Working With Data

General Outcome

• Collect, display and analyze data to solve problems.

Specific Outcomes

SP1 Demonstrate an understanding of central tendency and range by:
• determining the measures of central tendency (mean, median, mode) and range
• determining the most appropriate measures of central tendency to report findings.

SP2 Determine the effect on the mean, median and mode when an outlier is included in a data set.

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
12.1	✓ determine mean, median, and mode for a given set of data, and explain why these values may be the same or different
	✓ solve problems involving the measures of central tendency
12.2	✓ determine mean, median, and mode for a given set of data, and explain why these values may be the same or different
	✓ solve problems involving the measures of central tendency
12.3	✓ determine the range of given sets of data
	✓ analyse a set of data to identify any outliers
12.4	✓ explain the effect outliers have on the measures of central tendency for a given data set
	✓ identify outliers in a set of data and justify whether or not they are to be included in the reporting of the measures of central tendency
	✓ provide examples of situations in which outliers would and would not be used in reporting the measures of central tendency
12.5	✓ determine mean, median, and mode for a given set of data, and explain why these values may be the same or different
	✓ provide a context in which the mean, median, or mode is the most appropriate measure of central tendency to use when reporting findings
	✓ solve problems involving the measures of central tendency

Assessment *as* Learning	Supported Learning
Use the Before column of **BLM 12–1 Chapter 12 Self-Assessment** to provide students with the big picture for this chapter and to help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• As students complete each section of the chapter or complete the Chapter 12 Review, have them review the related parts of **BLM 12–1 Chapter 12 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Chapter 12 Planning Chart

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter Opener • 20−30 minutes		BLM 12–1 Chapter 12 Self-Assessment BLM 12–2 Working With Data	• stapler • ruler • scissors • notebook paper
12.1 Median and Mode • 80−100 minutes	**Essential:** 1–4, 6, 8, 10, 11, Math Link **Typical:** 1–4, 6, 8, 10−12, Math Link **Extension/Enrichment:** 1–3, 13–17	BLM 12–1 Chapter 12 Self-Assessment BLM 12–3 Section 12.1 Extra Practice BLM 12–4 Section 12.1 Math Link	• ruler
12.2 Mean • 80−100 minutes	**Essential:** 1–3, 5, 7, 10, 11, Math Link **Typical:** 1–3, 5, 7, 9–14, Math Link **Extension/Enrichment:** 1, 2, 9, 12, 14–16	Master 2 Two Stars and One Wish BLM 12–1 Chapter 12 Self-Assessment BLM 12–5 Section 12.2 Extra Practice BLM 12–6 Section 12.2 Math Link	• 35 centimetre linking cubes per pair of students • overhead or whiteboard (optional)
12.3 Range and Outliers • 80−100 minutes	**Essential:** 1–3, 5, 6, 8, 9, 12, Math Link **Typical:** 1–3, 5, 6, 8, 9–12, 16, Math Link **Extension/Enrichment:** 1, 2, 12–16	BLM 12–1 Chapter 12 Self-Assessment BLM 12–7 Section 12.3 Extra Practice BLM 12–8 Section 12.3 Math Link	• ruler • overhead or whiteboard (optional) • computer with Internet access (optional) • tape measure
12.4 The Effects of Outliers • 80−100 minutes	**Essential:** 1–3, 5, Math Link **Typical:** 1–3, 5–7, Math Link **Extension/Enrichment:** 1, 2, 6–8	BLM 12–1 Chapter 12 Self-Assessment BLM 12–9 Section 12.4 Extra Practice BLM 12–10 Section 12.4 Math Link	• ruler
12.5 Choose the Best Measure of Central Tendency • 80−100 minutes	**Essential:** 1–4, 6, 9, Math Link **Typical:** 1–4, 6, 8–10, Math Link **Extension/Enrichment:** 1, 2, 10–16	Master 2 Two Stars and One Wish BLM 12–1 Chapter 12 Self-Assessment BLM 12–11 Section 12.5 Extra Practice BLM 12–12 Section 12.5 Math Link	
Chapter 12 Review • 40−50 minutes	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	BLM 12–1 Chapter 12 Self-Assessment BLM 12–3 Section 12.1 Extra Practice BLM 12–5 Section 12.2 Extra Practice BLM 12–7 Section 12.3 Extra Practice BLM 12–9 Section 12.4 Extra Practice BLM 12–11 Section 12.5 Extra Practice	
Chapter 12 Practice Test • 40−50 minutes	Provide students with the number of questions that they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1–6, 9, 10, 12	BLM 12–1 Chapter 12 Self-Assessment BLM 12–13 Chapter 12 Test	• calculator (optional)
Chapter 12 Wrap It Up! • 60−75 minutes		Master 1 Project Rubric BLM 12–4 Section 12.1 Math Link BLM 12–6 Section 12.2 Math Link BLM 12–8 Section 12.3 Math Link BLM 12–10 Section 12.4 Math Link BLM 12–12 Section 12.5 Math Link BLM 12–14 Chapter 12 Wrap It Up!	• encyclopedia • computer with Internet access • poster board

Chapter 12 Planning Chart

Section Suggested Timing	Exercise Guide	Teacher's Resource Blackline Masters	Materials and Technology Tools
Chapter 12 Math Games • 40−50 minutes		BLM 12–15 Data Duel Spinner	• 4 six-sided dice • paper clip
Chapter 12 Challenge in Real Life • 80−100 minutes		Master 1 Project Rubric BLM 12−16 Sports Shoe Survey	• pictures of different shoe styles (optional) • calculator (optional)
Chapters 9–12 Review • 60−75 minutes	**Minimum:** 1–3, 5–8, 11–13, 15, 16, 18, 21		
Task • 60−75 minutes		Master 1 Project Rubric Master 9 0.5 Centimetre Grid Paper BLM 12–17 Chapter 12 *MathLinks 7* Student Resource Answers BLM 12–18 Chapter 12 BLM Answers	• ruler (optional) • calculator (optional) • cups and counters (optional)

Chapter 12 Assessment Planner

Assessment Options	Type of Assessment	Assessment Tool
Chapter Opener	Assessment *as* Learning (TR pages i, 421)	BLM 12–1 Chapter 12 Self-Assessment Chapter 12 Foldable
12.1 Median and Mode	Assessment *as* Learning (TR pages 423, 426, 427) Assessment *for* Learning (TR pages 424, 425, 426, 427)	Math Learning Log (TR page 427) BLM 12–1 Chapter 12 Self-Assessment
12.2 Mean	Assessment *as* Learning (TR pages 430, 431, 432) Assessment *for* Learning (TR pages 430, 432, 433)	Master 2 Two Stars and One Wish Math Learning Log (TR page 432) BLM 12–1 Chapter 12 Self-Assessment
12.3 Range and Outliers	Assessment *as* Learning (TR pages 435, 437, 439) Assessment *for* Learning (TR pages 436, 437, 438, 439)	Math Learning Log (TR page 439) BLM 12–1 Chapter 12 Self-Assessment
12.4 The Effects of Outliers	Assessment *as* Learning (TR pages 442, 443, 445) Assessment *for* Learning (TR pages 443, 444, 445)	Math Learning Log (TR page 445) BLM 12–1 Chapter 12 Self-Assessment
12.5 Choose the Best Measure of Central Tendency	Assessment *as* Learning (TR pages 448, 449, 451) Assessment *for* Learning (TR pages 448, 450, 451)	Master 2 Two Stars and One Wish Math Learning Log (TR page 451) BLM 12–1 Chapter 12 Self-Assessment
Chapter 12 Review	Assessment *for* Learning (TR page 452) Assessment *as* Learning (TR page 453)	Math Learning Log (TR page 453) BLM 12–1 Chapter 12 Self-Assessment
Chapter 12 Practice Test	Assessment *as* Learning (TR page 454) Assessment *of* Learning (TR page 455)	BLM 12–1 Chapter 12 Self-Assessment BLM 12–13 Chapter 12 Test
Chapter 12 Wrap It Up!	Assessment *of* Learning (TR page 454a)	Master 1 Project Rubric
Chapter 12 Math Games	Assessment *for* Learning (TR page 456)	
Chapter 12 Challenge in Real Life	Assessment *for* Learning (TR page 456a) Assessment *of* Learning (TR page 456a)	Master 1 Project Rubric
Chapters 9–12 Review	Assessment *for* Learning (TR page 460) Assessment *as* Learning (TR page 460)	Math Learning Log (TR page 460)
Task	Assessment *of* Learning (TR page 460a)	Master 1 Project Rubric

You may wish to use one or more of the following materials to help you assess student readiness for Chapter 12.

Assessment *for* Learning	Supported Learning
Method 1: Have students develop a journal entry to explain what they personally know about data analysis and how they use measures of central tendency in their daily lives. **Method 2:** Have students complete **BLM 12–2 Working With Data** to check their conceptual understanding. Remind students that you are looking for the scope of their knowledge.	• Students who require reinforcement of prerequisite skills may wish to complete the Get Ready materials available in the *MathLinks 7 Workbook* and at the **www.mathlinks7.ca** book site.

Chapter Opener

Key Words

measure of central tendency
median
mode
mean
range
outlier

What's the Math?

In this chapter, students explore data by looking at the measures of central tendency: mode, median, and mean (average). They learn that these single values can be used to represent whole sets of data. They also learn about the limitations of these numbers, depending on how the data is spread out.

The concept of an outlier is introduced in this chapter. An outlier is a data value that is significantly different from the rest of the data in the set. Students learn to determine how this outlier value can affect the measures of central tendency. They begin to justify whether an outlier should be discarded or kept in the data set.

Activity Planning Notes

Discuss the judging of sports events such as the snowboarder in the photograph. What is the purpose in having several judges score the performance? How are these scores collected and used to determine a winner?

Discuss with students where they have seen or used measures of central tendency in their lives, such as calculating an "average" mark.

Students have collected data many times in their previous years of math. A discussion of some of the surveys that they have conducted is a good starting point for this chapter. Ask students to identify reasons for conducting surveys. For example, why do companies survey their customers? Explain to students that in this chapter they will study ways to represent the data collected in a survey. They will learn about the limitations of these methods as well as their advantages.

Math Link

The Math Link gives students their first opportunity to think about collecting data or information that might be relevant for a specific question. This Math Link will help them work toward the chapter problem in the Wrap It Up! on page 455. As they work on the Math Link, have students orally explore and compare the types of data they think would be relevant for each question. Discussing different types of data and their purpose in specific scenarios will help students better understand what they will be encountering in the chapter.
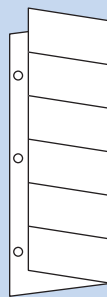
Have students make the Foldable in the student resource to keep track of the information in the chapter.

You may prefer to have students keep track of Key Words using a design specifically for that purpose. Students can make the following Foldable and write vocabulary terms on the front of each tab. Have them use the space beneath the tab to write definitions and to provide examples.

Step 1 Fold a sheet of notebook paper in half lengthwise with the crease to the right.

Step 2 Measure the height of the page and draw lines to divide the height into six equal parts. Cut every part as far as the fold, creating tabs as you go. This will create six tabs, one for each Key Word.

Step 3 Label each tab with a math term. Write definitions and give examples underneath the tabs.

Note: Since this will allow only a few lines per tab, if you have students with motor difficulties, you may wish to have them make two of these Foldables, each of them divided into thirds. This will allow enough room for sample data sets.

Remind students to take notes about the Key Ideas, examples, and Key Words on the appropriate page of their chapter Foldable.

Supported Learning

Learning Style
• Making the Foldable is a good hands-on activity for tactile and visual learners.

Learning Style
• You may wish to make a Foldable ahead of time to use as a model with students.

ESL and Language
• Some students may benefit from having new vocabulary written on study cards and also displayed in the classroom on a math word wall.

ESL and Memory
• Some students may choose to create their own vocabulary/picture dictionary in their notebook for the Key Words. Matching a picture and/or symbol with each Key Word and its definition helps students consolidate their understanding of the vocabulary. Alternatively, students may choose to add pictures to their vocabulary Foldable.

Meeting the Needs of All Learners
• Relate to students' experiences by quickly collecting some data about the class. Ask about shoe size or favourite food, colour, hockey team, or type of music.

Assessment *as* Learning	Supported Learning
Chapter 12 Foldable As students work on each section in Chapter 12, have them keep track of any problems they are having under the What I Need to Work On tab in their chapter Foldable.	• As students complete each section, have them review the list of items they need to work on and then have them check off any that have been handled.

12.1 Median and Mode

Suggested Timing
80–100 minutes

Materials
• ruler

Blackline Masters
BLM 12–1 Chapter 12 Self-Assessment

BLM 12–3 Section 12.1 Extra Practice

BLM 12–4 Section 12.1 Math Link

Mathematical Processes

☑ Communication

☑ Connections

☐ Mental Mathematics and Estimation

☑ Problem Solving

☑ Reasoning

☐ Technology

☐ Visualization

Specific Outcomes
SP1 Demonstrate an understanding of central tendency and range by:
• determining the measures of central tendency (mean, median, mode) and range
• determining the most appropriate measures of central tendency to report findings.

Warm-Up

1. Solve the equation modelled by the diagram. Check your answers.
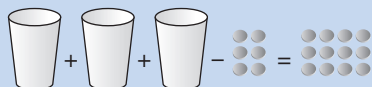
2. Model and solve this equation. Check your answer.
$15 = 2w + 7$

3. Solve for k, and then check your answer.
$6k + 4 = 28$

4. Calculate $(+4) - (-6)$.

5. For the table of values, use an expression to show the relationship between x and y.

x	1	2	3	4
y	2	4	6	8

Mental Math
Solve #6 to #9 by inspection.

6. $12 - y = 5$

7. $22 = a + 2$

8. $6t = 30$

9. $\dfrac{h}{8} = 8$

10. Estimate $228 + 371$. What method did you use?

Page Content (textbook excerpt)

2. Both Amir and Melanie have justified their decisions. They have each found a **measure of central tendency** to represent the "centre" of the data.
  a) Amir has calculated the **median** of the five scores. Describe how he found the median.
  b) Melanie has calculated the **mode** of the five scores. Describe how she found the mode.

