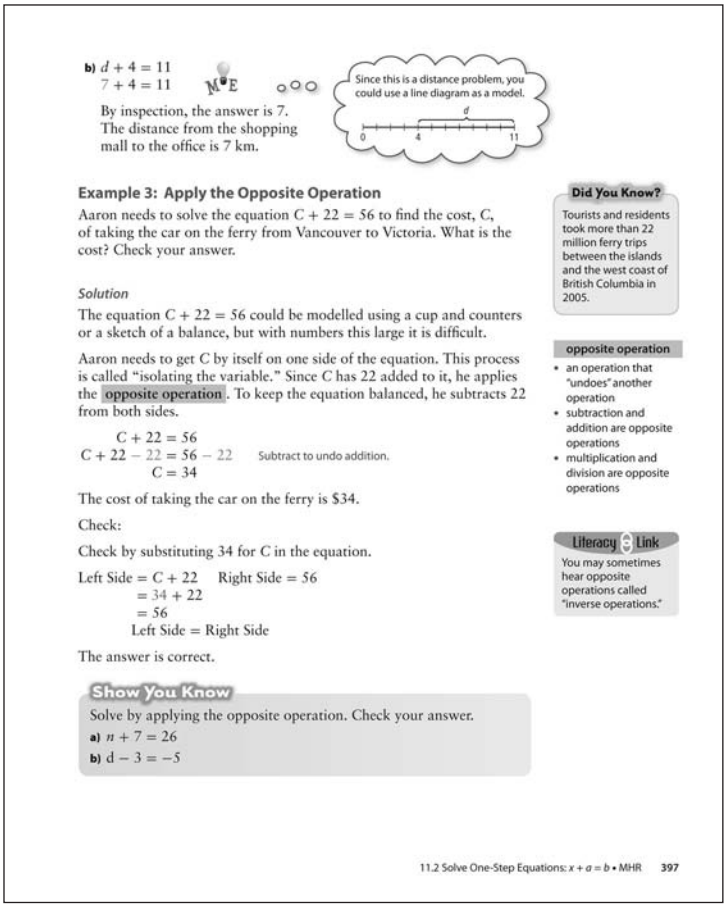
Explore the Math

In this investigation, a cup and coins are used to model a problem and develop an equation to represent the problem.

Method 1: Have students work in pairs or small groups. Give each pair or group of students a cup and 30 loonies. Have students model the situation, develop an equation, and solve the problem. Have students discuss their findings and highlight the importance of preserving equality. As a class, generalize the findings by listing the steps for solving an equation.

Method 2: Demonstrate how to model the situation by using cups and transparent coins on an overhead projector. Have students draw and use words to show the steps you took to model and solve the problem. Discuss the results as in Method 1.

Method 3: Consider modelling equations using a two-pan balance.



b) $d + 4 = 11$
$7 + 4 = 11$

By inspection, the answer is 7.
The distance from the shopping mall to the office is 7 km.

Since this is a distance problem, you could use a line diagram as a model.

Example 3: Apply the Opposite Operation

Aaron needs to solve the equation $C + 22 = 56$ to find the cost, C, of taking the car on the ferry from Vancouver to Victoria. What is the cost? Check your answer.

Solution

The equation $C + 22 = 56$ could be modelled using a cup and counters or a sketch of a balance, but with numbers this large it is difficult.

Aaron needs to get C by itself on one side of the equation. This process is called "isolating the variable." Since C has 22 added to it, he applies the opposite operation. To keep the equation balanced, he subtracts 22 from both sides.

$$C + 22 = 56$$
$$C + 22 - 22 = 56 - 22 \quad \text{Subtract to undo addition.}$$
$$C = 34$$

The cost of taking the car on the ferry is $34.

Check:

Check by substituting 34 for C in the equation.

Left Side $= C + 22$ Right Side $= 56$
$= 34 + 22$
$= 56$
Left Side = Right Side

The answer is correct.

Show You Know

Solve by applying the opposite operation. Check your answer.

a) $n + 7 = 26$
b) $d - 3 = -5$

Did You Know?

Tourists and residents took more than 22 million ferry trips between the islands and the west coast of British Columbia in 2005.

opposite operation

- an operation that "undoes" another operation
- subtraction and addition are opposite operations
- multiplication and division are opposite operations

Literacy Link

You may sometimes hear opposite operations called "inverse operations."

Answers

Show You Know: Example 1

a) $k = 9$ **b)** $t = 14$ **c)** $y = 2$

Show You Know: Example 3

a) $n = 19$ **b)** $d = -2$

Supported Learning

Meeting the Needs of All Learners

- Some students may need additional reinforcement for modelling problems with equations.
- English language learners may have difficulty with terms such as *ferry, piggy bank, inspection, life span, retire, decibels,* and *jet*. Have students add new terms to their dictionary.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss what they did during the Explore the Math, or read their explanations in #5.	• Make sure that students have a clear understanding of the importance of balancing an equation. • Some students may be able to demonstrate the steps they took to solve the problem, but have difficulty describing the process. As students demonstrate, verbalize what they are doing. Next, have them verbalize what they did and then record their explanation using a diagram and words.

In these examples, students develop three different methods of solving problems. Example 1 illustrates how to solve by inspection using mental math.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1 on page 396.	• Have students talk through their thinking with a partner. • You may wish to provide additional questions to students who would benefit from them: **a)** $12 = m + 4$ (What number added to 4 makes 12? $m = 8$) **b)** $l - 5 = 11$ (What number less 5 makes 11? $l = 16$) **c)** $15 - x = 9$ (What number do you subtract from 15 to get 9? $x = 6$) Coach students through a), and then have them try b) and c) on their own.

Example 2 illustrates how to solve a problem using a model. The method is the same as in Example 1, except that a model reinforces the scenario.

Answers

Communicate the Ideas

1. a) $x - 3 = 9$

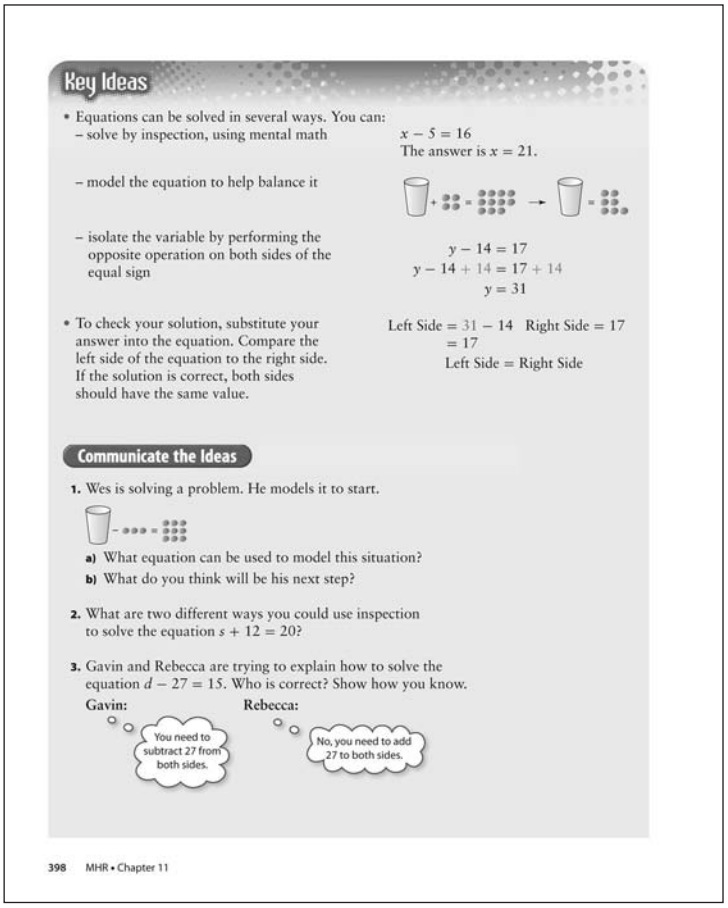
b) Answers may vary. For example: Wes could add 3 to both sides of the equation.

2. Answers may vary. For example: Method 1: Solve by inspection using mental math. What number added to 12 gives 20? Method 2: Model the situation and solve by inspection. How many counters should be placed in the cup on the left side to equal the number on the right side? The answer is 8.

3. Rebecca is correct.

$$d - 27 = 15$$
$$d - 27 + 27 = 15 + 27$$
$$d = 42$$

Left Side $= 42 - 27$ Right Side $= 15$
$= 15$
Left Side $=$ Right Side



Key Ideas

- Equations can be solved in several ways. You can:
  - solve by inspection, using mental math
    $x - 5 = 16$
    The answer is $x = 21$.
  - model the equation to help balance it
  - isolate the variable by performing the opposite operation on both sides of the equal sign
    $y - 14 = 17$
    $y - 14 + 14 = 17 + 14$
    $y = 31$
- To check your solution, substitute your answer into the equation. Compare the left side of the equation to the right side. If the solution is correct, both sides should have the same value.
  Left Side $= 31 - 14$ Right Side $= 17$
  $= 17$
  Left Side $=$ Right Side

Communicate the Ideas

1. Wes is solving a problem. He models it to start.
   a) What equation can be used to model this situation?
   b) What do you think will be his next step?
2. What are two different ways you could use inspection to solve the equation $s + 12 = 20$?
3. Gavin and Rebecca are trying to explain how to solve the equation $d - 27 = 15$. Who is correct? Show how you know.

Gavin: "You need to subtract 27 from both sides."
Rebecca: "No, you need to add 27 to both sides."

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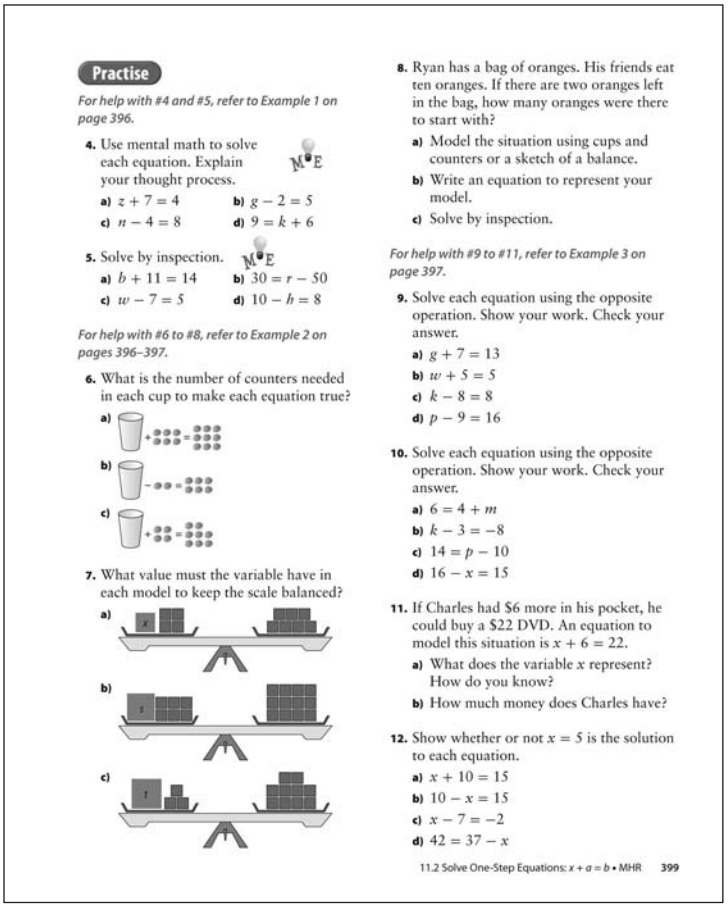
Example 3 illustrates applying the opposite operation. Emphasize the importance of keeping the equation balanced when isolating the variable on one side of the equation.

You might have students add a note to their chapter Foldable about inverse operations.

Assessment *for* Learning	Supported Learning
Example 3 Have students do the Show You Know related to Example 3 on page 397.	• Have students talk through their thinking with a partner. • You may wish to provide additional questions to students who would benefit from them: **a)** $s + 9 = 31$ (Isolate the variable by subtracting 9 from each side. $s + 9 - 9 = 31 - 9$; $s = 22$) **b)** $p - 5 = -8$ (Isolate the variable by adding 5 to each side. $p - 5 + 5 = -8 + 5$; $p = -3$) Sit down and coach students through a), and then have them try b) on their own.

Key Ideas

This section summarizes how to solve equations using three methods—solve by inspection, model the equation, and apply the opposite operation—and how to check a solution.



Practise

For help with #4 and #5, refer to Example 1 on page 396.

4. Use mental math to solve each equation. Explain your thought process.
   a) $z + 7 = 4$
   b) $g - 2 = 5$
   c) $n - 4 = 8$
   d) $9 = k + 6$

5. Solve by inspection.
   a) $b + 11 = 14$
   b) $30 = r - 50$
   c) $w - 7 = 5$
   d) $10 - h = 8$

For help with #6 to #8, refer to Example 2 on pages 396–397.

6. What is the number of counters needed in each cup to make each equation true?
   a)
   b)
   c)

7. What value must the variable have in each model to keep the scale balanced?
   a)
   b)
   c)

8. Ryan has a bag of oranges. His friends eat ten oranges. If there are two oranges left in the bag, how many oranges were there to start with?
   a) Model the situation using cups and counters or a sketch of a balance.
   b) Write an equation to represent your model.
   c) Solve by inspection.

For help with #9 to #11, refer to Example 3 on page 397.

9. Solve each equation using the opposite operation. Show your work. Check your answer.
   a) $g + 7 = 13$
   b) $w + 5 = 5$
   c) $k - 8 = 8$
   d) $p - 9 = 16$

10. Solve each equation using the opposite operation. Show your work. Check your answer.
   a) $6 = 4 + m$
   b) $k - 3 = -8$
   c) $14 = p - 10$
   d) $16 - x = 15$

11. If Charles had $6 more in his pocket, he could buy a $22 DVD. An equation to model this situation is $x + 6 = 22$.
   a) What does the variable x represent? How do you know?
   b) How much money does Charles have?

12. Show whether or not $x = 5$ is the solution to each equation.
   a) $x + 10 = 15$
   b) $10 - x = 15$
   c) $x - 7 = -2$
   d) $42 = 37 - x$

11.2 Solve One-Step Equations: $x + a = b$ • MHR 399

Communicate the Ideas

Students apply their understanding of solving one-step equations. In #1, students develop an equation and start to solve it. In #2, students reinforce their understanding of using mental math to solve an equation. In #3, students review a given solution for a problem.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students who need further help with their understanding do #1 to #3, which includes a concrete example. Other students could do #2 and #3.	• As you circulate, listen for student discussion about the processes that can be used to solve equations. • In #3, encourage students to share their answers with neighbouring students and listen to each other's explanations. • For #3, reinforce performing the opposite operation on both sides of the equal sign. Ask students why they are doing this (to isolate the variable).

Practise

Assessment *for* Learning	Supported Learning
Practise Have students do #4, #6, #8, #9, and #12. Students who have no problems with these questions can go on to the Apply questions	• In #4, reinforce using mental math instead of a calculator. • Students who have problems with #4 need to review Example 1, correct their errors, and then do #5 on their own. • Students who have problems with #6 and #8 need to review Example 2, correct their errors, and then do #7 on their own. • Students who have problems with #9 and #12 need to review Example 3, correct their errors, and then do #10 and #11 on their own. • Check back with them several times to make sure that they understand the concepts. Provide additional coaching where needed.