Reflect on Your Findings

3. a) Which measure, median or mode, do you think better represents how these students are doing in math this term? Explain why.
  b) Give a set of ten numbers where you think the median is a more accurate measure of central tendency.
  c) Give a set of ten numbers where you think the mode is a more accurate measure of central tendency.

Example 1: Determine the Mode and the Median

One month, the Edmonton Oilers won 7 out of 10 games.
As a team they scored the following number of goals in these 10 games:
5, 4, 3, 2, 1, 5, 3, 3, 7, 3
  a) What is the mode for the number of goals scored?
  b) What is the median number of goals scored?

Solution

a) Arrange the numbers in increasing order.
The mode is the most frequently occurring number in the list.
1 2 3 3 3 3 4 5 5 7
The mode is 3 since it occurs four times.

b) Arrange the numbers in increasing order. The median is the middle value. Cross off pairs of least and greatest values until you reach the middle one or two numbers.
1̶ 2̶ 3̶ 3̶ 3 3 4̶ 5̶ 5̶ 7̶
If one number remains, then it is the median value.
If two numbers remain then you must find the value halfway between them. In this case, since the two numbers are both 3, the median is 3.
Notice that both the mode and median are 3 in this example. The mode and median will sometimes be the same value, but not always.

measure of central tendency
- a value that represents the centre of a set of data
- can be the mean, median, or mode

median
- the middle number in a set of data after the data have been arranged in order
- median of 2, 5, 6, 8, and 9 is 6
- median of 1, 3, 6, 8, 9, and 10 is 7

mode
- the most frequently occurring number in a set of data
- mode of 3, 5, 7, 7, and 9 is 7
- modes of 2, 2, 4, 6, 6, 8, and 11 are 2 and 6
- the data 1, 3, 4, and 5 has no mode

Warm-Up

1. $3c - 6 = 12; c = 6$

2. Allow students to use a model of their choice. $w = 4$

3. $k = 4; 6(4) + 4 = 28$

4. $+10$

5. $2x$

6. $y = 7$

7. $a = 20$

8. $t = 5$

9. $h = 64$

10. front end: $200 + 300 = 500$; relative size: $230 + 370 = 600$

Discuss the Math

1. Answers will vary.

2. a) He placed the data in order and then found the middle number.
  b) She found the most frequently occurring number in the set of data.

3. a) Answers may vary.
  b) Answers may vary. For example: 3, 4, 4, 4, 5, 5, 6, 6, 6, 7
  c) Answers may vary. For example: 0, 4, 4, 4, 4, 4, 4, 4, 4, 5

Activity Planning Notes

Students are introduced to two different methods to use when analysing data: the median and the mode.

Discuss the Math

Read through the scenario about Melanie and Amir. Then have two students act out Amir's and Melanie's responses in the cartoon. Ensure that all students understand how Melanie and Amir arrived at their answers. Ask the class with whom they agree or if they agree with either of them.

Then ask students if there is a better method for determining a score that reflects their mark. Some students will probably mention calculating the average. Explain that this method will be looked at later.

Meeting the Needs of All Learners

- When teaching the terminology, show a lot of examples of each concept. Have students use small-group learning experiences and share key concepts and ideas.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss which method they prefer; they are generalizing what they have learned during the Discuss the Math.	• There are arguments for both Amir's and Melanie's methods. Make sure that students give reasons for their answers. • Students may wish to use statistics that they are familiar with to answer #3b) and c). These could be heights, masses, marks, or data discussed in the Math Link on page 420.

Show You Know: Example 1

a) mode: 4; median: 4

b) mode: 11; median: 10

c) mode: 5; median: 5

d) modes: 6, 10, and 14; median: 10

Common Errors

- Some students may get the terms *mean, mode,* and *median* mixed up.

R_x Try a mnemonic to help students keep these terms straight. For example, a median is also the strip of land in the *middle* of opposing lanes of traffic, and the median here is the *middle* number. Remind students that pie à la mode is pie with ice cream. Since mode is easy to determine, the statement "It's as easy as pie" can help make this connection for students.

- Some students may forget to place the numbers in a data set in order before determining the median or mode, which usually will lead to an incorrect answer.

R_x Show students by example why they must first order the numbers from smallest to largest to prevent errors.

- Some students may be puzzled by the fact that the mode does not have to be the same as or near the median.

R_x Show students by example that the mode is simply the most frequently occurring value, e.g., the mode of 1, 1, 1, 4, 5, 6, 8 is 1, while the median is 4.

- Some students may incorrectly count the number of values in the data set and then miscalculate the median.

R_x Remind students that the medians of an even and an odd number of values are determined differently, so they must first carefully count the number of values in the data set.

- Some students may feel certain that the median *must* be one of the numbers in the set of data, just like the mode is.

R_x This may be easier to remedy when students learn to calculate the mean of a set of data since neither the mean nor the median has to be a number in the data set.

- Some students may forget to determine the average of the two middle values for the median when the data set consists of an even number of values.

R_x Have students cross off values from both ends of the data set as shown in Example 1.

Show You Know

Find the mode(s) and median of each set of values.

a) 4, 2, 9, 6, 4   **b)** 11, 5, 8, 11, 10

c) 4, 5, 8, 5, 5, 8, 5, 7   **d)** 14, 6, 14, 8, 10, 6, 10, 12

Example 2: Determine the Mode and Median From a Frequency Table

The frequency table shows the hourly wages of employees at a recycling depot.

Hourly Wages ($)	Number of Employees
8	3
10	2
11	3
14	2

a) What is the mode for the hourly wages?
b) What is the median wage?
c) How would the mode and median be affected if one employee gets a raise from $10 per hour to $11 per hour?

Solution

a) *Method 1: List the Values in Order*
Record the hourly wages, in dollars, for each employee in increasing order.
8 8 8 10 10 11 11 11 14 14
There are three 8s and three 11s.
So, there are two modes: $8 and $11.

Since there are three people who earn $8 per hour, record three 8s.

If no number is repeated in a set of data then there is no mode.

Literacy Link
When there are two modes the data set is said to be *bimodal.* The prefix "bi" means two. For example, a bicycle has two wheels.

Method 2: Use the Frequency Table
From the frequency table, you can determine the mode by looking at the numbers in the second column. It corresponds to 3 people who earn $8 and 3 people who earn $11.
So, there are two modes: $8 and $11.

b) *Method 1: List the Values in Order*
Record the hourly wages, in dollars, for each employee in increasing order. The median is the middle value.
Cross off pairs of least and greatest values until you reach the middle one or two numbers.
8̶ 8̶ 8̶ 10 10 11 1̶1̶ 1̶1̶ 1̶4̶ 1̶4̶
Since two numbers remain, find the middle value between 10 and 11.
The median hourly wage is $10.50.

The median does not have to be a number in a set of data.

To calculate the middle value, you could add the numbers and then divide by two.
$$\frac{10 + 11}{2} = \frac{21}{2} = 10.5$$

Example 1 shows students how to work with median and mode using a sports example. You may wish to have them review which method Amir used and which method Melanie used.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know related to Example 1.	• Make sure that students understand the meanings of *mode* and *median*, and how to determine them. • Allow students to use whatever method they like to determine mode and median, although the method in Example 1 is probably the easiest. • For students who would benefit from it, coach them through a) and b), and then have them do c) and d) on their own.

In Example 2, students have to list the values from a frequency table. They need to realize that if three employees each earn $8 per hour, that should be shown as 8, 8, 8 in the related data set. To answer c), they will have to revise two values in the data set and then recalculate the median and mode.

Method 2: Use the Frequency Table

Since there are ten employees, the median must be halfway between the wages of the fifth and sixth employees when the salaries are arranged in order. From the frequency table, five people earn $8 or $10 and five people earn $11 or $14.

The median must be halfway between $10 and $11.

The median hourly wage is $10.50.

c) By changing one $10 to $11 the ordered values would now be

8 8 8 10 11 11 11 11 14 14

The only mode is $11, since $11 now occurs four times.

The fifth number is $11 and sixth number is $11, so the median is $11.

Show You Know

Find the mode and median prices of the baseball caps sold in the last week.

Baseball Cap Price ($)	Number of Sales
7	5
9	5
10	6
12	4

Key Ideas

- The mode is the most frequently occurring number in a set of data.
  - If no number repeats, there is no mode.
    1 2 3 4 5   No mode
  - There can be more than one mode.
    1 1 2 2 3   Two modes: 1 and 2 (bimodal)
- The median is the middle value in a set of data after the numbers have been arranged in order.
  1̶ 2̶ 3 4̶ 5̶   Median is 3
- If there is an even number of data values, then the median is the value halfway between the two middle numbers.
  2̶ 4̶ 6 8 10̶ 12̶

  $$\frac{6+8}{2} = \frac{14}{2} = 7$$

  The median here is the value halfway between 6 and 8, or 7.
- The median does not have to be a number in the set of data.

Show You Know: Example 2

mode: 10; median: 9.5

Supported Learning

Learning Style and Language

- Encourage students to share their understanding of the Key Ideas during group discussion.

Learning Style and Memory

- When dealing with data sets, have students count the number of values in the original list and in their list to make sure that they have written the correct number of values.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• Some students may have difficulties in deciding how to write out all the numbers in sequence. Explain that five caps were sold at $7, so you start with five 7s: 7, 7, 7, 7, 7. Another five caps were sold at $9, so you write five 9s. When you are finished you end up with 7, 7, 7, 7, 7, 9, 9, 9, 9, 9, 10, 10, 10, 10, 10, 10, 12, 12, 12, 12. • Suggest to more advanced students that they find the median and mode without listing the values in sequence. The price with the largest frequency ($10) is the mode. To find the median, find the middle value(s) by determining that $9 is 10 caps from the lower end and $10 is 10 caps from the higher end. The average of 9 and 10 is 9.5, which is the median. • Provide an additional example for students who will benefit from it. First, coach them as they correct any errors they made in the Show You Know, and then have them do the following exercise on their own: Find the mode and median prices of the T-shirts sold. <table><tr><td>T-Shirt Price ($)</td><td>8</td><td>10</td><td>15</td><td>20</td></tr><tr><td>Number of Sales</td><td>3</td><td>5</td><td>10</td><td>4</td></tr></table> (mode: 15; median: 15) Check how students are doing as they work on this question. At the end, point out that this is another example in which the median and mode are the same number.

Communicate the Ideas

1. Answers will vary. For example: Mode. It is easier to find the most frequently occurring number than to organize the numbers in order and then find the middle number.

2. Answers will vary. For example: 1, 2, 2, 3, 4. The number 2 occurs most frequently so it is the mode. It is also the middle number in the sequence so it is the median.

3. Answers may vary. For example: Yes. She found the middle number in the set of data after she arranged the data in order.

Supported Learning

Learning Style and Language

• Allow students to choose to complete the Communicate the Ideas either in writing, orally, or using a combination of both.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have all students complete all three questions. These questions allow students to show that they understand the concepts of median and mode and that they can determine both for a set of data.	• Encourage students to explain the difference(s) between median and mode. For example, the mode must be one of the values in the data set, while the median may not be. • Encourage students to share their explanation of #3. This should prompt comparative discussion.

Assessment *for* Learning	Supported Learning
Practise Have students do #4, #6, and #8. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #4, #6, and #8 will need additional coaching. Have these students explain what strategies they are using to identify the median and mode. Make sure that students are reading the frequency table correctly in #8. Clarify any misunderstandings and have students correct their work. Then coach students through #5a). Have them complete the rest of #5 and then #7 and #9 on their own. • Check back with them several times to make sure that they understand the concepts.

Communicate the Ideas

1. Over ten days, the following numbers of juice cans were collected by the school recycling team: 15, 20, 12, 16, 24, 20, 12, 20, 23, 17. Which value do you think is easier to determine, mode or median? Why?

2. Create a set of five numbers where the median and mode are the same. Explain why you chose the numbers you did.

3. Dana was asked to find the median of the following numbers: 3, 6, 5, 4, 2. She decided to order the numbers from greatest to least: 6, 5, 4, 3, 2. She removed outer pairs of numbers until she was left with the middle value, 4. Has she correctly found the median? Explain.

Practise

For help with #4 to #7, refer to Example 1 on page 423.

4. What are the median and mode of each set of data?
a) 4, 2, 9, 6, 4
b) 21, 15, 18, 21, 20, 18
c) 3, 8, 5, 12, 10, 8, 2

5. Determine the median and mode of each set of data.
a) 6, 4, 8, 6, 2, 9
b) 14, 5, 8, 11, 10
c) 18, 24, 16, 18, 24, 16, 18, 18

6. In one week, a store in the mall sold the following numbers of Nickelback CDs: 34, 42, 37, 44, 46, 42, 51. What were the mode and median for the CD sales that week?

7. In 12 hockey games, the Calgary Flames scored the following numbers of goals: 4, 4, 0, 2, 3, 3, 4, 1, 3, 0, 3, 2. What were the mode and median?

For help with #8 and #9, refer to Example 2 on pages 424–425.

8. A new T-shirt company sold 26 shirts on their first day. The table shows the number of T-shirts sold according to price. What were the mode and median for the T-shirt prices?

Price ($)	Number Sold
8	6
14	7
17	5
20	4
25	4

9. A coffee shop sold 36 beverages one hour. The prices of the beverages sold are shown in the table. What were the mode and median prices?

Price	Number Sold
$2	12
$3	10
$3.50	9
$4	5

Key Ideas

This section summarizes median and mode and reiterates how to determine them.

Communicate the Ideas

Some students may find #2 difficult because of its open-ended style. The easiest way to answer #2 is to make up a set of four numbers, and then include one of the middle two numbers twice. For example, start with 1, 2, 3, 4, and then add one of the two middle numbers: 1, 2, 2, 3, 4. The mode and median are both 2.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8, 10, 11, Math Link
Typical	1–4, 6, 8, 10−12, Math Link
Extension/Enrichment	1–3, 13–17

Practise

In #8 and #9, students must understand how to use a frequency table. Remind them to write out all the individual data values in sequence so that they grasp how large the set of data is.