Supported Learning

Learning Style and Memory

- Allow students to continue to use concrete materials such as cups and coins to model problems.

Learning Style

- Provide opportunities for students to share their strategies for solving a problem. Have partners take turns explaining the steps for solving a problem to each other.
- Once students have explored different ways to solve equations, allow them to choose their preferred method for answering questions.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8, 9, 12, 13, Math Link
Typical	2–4, 6, 8, 9, 12–18, Math Link
Extension/Enrichment	2, 3, 18–22, Math Link

Common Errors

- Some students may have difficulty developing an equation.

R_x Encourage students to continue using concrete or semi-concrete materials to model a problem until they have a better understanding of how to develop an equation.

- Some students may forget to perform the same operation on both sides of the equal sign. They mix up the signs or operations (e.g., subtracting on one side and adding on the other side of an equal sign).

R_x Have students make a diagram (e.g., pan balance) that reinforces the idea that equal numbers must be added to or subtracted from both sides of an equation.

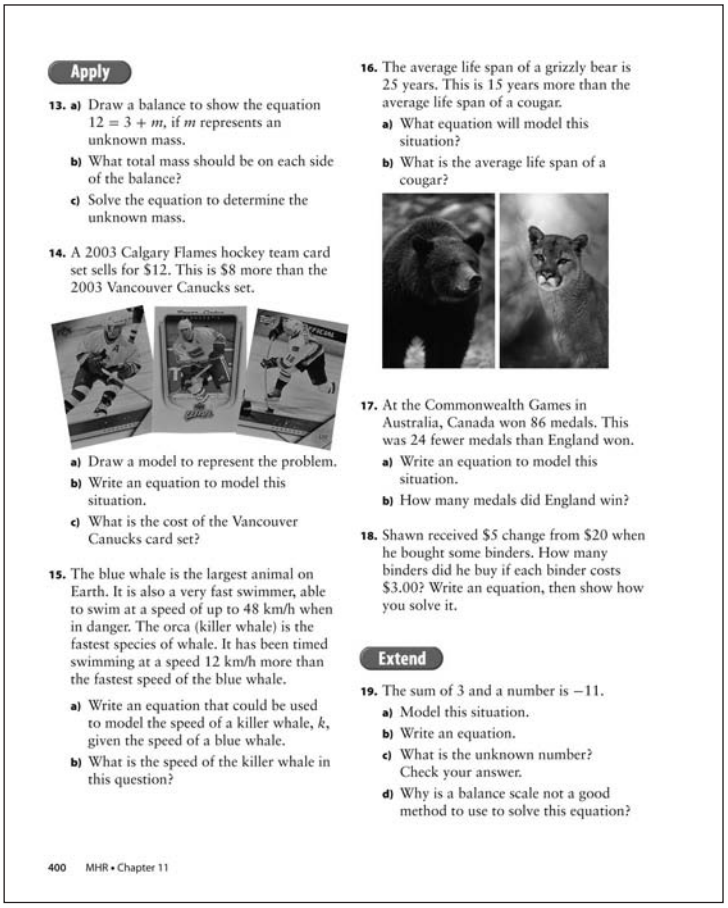
Answers

Math Link

a) $x - 3 = b$

b) $-2 = a$, $-1 = b$, $0 = c$, $1 = d$, $2 = e$, $3 = f$, $4 = g$, $5 = h$, $6 = i$, $7 = j$, $8 = k$, $9 = l$, $10 = m$, $11 = n$, $12 = o$, $13 = p$, $14 = q$, $15 = r$, $16 = s$, $17 = t$, $18 = u$, $19 = v$, $20 = w$, $21 = x$, $22 = y$, $23 = z$

c) Meet me at Steve's.



Apply

13. **a)** Draw a balance to show the equation $12 = 3 + m$, if m represents an unknown mass.
b) What total mass should be on each side of the balance?
c) Solve the equation to determine the unknown mass.

14. A 2003 Calgary Flames hockey team card set sells for $12. This is $8 more than the 2003 Vancouver Canucks set.

a) Draw a model to represent the problem.
b) Write an equation to model this situation.
c) What is the cost of the Vancouver Canucks card set?

15. The blue whale is the largest animal on Earth. It is also a very fast swimmer, able to swim at a speed of up to 48 km/h when in danger. The orca (killer whale) is the fastest species of whale. It has been timed swimming at a speed 12 km/h more than the fastest speed of the blue whale.

a) Write an equation that could be used to model the speed of a killer whale, k, given the speed of a blue whale.
b) What is the speed of the killer whale in this question?

16. The average life span of a grizzly bear is 25 years. This is 15 years more than the average life span of a cougar.
a) What equation will model this situation?
b) What is the average life span of a cougar?

17. At the Commonwealth Games in Australia, Canada won 86 medals. This was 24 fewer medals than England won.
a) Write an equation to model this situation.
b) How many medals did England win?

18. Shawn received $5 change from $20 when he bought some binders. How many binders did he buy if each binder costs $3.00? Write an equation, then show how you solve it.

Extend

19. The sum of 3 and a number is −11.
a) Model this situation.
b) Write an equation.
c) What is the unknown number? Check your answer.
d) Why is a balance scale not a good method to use to solve this equation?

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Supported Learning

Learning Style and Memory

- Provide **BLM 11–5 Section 11.2 Extra Practice** to students who require more practice.

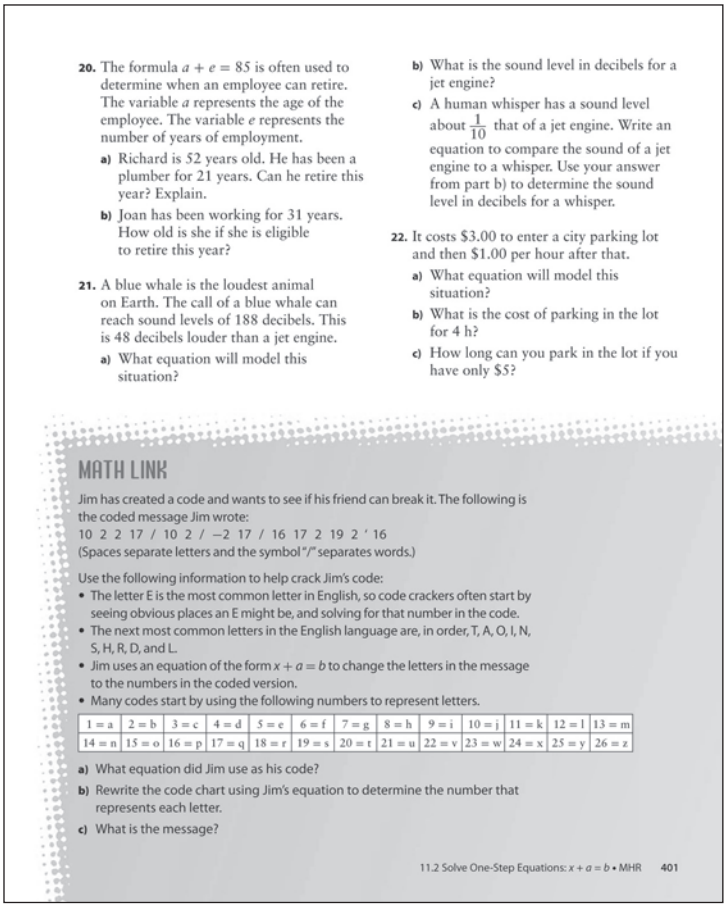
Apply and Extend

Hockey fans will particularly enjoy #14. Those interested in nature will appreciate #15 and #16.

The emphasis in Section 11.2 is on solving one-step problems with one variable. In #20, students are introduced to two variables.

In #21 and #22, you may wish to have students compare strategies for solving each problem. Some students may prefer using concrete materials to act out the problem or to check their answers.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question: • Why is it important to perform the same operation on both sides of the equal sign when solving a problem?	• Depending on students' learning style, have them provide verbal or written answers. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • You may wish to have students review the part related to Section 11.2 in **BLM 11–1 Chapter 11 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.



20. The formula $a + e = 85$ is often used to determine when an employee can retire. The variable a represents the age of the employee. The variable e represents the number of years of employment.

a) Richard is 52 years old. He has been a plumber for 21 years. Can he retire this year? Explain.

b) Joan has been working for 31 years. How old is she if she is eligible to retire this year?

21. A blue whale is the loudest animal on Earth. The call of a blue whale can reach sound levels of 188 decibels. This is 48 decibels louder than a jet engine.

a) What equation will model this situation?

b) What is the sound level in decibels for a jet engine?

c) A human whisper has a sound level about $\frac{1}{10}$ that of a jet engine. Write an equation to compare the sound of a jet engine to a whisper. Use your answer from part b) to determine the sound level in decibels for a whisper.

22. It costs $3.00 to enter a city parking lot and then $1.00 per hour after that.

a) What equation will model this situation?

b) What is the cost of parking in the lot for 4 h?

c) How long can you park in the lot if you have only $5?

MATH LINK

Jim has created a code and wants to see if his friend can break it. The following is the coded message Jim wrote:

10 2 2 17 / 10 2 / −2 17 / 16 17 2 19 2 ' 16

(Spaces separate letters and the symbol "/" separates words.)

Use the following information to help crack Jim's code:

- The letter E is the most common letter in English, so code crackers often start by seeing obvious places an E might be, and solving for that number in the code.
- The next most common letters in the English language are, in order, T, A, O, I, N, S, H, R, D, and L.
- Jim uses an equation of the form $x + a = b$ to change the letters in the message to the numbers in the coded version.
- Many codes start by using the following numbers to represent letters.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g	8 = h	9 = i	10 = j	11 = k	12 = l	13 = m
14 = n	15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u	22 = v	23 = w	24 = x	25 = y	26 = z

a) What equation did Jim use as his code?

b) Rewrite the code chart using Jim's equation to determine the number that represents each letter.

c) What is the message?

11.2 Solve One-Step Equations: $x + a = b$ • MHR 401

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 401 is intended to help students work toward the Wrap It Up! on page 417.	• This Math Link is recommended for all students. • If students are having trouble getting started, work together to get the solution for one letter. • Students who are having difficulty getting started could use **BLM 11–6 Section 11.2 Math Link**, which provides scaffolding.

Supported Learning

Gifted and Enrichment

- Encourage students to use the code in the Math Link to create messages or develop another equation and rewrite the code chart.

MATH LINK

This Math Link provides students with an opportunity to crack a coded message and determine the equation used to develop the code.

11.3 Solve One-Step Equations: $ax = b, \frac{x}{a} = b$

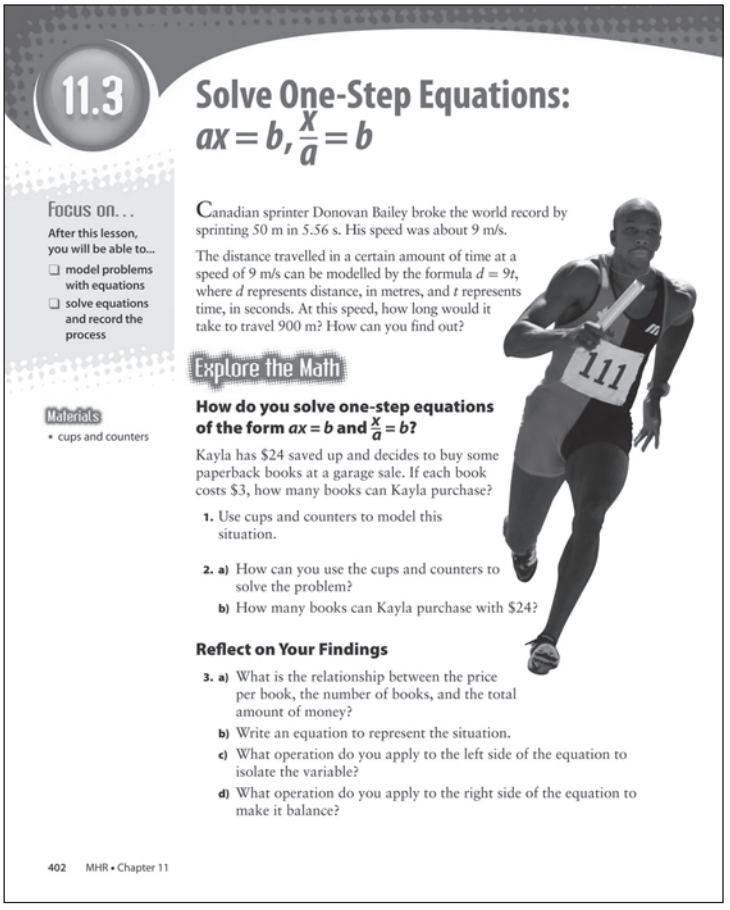
Suggested Timing
80–100 minutes

Materials
- cups and counters
- algebra tiles

Blackline Masters
BLM 11–1 Chapter 11 Self-Assessment
BLM 11–7 Section 11.3 Extra Practice
BLM 11–8 Section 11.3 Math Link

Mathematical Processes

- [ ] Communication
- [x] Connections
- [x] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



11.3 Solve One-Step Equations: $ax = b, \frac{x}{a} = b$

Focus on...
After this lesson, you will be able to...
- model problems with equations
- solve equations and record the process

Materials
- cups and counters

Canadian sprinter Donovan Bailey broke the world record by sprinting 50 m in 5.56 s. His speed was about 9 m/s.

The distance travelled in a certain amount of time at a speed of 9 m/s can be modelled by the formula $d = 9t$, where d represents distance, in metres, and t represents time, in seconds. At this speed, how long would it take to travel 900 m? How can you find out?

Explore the Math

How do you solve one-step equations of the form $ax = b$ and $\frac{x}{a} = b$?

Kayla has $24 saved up and decides to buy some paperback books at a garage sale. If each book costs $3, how many books can Kayla purchase?

1. Use cups and counters to model this situation.
2. a) How can you use the cups and counters to solve the problem?
   b) How many books can Kayla purchase with $24?

Reflect on Your Findings

3. a) What is the relationship between the price per book, the number of books, and the total amount of money?
   b) Write an equation to represent the situation.
   c) What operation do you apply to the left side of the equation to isolate the variable?
   d) What operation do you apply to the right side of the equation to make it balance?

402 MHR • Chapter 11

Specific Outcomes

PR7 Model and solve problems that can be represented by linear equations of the form:

- $ax + b = c$
- $ax = b$
- $\frac{x}{a} = b, a \neq 0$

concretely, pictorially and symbolically, where a, b and c are whole numbers.

Warm-Up

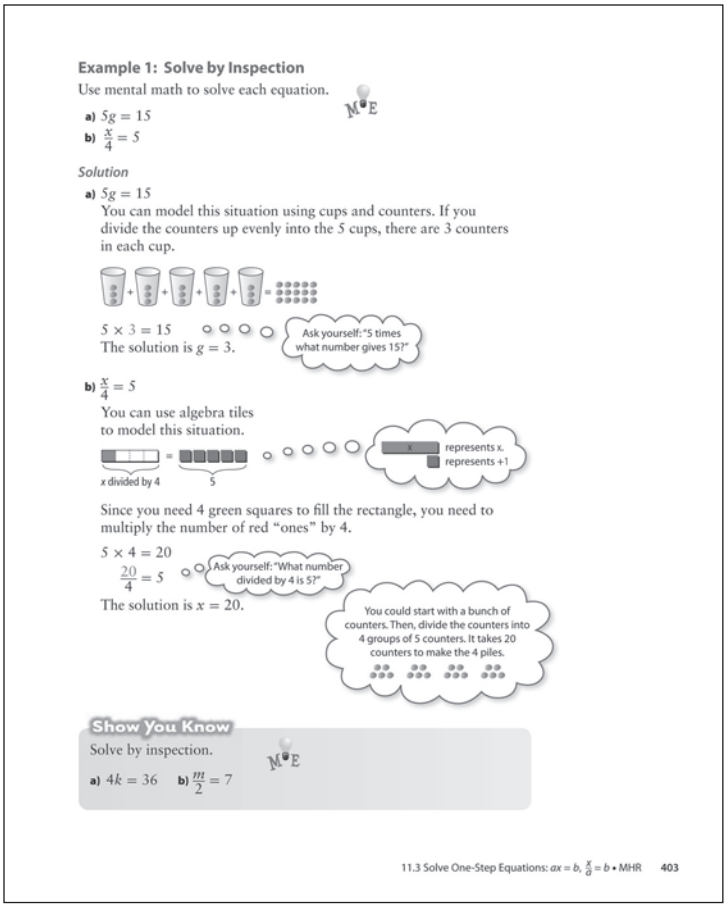
For #1 and #2, solve each equation using the opposite operation. Show your work.

1. $p - 5 = 20$
2. $21 + b = 30$
3. You get a 25% discount on a $36 item. How much do you pay?
4. Draw a pair of parallel lines. How do you know they are parallel?
5. What is the probability of having two Mondays in one week? Explain.

Mental Math

For #6 and #7, use mental math to solve the equation. Explain your thought process.

6. $p + 5 = 11$
7. $15 - r = 8$
8. Show $\frac{2}{5}$ as a decimal and a percent.
9. Show 85 out of 120 as a percent.
10. A circle has a radius of 5 m. Estimate the area.



Example 1: Solve by Inspection

Use mental math to solve each equation.

a) $5g = 15$

b) $\frac{x}{4} = 5$

Solution

a) $5g = 15$

You can model this situation using cups and counters. If you divide the counters up evenly into the 5 cups, there are 3 counters in each cup.

$5 \times 3 = 15$

The solution is $g = 3$.

Ask yourself: "5 times what number gives 15?"

b) $\frac{x}{4} = 5$

You can use algebra tiles to model this situation.

x represents x.
represents +1

x divided by 4 = 5

Since you need 4 green squares to fill the rectangle, you need to multiply the number of red "ones" by 4.

$5 \times 4 = 20$

$\frac{20}{4} = 5$

Ask yourself: "What number divided by 4 is 5?"

The solution is $x = 20$.

You could start with a bunch of counters. Then, divide the counters into 4 groups of 5 counters. It takes 20 counters to make the 4 piles.

Show You Know

Solve by inspection.

a) $4k = 36$ b) $\frac{m}{2} = 7$

11.3 Solve One-Step Equations: $ax = b$, $\frac{x}{a} = b$ • MHR 403

Answers

Warm-Up

1. $p - 5 + 5 = 20 + 5$
$p = 25$

2. $21 + b - 21 = 30 - 21$
$b = 9$

3. $36 - 9 = \$27$

4. Check that students draw parallel lines. The lines do not cross or intersect.

5. 0%. There is only one Monday each week.

6. $p = 6$; $5 + 6 = 11$

7. $r = 7$; $8 + 7 = 15$

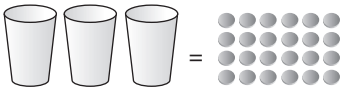
8. 0.4, 40%

9. 50% of 120 = 60
25% of 120 = 30
10% of 120 = 12
75% of 120 = 90 A little high
70% of 120 = 84 A little low
The answer is between 70% and 75%, but closer to 70%.

10. $3 \times 5 \times 5 = 75\ m^2$

Explore the Math

1.



2. a) Divide the counters equally among the cups. **b)** 8

3. a) The price multiplied by the number of books equals the total amount of money.
b) $3x = 24$ **c)** division **d)** division

Activity Planning Notes

This section focuses on solving one-step equations of the form $ax = b$ and $\frac{x}{a} = b$. Discuss how to model the formula for distance travelled, as described in the student resource. Discuss other instances where it is useful to use an equation (e.g., distance a car can travel in an hour). See if students can identify the operation needed in the example.

Explore the Math

Have students work in pairs to complete the activity, and discuss their findings and conclusions about the relationship between the price per book, the number of books, and the total amount of money. As a class, generalize the findings into procedures for solving equations.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss their conclusions.	• Have students discuss their conclusions. Ask them: – How do you know which operation to apply to the left and right side of the equation to isolate the variable?

Supported Learning

Meeting the Needs of All Learners

- Use visuals, models, and concrete materials to reinforce the concepts.
- Work through several examples of the problem presented in Explore the Math.
- Point out to students that $\frac{x}{a}$ is the same as *x* divided by *a*.

Common Errors

- Some students may have difficulty extracting relevant information from word problems and converting them into an equation.

R_x Highlight key words that signify operations (e.g., *twice/half, more/less*).

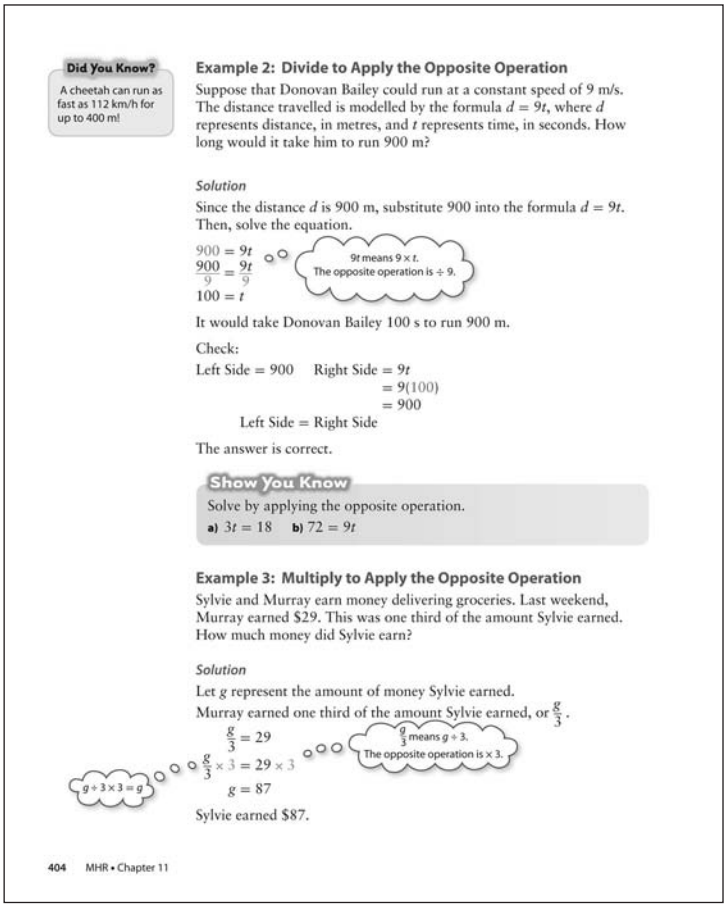
Answers

Show You Know: Example 1

a) $k = 9$ **b)** $m = 14$

Show You Know: Example 2

a) $t = 6$ **b)** $t = 8$



Did You Know?

A cheetah can run as fast as 112 km/h for up to 400 m!

Example 2: Divide to Apply the Opposite Operation

Suppose that Donovan Bailey could run at a constant speed of 9 m/s. The distance travelled is modelled by the formula $d = 9t$, where d represents distance, in metres, and t represents time, in seconds. How long would it take him to run 900 m?

Solution

Since the distance d is 900 m, substitute 900 into the formula $d = 9t$. Then, solve the equation.

$900 = 9t$

$\frac{900}{9} = \frac{9t}{9}$

$100 = t$

9t means 9 × t. The opposite operation is ÷ 9.

It would take Donovan Bailey 100 s to run 900 m.

Check:

Left Side = 900 Right Side = $9t$

$= 9(100)$

$= 900$

Left Side = Right Side

The answer is correct.

Show You Know

Solve by applying the opposite operation.

a) $3t = 18$ **b)** $72 = 9t$

Example 3: Multiply to Apply the Opposite Operation

Sylvie and Murray earn money delivering groceries. Last weekend, Murray earned $29. This was one third of the amount Sylvie earned. How much money did Sylvie earn?

Solution

Let g represent the amount of money Sylvie earned.

Murray earned one third of the amount Sylvie earned, or $\frac{g}{3}$.

$\frac{g}{3} = 29$

$\frac{g}{3} \times 3 = 29 \times 3$

$g = 87$

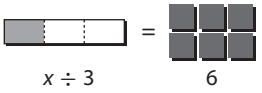
$\frac{g}{3}$ means $g \div 3$. The opposite operation is × 3.

$g \div 3 \times 3 = g$

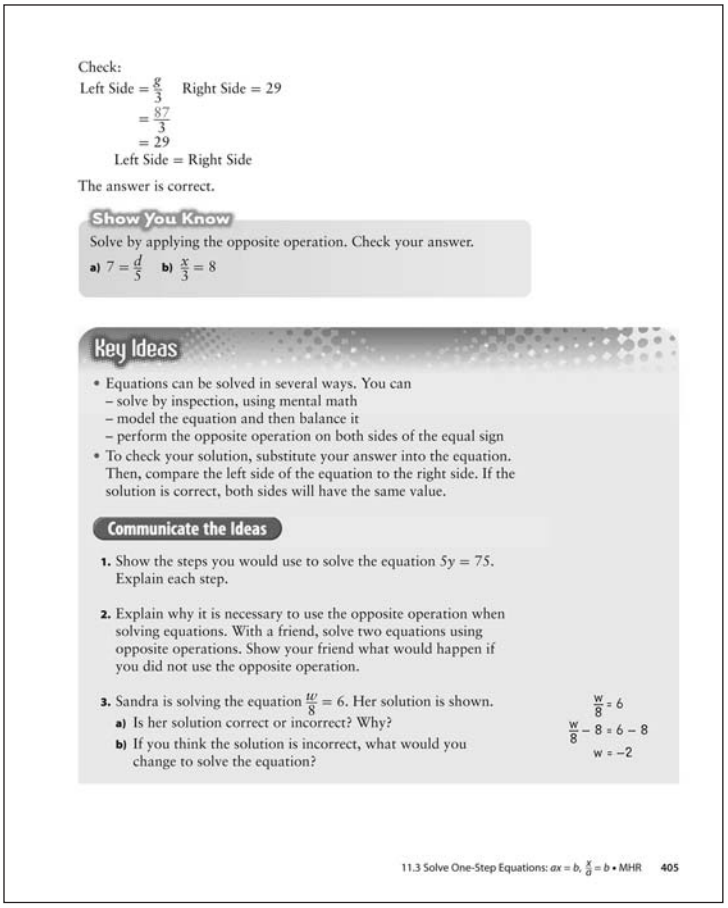
Sylvie earned $87.

404 MHR • Chapter 11

Example 1 demonstrates how to model equations using cups and counters and algebra tiles. Ensure students understand the importance of being able to use mental math to solve problems. Encourage students to use different methods to model the situations.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1 on page 403.	• You may wish to provide additional questions to students who would benefit from them: Model each equation and use mental math to solve it. **a)** $6x = 36$ (Model using cups and counters. If you divide the counters up evenly into 6 cups, there are 6 counters in each cup. $6 \times 6 = 36$; $x = 6$) **b)** $\frac{x}{3} = 6$ (Model using algebra tiles. It takes 3 squares to fill the rectangle in the model. You need to multiply the number of red "ones" by 3. $3 \times 6 = 18$; $\frac{18}{3} = 6$; $x = 18$.)  $x \div 3$ 6 Sit down and coach students through a), and then have them try b) on their own.

Example 2 demonstrates how to solve equations by dividing. Ensure students verify their work by checking the left side and right side of the equation.



Check:
Left Side $= \frac{g}{3}$ Right Side $= 29$
$= \frac{87}{3}$
$= 29$
Left Side = Right Side
The answer is correct.

Show You Know

Solve by applying the opposite operation. Check your answer.

a) $7 = \frac{d}{5}$ **b)** $\frac{x}{3} = 8$

Key Ideas

- Equations can be solved in several ways. You can
  - solve by inspection, using mental math
  - model the equation and then balance it
  - perform the opposite operation on both sides of the equal sign
- To check your solution, substitute your answer into the equation. Then, compare the left side of the equation to the right side. If the solution is correct, both sides will have the same value.

Communicate the Ideas

1. Show the steps you would use to solve the equation $5y = 75$. Explain each step.
2. Explain why it is necessary to use the opposite operation when solving equations. With a friend, solve two equations using opposite operations. Show your friend what would happen if you did not use the opposite operation.
3. Sandra is solving the equation $\frac{w}{8} = 6$. Her solution is shown.
   a) Is her solution correct or incorrect? Why?
   b) If you think the solution is incorrect, what would you change to solve the equation?

$\frac{w}{8} = 6$
$\frac{w}{8} - 8 = 6 - 8$
$w = -2$

Answers

Show You Know: Example 3

a) $d = 35$ **b)** $x = 24$

Communicate the Ideas

1. Divide each side by 5; $\frac{5y}{5} = \frac{75}{5}$; $y = 15$

2. Answers may vary. In order to isolate the variable, all the numbers surrounding it must be moved to the opposite side of the equation using opposite operations.

3. a) Her solution is incorrect because $-2 \div 8 \neq 6$.
b) Multiply each side of the equation by 8 instead of subtracting 8 from each side of the equation.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2 on page 404.	• You may wish to provide additional questions to students who would benefit from them: Solve the equation by applying the opposite operation. Verify your answer. **a)** $5x = 55$ ($5x \div 5 = 55 \div 5$; $x = 11$. Check: Left Side $= 5x = 5(11) = 55$. Right Side $= 55$. Right Side = Left Side. The answer is correct.) **b)** $56 = 8t$ ($56 \div 8 = 8t \div 8$; $t = 7$. Check: Left Side $= 56$. Right Side $= 8t = 8(7) = 56$. Right Side = Left Side. The answer is correct.) Sit down and coach students through a), and then have them try b) on their own.

Example 3 demonstrates how to solve equations by multiplying. Ensure students verify their work by checking each side of the equation.

Assessment *for* Learning	Supported Learning
Example 3 Have students do the Show You Know related to Example 3.	• You may wish to provide additional questions to students who would benefit from them: Solve the equation by applying the opposite operation. Verify your answer. **a)** $\frac{x}{6} = 4$ ($\frac{x}{6} \times 6 = 4 \times 6$; $x = 24$. Check: Left Side $= \frac{x}{6} = \frac{24}{6} = 4$. Right Side $= 4$. Left Side = Right Side. The answer is correct.) **b)** $9 = \frac{t}{3}$; $9 \times 3 = \frac{t}{3} \times 3$; $27 = t$. Check: Left Side $= 9$. Right Side $= \frac{t}{3} = \frac{27}{3} = 9$. Left Side = Right Side. The answer is correct.)