Apply

10. A cat gives birth to a litter of six kittens. The masses of the kittens are:

95 g, 100 g, 100 g, 105 g, 110 g, 110 g

a) What is the mode?

b) What is the median mass?

c) The mother cat has a second litter. The masses of these kittens are:

90 g, 95 g, 100 g, 105 g, 110 g, 115 g

What are the median and mode of the masses of all 12 kittens?

11. The following tally chart shows the heights of a grade 7 boys' basketball team.

Height (cm)	Number of Players
150	II
155	I
160	II
165	IIII
170	IIII
180	I

a) What is the mode for the heights? How can you determine the mode(s) quickly from the chart?

b) What is the median height?

12. What is one possible set of four numbers that have a mode of 7 and a median of 11?

Extend

13. Five whole numbers have a mode of 4 and a median of 3. What are the five numbers?

14. If the only mode is 4, describe all possible whole number values for n.

a) 5, 6, 7, 4, 2, n

b) 5, 6, 4, 4, n

15. If the median is 4, describe all possible whole number values for n.

a) 3, 2, 6, 7, n

b) 1, 3, 4, 5, n

16. The median of the set of numbers 3, 4, 5, x, and y is 5. What are the possible whole number values for x and y?

17. Five classmates have birthdays in March. The median of their birth dates is 12 (March 12) and the mode is 8. If the sum of the five birth dates is 56, find all possible sets of birth dates.

MATH LINK

David surveyed ten friends about their shoe size. He recorded the following sizes:

6, 7, 5, 8, 8, 7, 7, 6, 9, 8

a) What is the median shoe size?

b) What is the mode?

Apply and Extend

One way of finding the median in the tally chart in #11 is to count from top down and left to right. If students count from the bottom, they must count right to left. Alternatively, students can write out the numbers, as shown in the examples.

Have students discuss what strategies they used to answer #13 to #17. Some of them may use Guess and Check. Find out what methods other students used.

Assessment *as* Learning	Supported Learning
Math Learning Log Ask students to answer the following questions: • How can you tell the difference between median and mode? • Which value do you find the easiest to calculate?	• Encourage concrete learners to provide examples of medians and modes in everyday life as they answer these questions. • Depending on students' learning style, have them provide oral or written answers. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Have students review the part related to section 12.1 in **BLM 12–1 Chapter 12 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Answers

Math Link

a) median: 7

b) modes: 7 and 8

Supported Learning

ESL

• English language learners may need help in understanding the term *litter of kittens* in #10.

Learning Style and Memory

• Provide **BLM 12–3 Section 12.1 Extra Practice** to students who require more practice.

Math Link

Ask students whether they think the mode or the median shoe size would be more useful to a shoe store owner. Note that the mode is likely more useful since it will tell how many shoes of each size should be ordered.

Assessment *for* Learning	Supported Learning
Math Link The Math Link is intended to help students work toward the Wrap It Up! on page 455.	• Give to students who are having difficulty **BLM 12–4 Section 12.1 Math Link**, which provides scaffolding for this Math Link.

12.2 Mean

Suggested Timing
80–100 minutes

Materials
- 35 centimetre linking cubes per pair of students
- overhead or whiteboard (optional)

Blackline Masters
Master 2 Two Stars and One Wish
BLM 12–1 Chapter 12 Self-Assessment
BLM 12–5 Section 12.2 Extra Practice
BLM 12–6 Section 12.2 Math Link

Mathematical Processes

✓	Communication
✓	Connections
☐	Mental Mathematics and Estimation
✓	Problem Solving
✓	Reasoning
✓	Technology
✓	Visualization

12.2 Mean

Focus on...

After this lesson, you will be able to...
- ☐ determine the mean for a set of data
- ☐ solve problems by finding the mean

Ms. Fermat was not satisfied with the way Amir and Melanie calculated their math midterm reports. She did not feel that the median and mode provided a correct view of their performances. Ms. Fermat asked the students to explore another way of representing the centre of the data.

Explore the Math

How do you calculate the mean of a set of data?

Materials
- 35 centimetre linking cubes

We need to find a value that the test scores are centred around.

Let's try levelling out the towers without adding any more blocks.

1. Build a tower that represents each score that Amir and Melanie received on their weekly math quizzes: 4, 5, 8, 9, 9. A tower 4 cubes high represents a score of 4 out of 10.

2. Sketch a picture of your towers for each of the five scores.

3. Move cubes from the taller towers to the shorter towers to create five identical towers with the same height. Use only the cubes that you used in #1.
   a) What is the new height of each tower?
   b) How does this value represent the centre of the data?

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Specific Outcomes

SP1 Demonstrate an understanding of central tendency and range by:
- determining the measures of central tendency (mean, median, mode) and range
- determining the most appropriate measures of central tendency to report findings.

Warm-Up

1. Determine the median and mode of the following set of data: 23, 17, 20, 23, 22, 20.

2. A farmer owns 16 hens. The table shows the number of eggs laid by each hen in one week. What is the mode and median for this data?

Number of Hens	Number of Eggs Laid
3	5
4	6
8	7
1	8

3. What is the value of $4h + 8$ when $h = 5$?

4. $\frac{1}{9} = 0.1111 ...$, $\frac{2}{9} = 0.2222 ...$, $\frac{3}{9} = 0.3333 ...$

   a) Predict what $\frac{7}{9}$ will be as a repeating decimal.

   b) Write $\frac{7}{9}$ as a repeating decimal using bar notation.

5. Calculate $(-5) + (+8)$.

Mental Math

6. The diameter of a circle is 3 m. Estimate the circumference.

7. The radius of a circle is 5.5 cm. Estimate the circumference.

8. The radius of a circle is 3 cm. Estimate the area.

9. The diameter of a circle is 12 m. Estimate the area.

10. 35% of grade 7 students own a cat. You want to show this on a circle graph. Estimate the size of the interior angle.

The textbook page (left panel)

4. a) What is the sum of the five original scores?

b) Divide the sum you found in part a) by the number of quizzes.

c) How does your answer compare to the height of each tower in #3?

Reflect on Your Findings

5. The value you determined in #3 and #4 above is called the mean .

a) How does this value compare with the median and the mode?

b) Do you think the mean is the best measure of how Amir and Melanie are doing in math this term? Explain why.

mean
- a measure of central tendency
- the sum of a set of values divided by the number of values in the set
- for example, Mean $= \frac{6 + 8 + 4}{3}$ $= 6$

Example 1: Calculate the Mean

The daily sales at More Movies for the first week after the DVD release of *The Threat of the Brain Snatchers* are shown in the table.

a) What is the daily mean number of sales from Monday to Saturday?

b) How many sales will be needed on Sunday if More Movies hopes to have a daily mean of 100 sales for the first week?

Day	Number of Daily Sales
Monday	140
Tuesday	90
Wednesday	80
Thursday	90
Friday	110
Saturday	120

Solution

a) Calculate the sum of the six numbers.
$140 + 90 + 80 + 90 + 110 + 120 = 630$
Divide the sum by the number of days, 6.
$630 \div 6 = 105$
The daily mean number of sales is 105 from Monday to Saturday.

Look for numbers that are easy to add.
$90 + 110 = 200$
$80 + 120 = 200$
So, $140 + 90 + 200 + 200 = 630$

b) Calculate the total number of sales that will be necessary in order to have a daily mean number of sales of 100 for 7 days (one week). Since the mean needs to be 100, multiply 100 by 7.
$100 \times 7 = 700$
From part a), the sum of the sales for the first 6 days was 630. Subtract to calculate the number of sales needed on Sunday.
Total Sales $= 700 - 630$
$= 70$
70 sales need to be made on Sunday.

Strategies
Work Backwards
Refer to page xvi.

Show You Know

What is the mean of each set of values?

a) 7, 8, 6, 9, 9, 5, 7, 7, 8, 4 **b)** 300, 250, 400, 300, 250

Literacy Link
You often see the word "average" used instead of the word "mean."

12.2 Mean • MHR 429

Activity Planning Notes

In this section, students learn how to determine the mean of a set of data. The mean is the most used of the three measures of central tendency. However, even if students already know how to calculate the average of a few numbers, they will have a better understanding of the concept of mean if they take the time to visualize this process.

Explore the Math

Method 1: Divide the class into pairs. Provide each pair of students with 35 cubes. Ask students to build five towers that represent the scores for Amir and Melanie. Have them level the heights of the towers by moving cubes from taller towers to shorter towers. Ask students, "What is the height of the identical towers?" Provide another set of scores and have students go through the same process. For example, have them create towers that correspond to the following six scores: 5, 4, 8, 3, 6, 4.

Supported Learning

Learning Style
- Kinesthetic and concrete learners will benefit from the tower-building activity in the Explore the Math.

Motor
- Some students who have motor challenges may find it difficult to manipulate the cubes. Consider allowing them to use virtual manipulatives or larger cubes.

Meeting the Needs of All Learners
- Do not skip working with the linking cubes. They will help students to understand the process for finding the mean.

ESL

- English language learners may be unfamiliar with the terms *spawning* and *salmon*.

Motor

- The buttons on most calculators are typically too small and close together for students with motor difficulties to use accurately. These students may benefit from the use of a calculator with oversized keys.

Common Errors

- Students may make arithmetic miscalculations when determining the mean.
- R_x If a calculator is being used, students should place brackets around the addition of all the data values before dividing by the number of values.
- Some students may not calculate the total sum correctly when data values are displayed in a frequency table.
- R_x For practice, have students list all data values from the frequency tables (as they did when determining the median and the mode).

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Encourage discussion among student groups or pairs. As they answer these questions, check that students understand the concepts.	• Listen to group discussions to see who may be having difficulty with the concepts and give them other data sets to try. • Suggest that students compare the median and the mode they determined in section 12.1 for Amir's and Melanie's data with this mean.

Assessment *for* Learning	Supported Learning
Example 1 Have students do each question in the Show You Know on page 429 related to Example 1.	• Encourage students to use the method outlined in Example 1. Discuss how the numerical value is important for questions such as Example 1, since it would be too time-consuming to determine the answers using the tower method. • Work through the answers to a) and b) with students and help them correct any errors. Provide an additional set of data for students who will benefit from it: 400, 520, 300, 350, 470. (Discuss how the mean must be between 300 and 520. Have them add the values and then remind them to divide by the number of values. Mean: 408) • Make sure that students know how to do this type of calculation before going on to Example 2.

Example 2: Use the Mean to Make Predictions

A scientist recorded the daily distance travelled by a spawning salmon for five days in a row.

Day	Distance Travelled (km)
Monday	44
Tuesday	52
Wednesday	51
Thursday	46
Friday	57

a) What is the mean distance travelled by the salmon each day?
b) How far would you expect the salmon to travel in the next ten days? Assume the salmon has not yet reached its spawning destination.
c) A Chinook salmon may travel 8000 km from its feeding grounds in the North Pacific Ocean! How many days would you predict this journey to take based on the mean daily travel distance you calculated in part a)?

Solution

a) Calculate the sum of the five distances.
$44 + 52 + 51 + 46 + 57 = 250$
Divide the sum by the number of days.
$250 \div 5 = 50$

> Add the tens.
> $40 + 50 + 50 + 40 + 50 = 230$
> Add the ones.
> $4 + 2 + 1 + 6 + 7 = 20$
> Add the subtotals.
> $230 + 20 = 250$

C ((44 + 52 + 51 + 46 + 57) ÷ 5 = 50.
The mean distance travelled each day is 50 km.

b) The mean distance travelled each day is 50 km.
Multiply to calculate the expected distance travelled over the next ten days.
$50 \times 10 = 500$
You would expect the salmon to travel about 500 km over the next ten days.

c) The total distance travelled is 8000 km.
The mean distance travelled each day is 50 km.
Divide to find the length of time the total journey would take.
$8000 \div 50 = 160$
C 8000 ÷ 50 = 160.
The total journey would take about 160 days.

Did You Know?
Canada is about 5000 km from east to west, and also about 5000 km from north to south. The geographical centre of Canada is located just outside Baker Lake, Nunavut.

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Method 2: Demonstrate Method 1 at the front of the class where every student can easily see. Have one or two students come up to build and level the towers.

Method 3: Demonstrate Method 1 on the overhead with cubes by arranging them in lines so that students can see their heights on the screen. Alternatively, draw the towers on a whiteboard and erase/add squares to show levelling.

In the Explore the Math, students find the mean by manipulating the height of towers representing each value in the data set. In Example 1, they will use a numerical method to calculate mean.
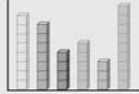
In Example 2, students use their knowledge of mean to solve a real-life problem. Read the Did You Know? at the bottom of page 430, and have students calculate how long it would take to bike across Canada from west to east if they travelled at the same speed as the salmon ($5000 \div 50 = 100$ days). Encourage a discussion in which students discuss people they know of who have made a trip across Canada.

Key Ideas

- To calculate the mean, add all of the numbers in a set of data and then divide by the number of numbers.
- The mean does not have to be a number in the set of data.

$$\text{Mean} = \frac{4 + 6 + 8 + 10 + 6 + 7 + 15}{7}$$
$$= \frac{56}{7}$$
$$= 8$$

Communicate the Ideas

1. Describe to a classmate who missed the lesson how to calculate the mean of the following set of five numbers:
   2, 6, 8, 9, 10

2. A toy store has six bins of stuffed animals. These bins contain 8, 7, 4, 5, 3, and 9 stuffed animals each.
   a) What is the mean number of stuffed animals?
   b) How could the vertical towers of linking cubes be levelled to determine the mean number of stuffed animals in a bin?

Practise

Round all answers to the nearest tenth where necessary.

For help with #3 to #6, refer to Example 1 on page 429.

3. What is the mean of each set of data?
   a) 5, 4, 10, 5, 6
   b) 2.2, 1.6, 1.9, 2.3, 2.1, 1.9
   c) 30, 85, 50, 105, 100, 65, 20, 25

4. Determine the mean of each set of data.
   a) 6, 7, 8, 9, 4, 11
   b) 3.4, 2.2, 1.4, 4.6, 2.2, 1.4, 1.6, 1.6
   c) 120, 72, 100, 110, 150, 75, 73

5. A store's sales of projection TVs on four Saturdays in February were 8, 7, 9, and 10. What was the mean number of Saturday sales in February?

6. Faith picked the following amounts of cranberries over seven days:
   2 L, 1 L, 1.5 L, 3 L, 1 L, 0.5 L, 1.5 L
   What is the mean volume of cranberries she picked?

For help with #7 and #8, refer to Example 2 on page 430.

7. Juanita scored the following points in her first six basketball games:
   12, 10, 11, 12, 14, 13
   a) What is the mean number of points scored?
   b) How many points would she need to score in her next game to increase her mean by 1 point for the seven games?

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Key Ideas

The points in the Key Ideas summarize mean and how to calculate it. Ensure that students understand mean, median, and mode and how they differ.

Communicate the Ideas

You will get a good insight into whether students have grasped the concept of mean from their answers to #1.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students do both questions, which help them to explain their understanding of mean and its calculation.	• Encourage students to explain the differences among mean, median, and mode. If they wrote about the differences between median and mode while studying section 12.1, have them add to that explanation. Encourage students to share their lists of differences in order to prompt a discussion. • Have students use **Master 2 Two Stars and One Wish** to critique other students' explanations for #1. With the class, list a set of criteria that answers should meet. Students can write two things they like about an explanation and one thing they would improve.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following questions: • How can you tell the difference between mean, median, and mode? • What do you find most difficult about working with mean, median, and mode?	• Encourage students to develop their own mnemonic for mean. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Have students review the part related to section 12.2 in **BLM 12–1 Chapter 12 Self-Assessment,** fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

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8. The chart shows the growth of a seed planted indoors in January.

Month	Height (cm)
Jan	3
Feb	4
Mar	4
Apr	3
May	5
June	5

a) What is the mean monthly growth?

b) How much will the plant have to grow in July for the mean monthly growth to be 5 cm for the seven-month period?

c) Predict the height of the plant after one year.

Apply

9. A survey company collected information about the amount of television teens watch each week.

Province	Television Viewing (hours per week)
British Columbia	12.4
Alberta	13.4
Saskatchewan	14.6
Manitoba	14.7

a) What is the mean for the four provinces listed? Round your answer to the nearest tenth of an hour.

b) The number of hours per week of television watched by teens for all of Canada was given as 14.0. How does the mean for the four provinces compare with the value given for all of Canada?

c) Would you predict the mean for the provinces not listed to be more or less than 14.0? Explain your reasoning.

d) How many hours of TV would you expect a typical Canadian teen to watch in one day?

e) How many hours of TV would you expect a typical Canadian teen to watch in ten weeks?

10. The graph shows the number of homes cleaned by Quick & Clean Housecleaning. What is the mean number of homes cleaned for the months shown?
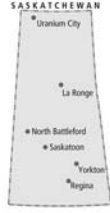

Quick & Clean Housecleaning

11. Here are Sasha's test scores in math this term:
78%, 68.5%, 82%, 87%, 91%, 88%, 74.5%, 71%

a) What is the mean of Sasha's test scores?

b) Sasha wants to raise her mean mark by 1% after the next test. What mark does she need to get on the next test?

12. The table gives the maximum daytime temperatures for five cities in Saskatchewan during August.

City	Maximum Temperature (°C)
La Ronge	22
North Battleford	24
Regina	26
Uranium City	19
Yorkton	24

a) What is the mean daytime temperature?

b) Predict the maximum daily temperature for Saskatoon, SK, in August. Explain your reasoning.

Practise

Once students have completed #7, discuss how mean can be used to make predictions. This is an important reason for learning about the measures of central tendency.

Assessment *for* Learning	Supported Learning
Practise Have students do #3, #5, and #7. Students who have no problems with these questions can go on to the Apply questions.	• Students who have problems with #3 and #5 will need additional coaching with Example 1. Have these students explain their strategies for calculating mean. Clarify any misunderstandings. Have students correct their answers to #3 and #5. Then coach them through #4a). Have students complete the rest of #4 and then #6 on their own. • Students who have problems with #7 will need additional coaching with Example 2. Have these students explain their strategies for using mean to make predictions. Clarify any misunderstandings. Have students correct their answers to #7. Then discuss how to use the frequency table in #8. Have students complete the rest of #8 on their own. • Check back with them several times to make sure that they understand the concepts.

13. Brennan recorded his test scores for his English course in a table.

Test	My Score	Out of (Total)
Grammar	6	10
Spelling	11	15
Novel Study	27	35
Poetry	31	40

a) What is the sum of Brennan's four scores?

b) What is the total possible score Brennan could have received?

c) Calculate Brennan's overall mean score, to the nearest whole percent.

14. The table shows the percent of people who take their own bags shopping in order to minimize the waste of plastic and paper. What is the mean percent? Round your answer to the nearest tenth of a percent.

Province	People Who Bring Their Own Shopping Bags
Alberta	31%
British Columbia	31%
Manitoba	35%
Ontario	25%
Québec	18%
Saskatchewan	33%

Extend

15. The table gives the ticket prices and number of seats available at each seat location for a soccer stadium.

Seat Location	Ticket Price	Number of Seats
Lower Centre	$12	500
Upper Centre	$10	1000
End Zone	$6	500

What is the mean ticket price for a sell-out?

16. During one month, major league baseball players Joe McEwing and Mike Lieberthal each had 45 hits in 132 attempts.

a) Determine the *batting average* for each player by dividing the number of hits by the number of attempts. A *batting average* is always expressed as a decimal to 3 places.

b) Suppose these players played against each other. Joe hit .800 by hitting 4 times in 5 attempts. Mike hit 1.000 after successfully hitting 3 times in 3 attempts. Who do you think has the better updated *batting average* for the year? Explain your prediction.

c) Calculate each player's new *batting average*. Was your prediction correct?

MATH LINK

Leah interviewed ten friends about the number of cousins they have.

Name	Number of Cousins	Name	Number of Cousins
Danika	18	Kyle	20
Jerome	3	Nicole	8
Paula	9	Vishal	22
Sam	14	Michelle	6
Janice	12	Jonah	10

What is the mean number of cousins among Leah's friends? Round your answer to the nearest whole number.

Apply and Extend

You may wish to replace the data in #9 with data collected from your class. Survey class members to see how many hours of television each student watches per week, place the results in a frequency table, and have students calculate the mean.

You may also wish to replace the scores used in #11 and #13 with actual sets of scores from your class. To prevent embarrassment, no student identities should be connected with the scores.

Assessment *for* Learning	Supported Learning
Math Link The Math Link is intended to help students work toward the Wrap It Up! on page 455.	• If any students have difficulty with calculating the mean, have them use **BLM 12–6 Section 12.2 Math Link,** which provides scaffolding for this activity. • As they work on the Math Link, circulate and check their understanding and their rationale for the mean number of cousins. Ask why they round their answer to the nearest whole number (you cannot have 0.2 of a cousin).

Math Link

12

Supported Learning

ESL

• English language learners may need help in understanding the term *sell-out* in #15.

Learning Style and Memory

• Provide **BLM 12–5 Section 12.2 Extra Practice** to students who require more practice.

Math Link

This Math Link provides a real-life situation in which students might use mean. You may wish to have students collect their own data by surveying ten friends. They can then develop a frequency table and find the mean of that data.

12.3 Range and Outliers

Mathematical Processes	
✓	Communication
✓	Connections
☐	Mental Mathematics and Estimation
☐	Problem Solving
✓	Reasoning
☐	Technology
✓	Visualization

Specific Outcomes

SP1 Demonstrate an understanding of central tendency and range by:
• determining the measures of central tendency (mean, median, mode) and range
• determining the most appropriate measures of central tendency to report findings.
SP2 Determine the effect on the mean, median and mode when an outlier is included in a data set.

Warm-Up

1. Calculate the mean, median, and mode of the following set of data: 124, 74, 104, 116, 154, 79, 77.

2. The circle graph shows the results when students were asked which school library books they prefer.

   Favourite Books (200 students)
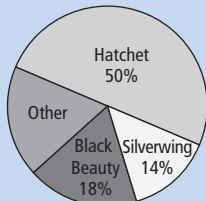
   a) What is the favourite book? How many students prefer it?
   b) Estimate and then calculate the interior angle needed to show 14%.
   c) What percent of students prefer an unnamed book?

For #3 to #5, solve the subtraction statement.

3. $(4) - (-5)$ **4.** $\dfrac{3}{4} - \dfrac{2}{5}$ **5.** $1\dfrac{2}{3} - \dfrac{5}{6}$

Mental Math

For #6 to #8, decide whether the answer is closest to 0, $\dfrac{1}{2}$, or 1. Explain your choice.

6. $\dfrac{1}{3} + \dfrac{1}{6}$ **7.** $\dfrac{1}{3} - \dfrac{1}{6}$ **8.** $\dfrac{9}{10} + \dfrac{1}{5}$

For #9 and #10, estimate the answer.

9. $2\dfrac{9}{10} + 1\dfrac{3}{4}$ **10.** $1\dfrac{1}{5} + 2\dfrac{5}{6}$

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2. Fill in the elevations at each point.

3. a) What is the largest value?
   b) What is the smallest value?
   c) What is the **range** of heights for the roller coaster?

4. Do you think there are any **outliers** in the set of 7 locations on the roller coaster? If so, what are they?

range
- the positive difference between the largest and smallest values in a data set

outlier
- a value that is much smaller or larger than the other data values
- a data set may have one or more outliers or no outliers

Reflect on Your Findings

5. How would you use the highest and lowest values of a data set to help you describe the data?

Example 1: Calculate the Range

A hospital recorded the following number of births one week.

Day	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Number of Births	7	10	4	6	7	8	3

a) What is the highest number of births?
b) What is the lowest number of births?
c) What is the range of the data?

Solution

a) The highest number of births is 10 births on Tuesday.

b) The lowest number of births is 3 births on Sunday.

c) The range is calculated by subtracting the lowest value from the highest value.

Range = highest value − lowest value
$= 10 - 3$
$= 7$

Show You Know

What is the range of each set of values?
a) 7, 4, 3, 6, 2, 5, 8, 3
b) 11, 8, 8, 5, 7, 6, 7, 6, 6
c) −4, 3, −1, 0, 2

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[Answers — right panel]

Warm-Up

1. mean: 104; median: 104; mode: none

2. a) Hatchet. $50\% = \frac{1}{2}$. $200 \div 2 = 100$
   b) Estimate: $10\% = 36°$; $5\% = 18°$; $15\% = 54°$
      Calculate: $0.14 \times 360 = 50.4°$
   c) $50\% + 18\% + 14\% = 82\%$; $100\% - 82\% = 18\%$

3. 9

4. $\frac{15}{20} - \frac{8}{20} = \frac{7}{20}$ 5. $1\frac{4}{6} - \frac{5}{6} = \frac{10}{6} - \frac{5}{6} = \frac{5}{6}$

6. Close to $\frac{1}{2}$. $\frac{1}{3}$ is a little less than $\frac{1}{2}$, and then you added a bit.

7. Close to 0. $\frac{1}{3}$ is less than $\frac{1}{2}$, and then you subtracted from it.

8. Close to 1. $\frac{9}{10}$ is close to 1, and then you added a bit.

9. $3 + 2 = 5$ 10. $1 + 3 = 4$

Discuss the Math

2. $0, -6, 32, -3, 18, -21, 0$

3. a) 32 b) −21 c) 53
4. Answers may vary. For example: −21 and 32

5. They give upper and lower limits to the data. No data exists outside these values.

Show You Know: Example 1

a) 6 b) 6 c) 7

Activity Planning Notes

Knowing the range of a set of data can be very useful when analysing the data. If the range appears large, it is a good idea to look for outliers, which are values that are much smaller or larger than the other data values.

Discuss the Math

Most students will find this section of the chapter quite straightforward.

Method 1: Have students read through the passage and complete the chart on their own. They may check their answers against a classmate's.

Method 2: Lead students through completing the chart by filling it in with them on an overhead or a whiteboard.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss this question. During this process, they are generalizing what they have learned from the Discuss the Math.	• Ensure that students understand that range refers to the highest and lowest values in a data set. If they were talking about the range of age of their friends, they might refer to having friends between 5 years old and 85 years old, depending on their social group. This would be a range of 80 years. Encourage students to use a personal example to show range.

Supported Learning

ESL

• Many students may not have had the chance to go on a roller coaster. This word, as well as the experience, is worth a small discussion as a class. If any students have had the opportunity to ride a roller coaster, have them explain what it is like.

Motor

• Some students who have motor challenges may find copying the table into their notebook to be a difficult task. Ensure that they use a ruler and are given adequate time.

Show You Know: Example 2

a) 79

b) 1

c) 202, 2

Common Errors

- Some students may experience difficulties with subtracting the negative integer in the Example 1 Show You Know.

R_x Remind students that when subtracting integers, they can add the opposite instead. Note that students will use this skill in this section.

- Some students may think that all data sets begin at zero.

R_x Remind students that sets of data have a lowest and a highest value. When they graph data, for example, they do not always begin the graph at zero.

Supported Learning

ESL

- Some English language learners may need help in understanding the word *season* in relation to NHL hockey.

Learning Style

- Most of the questions in the Example 1 Show You Know are simple enough for students to do mentally. If they are having difficulties, suggest that they write down the highest value and the lowest value, and then subtract them.

Example 2: Identify Outliers in a Data Set

In the 1985–1986 NHL hockey season, Wayne Gretzky set a record for the most points (goals + assists) recorded during a single regular season. The following table shows the top five point leaders for the 1985–1986 season and the 2005–2006 season.