Supported Learning

Learning Style

- After students have explored solving equations using all three methods, allow them to choose their preferred method.

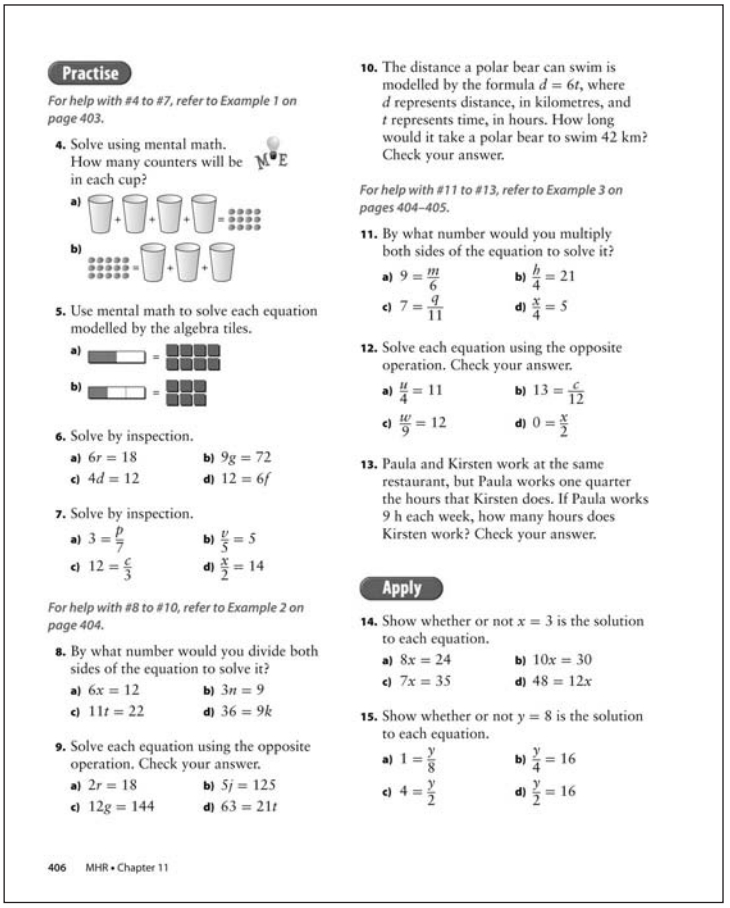
Learning Style and Memory

- Encourage students to model the equations using concrete materials such as cups and counters.
- Provide **BLM 11–7 Section 11.3 Extra Practice** to students who require more practice.

ESL

- Consider having students work in pairs for Communicate the Ideas.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8, 10, 11, 13–15, Math Link
Typical	1–4, 6, 8, 10, 11, 13–18, 20, Math Link
Extension/Enrichment	1–3, 16–21, Math Link



Practise

For help with #4 to #7, refer to Example 1 on page 403.

4. Solve using mental math. How many counters will be in each cup?

a)

b)

5. Use mental math to solve each equation modelled by the algebra tiles.

a)

b)

6. Solve by inspection.

a) $6r = 18$ b) $9g = 72$

c) $4d = 12$ d) $12 = 6f$

7. Solve by inspection.

a) $3 = \frac{p}{7}$ b) $\frac{v}{5} = 5$

c) $12 = \frac{c}{3}$ d) $\frac{x}{2} = 14$

For help with #8 to #10, refer to Example 2 on page 404.

8. By what number would you divide both sides of the equation to solve it?

a) $6x = 12$ b) $3n = 9$

c) $11t = 22$ d) $36 = 9k$

9. Solve each equation using the opposite operation. Check your answer.

a) $2r = 18$ b) $5j = 125$

c) $12g = 144$ d) $63 = 21t$

10. The distance a polar bear can swim is modelled by the formula $d = 6t$, where d represents distance, in kilometres, and t represents time, in hours. How long would it take a polar bear to swim 42 km? Check your answer.

For help with #11 to #13, refer to Example 3 on pages 404–405.

11. By what number would you multiply both sides of the equation to solve it?

a) $9 = \frac{m}{6}$ b) $\frac{h}{4} = 21$

c) $7 = \frac{q}{11}$ d) $\frac{x}{4} = 5$

12. Solve each equation using the opposite operation. Check your answer.

a) $\frac{u}{4} = 11$ b) $13 = \frac{c}{12}$

c) $\frac{w}{9} = 12$ d) $0 = \frac{x}{2}$

13. Paula and Kirsten work at the same restaurant, but Paula works one quarter the hours that Kirsten does. If Paula works 9 h each week, how many hours does Kirsten work? Check your answer.

Apply

14. Show whether or not $x = 3$ is the solution to each equation.

a) $8x = 24$ b) $10x = 30$

c) $7x = 35$ d) $48 = 12x$

15. Show whether or not $y = 8$ is the solution to each equation.

a) $1 = \frac{y}{8}$ b) $\frac{y}{4} = 16$

c) $4 = \frac{y}{2}$ d) $\frac{y}{2} = 16$

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Common Errors

- Some students may forget to apply the same operation to both sides of the equal sign.

R_x Have students use words and diagrams to explain their solution. As they do, reinforce applying the same operation to both sides of the equation.

Key Ideas

This section emphasizes using different methods to solve equations.

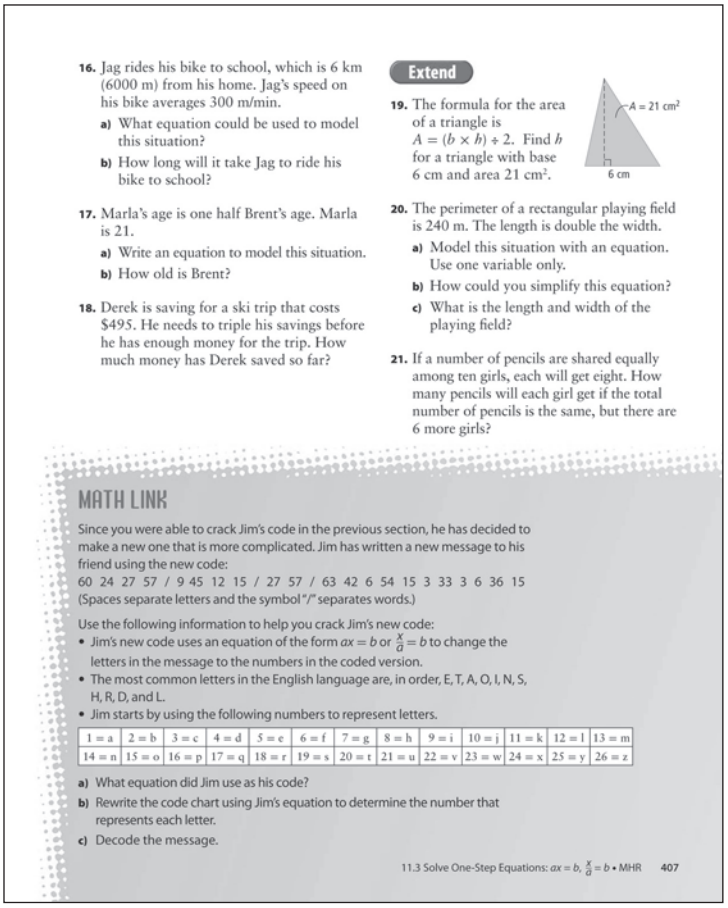
Communicate the Ideas

Have students work in groups and explain their answers orally.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students work individually on #1 and #3, and do #2 in pairs.	• For #1, have students explain each step after they solve the equation. • For #2, students explain using the opposite operation to solve equations. Have them explain the method and then work in pairs to solve two equations using this method. • For #3, students analyse the solution to a problem. • As you circulate, listen to students' explanations and assess how well they understand different ways to solve a given problem.

Practise

Encourage students to use counters and algebra tiles to help solve #4 and #5.



16. Jag rides his bike to school, which is 6 km (6000 m) from his home. Jag's speed on his bike averages 300 m/min.
   a) What equation could be used to model this situation?
   b) How long will it take Jag to ride his bike to school?

17. Marla's age is one half Brent's age. Marla is 21.
   a) Write an equation to model this situation.
   b) How old is Brent?

18. Derek is saving for a ski trip that costs $495. He needs to triple his savings before he has enough money for the trip. How much money has Derek saved so far?

Extend

19. The formula for the area of a triangle is $A = (b \times h) \div 2$. Find h for a triangle with base 6 cm and area 21 cm^2.

20. The perimeter of a rectangular playing field is 240 m. The length is double the width.
   a) Model this situation with an equation. Use one variable only.
   b) How could you simplify this equation?
   c) What is the length and width of the playing field?

21. If a number of pencils are shared equally among ten girls, each will get eight. How many pencils will each girl get if the total number of pencils is the same, but there are 6 more girls?

MATH LINK

Since you were able to crack Jim's code in the previous section, he has decided to make a new one that is more complicated. Jim has written a new message to his friend using the new code:

60 24 27 57 / 9 45 12 15 / 27 57 / 63 42 6 54 15 3 33 3 6 36 15

(Spaces separate letters and the symbol "/" separates words.)

Use the following information to help you crack Jim's new code:
- Jim's new code uses an equation of the form $ax = b$ or $\frac{x}{a} = b$ to change the letters in the message to the numbers in the coded version.
- The most common letters in the English language are, in order, E, T, A, O, I, N, S, H, R, D, and L.
- Jim starts by using the following numbers to represent letters.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g	8 = h	9 = i	10 = j	11 = k	12 = l	13 = m
14 = n	15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u	22 = v	23 = w	24 = x	25 = y	26 = z

   a) What equation did Jim use as his code?
   b) Rewrite the code chart using Jim's equation to determine the number that represents each letter.
   c) Decode the message.

Answers

Math Link

a) $3x = b$

b) 3 = a, 6 = b, 9 = c, 12 = d, 15 = e, 18 = f, 21 = g, 24 = h, 27 = i, 30 = j, 33 = k, 36 = l, 39 = m, 42 = n, 45 = o, 48 = p, 51 = q, 54 = r, 57 = s, 60 = t, 63 = u, 66 = v, 69 = w, 72 = x, 75 = y, 78 = z

c) This code is unbreakable.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • What are some ways you can use to solve equations? • How can you check your solution once you have solved the equation?	• Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • You may wish to have students review the part related to Section 11.3 in **BLM 11–1 Chapter 11 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Assessment *for* Learning	Supported Learning
Practise Have students do #4, #6, #8, #10, and #11. Students who have no problems with these questions should do #13 and then go on to the Apply questions.	• Students who have problems with #4 or #6 will need additional coaching. Review Example 1, coach them through corrections, and then have them complete #5 and #7 on their own. • Students who have problems with #8 and #10 will need additional coaching. Review Example 2, coach them through corrections, and then have them complete #9 on their own. • Students who have problems with #11 will need additional coaching. Review Example 3, coach them through corrections, and then have them complete #12 on their own. • Check back with students several times to make sure that they understand the concepts.

Apply and Extend

Most students will be able to do #20.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 407 leads toward the chapter problem wrap-up titled Wrap It Up! on page 417.	• All students should do this Math Link. • As you circulate, coach students who seem confused. Pair them with someone who has cracked the first letter and knows how to use that to crack the code. • Students who are having difficulty could use **BLM 11–8 Section 11.3 Math Link**, which provides scaffolding.

MATH LINK

The Math Link provides information to help students crack a code, including how to use equations to change letters into numbers, and examples of numbers used to represent letters.

11.4 Solve Two-Step Equations: $ax + b = c$

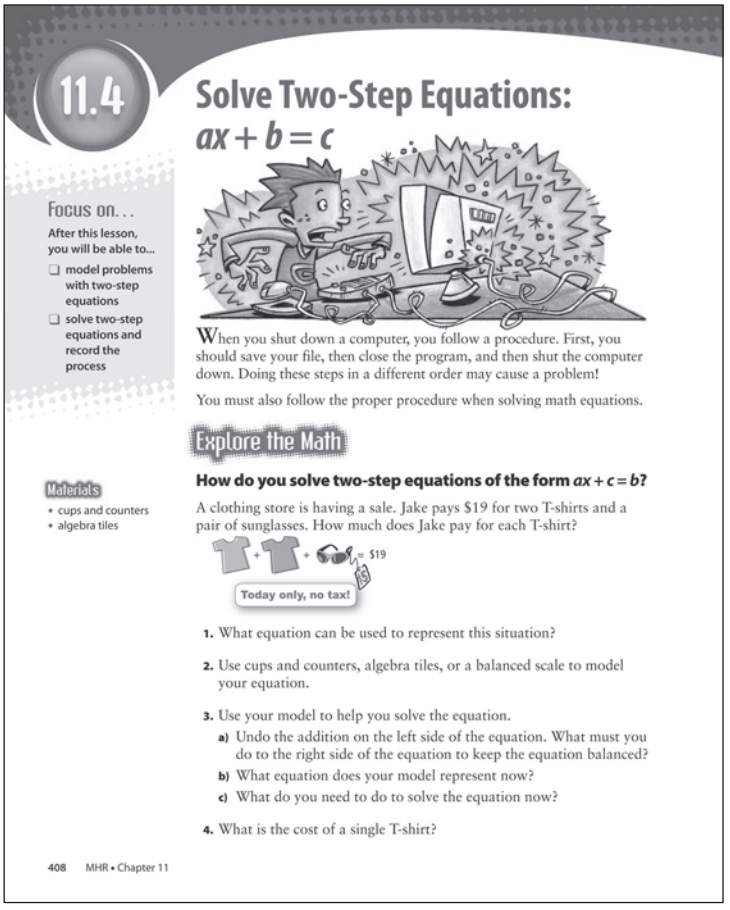
Suggested Timing
80–100 minutes

Materials
- cups and counters
- algebra tiles
- 2-pan balance with masses, including the sizes needed in Example 1
- blocks

Blackline Masters
BLM 11–1 Chapter 11 Self-Assessment
BLM 11–9 Section 11.4 Extra Practice
BLM 11–10 Section 11.4 Math Link

Mathematical Processes

- [ ] Communication
- [x] Connections
- [ ] Mental Mathematics and Estimation
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



11.4 Solve Two-Step Equations: $ax + b = c$

Focus on...
After this lesson, you will be able to...
- model problems with two-step equations
- solve two-step equations and record the process

When you shut down a computer, you follow a procedure. First, you should save your file, then close the program, and then shut the computer down. Doing these steps in a different order may cause a problem!

You must also follow the proper procedure when solving math equations.

Explore the Math

Materials
- cups and counters
- algebra tiles

How do you solve two-step equations of the form $ax + c = b$?

A clothing store is having a sale. Jake pays $19 for two T-shirts and a pair of sunglasses. How much does Jake pay for each T-shirt?

Today only, no tax!

1. What equation can be used to represent this situation?
2. Use cups and counters, algebra tiles, or a balanced scale to model your equation.
3. Use your model to help you solve the equation.
   a) Undo the addition on the left side of the equation. What must you do to the right side of the equation to keep the equation balanced?
   b) What equation does your model represent now?
   c) What do you need to do to solve the equation now?
4. What is the cost of a single T-shirt?

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Specific Outcomes

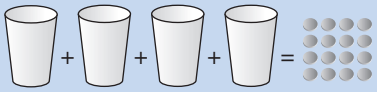
PR7 Model and solve problems that can be represented by linear equations of the form:

• $ax + b = c$ • $ax = b$ • $\frac{x}{a} = b$, a $\neq 0$

concretely, pictorially and symbolically, where a, b and c are whole numbers.

Warm-Up

Use the following model to answer #1 to #3.



1. Write the model as an equation.
2. Identify the variables, numerical coefficients, and constants in the equation for #1.
3. Solve the equation.
4. Draw a line segment 10 cm long. Draw another line perpendicular to it.
5. Estimate and then calculate 24×9.78.

Mental Math

For #6 to #8, put the decimal point in the correct place in the answer without calculating. Show your reasoning.

6. $1.56 + 2.34 + 0.567 = 4467$
7. $35.89 - 12.256 = 23634$
8. $3.7 \times 15.2 = 5624$
9. Show $\frac{1}{8}$ as a decimal and a percent.
10. Estimate 45 out of 92 as a percent.

Reflect on Your Findings

5. a) What two mathematical operations did you use to solve the equation?

b) Why did you use these operations when you did?

c) What is the relationship between the order of operations used to evaluate an expression and the order you used to solve the equation?

Example 1: Model Equations

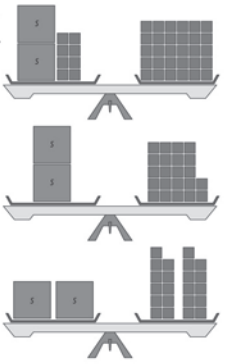
Maurie saw this sign advertising T-shirts and socks. He pays $30 for two T-shirts and four pairs of socks. What is the price of one T-shirt?

Solution

Let s represent the cost of one T-shirt.

The cost of the socks is $4 \times \$2$, or $8.

The equation that represents this situation is $2s + 8 = 30$.



Strategies
Model It
Refer to page xvi.

To isolate the variable, first remove the eight blocks on the left side of the scale. To keep the scale balanced you must remove the same number of blocks from the right side of the scale.

There are two s-variable blocks on the left side of the scale. There are 22 blocks on the right side of the scale. For the scale to balance, each s-variable block must have a mass of 11 blocks.

The cost of one T-shirt is $11.

Check:

Left Side $= 2s + 8$
$= 2(11) + 8$
$= 22 + 8$
$= 30$
Left Side = Right Side

Right Side $= 30$

The answer is correct.

Activity Planning Notes

This section focuses on following the proper procedure when solving math equations. As a class, review the steps to shut down a computer. Discuss why it is important to follow these steps in order (e.g., if you don't follow the order, you might lose data). Ask students if they know why it is important to follow the steps for solving an equation (e.g., otherwise the solution will be incorrect).

Explore the Math

Have students work in groups of three. Have each student model the same problem in a different way (e.g., cups and counters, algebra tiles, balance). Have students explain to the group why they modelled the problem the way they did. Ask students to identify the common aspects of the models (e.g., variables, constants, coefficients). As a class, generalize the findings into procedures for solving equations. Ensure students understand how to apply reverse order of operations.

Answers

Warm-Up

1. $4c = 16$

2. variable: c; numerical coefficient: 4; constant: 16

3. $4c \div 4 = 16 \div 4$; $c = 4$

4. Check that the line segment is the correct length within 2 mm, and that the second line is at 90° to it.

5. Estimate: $24 \times 10 = 240$. Calculate: $24 \times 9.78 = 234.72$

6. $2 + 2 + 1 = 5$; 4.467

7. $36 - 12 = 24$; 23.634

8. $4 \times 15 = 60$; 56.24

9. 0.125, 12.5%

10. 50% of $92 = 46$; close to 50%

Explore the Math

1. $2x + 5 = 19$

2. Models will vary.

3. a) Subtract 5. **b)** $2x = 14$ **c)** Divide by 2.

4. $7

5. a) subtract, divide

b) To isolate the term with x in it on the left side, the constant 5 had to be subtracted first from both sides of the equation. Then x could be isolated by dividing by 2.

c) The order of operations for solving an equation is the reverse order used to evaluate an expression.

Supported Learning

Meeting the Needs of All Learners

- Use visuals, models, and concrete materials to reinforce the concepts and ensure understanding. Encourage talking between you and students, and between students and peers. Remember to use proper terms and complete explanations to help them understand new concepts.
- Emphasize that two steps are required to solve the problems in this section. Provide several problems similar to those in Explore the Math and have students identify the two steps needed to solve each problem. Include questions that use cups and coins, equations such as $3x + 8 = 17$, and word problems.

Answers

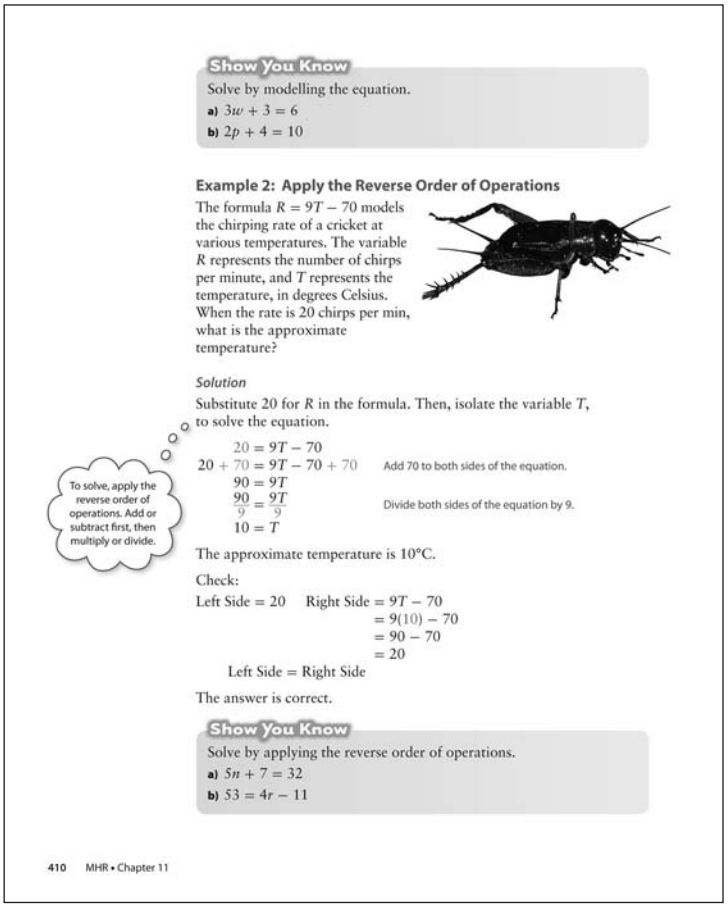
Show You Know: Example 1

a) $w = 1$ **b)** $p = 3$

Show You Know: Example 2

a) $n = 5$ **b)** $r = 16$

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss their conclusions.	• Check that students have written the equation to represent the situation accurately. • Have students discuss their conclusions. Ask the following questions: – How do you know which operation to use to solve the equation? – What order of operations should you follow when solving equations?



Show You Know

Solve by modelling the equation.

a) $3w + 3 = 6$

b) $2p + 4 = 10$

Example 2: Apply the Reverse Order of Operations

The formula $R = 9T - 70$ models the chirping rate of a cricket at various temperatures. The variable R represents the number of chirps per minute, and T represents the temperature, in degrees Celsius. When the rate is 20 chirps per min, what is the approximate temperature?

Solution

Substitute 20 for R in the formula. Then, isolate the variable T, to solve the equation.

To solve, apply the reverse order of operations. Add or subtract first, then multiply or divide.

$20 = 9T - 70$

$20 + 70 = 9T - 70 + 70$ Add 70 to both sides of the equation.

$90 = 9T$

$\frac{90}{9} = \frac{9T}{9}$ Divide both sides of the equation by 9.

$10 = T$

The approximate temperature is 10°C.

Check:

Left Side = 20 Right Side $= 9T - 70$

$= 9(10) - 70$

$= 90 - 70$

$= 20$

Left Side = Right Side

The answer is correct.

Show You Know

Solve by applying the reverse order of operations.

a) $5n + 7 = 32$

b) $53 = 4r - 11$

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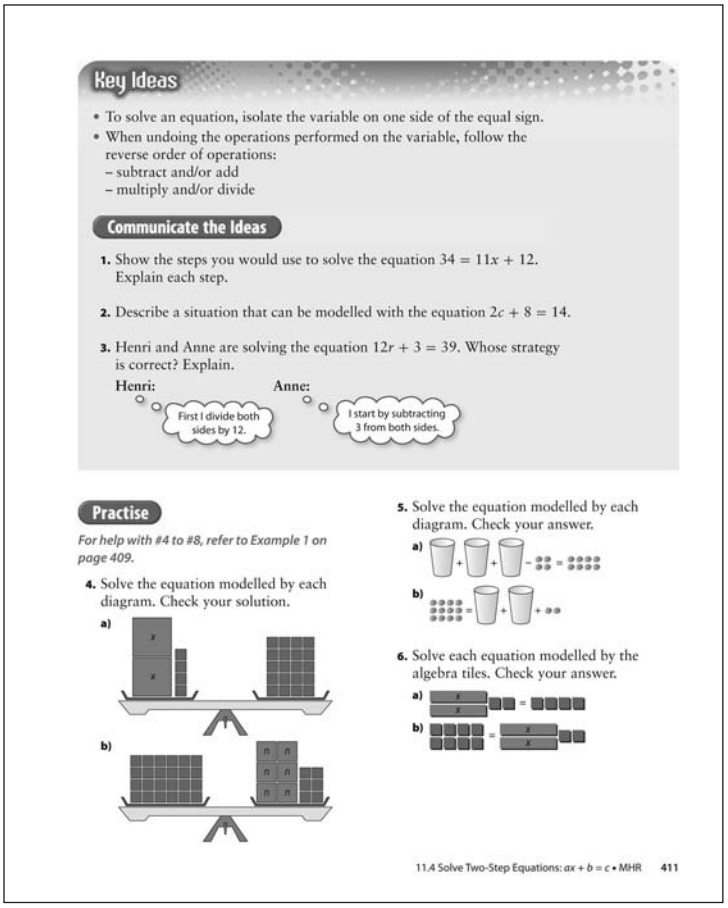
Example 1 shows students how to model equations using a balance. If possible, have students use a balance to work through the solution to the equation. Remind students to verify their solution.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• You may wish to provide additional questions to students who would benefit from them: Solve by modelling the equation. Verify your answer. **a)** $4t + 5 = 29$ (Use a balance or cups and counters to model the equation. Isolate the variable by removing 5 unit blocks or counters on the left side. Remove the same number of blocks or counters from the right side. $t = 6$. Check: Left Side $= 4t + 5 = 4(6) + 5 = 24 + 5 = 29$. Right Side $= 29$. Left Side $=$ Right Side.) **b)** $5t + 10 = 20$ (Use a balance or cups and counters to model the equation. Isolate the variable by removing 10 unit blocks or counters on the left side. Remove the same number of blocks or counters from the right side. $t = 2$. Check: Left Side $= 5t + 10 = 5(2) + 10 = 10 + 10 = 20$. Right Side $= 20$. Left Side $=$ Right Side.) Sit down and coach students through a), and then have them try b) on their own.



Key Ideas

- To solve an equation, isolate the variable on one side of the equal sign.
- When undoing the operations performed on the variable, follow the reverse order of operations:
  – subtract and/or add
  – multiply and/or divide

Communicate the Ideas

1. Show the steps you would use to solve the equation $34 = 11x + 12$. Explain each step.
2. Describe a situation that can be modelled with the equation $2c + 8 = 14$.
3. Henri and Anne are solving the equation $12r + 3 = 39$. Whose strategy is correct? Explain.

Henri: First I divide both sides by 12.

Anne: I start by subtracting 3 from both sides.

Practise

For help with #4 to #8, refer to Example 1 on page 409.

4. Solve the equation modelled by each diagram. Check your solution.

a)

b)

5. Solve the equation modelled by each diagram. Check your answer.

a)

b)

6. Solve each equation modelled by the algebra tiles. Check your answer.

a)

b)

11.4 Solve Two-Step Equations: $ax + b = c$ • MHR 411

Answers

Communicate the Ideas

1.

$$34 = 11x + 12$$
$$34 - 12 = 11x + 12 - 12$$ Subtract 12 from both sides of the equation.
$$22 = 11x$$
$$\frac{22}{11} = \frac{11x}{11}$$ Divide both sides of the equation by 11.
$$2 = x$$

2. Answers may vary. For example: Garr went to the store to purchase two comic books and a roll of film that costs $8. He spent $14 altogether.

3. Anne's strategy is correct. Answers may vary. For example: When solving an equation, the order of operations to form the equation is reversed, so subtraction must occur before division.

Example 2 demonstrates how to apply the reverse order of operations. Make sure students understand that they must add or subtract first, and then multiply or divide.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2 on page 410.	• You may wish to provide additional questions to students who would benefit from them: Solve the equation by applying the reverse order of operations. Verify your answer. **a)** $5x + 12 = 57$ ($5x + 12 - 12 = 57 - 12$; $5x = 45$; $5x \div 5 = 45 \div 5$; $x = 9$. Check: Left Side: $5x + 12 = 5(9) + 12 = 45 + 12 = 57$. Right Side $= 57$. Right Side $=$ Left Side. The answer is correct.) **b)** $55 = 8t - 9$ ($55 + 9 = 8t - 9 + 9$; $64 = 8t$; $64 \div 8 = 8t \div 8$; $8 = t$. Check: Left Side $= 55$. Right Side $= 8(8) = 64$. Right Side $=$ Left Side. The answer is correct.) Sit down and coach students through a), and then have them try b) on their own.

Key Ideas

The Key Ideas emphasize solving equations by isolating the variable. Another focus is on following the order of operations.

Communicate the Ideas

Have students work in groups. Encourage them to explain their answers orally to each other.

Supported Learning

Learning Style, ESL, and Language

- Encourage students to share their understanding of the Key Ideas.

Learning Style and Memory

- Encourage students to use concrete models of the equations such as cups and counters, algebra tiles, or a balance with masses.

Memory

- Have students use their own words to summarize the steps for solving two-step equations. Consider allowing students to use their summary to answer questions throughout the chapter.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 2 *or* 3, 4, 6, 7, 9–11, 14, 15, Math Link
Typical	1, 2 *or* 3, 4, 6, 7, 9–11, 14–19, Math Link
Extension/Enrichment	1, 2 *or* 3, 16–21, Math Link

Supported Learning

Learning Style and Memory

- Provide **BLM 11–8 Section 11.4 Extra Practice** to students who require more practice.