NHL Point Leaders			
1985–1986 Season	Points	2005–2006 Season	Points
Wayne Gretzky	215	Joe Thornton	125
Mario Lemieux	141	Jaromir Jagr	123
Paul Coffey	138	Alexander Ovechkin	106
Jari Kurri	131	Dany Heatley	103
Mike Bossy	123	Daniel Alfredsson	103

a) Determine the range for each season. Compare the two ranges.
b) Which data value appears to be an outlier in one of the two seasons?
c) Remove the outlier and determine the new range for that season. Compare the new range with the range from the other season.

Solution
a) 1985–1986 Season:
215 is the highest value, and 123 is the lowest value
Range = 215 − 123
= 92

2005–2006 Season:
125 is the highest value, and 103 is the lowest value
Range = 125 − 103
= 22

The ranges are very different: 92 and 22.

b) The outlier appears to be Gretzky's 215 points in the 1985–1986 season.

c) The new highest value is 141.
The new range = 141 − 123
= 18
The two ranges are more comparable in size: 22 and 18.

Show You Know
What value(s) appear to be outliers in each set of data?
a) 18, 19, 79, 17, 20, 12
b) 14, 14, 16, 15, 1, 16
c) 98, 202, 99, 91, 2, 95, 89, 88, 94

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Example 1 uses a small number of births to have students consider range.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know on page 435 related to Example 1.	• If students get c) incorrect, but have the correct answers for a) and b), their problem likely is with subtracting integers. Review that process. • If students get a) or b) incorrect, review the example with them and then provide another set of data for them to do on their own: The following data set shows the number of birds at a feeder from Monday to Sunday. What is the range? | M	T	W	T	F	S	S
---	---	---	---	---	---	---
3	4	6	9	2	11	13	 (highest value = 13; lowest value = 2; range = 13 − 2 = 11) • You may wish to have students do the Math Games on page 456. This activity provides a fun way to practise calculating median, mean, mode, and range.

In Example 2, students examine sets of data to see if any numbers appear extremely different from the rest of the data.

You may wish to discuss this idea with students: If they have 25 friends between 5 and 25, and one friend who is 85, they might wish to remove the 85-year-old when talking about the age range of their friends because *most* of their friends are between 5 and 25.

Key Ideas

- The range provides information about the spread of the data.
  Range = highest value − lowest value
- Outliers are values that are very different from the rest of the numbers in a set of data. A data set can have no outliers, or one or more outliers.

Communicate the Ideas

1. In your own words, explain the meaning of range and outliers.

2. How can you determine the smallest value in a data set if you are given the range and the largest value? Use an example to explain your response.

Practise

For help with #3 to #5, refer to Example 1 on page 435.

3. What is the range of each set of data?
a) 11, 8, 7, 10, 6, 5, 18, 13
b) 4, −5, 3, −2, 0, 7

4. Determine the range of each set of data.
a) 16, 11, 7, 29, 31, 18, 21, 18
b) 7, 1, −1, −2, 9, −7, 4

5. Leanne timed how long she could hold her breath while swimming under water. She recorded the following timings, in seconds:
47, 55, 42, 65, 61
What is the range of the data?

For help with #6 to #8, refer to Example 2 on page 436.

6. What value(s) appear to be outliers in each set of data?
a) 6, 9, 9, 37, 8, 7
b) 24, 34, 46, 26, 32, 43, 115
c) 48, 32, 67, 61, 47, 95, 89, 888, 1

7. Identify any possible outlier(s) in each set of data.
a) 666, 11, 9, 12, 8, 13, 10
b) 43, 54, 62, 64, 0, 211, 45
c) 82, 75, 76, 85, 89, 95, 92, 88, 80

8. Curb-cut ramps are often built at intersections to allow wheelchairs and strollers to cross streets more easily. In one town, a study was done to count the number of curb-cuts at ten random intersections. The results for five different neighbourhoods are given:
9, 8, 8, 1, 7
a) What is the range?
b) Which data value may be an outlier?
c) If you remove the outlier, what is the new range?

Communicate the Ideas

1. Answers may vary. For example: The range is the difference between the largest and smallest values in a set of data. An outlier is a value that is much smaller or larger than the other data values.

2. Answers may vary. For example: Subtract the range from the largest value. If the range of a set of data is 15 and the largest value is 8, then the smallest value is $8 - 15 = -7$.

Supported Learning

ESL

- Some English language learners may need assistance in understanding the term *curb-cut ramps* in #8.

Meeting the Needs of All Learners

- Repeat key concepts and show a lot of examples until you see that students can grasp the ideas. Then have them work in small groups to show their understanding.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know on page 436 related to Example 2.	• Have students examine the data sets and identify any values that seem very different from the rest. Reinforce that data sets can have more than one outlier, as in c). • For students who will benefit from it, coach them through a) to c), and then have them identify the outlier(s) in another data set: 24, 30, 26, 54, 28, 19. (54 is the outlier as it is a much larger number.)

Key Ideas

Ensure that students understand the concepts of range and outliers in a set of data. Ask them to think of a time when they may have noticed an outlier in their own sports scores, test marks, or game results.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students do both Communicate the Ideas questions.	• If students have problems with #2, you might wish to provide them with a concrete example. Without telling them, use the data from Example 1. The range for the number of babies born in one week at a hospital is 7. The highest value is 10. What is the lowest value? Discuss how to solve this question. Then have students come up with their own example and explain how to solve it. • Encourage students to share their processes and rationales with a partner.

ESL

- English language learners may not be familiar with the term *jigsaw puzzle* in #9.

Learning Style and Memory

- Provide **BLM 12–7 Section 12.3 Extra Practice** to students who require more practice.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 6, 8, 9, 12, Math Link
Typical	1–3, 5, 6, 8, 9–12, 16, Math Link
Extension/Enrichment	1, 2, 12–16

Assessment *for* Learning	Supported Learning
Practise Have students do #3 and #6. Students who have no problems with these questions can go on to complete #5 and #8, and then the Apply questions.	• Students who have problems with #3 and #6 will need additional coaching. Have these students explain their understanding of range and outliers. Clarify any misunderstandings and have them correct their work. Coach students through #4a) and #7a). Then have them complete #4 and #7, and do #5 and #8 on their own. • Check back with them several times to make sure that they understand and can use the concepts.

Apply

9. Vincent received a 200-piece jigsaw puzzle for his birthday. The following times indicate how long it took him to complete the puzzle the first six times he did it:

54 min, 36 min, 34 min, 29 min, 27 min, 28 min

a) What is the range?
b) Which time may be an outlier?
c) Why might this particular value be so different from the others?
d) If you remove the outlier, what is the new range?

10. The following table shows the mean high temperature for each month in Whitehorse, Yukon Territory.

Month	Mean Temp.
January	−13°C
February	−7°C
March	−1°C
April	6°C
May	13°C
June	18°C
July	20°C
August	18°C
September	12°C
October	4°C
November	−5°C
December	−11°C

a) What is the highest value?
b) What is the lowest value?
c) What is the range of the data?

11. The table gives the mean number of wet days per year for several locations across Canada. What is the range of wet days?

Location	Mean Number of Wet Days
St. John's	217
Halifax	170
Fredericton	156
Toronto	139
Winnipeg	119
Regina	109
Edmonton	123
Vancouver	164
Whitehorse	122
Yellowknife	118

12. The table gives the magnitudes of five of the largest earthquakes that have occurred in western Canada.

Location	Date	Magnitude
West of Vancouver Island, BC	Jan 26, 1700	9.0
South of Hope, BC	Dec 14, 1872	7.4
West Coast of Vancouver Island	Dec 6, 1918	7.0
Central Vancouver Island	June 23, 1946	7.3
Queen Charlotte Islands, BC	Aug 22, 1949	8.1

Magnitude refers to the relative size of an earthquake. Magnitude measurements are recorded using the Richter scale, which is a special scale that goes from 0 to 10. What is the range of magnitudes of the five earthquakes in the table?

WWW Web Link
To learn more about earthquakes in Canada, go to www.mathlinks7.ca and follow the links.

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Communicate the Ideas

The Communicate the Ideas is intended to allow students to explain their understanding of a range and an outlier. In addition, students are challenged with finding the smallest data value when given the range and the largest data value of a set. You may wish to have students set this up as an algebraic equation: largest value $- s =$ range, where s is the smallest value.

Practise

In #3b), students subtract negative numbers. If you did not review this process during the Warm-Up, you may wish to do so before assigning this question.

Apply and Extend

For #9, you may wish to use an activity that is popular with students, and provide data related to that activity. Consider providing local data for #10.

Many students are interested in earthquakes. Have them research Canadian earthquakes using the Web Link and use the data to make up a question of their own, modelling it after #12. They can exchange their question with a partner's.

13. Final times, in minutes, for the top ten contestants in a race were 55.2, 56.7, 56.9, 57.3, 58.8, 58.9, 59.2, 59.3, 59.3, and 59.7.

a) What is the range of times?

b) Do you think this set of data contains any outliers? Explain.

Extend

14. A data set contains five numbers: 12, 20, 18, 15, 5

a) Determine the range, median, mode, and mean of the data set.

b) Add 10 to each of the five numbers and recalculate the range, median, mode, and mean. How do your answers compare with your answers to part a)?

c) Multiply the numbers in the original data set by 10 and calculate the new measures of central tendency. How do your answers compare with your answers to part a)?

15. Five students competed in a pie-eating contest.

• The range of times for the group to finish eating their pies was 9 min.
• The least amount of time it took for someone to finish their pie was 1 min.
• The mode was 4 min.
• One boy took 2 min more than the median.

a) What are the finishing times for the five contestants?

b) What was the mean number of minutes it took the five contestants to eat their pies?

c) What strategy did you use to determine the mean?

16. Conduct the following experiment to simulate rolling two 10-sided dice each numbered from 0 to 9.

• Randomly choose a page from your textbook.
• Calculate the sum of the last two digits of the page number. For example, for page 108 you would add 0 and 8 to get 8.
• Randomly choose 49 more pages for a total of 50 pages.
• Copy and complete the frequency table.

Sum	Tally	Freq.	Sum	Tally	Freq.
0			10		
1			11		
2			12		
3			13		
4			14		
5			15		
6			16		
7			17		
8			18		
9			19		

a) What is the mode?

b) What is the mean?

c) What is the median?

d) What is the range?

e) Why are there no tally marks on 19?

MATH LINK

Measure and record the heights of ten people in your class, including your teacher.

a) What is the range of heights?

b) Identify any possible outliers.

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You may wish to have most students do #16, as this is a good hands-on activity.

Math Link

Many students know their height in feet and inches, rather than in centimetres, so they may find measuring themselves in centimetres an interesting activity. A tape measure could be taped to the wall.

Assessment *for* Learning	Supported Learning
Math Link This Math Link prepares students for the Wrap It Up! on page 455, in which they will collect data on a larger scale, based on a topic of their own choosing.	• If students have difficulty with this Math Link, give them **BLM 12–8 Section 12.3 Math Link,** which provides scaffolding for this activity. • Make sure that students measure heights accurately. • As alternatives, measure hand span or head circumference.

Assessment *as* Learning	Supported Learning
Math Learning Log Ask students to answer the following questions: • How does an outlier affect the range in a data set? Use an example in your explanation. • What did you find most interesting and useful in this section? • What did you find least useful?	• Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Work with students to develop a plan for dealing with the areas in which they are having difficulty. • Have students review the part related to section 12.3 in **BLM 12–1 Chapter 12 Self-Assessment,** fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

12.4 The Effects of Outliers

Suggested Timing
80–100 minutes

Materials
• ruler

Blackline Masters
BLM 12–1 Chapter 12 Self-Assessment
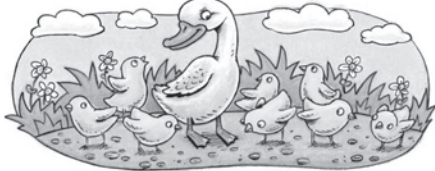BLM 12–9 Section 12.4 Extra Practice
BLM 12–10 Section 12.4 Math Link

Mathematical Processes

- ☑ Communication
- ☑ Connections
- ☐ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☐ Technology
- ☑ Visualization

12.4 The Effects of Outliers

Focus on...

After this lesson, you will be able to...
- ☐ explain the effects of outliers on measures of central tendency
- ☐ justify whether outliers should be included when determining measures of central tendency

Can you spot the outlier in the cartoon shown?
Suppose you are asked to determine the mean mass of these babies. Should this outlier be removed from the data set?

Some outliers are caused by mistakes in data collection. Other outliers are just as important as the other data values. When there are outliers in a data set, the mean, median, and range can be different from what they are when the outliers are removed. People who work with data need to decide when outliers should and should not be used when calculating measures of central tendency.

Explore the Math

How do outliers affect measures of central tendency?

For a grade 7 science project, students were asked to plant and observe the growth of six bean seedlings. Students were given specific instructions to give their planted seeds 1 h of light and 30 mL of water per day. After two weeks the students brought their plants back to school.

Materials
• ruler

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Specific Outcomes

SP2 Determine the effect on the mean, median and mode when an outlier is included in a data set.

Warm-Up

1. Show the mean, median, mode, and range of the following set of data: 26, 14, 23, 17, 32, 41, 19, 31, 21, 32.

2. Your weekly math test is out of 10. You receive the following grades: 9, 4, 9, 10, 7, 9, 8.
   a) Show the mean, median, mode, and range.
   b) Identify the outlier.
   c) If you remove the outlier, what is the new range?

For #3 to #5, show the answers in lowest terms.

3. $\frac{3}{5} + \frac{4}{10}$ 4. $\frac{5}{6} + 2\frac{1}{8}$ 5. $5\frac{1}{4} + 2\frac{2}{3}$

Mental Math

Use the following list for #6 to #8. Explain your reasoning.

90, 198, 256, 389, 456, 563

6. List the numbers divisible by 9.

7. List the numbers divisible by 4.

8. List the numbers divisible by 3.

9. Estimate 22% of 532.

10. Estimate 349 out of 460 as a percent.

Activity Planning Notes

This section introduces students to how outliers can affect their data. They have probably experienced exceptionally good or poor scores in relations to tests, sports, or games. People naturally want to keep those exceptionally good ones and forget the poor ones. As students work on outliers that have nothing to do with themselves, remind them of how they might analyse their own data fairly.