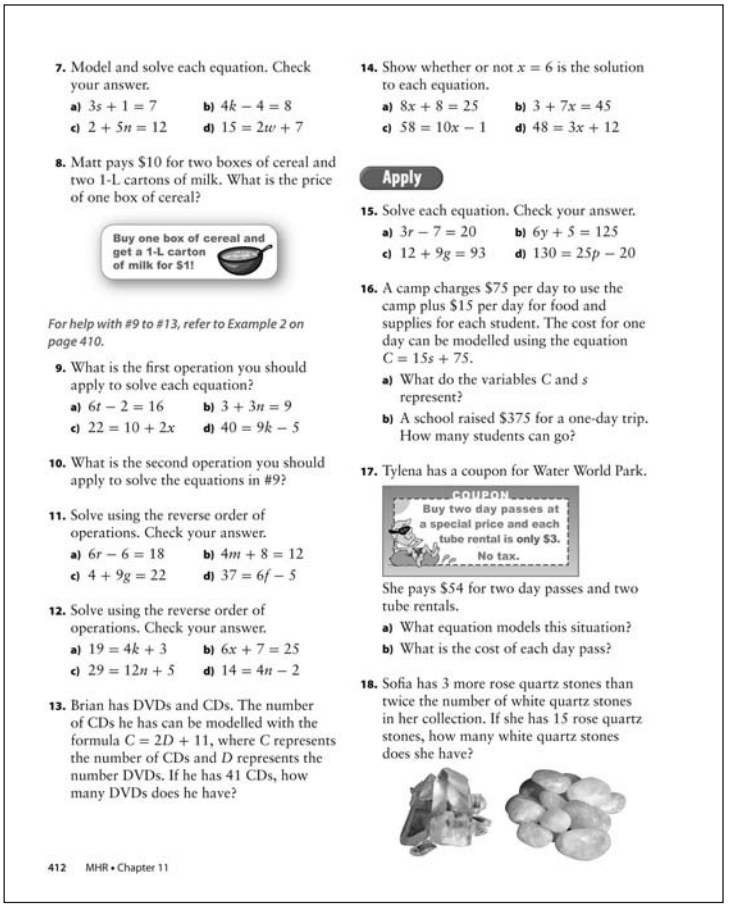
Common Errors

- Some students may forget to apply the same operation on both sides of the equal sign.

R$_x$ Have students identify the operations used in the original equation, the required reverse operations, and the order of operations, and then solve the equation.

- Some students may mix up the signs or operation (e.g., multiplying on one side of the equal sign while dividing on the other).

R$_x$ Have students describe their solution processes using diagrams (e.g., of cups and counters) to reinforce their understanding that equal numbers must be multiplied or divided on both sides of the equation.



7. Model and solve each equation. Check your answer.
a) $3s + 1 = 7$ **b)** $4k - 4 = 8$
c) $2 + 5n = 12$ **d)** $15 = 2w + 7$

8. Matt pays $10 for two boxes of cereal and two 1-L cartons of milk. What is the price of one box of cereal?

For help with #9 to #13, refer to Example 2 on page 410.

9. What is the first operation you should apply to solve each equation?
a) $6t - 2 = 16$ **b)** $3 + 3n = 9$
c) $22 = 10 + 2x$ **d)** $40 = 9k - 5$

10. What is the second operation you should apply to solve the equations in #9?

11. Solve using the reverse order of operations. Check your answer.
a) $6r - 6 = 18$ **b)** $4m + 8 = 12$
c) $4 + 9g = 22$ **d)** $37 = 6f - 5$

12. Solve using the reverse order of operations. Check your answer.
a) $19 = 4k + 3$ **b)** $6x + 7 = 25$
c) $29 = 12n + 5$ **d)** $14 = 4n - 2$

13. Brian has DVDs and CDs. The number of CDs he has can be modelled with the formula $C = 2D + 11$, where C represents the number of CDs and D represents the number DVDs. If he has 41 CDs, how many DVDs does he have?

14. Show whether or not $x = 6$ is the solution to each equation.
a) $8x + 8 = 25$ **b)** $3 + 7x = 45$
c) $58 = 10x - 1$ **d)** $48 = 3x + 12$

Apply

15. Solve each equation. Check your answer.
a) $3r - 7 = 20$ **b)** $6y + 5 = 125$
c) $12 + 9g = 93$ **d)** $130 = 25p - 20$

16. A camp charges $75 per day to use the camp plus $15 per day for food and supplies for each student. The cost for one day can be modelled using the equation $C = 15s + 75$.
a) What do the variables C and s represent?
b) A school raised $375 for a one-day trip. How many students can go?

17. Tylena has a coupon for Water World Park.

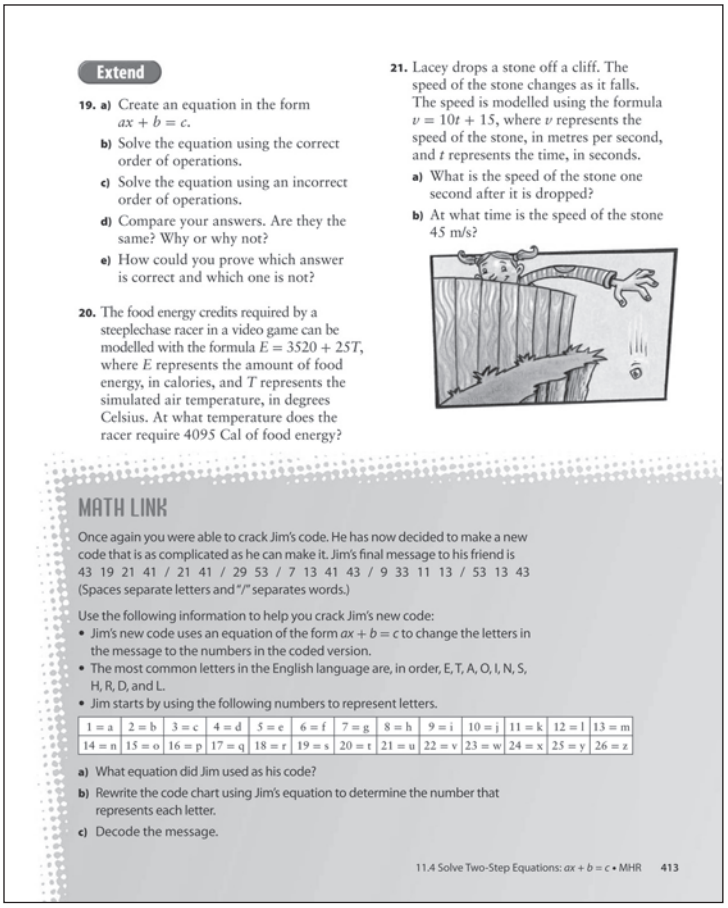
She pays $54 for two day passes and two tube rentals.
a) What equation models this situation?
b) What is the cost of each day pass?

18. Sofia has 3 more rose quartz stones than twice the number of white quartz stones in her collection. If she has 15 rose quartz stones, how many white quartz stones does she have?

Assessment *as* Learning	Supported Learning
Communicate the Ideas Consider assigning #1 to all students and having them choose one other question to answer.	• In #1, students explain each step in solving an equation. • In #2, students describe a situation that can be modelled with an equation. Have students share their ideas with others. • In #3, students analyse a solution to determine if it is correct or not. Students demonstrate an understanding of the order of operations if they correctly answer this question. • As you circulate, listen for student explanations and assess whether they have a basic understanding or one that provides multiple ways to solve the given problem.

Practise

Assessment *for* Learning	Supported Learning
Practise Have students do #4, #6, #7, and # 9 to #11. Students who have no problems with these questions can complete #14 and then go on to the Apply questions.	• Students who have problems with #4 or #6 will need additional coaching with Example 1. Review this material with them, coach them through corrections to their answers, and then have them complete #5 and #8 on their own. • Students who have problems with #9 to #11 will need additional coaching with Example 2. Review this material with them, coach them through corrections to their answers, and then have them complete #12 to #14 on their own. • Check back with students several times to make sure that they understand the concepts.



Extend

19. a) Create an equation in the form $ax + b = c$.
b) Solve the equation using the correct order of operations.
c) Solve the equation using an incorrect order of operations.
d) Compare your answers. Are they the same? Why or why not?
e) How could you prove which answer is correct and which one is not?

20. The food energy credits required by a steeplechase racer in a video game can be modelled with the formula $E = 3520 + 25T$, where E represents the amount of food energy, in calories, and T represents the simulated air temperature, in degrees Celsius. At what temperature does the racer require 4095 Cal of food energy?

21. Lacey drops a stone off a cliff. The speed of the stone changes as it falls. The speed is modelled using the formula $v = 10t + 15$, where v represents the speed of the stone, in metres per second, and t represents the time, in seconds.
a) What is the speed of the stone one second after it is dropped?
b) At what time is the speed of the stone 45 m/s?

MATH LINK

Once again you were able to crack Jim's code. He has now decided to make a new code that is as complicated as he can make it. Jim's final message to his friend is
43 19 21 41 / 21 41 / 29 53 / 7 13 41 43 / 9 33 11 13 / 53 13 43
(Spaces separate letters and "/" separates words.)

Use the following information to help you crack Jim's new code:
- Jim's new code uses an equation of the form $ax + b = c$ to change the letters in the message to the numbers in the coded version.
- The most common letters in the English language are, in order, E, T, A, O, I, N, S, H, R, D, and L.
- Jim starts by using the following numbers to represent letters.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g	8 = h	9 = i	10 = j	11 = k	12 = l	13 = m
14 = n	15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u	22 = v	23 = w	24 = x	25 = y	26 = z

a) What equation did Jim used as his code?
b) Rewrite the code chart using Jim's equation to determine the number that represents each letter.
c) Decode the message.

11.4 Solve Two-Step Equations: $ax + b = c$ • MHR 413

Apply and Extend

You may wish to change the prices in #17 to reflect those at a local attraction. Students may find it useful to model the scenario in #18. Many students will be able to do #19 alone, but you may wish to have students work in pairs on this question.

Assessment *for* Learning	Supported Learning
Math Link The Math Link on page 413 is intended to allow students to solve equations to crack a code, which is part of the chapter problem wrap-up titled Wrap It Up! on page 417.	• It is recommended for all students to do this Math Link. You may wish to have students who experienced difficulty in the previous Math Link(s) work with a partner. • Have students use **BLM 11–10 Section 11.4 Math Link** to assist them with completing the activity.

Answers

Math Link

a) $2x + 3 = c$

b) 5 = a, 7 = b, 9 = c, 11 = d, 13 = e, 15 = f, 17 = g, 19 = h, 21 = i, 23 = j, 25 = k, 27 = l, 29 = m, 31 = n, 33 = o, 35 = p, 37 = q, 39 = r, 41 = s, 43 = t, 45 = u, ÷47 = v, 49 = w, 51 = x, 53 = y, 55 = z

c) This is my best code yet.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • What is the first step when solving an equation? • What is the order of operations when solving an equation?	• Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Keep a record of student reflections in their learning portfolio. You may wish to have them return to these reflections at the end of the chapter. • You may also wish to have students review the part related to Section 11.4 in **BLM 11–1 Chapter 11 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

MATH LINK

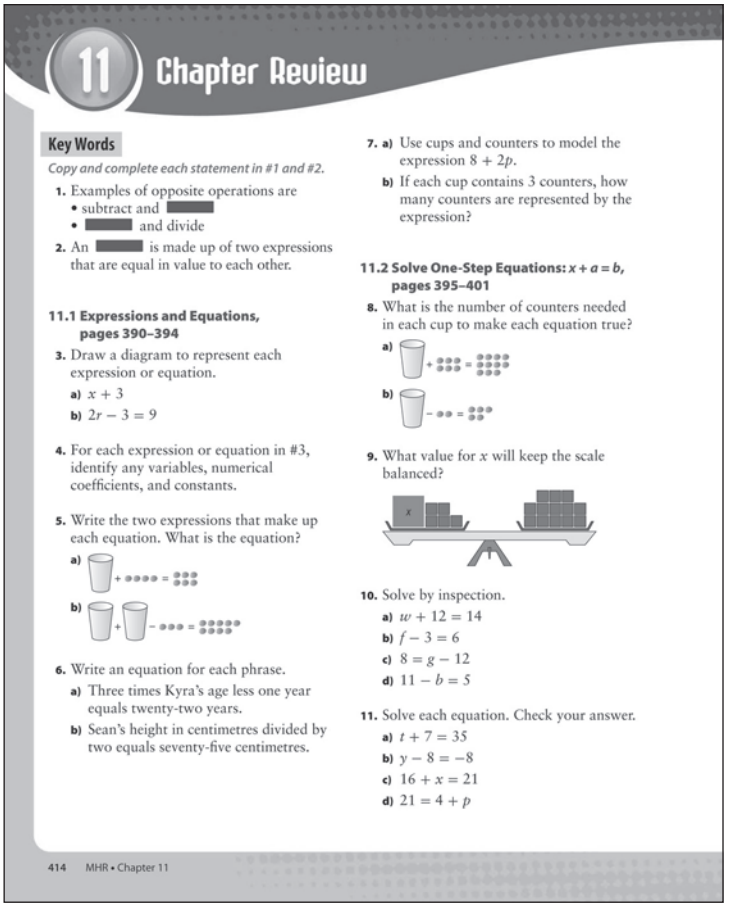
The Math Link supplies students with information to help them crack the code, including how to use equations to change letters into numbers, and examples of numbers used to represent letters.

11 Chapter Review

Suggested Timing
40–50 minutes

Materials
- cups and counters

Blackline Masters
BLM 11–1 Chapter 11 Self-Assessment
BLM 11–3 Section 11.1 Extra Practice
BLM 11–5 Section 11.2 Extra Practice
BLM 11–7 Section 11.3 Extra Practice
BLM 11–9 Section 11.4 Extra Practice



11 Chapter Review

Key Words

Copy and complete each statement in #1 and #2.

1. Examples of opposite operations are
   - subtract and [redacted]
   - [redacted] and divide
2. An [redacted] is made up of two expressions that are equal in value to each other.

11.1 Expressions and Equations, pages 390–394

3. Draw a diagram to represent each expression or equation.
   a) $x + 3$
   b) $2r - 3 = 9$
4. For each expression or equation in #3, identify any variables, numerical coefficients, and constants.
5. Write the two expressions that make up each equation. What is the equation?
   a)
   b)
6. Write an equation for each phrase.
   a) Three times Kyra's age less one year equals twenty-two years.
   b) Sean's height in centimetres divided by two equals seventy-five centimetres.
7. a) Use cups and counters to model the expression $8 + 2p$.
   b) If each cup contains 3 counters, how many counters are represented by the expression?

11.2 Solve One-Step Equations: $x + a = b$, pages 395–401

8. What is the number of counters needed in each cup to make each equation true?
   a)
   b)
9. What value for x will keep the scale balanced?

10. Solve by inspection.
    a) $w + 12 = 14$
    b) $f - 3 = 6$
    c) $8 = g - 12$
    d) $11 - b = 5$
11. Solve each equation. Check your answer.
    a) $t + 7 = 35$
    b) $y - 8 = -8$
    c) $16 + x = 21$
    d) $21 = 4 + p$

414 MHR • Chapter 11

Supported Learning

Learning Style and Memory

- Students who require more practice on a particular topic may refer to **BLM 11–3 Section 11.1 Extra Practice**, **BLM 11–5 Section 11.2 Extra Practice**, **BLM 11–7 Section 11.3 Extra Practice**, and **BLM 11–9 Section 11.4 Extra Practice**.

Learning Style

- Allow students to complete the chapter review using any combination of verbal description, diagrams for explanation, and written answers.
- Allow students to use cups and counters to model the expressions and equations.

ESL, Language, and Memory

- As students study, have them create their own vocabulary/ picture dictionary in their notebook. Matching a picture or symbol with each definition helps students consolidate their understanding of the vocabulary.
- Allow students to practise the vocabulary terms using flash cards. Have students work together to quiz each other on the Key Words for the chapter. Consider allowing students to use their Foldable notes during the practice test.

Gifted and Enrichment

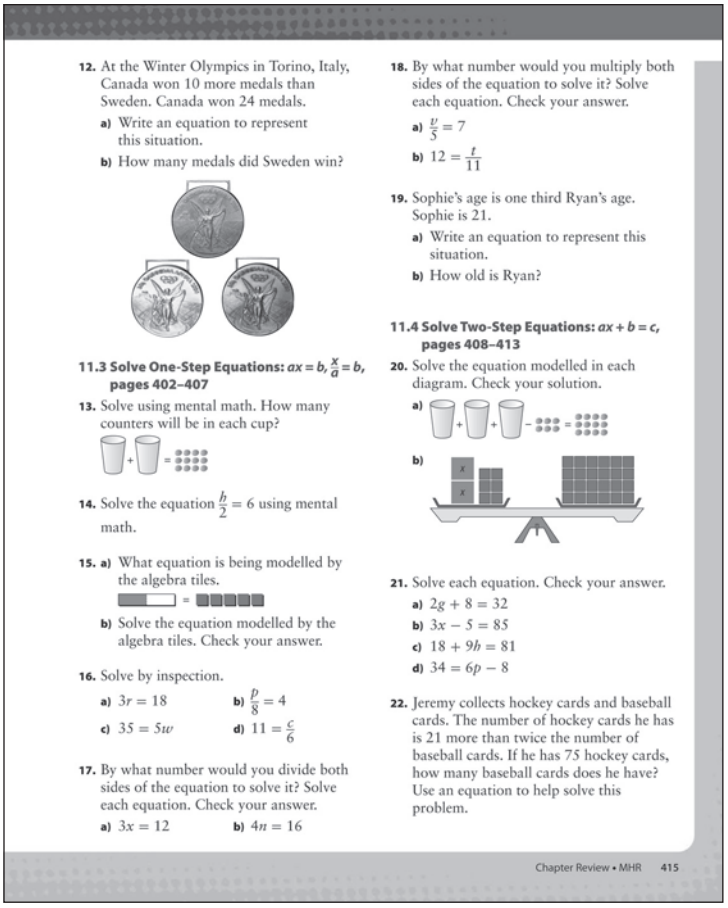
- Students may already be familiar with the skills handled in this review. To provide additional questions, go to **www.mathlinks7.ca** and follow the links.

Activity Planning Notes

Have students work independently or in pairs to complete the review questions. If students encounter difficulties, they could discuss strategies with other students.

Have students write the numbers from 3 to 22 in two columns in their notebooks. Then have them scan the chapter review question related to each number. Have students use the colours that they used on **BLM 11–1 Chapter 11 Self-Assessment** to circle the questions they need help with or do not yet understand.

Assessment *for* Learning	Supported Learning
Chapter 11 Review The chapter review provides an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On tab of their chapter Foldable. Have students do at least one question related to any concept, skill, or process that has been giving them trouble. • Have students revisit any section they are having difficulty with prior to working on the Chapter 11 PracticeTest.



12. At the Winter Olympics in Torino, Italy, Canada won 10 more medals than Sweden. Canada won 24 medals.
a) Write an equation to represent this situation.
b) How many medals did Sweden win?

11.3 Solve One-Step Equations: $ax = b$, $\frac{x}{a} = b$, pages 402–407

13. Solve using mental math. How many counters will be in each cup?

14. Solve the equation $\frac{h}{2} = 6$ using mental math.

15. a) What equation is being modelled by the algebra tiles.

b) Solve the equation modelled by the algebra tiles. Check your answer.

16. Solve by inspection.
a) $3r = 18$ b) $\frac{p}{8} = 4$
c) $35 = 5w$ d) $11 = \frac{c}{6}$

17. By what number would you divide both sides of the equation to solve it? Solve each equation. Check your answer.
a) $3x = 12$ b) $4n = 16$

18. By what number would you multiply both sides of the equation to solve it? Solve each equation. Check your answer.
a) $\frac{v}{5} = 7$
b) $12 = \frac{t}{11}$

19. Sophie's age is one third Ryan's age. Sophie is 21.
a) Write an equation to represent this situation.
b) How old is Ryan?

11.4 Solve Two-Step Equations: $ax + b = c$, pages 408–413

20. Solve the equation modelled in each diagram. Check your solution.
a)

b)

21. Solve each equation. Check your answer.
a) $2g + 8 = 32$
b) $3x - 5 = 85$
c) $18 + 9h = 81$
d) $34 = 6p - 8$

22. Jeremy collects hockey cards and baseball cards. The number of hockey cards he has is 21 more than twice the number of baseball cards. If he has 75 hockey cards, how many baseball cards does he have? Use an equation to help solve this problem.

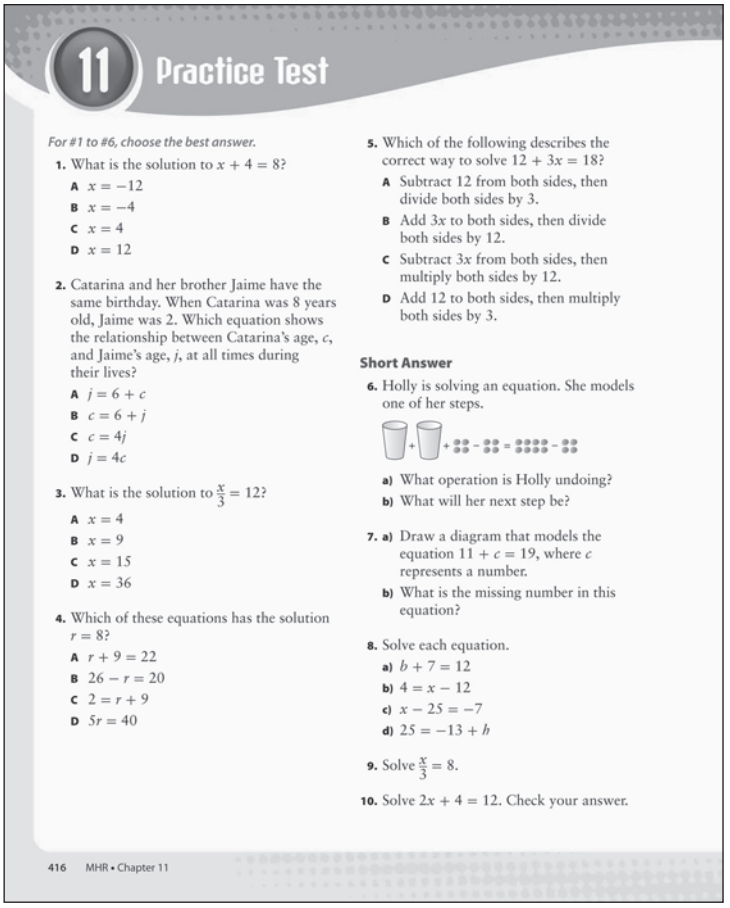
Assessment *as* Learning	Supported Learning
Math Learning Log Once students have completed the chapter review and prior to the Chapter 11 Test, have them reflect on their progress and complete a journal entry for each statement: – I am comfortable with the following parts of the chapter … – I am having difficulty with … – Here's how I worked on some of the areas I originally had difficulty with … – Here's how I plan to address the areas I am still having difficulty with …	• Have students refer back to the What I Need to Work On section of their chapter Foldable and answer these questions from the contents of that section. • You may wish to have students refer to **BLM 11–1 Chapter 11 Self-Assessment** when they report on what they are comfortable with, what they continue to have difficulty with, and what they plan to do about it.

11 Practice Test

Suggested Timing
40–50 minutes

Blackline Masters
BLM 11–1 Self-Assessment
BLM 11–11 Chapter 11 Test

Assessment *as* Learning	Supported Learning
Chapter 11 Self-Assessment Have students review their earlier responses on **BLM 11–1 Chapter 11 Self-Assessment**.	• Have students use their responses on the practice test and work they completed earlier in the chapter to complete the After column of this self-assessment. • Before students do the **BLM 11–11 Chapter 11 Test**, coach students in the areas in which they are having problems.



11 Practice Test

For #1 to #6, choose the best answer.

1. What is the solution to $x + 4 = 8$?
   - A $x = -12$
   - B $x = -4$
   - C $x = 4$
   - D $x = 12$

2. Catarina and her brother Jaime have the same birthday. When Catarina was 8 years old, Jaime was 2. Which equation shows the relationship between Catarina's age, c, and Jaime's age, j, at all times during their lives?
   - A $j = 6 + c$
   - B $c = 6 + j$
   - C $c = 4j$
   - D $j = 4c$

3. What is the solution to $\frac{x}{3} = 12$?
   - A $x = 4$
   - B $x = 9$
   - C $x = 15$
   - D $x = 36$

4. Which of these equations has the solution $r = 8$?
   - A $r + 9 = 22$
   - B $26 - r = 20$
   - C $2 = r + 9$
   - D $5r = 40$

5. Which of the following describes the correct way to solve $12 + 3x = 18$?
   - A Subtract 12 from both sides, then divide both sides by 3.
   - B Add $3x$ to both sides, then divide both sides by 12.
   - C Subtract $3x$ from both sides, then multiply both sides by 12.
   - D Add 12 to both sides, then multiply both sides by 3.

Short Answer

6. Holly is solving an equation. She models one of her steps.

   a) What operation is Holly undoing?
   b) What will her next step be?

7. a) Draw a diagram that models the equation $11 + c = 19$, where c represents a number.
   b) What is the missing number in this equation?

8. Solve each equation.
   a) $b + 7 = 12$
   b) $4 = x - 12$
   c) $x - 25 = -7$
   d) $25 = -13 + h$

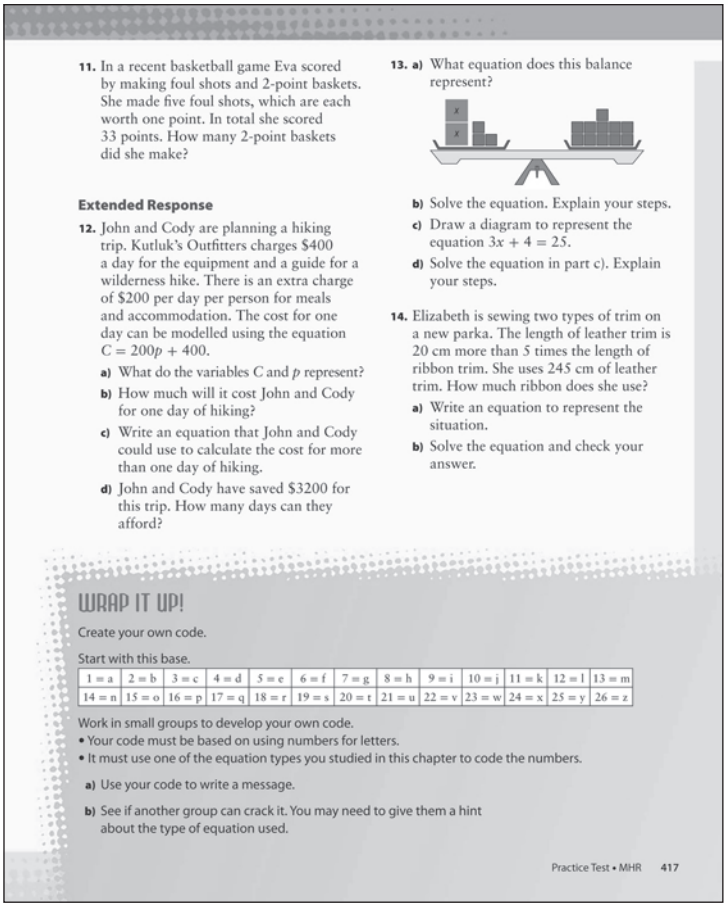
9. Solve $\frac{x}{3} = 8$.

10. Solve $2x + 4 = 12$. Check your answer.

416 MHR • Chapter 11

Study Guide

Question(s)	Section(s)	Refer to	I can …
1, 4, 7, 8	11.2	Examples 1, 2, 3	✓ model problems with equations ✓ solve equations and record the process ✓ verify solutions to equations
2	11.1	Examples 1, 2	✓ identify constants, coefficients, and variables in expressions and equations ✓ describe the difference between an expression and an equation
3, 9	11.3	Examples 1, 2, 3	✓ model problems with equations ✓ solve equations and record the process
5, 6, 10, 11, 13	11.4	Example 2	✓ model problems with two-step equations ✓ solve two-step equations and record the process
12, 14	11.1 11.4	Example 2 Example 2	✓ identify constants, coefficients, and variables in expressions and equations ✓ model problems with two-step equations ✓ solve two-step equations and record the process



11. In a recent basketball game Eva scored by making foul shots and 2-point baskets. She made five foul shots, which are each worth one point. In total she scored 33 points. How many 2-point baskets did she make?

Extended Response

12. John and Cody are planning a hiking trip. Kutluk's Outfitters charges $400 a day for the equipment and a guide for a wilderness hike. There is an extra charge of $200 per day per person for meals and accommodation. The cost for one day can be modelled using the equation $C = 200p + 400$.

a) What do the variables C and p represent?

b) How much will it cost John and Cody for one day of hiking?

c) Write an equation that John and Cody could use to calculate the cost for more than one day of hiking.

d) John and Cody have saved $3200 for this trip. How many days can they afford?

13. a) What equation does this balance represent?

b) Solve the equation. Explain your steps.

c) Draw a diagram to represent the equation $3x + 4 = 25$.

d) Solve the equation in part c). Explain your steps.

14. Elizabeth is sewing two types of trim on a new parka. The length of leather trim is 20 cm more than 5 times the length of ribbon trim. She uses 245 cm of leather trim. How much ribbon does she use?

a) Write an equation to represent the situation.

b) Solve the equation and check your answer.

WRAP IT UP!

Create your own code.

Start with this base.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g	8 = h	9 = i	10 = j	11 = k	12 = l	13 = m
14 = n	15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u	22 = v	23 = w	24 = x	25 = y	26 = z

Work in small groups to develop your own code.

- Your code must be based on using numbers for letters.
- It must use one of the equation types you studied in this chapter to code the numbers.

a) Use your code to write a message.

b) See if another group can crack it. You may need to give them a hint about the type of equation used.

Practice Test • MHR 417

Supported Learning

Learning Style and Memory

- As an alternative to a chapter test, consider using the oral presentation from Wrap It Up! to assess the knowledge and skills of students who have difficulty with tests.

ESL, Language, and Memory

- Consider allowing students to use their chapter Foldable during the practice test.

Activity Planning Notes

This practice test can be assigned as an in-class or take-home assignment. These are the minimum questions that will meet the related curriculum outcomes: #2, #6, #7, #9, #10, #12, and #13.