Explore the Math

Method 1: Have students complete the activity in pairs. Ensure that students can measure correctly to the nearest millimetre.

Method 2: Lead the class through the activity. Ask individual students to measure each plant length on the diagram.

Explore the Math

8. a) Answers will vary. For example: The mean is affected more because the outlier is no longer included in the calculation of the mean. The median without the outlier included simply shifts one number to the left in the ordered list of values.

b) Answers will vary. For example: The outlier should be included because it is just as important as the other measurements. It did not result from a mistake in measuring or in recording the data.

Supported Learning

Memory

• Encourage students to refer to their vocabulary study cards or Foldables to remember the definitions of *median, mode, mean, range,* and *outliers.*

Common Errors

• Some students may struggle with determining outliers.

R$_x$ Suggest that students order the numbers from smallest to largest and then look at the numbers at both ends of the list (smallest and largest) for possible outliers.

Example 1: Identify Outliers

Shannon practised shooting baskets every day last week to prepare for a basketball tournament. She recorded the number of baskets she made each day out of 25 shots.

a) What is the range of baskets scored?
b) What are the median and mean numbers of baskets scored?
c) Identify any possible outliers. Should the outlier(s) be removed from the data set? Explain why or why not.

Day	Number of Baskets
Sunday	14
Monday	17
Tuesday	16
Wednesday	20
Thursday	5
Friday	22
Saturday	18

Solution

a) The highest and lowest values are 22 and 5.   Range = 22 − 5
                                                                                          = 17

b) Arrange the data in order: 5, 14, 16, 17, 18, 20, 22
The median number of baskets scored is 17.

$$\text{Mean} = \frac{5 + 14 + 16 + 17 + 18 + 20 + 22}{7}$$
$$= \frac{112}{7}$$
$$= 16$$

The mean number of baskets scored is 16.

c) The value 5 could be considered an outlier. This value is significantly different from the other values. But this value should not be removed from the data set because it is just as important as the other data values. It is probably not an error in measurement. It may simply represent a poorer performance that day.

Example 2: Identify Outliers and Determine Their Effects

In a science experiment, students were asked to measure the length of their right thumb from the first knuckle to the end of their thumb. The table shows the lengths that were measured by ten different students.
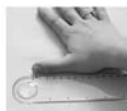
Student	A	B	C	D	E	F	G	H	I	J
Length (cm)	3.2	2.7	3.1	2.8	2.8	2.7	31	3.3	2.6	3.0

a) What is the range?
b) What are the median and the mean?
c) Identify any possible outlier(s). Should the outlier(s) be removed from the data set? Explain why or why not.
d) How would removing the outlier affect the median and the mean?

Assessment *as* Learning	Supported Learning
Reflect on Your Findings Listen as students discuss how the outlier affects the median and mean and consider whether or not it should be included in the data set. During this process, students are generalizing what they have learned during the Explore the Math	• Encourage students to explain their answer. The explanation is important here because either answer is correct, depending on the justification.

In Example 1, students identify outliers. This should be familiar as it is similar to what they did in section 12.3. In this example, however, students work with range, median, and mode, and assess whether or not the outlier(s) should be removed from the data set. Emphasize that the justification for whether or not an outlier should be removed is just as important as the decision to keep or remove it from the data set.
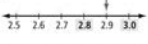
Example 2 takes the discussion of outliers to the next level. In this example, students assess whether or not to remove the outlier(s) and then justify their response by noting how removing the outlier(s) would affect the measures of central tendency.

Solution

a) The highest value is 31 cm.
The lowest value is 2.6 cm.
The range 28.4 cm.

$$\text{Range} = 31 - 2.6 = 28.4$$

b) To find the median, arrange the numbers in order.
2.6, 2.7, 2.7, 2.8, 2.8, 3.0, 3.1, 3.2, 3.3, 31
With ten values, the median will be halfway between the fifth and sixth values.
The median is 2.9 cm.

$$\text{Mean} = \frac{3.2 + 2.7 + 3.1 + 2.8 + 2.8 + 2.7 + 31 + 3.3 + 2.6 + 3.0}{10}$$
$$= \frac{57.2}{10}$$
$$= 5.72$$

The mean is 5.72 cm.

c) The value of 31 is an outlier. It is about ten times greater than the other measurements and is most likely an error in either measurement or recording. The student may have measured in millimetres instead of centimetres. Or, it is possible that the student forgot to place the decimal point between the digits 3 and 1. In this case, the outlier should be removed.

d) Remove the outlier value of 31 from the ordered list of values from part b). Recalculate the median and the mean.
2.6, 2.7, 2.7, 2.8, 2.8, 3.0, 3.1, 3.2, 3.3
Since there are only nine values now, the median will be the fifth value. The median is 2.8 cm.

$$\text{Mean} = \frac{2.6 + 2.7 + 2.7 + 2.8 + 2.8 + 3.0 + 3.1 + 3.2 + 3.3}{9}$$
$$\approx 2.9$$

The mean is approximately 2.9.

The median changes from 2.9 to 2.8. The mean changes from 5.7 to 2.9. The mean is affected more by removing the outlier.

Show You Know

The following times were recorded, in seconds, for the runners in a race:
20.2, 16.5, 40.4, 18.5, 21.4, 20.5, 17.1, 24.5, 19.0

a) What is the range of times?
b) What are the median and mean times?
c) Identify any possible outliers. Should the outlier(s) be removed from the data set? Explain why or why not.

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Answers

Show You Know: Example 2

a) 23.9

b) median: 20.2 s; mean: approximately 22.0 s

c) Answers will vary. For example: 40.4 s. No. Even though one runner had a much slower time than the other runners, the value is still part of the data from the race.

Key Ideas

Have students read and review the Key Ideas. Make sure they understand what outliers are and how they can affect the measures of central tendency in a set of data. Hold a class discussion on examples of outliers in students' own lives.

Communicate the Ideas

The answer to #1 depends on whether Brian is playing five-pin or ten-pin bowling. The top score in five-pin is 300, which makes 316 a definite outlier. The top score in 10-pin is 450, so he may just have had a bit of luck. Ask any bowlers in the class how they would treat this score.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students answer both questions on page 444. They can do #2 as a group, but should write individual answers.	• Have each group present their situations from #2a) and b) to the class. The class could discuss each group's choice of situations and then discuss the group's decision to include or exclude the outlier. • Ask students how they treat outliers in their own data from sports, games, and test scores.

Assessment *for* Learning	Supported Learning
Example 2 Have students do the Show You Know related to Example 2.	• For students who will benefit from it, discuss their analysis and clarify any misunderstandings. Have them consider whether or not the outlier should be removed and how removing it would affect the measures of central tendency. • Provide additional practice by having them do the analysis using one or more of the questions from section 12.3. For example, have them answer a) to c) below for #12 on page 438. **a)** What is the range? $(9.0 - 7.0 = 2.0)$ **b)** What are the median and mean? (Data set: 7.0, 7.3, 7.4, 8.1, 9.0; median: 7.4; mean: approximately 7.8) **c)** Identify any possible outlier(s). Should the outlier(s) be removed from the data set? Explain why or why not. (9.0 is an outlier. This outlier should not be removed because it is a recorded magnitude of an earthquake.) • You may wish to have students work with other data sets they examined in section 12.3 and answer the same questions.

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Communicate the Ideas

1. Answers may vary. For example: 316. The outlier should be removed from the data set. It is much higher than the other scores. It may have resulted from an error in recording the score if the scorekeeper intended to write 136.

2. a) Answers will vary.
   b) Answers will vary.

Meeting the Needs of All Learners

• Repeat the Key Ideas from section 12.3 and explain the ones in 12.4 thoroughly, giving plenty of illustrations. Allow a lot of discussion between you and the students, and between one student and another student.

Learning Style, ESL, and Language

• Consider using the Communicate the Ideas to allow students to work on improving their verbal skills and vocabulary development through group discussion.

• Some students may have trouble with the amount of reading required in the Practise and Apply questions. You may wish to read each question aloud with students and clarify any difficult wording.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, Math Link
Typical	1–3, 5–7, Math Link
Extension/Enrichment	1, 2, 6–8

Assessment *for* Learning	Supported Learning
Practise Have students do #3. Students who have no problems with this question can go on to #5 and then do the Apply questions.	• Coach students who have problems with #3, helping to correct any misconceptions. Assign #4 and discuss how it will be done. Have students do this question and discuss their analysis.

Key Ideas

• Outliers can affect all measures of central tendency.
• When a small set of data has an outlier, the mean is usually affected more by the outlier than the median.
• Some outliers are just as important as the other data values, while others are better removed from the data set.

Communicate the Ideas

1. Brian's bowling scores are 135, 132, 128, 316, 135, and 138. Identify a possible outlier in his scores. Should you remove it from the data set? Explain your reasoning.

2. a) Give an example of a situation where an outlier would be used when reporting on measures of central tendency.
   b) Give an example of a situation where an outlier would not be used when reporting on measures of central tendency.

Practise

For help with #3 to #5, refer to Examples 1 and 2 on pages 442–443.

3. The table shows the percent of students surveyed that had at least one song on their MP3 players by the musicians listed.

Musician	Students With at Least One Song
Snoop Dogg	42%
Shania Twain	38%
Britney Spears	6%
Kanye West	40%
Led Zeppelin	41%
Avril Lavigne	38%
U2	88%

a) What is the range?
b) What are the median and mean?
c) Identify any possible outliers. Should the outlier(s) be removed from the data set? Explain why or why not.

4. Two grade 7 students randomly ask the ages of the first eight people to pass them in the hallway. They record the following ages:
7, 11, 8, 8, 52, 9, 9, 10
a) What is the range?
b) What are the median and mean age?
c) Identify any possible outliers. Should the outlier(s) be removed from the data set? Explain why or why not.

5. Sharon recorded the following prices for five different brands of canned tomatoes on the grocery store shelf:
$1.29, $1.69, $1.59, $9.61, $1.39
a) What is the range?
b) What are the median and the mean?
c) Identify any possible outlier(s). Should the outlier(s) be removed from the data set? Explain why or why not.
d) How would removing the outlier(s) affect the median and the mean?

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Practise

Discuss with students that if an outlier represents a mistake in measurement, it should be removed. However, it may not always be clear whether or not it should be removed. The context of the question is important, which is why students should justify removing or leaving the outliers.

In #4, if the desire was to find the ages of students only, the value of 52 (which represents a staff member) should be excluded. On the other hand, if the study is looking at ages of any members of the school community, it should be included. Allow students to defend their answers.

In #5, the sizes of the cans of tomatoes are not mentioned. Remind students that one has to know as much as possible about the data before making a choice whether or not to remove an outlier. This would then be an "informed decision."

Math Link

a) median: 8.13; mean: 8.06; highest score: 9.75; lowest score: 6.50; range: 3.25

b) median: 8.13; mean: 8.04; range: 1

c) range

d) Answers may vary. For example: Yes. The outlier scores are not consistent with the rest of the data.

Supported Learning

Learning Style and Memory

• Provide **BLM 12–9 Section 12.4 Extra Practice** to students who require more practice.

Apply

6. A medical study was conducted to learn the effect of caffeine on heart rate. Participants were asked to drink one 250-mL cup of coffee and record their number of heartbeats over a 15-second interval. The following data were collected.

Heartbeats in 15 seconds:
33, 35, 30, 70, 33, 31, 36, 40, 37, 29

a) What is the range?
b) What are the median and the mean?
c) Identify any possible outlier(s). Should the outlier(s) be removed from the data set? Explain why or why not.
d) How would removing the outlier(s) affect the median and the mean?

7. David had the following scores on his eight weekly spelling tests:
70%, 80%, 80%, 70%, 100%, 80%, 20%, 90%

a) What is the range?
b) What are his median and mean scores?
c) Identify any possible outlier(s). Should the outlier(s) be removed from the data set? Explain why or why not.
d) How would removing the outlier(s) affect the median and the mean?
e) Describe two different ways that you could determine David's overall mark to be between 75% and 85%.

Extend

8. A set of nine numbers has two outliers. The mean and median both equal 50 if you include or exclude the outliers. What are the possible nine numbers?

MATH LINK

In a gymnastics competition, each performance was judged by eight judges on a scale from 0.25 to 10.00. In order to calculate the gymnast's overall performance, the top score and bottom score were removed and the mean of the remaining scores was determined. This value is called the trimmed mean.

Jordan recorded the following scores for her friend's performance.

Judge	A	B	C	D	E	F	G	H
Score	8.25	7.50	9.75	8.50	6.50	7.75	8.00	8.25

Round your answers to 2 decimal places.

a) Using all the scores, what is the median? mean? highest score? lowest score? range?
b) Remove the top and bottom scores. What is the new median? mean? range?
c) Which value in part b) has changed the most?
d) Would you consider the highest and lowest scores to be outliers in this example? Explain.

12.4 The Effects of Outliers • MHR **445**

Apply and Extend

For #7, students might consider possibilities such as David was not feeling well and could not finish the test on the day he scored 20%. Ask if such a value should be counted.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students answer the following question in their journal: • What strategies do you know that will help you decide whether to remove or include an outlier?	• You may wish to review each of the questions in the exercises and discuss what needs to be considered in order to decide whether or not to include an outlier. • Ensure that students realize this decision is easier if they have as much information as possible about the set of data. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Have students review the part related to section 12.4 in **BLM 12–1 Chapter 12 Self-Assessment**, fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Math Link

In many competitions that include scores by a group of judges, the highest and lowest scores are considered to be outliers. As a class, you might wish to discuss the advantages and disadvantages of this approach and whether students agree it should be done.

Assessment *for* Learning	Supported Learning
Math Link The Math Link is intended to help students work toward the Wrap It Up! on page 455.	• Use **BLM 12–10 Section 12.4 Math Link** with students who are having difficulty getting started. • Encourage students to justify their responses.

12.5 Choose the Best Measure of Central Tendency

12.5 Choose the Best Measure of Central Tendency

Focus on...
After this lesson, you will be able to...
☐ determine when to use the mean, median, or mode to best describe a set of data
☐ solve problems using mean, median, and mode

The table below shows school T-shirt sales for the past ten weeks. The school wants to make one more order for the next 30 weeks. How could the school decide how many T-shirts to order?

Date	Sept 10	Sept 17	Sept 24	Oct 1	Oct 8	Oct 15	Oct 22	Oct 29	Nov 5	Nov 12
Sales	7	50	8	9	10	12	7	7	9	11

Discuss the Math

Which measure of central tendency is most appropriate?

1. What are the mean, median, mode, and range of the T-shirt data shown above?

2. Compare the mean, median, and mode. Which measure seems to best represent the ten numbers? Explain.

3. Share your answer from #2 with a classmate. Do your answers agree? If not, explain the reasons for your selection.

4. How many T-shirts should be purchased for the next 30 weeks? Explain your thinking.

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Specific Outcomes

SP1 Demonstrate an understanding of central tendency and range by:
• determining the measures of central tendency (mean, median, mode) and range
• determining the most appropriate measures of central tendency to report findings.

Warm-Up

1. Your weekly math test is out of 25. You receive the following scores: 15, 25, 16, 17, 16, 14, 15, 4, 17.
   a) Show the mean, median, mode, and range. Round to the closest whole number.
   b) Identify any possible outliers.
   c) Should the outlier(s) be removed from the data set? Explain why or why not.

2. A toolbox contains three hammers, two wrenches, and a screwdriver. You grab a tool at random. What is the probability of grabbing a wrench? Show as a fraction, ratio, and percent.

3. Model trains are on sale for 25% off. The regular price is $25.90. Estimate and then calculate the sale price, before tax.

4. A triangle has a base of 8.9 m and a height of 4.1 m. Estimate and then calculate the area of the triangle. Show your answer to the nearest tenth.

5. Rewrite the statement using brackets to make it true:
   $4.5 + 7.2 \times 3.1 = 36.27$

Mental Math

6. List the numbers divisible by 4 between 250 and 285. How do you know?

7. List the numbers divisible by 6 between 299 and 350.

8. Calculate 35% of 345.

9. Mentally calculate 95% of 250 in two ways.

10. Estimate 126 of 380 as a percent.

Reflect on Your Findings

5. a) Why do you think the sales in the second week were so high compared to the other nine weeks?

b) Which measure of central tendency was most affected by the large number of sales in the second week?

c) Which measure of central tendency do you think is not a good measure of the centre of these data? Explain why.

Example 1: Compare Measures of Central Tendency

Amir and Melanie's weekly quiz scores were 4, 5, 8, 9, 9.

Which measure(s) of central tendency best describe the data? Explain why.

The median is 8.

The mode is 9.

The mean is 7.
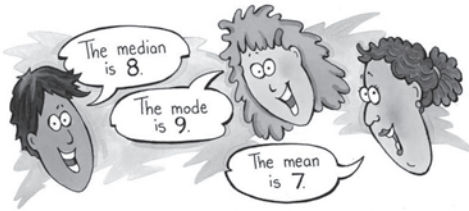
Solution

Since the mode represents the highest score, it is not the best representation of the five scores. The other two measures, median and mean, are both acceptable.

Show You Know

a) What are the mean, median, and mode for the following data set? Round your answers to the nearest tenth, if necessary.

16, 53, 14, 16, 11, 11, 12, 13, 11

b) Which measure(s) of central tendency best describe the data? Explain.

12.5 Choose the Best Measure of Central Tendency • MHR   447

Activity Planning Notes

In this section, students compare the measures of central tendency to determine which is best in a variety of situations. They also use them to solve problems.

Discuss the Math

Method 1: Have each student calculate the range and the three measures of central tendency individually. Circulate and see how each student is handling this task. Let students pair up to share and compare their answers.

Method 2: Calculate the range and measures of central tendency together as a class. Ask the class to vote on which measure of central tendency best represents the data. Discuss their choices. Ask individuals to justify their answers. Have students think of reasons for the peak in sales in the second week and then a steady sale of 7 to 12 shirts per week thereafter.

Warm-Up

1. a) mean = 15; median = 16; mode = 15, 16, 17; range = 21

b) 4, 25

c) Answers may vary. Removing both outliers would provide a mean of 16. Students might argue for keeping the 25, which represents a lot of hard work on their part, giving a mean of 17. This would make an interesting class discussion.

2. $P(\text{wrench}) = \frac{2}{6} = \frac{1}{3}$; 1 : 3; 33.$\overline{3}$%

3. Estimate: 25% = $\frac{1}{4}$; 25.90 is close to 26; 26 ÷ 4 = 6.50;

26 − 6.50 = \$19.50

Calculate: 0.25 × 25.90 = 6.48; 25.90 − 6.48 = \$19.42

4. Estimate: 9 × 4 ÷ 2 = 18 m^2

Calculate: 8.9 × 4.1 ÷ 2 = 18.2 m^2

5. (4.5 + 7.2) × 3.1 = 36.27

6. 252, 256, 260, 264, 268, 272, 276, 280, 284. The number formed by the last two digits is divisible by 2 at least twice. I found the first number, added 4 to get the second one, and then added 4 to each consecutive number to get the others.

7. 300, 306, 312, 318, 324, 330, 336, 342, 348

8. 10% of 345 = 34.5; 5% of 345 = 17.25; 3 × 34.5 + 17.25 = 120.75; 35% of 345 = 120.75

9. 10% of 250 = 25

5% = 12.5

95% = 100% − 5% = 250 − 12.5 = 237.5

95% = 9 × 10% + 5% = 225 + 12.5 = 237.5

10. 50% of 380 = 190; 25% of 380 = 95;

10% of 380 = 38; 5% = 19

35% of 380 = 133 Too high

30% of 380 = 114 Too low

The answer is between 30% and 35%.

Discuss the Math

1. mean: 13; median: 9; mode: 7; range: 43

2. Answers may vary. For example: Median. The mean includes the outlier of 50 and the mode is representative of the low end of the set of data.

3. Answers will vary.

4. Answers will vary. For example: 270. Multiply the median by the number of weeks: 9 × 30 = 270.

5. a) Answers will vary. For example: Perhaps the T-shirts were advertised the most that week.

b) mean

**c)** Answers may vary. For example: Mean. It is affected more by the higher number of orders in the second week.

Show You Know: Example 1

a) mean: 17.4; median: 13; mode: 11

b) Answers will vary. For example: Median. The median is not affected by the presence of the outlier 53.

Assessment *as* Learning	Supported Learning
Reflect on Your Findings These questions require students to analyse the data and to thoroughly understand measures of central tendency. During this process, they are generalizing what they have learned during the Discuss the Math.	• For #5a) and c), students must make judgments. Discuss the various possibilities and which one seems most feasible. • Make sure that students calculate the mean, median, mode, and range with and without the outlier, so that they can answer #5b).

Example 2: Choose Mode as the Best Measure of Central Tendency

Students have taken a vote on the new official school colours for sports uniforms. The number of votes for each colour is shown. Which measure of central tendency would you use to decide the winning colour?

7 4 1 5 6 10
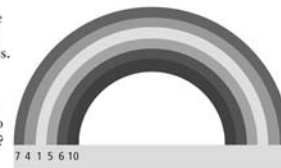
Solution

The data collected involve the frequency of colour choices. The most popular choice wins.

In this case, the median and mean do not provide any meaningful information about colour choice. The best measure to use is the mode. The mode is purple since purple is the most popular choice.

Example 3: Compare Median and Mean

Suki paid the following amounts for her last six pairs of jeans before tax:

$44, $38, $45, $49, $125, $50

a) What are the median and the mean jean prices?
b) Which measure of central tendency best describes these data?

Solution

a) Arrange the numbers in order. The median is the middle value.
38 44 45 49 50 125
The median is halfway between the values of 45 and 49 at 47.
The median price is $47.00.

$$\text{Mean} = \frac{38 + 44 + 45 + 49 + 50 + 125}{6}$$

(The number of pairs of jeans is 6.)

$$= 58.50$$
The mean price is $58.50.

b) The value of $125 is very different from the other five values. The single value, $125, alters the mean much more than the median. The median is a better measure of central tendency for the six prices.

Example 1 repeats the process of the Discuss the Math using a different set of data. Have students consider the median, mode, and mean and discuss how well each of them portrays the data set.

Assessment *for* Learning	Supported Learning
Example 1 Have students do the Show You Know on page 447 related to Example 1.	• Part a) requires students to calculate mean, median, and mode. Since earlier sections of the chapter dealt with these skills, students will not likely have problems with this part of the question. If they do, you may wish to review some of the skills from earlier in the chapter. • Part b) requires students to understand how measures of central tendency should be used in a given situation. You may wish to discuss which measures of central tendency are most affected by the outlier (53) and whether these measures of central tendency are good ones to use to represent the data set.

Hold a class discussion about Example 2. It is important for students to realize that median and mean have no value in this type of data set because the data values are not numerical. As a class, draw a frequency table for these data. Have students compare this table to the one on page 436. In the Example 2 table, the numbers are frequencies, and the median and mean are meaningless. In the page 436 table, the numbers are data values, and the mean, median, and mode are all meaningful.

For Example 3, students compare the median and mean of a set of data. Discuss why there is a great difference between them ($125 is much higher than other jeans prices but should be left in because it is an actual price).

Answers

Communicate the Ideas

1. Mode. The data collected involves the frequency of singer choices. The mode is the only measure that provides meaningful information.

2. median: 21.5; mean: 27. Answers may vary. For example: Median. The outlier has been included when calculating the mean.

Common Errors

- Some students may need clarification about when the mode is most useful.

R_x Mode is used when people are surveyed about their favourite colour, food, city, etc. No numerical values can be attributed to these categories. Therefore, mean and median cannot be determined.

- Some students may not identify outliers in the data sets that would lead to a preference of median or mean.

R_x This is a good opportunity to re-teach the concept of outlier to those students.

Discuss that the median price would be a better measure of central tendency because it is in the middle of the data set. Due to the one high value, the mean is higher than all but one of the jeans prices.

Key Ideas

The Key Ideas summarize the concepts in this section. Suggest that students write their own summary of the three measures of central tendency and when to use each one.

Communicate the Ideas

When working on #1, have students discuss which example it is closest to (Example 2). This comparison will help them with their assessment. In #2, emphasize that an outlier frequently makes the median a better measure of central tendency than the mean.

Assessment *as* Learning	Supported Learning
Communicate the Ideas Have students do both questions. The communication part of each question will help them generalize their understanding of which measure of central tendency is the best for the given examples.	• Observe and encourage students to give complete rationales for their choices. • Have students explain #1 to each other. • Use **Master 2 Two Stars and One Wish** to have students critique other students' explanations for either #1 or #2. As a class, make a list of criteria to consider during this critique. Students can write two things they like about an explanation and one thing they would improve.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 9, Math Link
Typical	1–4, 6, 8–10, Math Link
Extension/Enrichment	1, 2, 10–16

For help with #4 and #5, refer to Example 2 on page 448.

4. Which measure of central tendency best represents the following data? What is its value?

Favourite Hockey Player	Number of Votes
Sidney Crosby	8
Jarome Iginla	5
Alexander Ovechkin	8
Ryan Smyth	7

5. The following tally chart represents the sizes of running shoes that were sold last Saturday.

Size	7	8	9	10
Number Sold	IIII	IIII		II

a) What are the mean and the mode size of shoe?

b) If you are restocking the shoes at the end of the day, which measure of central tendency is more meaningful? Why?

For help with # 6 and #7, refer to Example 3 on page 448.

6. A realtor in Rainbow Town sold the following houses in the past month.

House Description	Selling Price
Red starter house	$80 000
Blue house	$140 000
Green house	$145 000
Grey house	$150 000
Pink mansion	$2 100 000

a) What are the median and mean?

b) Which measure of central tendency is more representative of the house prices in Rainbow Town?

7. In a grade 5 class, 16 students are 10 years old, 2 students are 9 years old, and 2 students are 11 years old. The teacher and her assistant are both 50 years old.

a) What are the mean, median, and mode ages of all students and teachers?

b) Which measure(s) of central tendency best describes the data? Explain why.

Apply

8. The following table shows survey results for the percent of radio listening time by music type among 100 Canadian teens.

Music Type	Listening Time (%)
Pop	19.0
Contemporary rock	31.0
Rap	14.7
Album rock	10.6
Country	8.7
Other	16.0

Which single music type best represents Canadian teenagers? Which measure of central tendency did you use to find your answer? Explain why.

9. Juan's Cleaners had developed a new disinfectant to kill germs. Ten tests were performed with the following results.
Percent of germs eliminated:
67, 99, 91, 87, 99, 70, 99, 69, 92, 61

a) If you were the owner of the company, which measure of central tendency would you use for advertising? Why?

b) If you were working for the Centre for Disease Control, which measure of central tendency would be best for the public to use in evaluating the product? Why?

Practise

Note that #4 is similar to Example 2, with a meaningless median and mean. Also, #5 is a combination of Examples 1 and 2. Part a) is like Example 1: students must determine the mean and mode, and part b) is like Example 2: students must determine that the mode is the most meaningful measure of central tendency.

Assessment *for* Learning	Supported Learning
Practise Have students do #3, #4, and #6. Students who have no difficulty with these questions can go on to the Apply questions.	• Students who have difficulty with #3 need to review Example 1. Coach them as they correct their answer. • Students may have trouble with #4. If they do, have them review Example 2, discuss how this type of data is different from the data in #3 and #6, correct their answer, and then have them do #5 on their own. You may need to use peer coaching to help students understand how this type of question works. • Students who have difficulty with #6 need to review Example 3. Coach them as they correct their answer, and then have them do #7 on their own.

Apply

Note that #8 is similar to Example 2, but worded a little differently. Discuss with students the meaning of "best represents."

10. A school held a canned food drive for holiday season food baskets in their community. The following numbers of cans were collected.