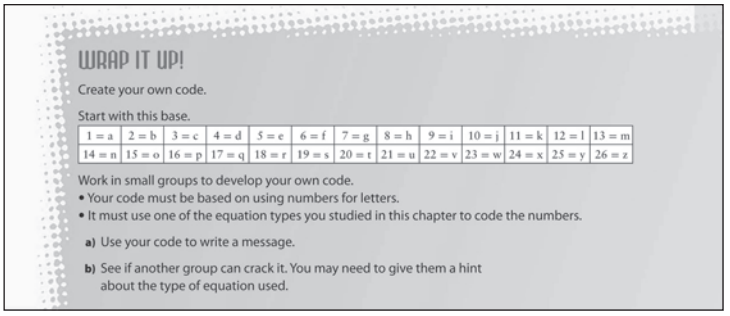
Answers to the Chapter 11 Practice Test are provided on **BLM 11–13 Chapter 11 *MathLinks 7* Student Resource Answers**.

Assessment *for* Learning	Supported Learning
Chapter 11 Test After students complete the practice test, you may wish to use **BLM 11–11 Chapter 11 Test** as a summative assessment.	• Consider allowing students to use their chapter Foldable. • Consider using the Math Games on page 418 or the Challenge in Real Life on page 419 to assess the knowledge and skills of students who have difficulty with tests.

Wrap It Up!

Suggested Timing
40–50 minutes

Blackline Masters
Master 1 Project Rubric
BLM 11–4 Section 11.1 Math Link
BLM 11–6 Section 11.2 Math Link
BLM 11–8 Section 11.3 Math Link
BLM 11–10 Section 11.4 Math Link
BLM 11–12 Chapter 11 Wrap It Up!



WRAP IT UP!

Create your own code.

Start with this base.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g	8 = h	9 = i	10 = j	11 = k	12 = l	13 = m
14 = n	15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u	22 = v	23 = w	24 = x	25 = y	26 = z

Work in small groups to develop your own code.
- Your code must be based on using numbers for letters.
- It must use one of the equation types you studied in this chapter to code the numbers.

a) Use your code to write a message.

b) See if another group can crack it. You may need to give them a hint about the type of equation used.

Specific Outcomes

PR7 Model and solve problems that can be represented by linear equations of the form:

- $ax + b = c$
- $ax = b$
- $\frac{x}{a} = b$, a $\neq 0$

concretely, pictorially and symbolically, where a, b and c are whole numbers.

Common Errors

- Some students may try to make their codes too complex.

R_x You may wish to limit students to the types of equations they studied in this chapter when developing their code.

Activity Planning Notes

Introduce the problem and clarify the assessment criteria.

Assessment *of* Learning	Supported Learning
Wrap It Up! This chapter problem wrap-up allows students to apply their knowledge of codes and the mathematical applications associated with the process. Have students use their code to write a message and have another group try to crack the code. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this Wrap It Up! Page 417a provides notes on how to use this rubric for this Wrap It Up!	• You may wish to have students who have had difficulty with the Math Links throughout the chapter work with a partner. • Ensure that students provide other groups with a hint about the type of equation to use when cracking the code. • You may wish students to review the Math Links they completed earlier by referring to **BLM 11–4 Section 11.1 Math Link**, **BLM 11–6 Section 11.2 Math Link**, **BLM 11–8 Section 11.3 Math Link**, and **BLM 11–10 Section 11.4 Math Link**. • Some students may benefit from using **BLM 11–12 Chapter 11 Wrap It Up!**, which provides scaffolding for the chapter problem wrap-up.

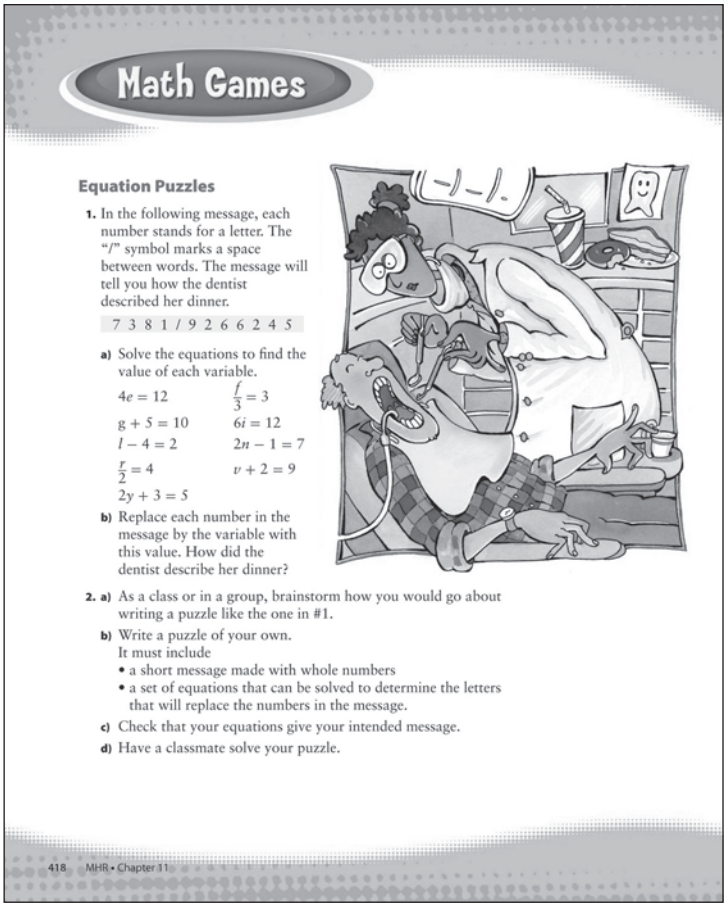
The chart below shows **Master 1 Project Rubric** for tasks such as that in the Wrap It Up!, and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 (Standard of Excellence)	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• correctly cracks the code and solves the message providing supporting work and equation
4 (Above Acceptable)	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• correctly cracks the code, solves the message, and states the equation, but does not provide supporting work
3 (Meets Acceptable)	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• identifies the message and an equation, but the equation is only a partial solution (contains an error); supporting work is shown for some of the calculations *or* • partially solves the message, but has the correct equation
2 (Below Acceptable)	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• through trial and error, identifies some values of the code correctly, but does not complete it
1 (Beginning)	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• identifies some of the values (one or two), but does not proceed any further

Math Games

Suggested Timing
40–50 minutes

Materials
- cups and counters
- algebra tiles


Math Games

Equation Puzzles

1. In the following message, each number stands for a letter. The "/" symbol marks a space between words. The message will tell you how the dentist described her dinner.

7 3 8 1 / 9 2 6 6 2 4 5

a) Solve the equations to find the value of each variable.

$4e = 12$ $\frac{f}{3} = 3$
$g + 5 = 10$ $6i = 12$
$l - 4 = 2$ $2n - 1 = 7$
$\frac{r}{2} = 4$ $v + 2 = 9$
$2y + 3 = 5$

b) Replace each number in the message by the variable with this value. How did the dentist describe her dinner?

2. a) As a class or in a group, brainstorm how you would go about writing a puzzle like the one in #1.
b) Write a puzzle of your own. It must include
- a short message made with whole numbers
- a set of equations that can be solved to determine the letters that will replace the numbers in the message.

c) Check that your equations give your intended message.
d) Have a classmate solve your puzzle.

418 MHR • Chapter 11

Supported Learning

ESL and Language

- Review terms such as *variable*, *solve*, and *equation*.

Gifted and Enrichment

- Encourage students to create a more complex puzzle, possibly using a computer.
- Have students research applications of secret codes in the real world.

Assessment *for* Learning	Supported Learning
Equation Puzzles Observe students as they solve the equations. Make sure that they keep both sides of the equation balanced.	• Use manipulatives to help students understand the concept of a balanced equation. • Have students verify the solutions.

Specific Outcomes

PR3 Demonstrate an understanding of preservation of equality by:
- modelling preservation of equality, concretely, pictorially and symbolically
- applying preservation of equality to solve equations.

PR5 Evaluate an expression given the value of the variable(s).

PR6 Model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially and symbolically, where a and b are integers.

PR7 Model and solve problems that can be represented by linear equations of the form:

- $ax + b = c$
- $ax = b$
- $\frac{x}{a} = b$, a $\neq 0$

concretely, pictorially and symbolically, where a, b and c are whole numbers.

Common Errors

- Some students may have trouble solving the equations.

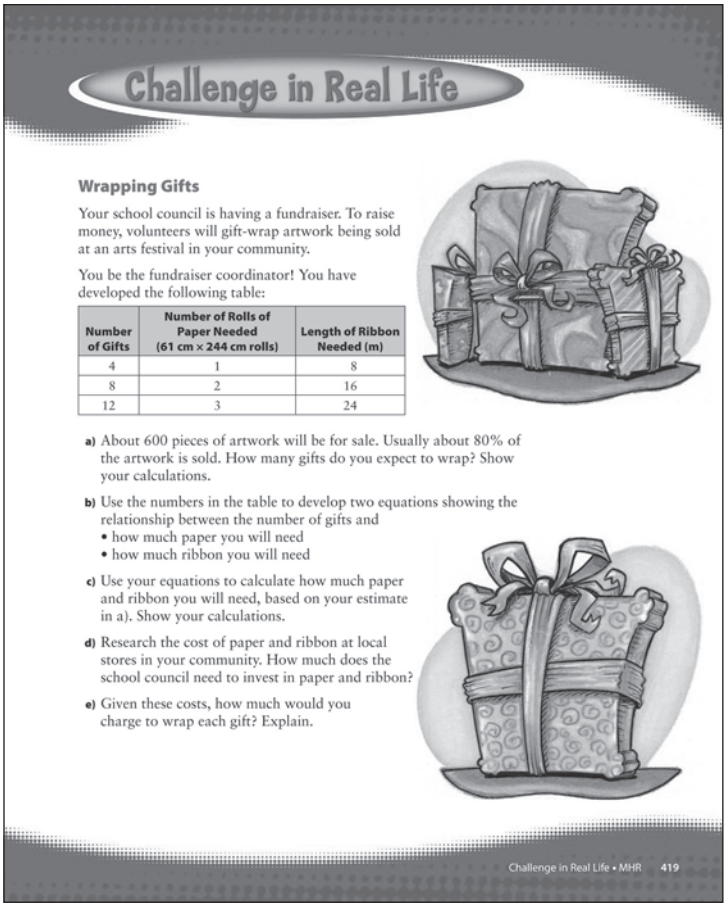
R_x Illustrate the process of solving equations using concrete materials.

Activity Planning Notes

Students should record every step in the equation-solving process. Encourage them not to skip the steps that show an operation being performed on each side of the equation.

For #2a), suggest that students work backward from the intended message. Discuss the following example:
- Suppose one letter in the message is t. Assign any value to t, e.g., 7.
- Replace t by 7 in the puzzle.
- Create an equation that includes 7 (e.g., $7 + 5 = 12$).
- Since $t = 7$, replace 7 in the equation with t (e.g., $t + 5 = 12$).

Challenge in Real Life



Challenge in Real Life

Wrapping Gifts

Your school council is having a fundraiser. To raise money, volunteers will gift-wrap artwork being sold at an arts festival in your community.

You be the fundraiser coordinator! You have developed the following table:

Number of Gifts	Number of Rolls of Paper Needed (61 cm × 244 cm rolls)	Length of Ribbon Needed (m)
4	1	8
8	2	16
12	3	24

a) About 600 pieces of artwork will be for sale. Usually about 80% of the artwork is sold. How many gifts do you expect to wrap? Show your calculations.

b) Use the numbers in the table to develop two equations showing the relationship between the number of gifts and
- how much paper you will need
- how much ribbon you will need

c) Use your equations to calculate how much paper and ribbon you will need, based on your estimate in a). Show your calculations.

d) Research the cost of paper and ribbon at local stores in your community. How much does the school council need to invest in paper and ribbon?

e) Given these costs, how much would you charge to wrap each gift? Explain.

Challenge in Real Life • MHR 419

Suggested Timing
60–75 minutes

Materials
- shoe box or similar box, wrapping paper, ribbon
- flyers and catalogues, phone numbers of local stores, or phone books (optional)
- calculator

Blackline Masters
Master 1 Project Rubric

Mathematical Processes

- ☑ Communication
- ☑ Connections
- ☑ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☑ Technology
- ☑ Visualization

Specific Outcomes

PR7 Model and solve problems that can be represented by linear equations of the form:

• $ax + b = c$ • $ax = b$ • $\frac{x}{a} = b$, a $\neq 0$

concretely, pictorially and symbolically, where a, b and c are whole numbers.

N3 Solve problems involving percents from 1% to 100%.

Activity Planning Notes

You may wish to use the following steps to introduce and complete this challenge:

1. Read through Wrapping Gifts. Using a box, wrapping paper, and ribbon, wrap a gift or invite a student to do so. Follow up with tips on doing a good wrapping job.
2. Invite students to describe an event that they may have attended that included a gift-wrapping service. (This activity could be used for an actual school fundraising event at a local art and crafts festival. Your class could run the fundraiser or just be involved in preparing for the event by doing this activity. Students would have to contact the festival coordinator for information to help them estimate the number of items that might be sold, e.g., expected number of attendees, number of vendors.)

Supported Learning

Learning Style

- Consider allowing students to work with a partner.
- Encourage kinesthetic learners to try gift-wrapping so that they get a hands-on understanding of how much paper and ribbon is needed.
- You may wish to allow students to use calculators for parts c), d), and e).

Gifted and Enrichment

- Encourage students to come up with alternative materials for wrapping paper and ribbon that are more environmentally friendly (e.g., newspaper, magazine pages, paper bags, pine cones, dried flowers, leaves). They will then need to determine the quantity of materials needed to wrap a gift and complete the activity based on these values.

3. Have students work individually to write the equations and do the calculations.
4. Clarify that the task is to
   - calculate how many gifts they expect to wrap
     Note: Remind students that they worked with percents in Chapter 4. Point out that the value of 85% is the usual number of art pieces sold. Have students consider the number of expected sales at this particular event and whether they think they will wrap every art piece that is sold.
   - write an equation showing the relationship between the number of gifts and how much paper they will need
   - write an equation showing the relationship between the number of gifts and how much ribbon they will need
   - use their equations to calculate how much paper and ribbon they will need
   - determine how much the paper and ribbon will cost
   - decide how much they will charge to wrap each gift
5. Review **Master 1 Project Rubric** with students so they will know how their work will be assessed.

This challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment *for* Learning	Supported Learning
Wrapping Gifts Discuss the challenge and the requirements for the activity with the class. Students will estimate, develop equations, calculate amounts based on equations, do research, and determine costs. Students may work individually, in pairs, or in small groups.	• Review with students how to develop equations and use them to calculate how much paper and ribbon they will need. • As a class or a small group, discuss the factors that students considered to make estimates and develop equations.

Assessment *of* Learning	Supported Learning
Wrapping Gifts Discuss the challenge and the requirements for the activity with the class. Students will estimate, develop equations, calculate amounts based on equations, do research, and determine costs. Students may work individually, in pairs, or in small groups.	• Use **Master 1 Project Rubric** to assist you in assessing student work. Page 419a provides notes on how to use this rubric for this challenge. • To view student exemplars, go to w**ww.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as the Challenge in Real Life and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution with logical/reasonable explanations
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete response to parts a), b), c), and d) *or* • provides a complete response to all parts with no explanations *or* • provides a complete and correct solution based on an incorrect part a)
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides correct parts a), b), and c) *or* • provides correct parts b) and c) based on an incorrect part a)
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a correct part a) and one correct equation in part b) *or* • provides a correct part b) *or* • provides correct parts a) and b)
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides a correct response to part a) with or without justification *or* • provides a correct start to any part of a question