Grade	Number of Students	Total Cans Collected
1	20	100
2	24	96
3	22	99
4	20	120
5	18	90
6	26	130
7	25	125

a) Which grade collected the most cans?

b) What was the mean number of cans collected for each student in each grade?

c) Which grade should win a prize for most cans collected? Explain your choice.

Extend

11. A set of five distinct natural numbers has a mean of 6 and a median of 6. What is the largest possible number in the set?

12. The mean of four natural numbers is 28. If there is only one mode, what is a possible set of numbers?

13. A set of three whole numbers has a median of 17. If the smallest number is 10 and the mean is 15, what is the largest number?

14. A set of four whole numbers has a mode of 7. The smallest number is 2. What is the mean if the largest number is 8?

15. On a recent class test, the mean score was 80%, the mode score was 70%, and the median score was 90%. Kyle scored 80%. Which measure of central tendency would you use to make Kyle feel

a) confident about himself? Why?

b) like he could do better? Why?

16. Work with a partner to create a scenario where there are two groups that take opposite positions (labour and management, children and parents, etc.).

a) Provide data that supports using the mode as the best measure of central tendency.

b) Provide data that supports using the median as the best measure of central tendency.

MATH LINK

A set of seven judges gave the following scores to Susan's diving performance:
7.2, 6.8, 7.3, 8.0, 8.5, 8.2, 6.8

a) What is the mean? Round your answer to the nearest tenth.

b) What is the median?

c) What is the mode?

d) Which measure(s) of central tendency best represent the centre of the data? Explain why.

12.5 Choose the Best Measure of Central Tendency • MHR   451

Extend

In #11, students may need a reminder about the meaning of distinct and natural numbers. Distinct means "different," and natural numbers include 1, 2, 3, etc.

You may wish to have most students do #16. Developing a scenario like this will provide them with further understanding of Example 2.

Assessment *as* Learning	Supported Learning
Math Learning Log Have students write a response based on the following prompt: • Develop three different situations. In each one, a different measure of central tendency should be the best choice to describe the centre of the data set.	• Refer students to the three examples in this section. Example 1 shows the mean or median as the preferred choice, Example 2 shows the mode, and Example 3 shows the median. Students may wish to use the examples as models as they work on their responses. • Have students check the What I Need to Work On tab of their chapter Foldable. Encourage them to keep track of the items that are giving them difficulty and to check off each item as the problem is resolved. • Have students review the part related to section 12.5 in **BLM 12–1 Chapter 12 Self-Assessment,** fill in the appropriate part of the During column, and report what they might do about any items that they have marked either red or yellow.

Supported Learning

Learning Style and Memory

• Provide **BLM 12–11 Section 12.5 Extra Practice** to students who require more practice.

Meeting the Needs of All Learners

• Depending on students' interests, you could change the Math Link activity to one involving a fishing derby and the size of the fish caught (in kg).

Math Link

The Math Link provides to students additional practice with rounding decimals as they work on measures of central tendency. Note that there is no outlier in this set of data. In this scoring, the high and low scores are not removed. You may wish to have students analyse what would happen to the measures of central tendency if these scores were removed.

Assessment *for* Learning	Supported Learning
Math Link The Math Link is intended to help students work toward the Wrap It Up! on page 455.	• Observe students to see the processes they use to find the measures of central tendency, and note the conclusions they draw. • Encourage students to justify their response to d); check that their rationale is supported by their calculations. • Provide **BLM 12–12 Section 12.5 Math Link** to students who are having difficulty getting started on this Math Link.

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Suggested Timing
40–50 minutes

Blackline Masters
BLM 12–1 Chapter 12 Self-Assessment
BLM 12–3 Section 12.1 Extra Practice
BLM 12–5 Section 12.2 Extra Practice
BLM 12–7 Section 12.3 Extra Practice
BLM 12–9 Section 12.4 Extra Practice
BLM 12–11 Section 12.5 Extra Practice

Supported Learning

Learning Style and Memory
- Students who require more practice on a particular topic may refer to **BLM 12–3 Section 12.1 Extra Practice, BLM 12–5 Section 12.2 Extra Practice, BLM 12–7 Section 12.3 Extra Practice, BLM 12–9 Section 12.4 Extra Practice,** and **BLM 12–11 Section 12.5 Extra Practice**.

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12 Chapter Review

Key Words

For #1 to #4, unscramble the letters to identify the appropriate key word.

1. A E I M N D
   the middle number in a set of data arranged from smallest to largest

2. N A M E
   the sum of a data set divided by the number of values in the set

3. L R U O I T E
   a value that is significantly smaller or larger than the other values in a data set

4. A G R E N
   the positive difference between the largest and smallest values in a data set

12.1 Median and Mode, pages 422–427

5. The number of phone calls received at Jessica's house in the last seven days were 5, 9, 8, 5, 6, 3, and 4. What is the mode and median?

6. The table shows the mean wind speed in kilometres per hour from weather readings taken at the Blood Reserve near Lethbridge, Alberta.

Month	Wind Speed (km/h)	Month	Wind Speed (km/h)
Jan	22	July	16
Feb	20	Aug	15
Mar	19	Sept	17
Apr	21	Oct	21
May	20	Nov	21
June	19	Dec	22

What is the mode and median?

7. Make a data set with six numbers for which the values of the median and mode differ by 2. Explain how you found your answer.

12.2 Mean, pages 428–433

8. Between January and March one year, school was cancelled in Chesterfield Inlet seven times due to blizzards. The following data give the number of days each blizzard lasted:
   1, 3, 2, 3, 5, 2, 3
   What was the mean length of a blizzard that year?

9. The Wong family took a family vacation to California. The one-way driving trip was 2100 km. It took them 3 days to drive there.
   a) What was the mean distance they travelled each day?
   b) Predict how many days it would take them next summer to drive one way to eastern Canada, a distance of 4900 km. Assume they can travel the same mean distance each day.

10. A set of five numbers has a sum of 35.
    a) What is the mean of the five numbers?
    b) If one of the numbers is removed, the mean is 6. What is the sum of the remaining four numbers?
    c) Which number was removed?

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Activity Planning Notes

Allow students to work independently or in pairs. Provide an opportunity for students to discuss any questions, consider alternative methods of solving, and ask about questions they found difficult.

Have students place the numbers 5 to 14 in two columns in their notebooks. Tell them to look at the question related to the number in their student resource. Students can then use the colours that they used on **BLM 12–1 Chapter 12 Self-Assessment** to circle the questions that they need help with or do not yet understand.

Assessment *for* Learning	Supported Learning
Chapter 12 Review The chapter review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Suggest that students check the contents of the What I Need to Work On tab of their chapter Foldable. Have students do at least one question related to any concept, skill, or process that has been giving them trouble. • Once they have completed the Chapter 12 Review, encourage students to list in their chapter Foldable any questions they found difficult. • Have students revisit any section they are having difficulty with prior to working on the Chapter 12 Test.

12.3 Range and Outliers, pages 434–439

11. The following table shows the number of forest fires that were caused by humans in one year.

Region	Fires Caused by Humans in One Year
Manitoba	203
Saskatchewan	239
Alberta	336
British Columbia	644
Yukon Territory	25
Northwest Territories	17

a) What are the highest and lowest values?

b) What is the range?

c) Identify any possible outlier(s). What might be a possible explanation for the outlier(s)?

12. The following table shows the mean hourly wages of six different categories of employment in British Columbia, Manitoba, and Alberta in 2006.

Type of Occupation	Hourly Wages ($)		
	MB	AB	BC
processing and manufacturing	13.62	17.68	17.53
health related	20.41	22.72	25.16
trades, transport, and equipment	16.87	19.98	20.73
science related	23.99	28.84	25.71
business, finance, and administration	16.29	18.77	18.94
sales and service	11.37	12.68	13.60

a) What is the range of wages in each province?

b) What is the range for the three highest paying categories in each province?

12.4 The Effects of Outliers, pages 440–445

13. The following table shows the October rainfall in several locations.

Location	October Rainfall (mm)
La Ronge, SK	22
Regina, SK	14
Jasper, AB	24
Edmonton, AB	10
Slave Lake, AB	18
Terrace, BC	197
Kelowna, BC	21

a) What are the median and mean October rainfalls?

b) Identify any possible outlier(s). Should the outlier(s) be removed from the data set? Explain why or why not.

c) How would removing the outlier(s) affect the median and the mean?

Geography Link

The rainfall recorded for Terrace, BC is correct. You may wish to learn more about the geography of Terrace that makes it such a wet place.

12.5 Choose the Best Measure of Central Tendency, pages 446–451

14. Two grade 7 classes collected cans of food for a local food bank. The table shows the number of cans of food that were collected by each student.

	Number of Cans Per Student						
	0	1	2	3	4	5	37
Class A	IIII	IIII II	IIII III				I
Class B	IIII	IIII	IIII II	II	I		

a) What is the median for each class?

b) What is the mean for each class?

c) If a tie is not allowed, which class should get the prize? Explain.

Learning Style

• Allow students to complete the chapter review using any combination of oral description, diagrams for explanation, and written answers.

ESL

• English language learners may still be having difficulty with the words *mean, median, mode,* etc.

ESL, Language, and Memory

• Allow students to practise the vocabulary terms using flash cards. Have students work together to quiz each other on the Key Words for the chapter.

• Encourage students to use their chapter Foldable during the chapter review and to add any notes into the pertinent sections.

Gifted and Enrichment

• Some students may already be familiar with the skills handled in this review. To provide extra questions, go to **www.mathlinks7.ca** and follow the links.

Assessment *as* Learning	Supported Learning
Math Learning Log Once students have completed the chapter review, have them reflect on their progress and complete a journal entry for each statement: – I am comfortable with the following parts of the chapter … – I can calculate median, mode, and mean by ... – I can identify outliers and understand their effect by ... – I am having difficulty with … – Here's how I plan to address the areas I am having difficulty with …	• Have students refer back to the What I Need to Work On tab of their chapter Foldable and complete the journal statements using the contents of that tab. • You may wish to have students refer to **BLM 12–1 Chapter 12 Self-Assessment** when they report on what they are comfortable with, what they continue to have difficulty with, and what they plan to do about it.

Suggested Timing
40–50 minutes

Materials
• calculator (optional)

Blackline Masters
BLM 12–1 Chapter 12 Self-Assessment

BLM 12–13 Chapter 12 Test

Assessment *as* Learning	Supported Learning
Chapter 12 Self-Assessment Have students review their earlier responses on **BLM 12–1 Chapter 12 Self-Assessment.**	• Have students use their responses on the Chapter 12 Practice Test and the work they completed earlier in the chapter to complete the After column of this self-assessment. Before the chapter test, coach them in the areas in which they are having problems.

Practice Test (reproduced page)

12 Practice Test

For #1 to #3, choose the best answer.

Use the following information to answer #1 to #3.
In April, 2003, Canadian Mike Weir won the Masters golf tournament. His scores for the four-day event appear in the table.

Day	Score
Thursday	70
Friday	68
Saturday	75
Sunday	68

1. What is the mode of the set of golf scores?
  A 7   B 68   C 69   D 70.25

2. What is the mean of the set of golf scores?
  A 7   B 68   C 69   D 70.25

3. What is the range of the set of golf scores?
  A 7   B 68   C 69   D 70.25

Use the following information to answer #4 to #6.
In the past year the following sizes of litters of kittens were born in one neighbourhood:
5, 6, 3, 5, 6, 3, 4, 5, 5, 4

4. The mode is ■.

5. The median is ■.

6. The mean is ■.

Short Answer

7. Suppose every piece of data in a set is increased by 20. How does this affect the values of the mean, median, and mode?

8. a) Give an example of a situation where the mode is the best measure of central tendency.
  b) Give an example of a situation where the median is the best measure of central tendency.
  c) Give an example of a situation where the mean is the best measure of central tendency.

9. The table shows the percent of homes in four provinces that have water-saving, low-flow shower heads installed.

Province	Homes With Low-Flow Shower Heads (%)
Manitoba	34.0
Saskatchewan	26.6
Alberta	32.2
British Columbia	43.4

  a) What is the range of percents?
  b) What is the mean of percents?

10. Mary and Iblauk went sledding after school. They both went down the hill six times on the same sled. The following list gives the distances they slid, in metres:
25, 42, 53, 47, 41, 51

  a) What is the mean distance travelled, to the nearest tenth of a metre?
  b) Identify any outlier(s). Should the outlier(s) be included in the calculation of the mean? Explain why.

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Study Guide

Question(s)	Section(s)	Refer to	I can …
1–7	12.1 12.2	Example 1 Example 1	✓ determine the median and mode for a given set of data ✓ determine the mean for a given set of data
8, 11	12.5	Examples 1, 2, 3	✓ determine when it is best to use the mean, median, or mode to describe a set of data ✓ solve problems using mean, median, and mode
9	12.2 12.3	Example 1 Example 1	✓ determine the mean for a set of data ✓ determine the range for data sets
10	12.2 12.3 12.4	Example 2 Example 2 Example 2	✓ solve problems by finding the mean ✓ identify outliers in data sets ✓ justify whether outliers should be included when determining measures of central tendency
12, 13	12.4	Example 2	✓ explain the effects of outliers on measures of central tendency ✓ justify whether outliers should be included when determining measures of central tendency

11. The following times, in seconds, were recorded for a mouse to run through a maze:

34, 12, 10, 11, 12, 13, 9

  a) What are the median and mean times?
  b) Which measure of central tendency best represents the data?
  c) Identify any possible outlier(s). Should the outlier(s) be removed from the data set? Explain why or why not.
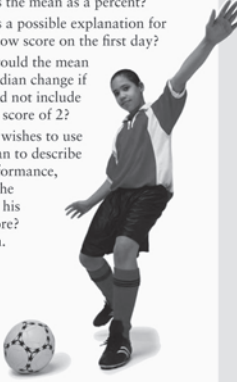 d) How would removing the outlier(s) affect the median and the mean?

Extended Response

12. Laura tracks the number of hours she spends playing games on the computer each night for one week:

2.5, 2.0, 1.0, 2.5, 2.0, 15, 3.5

  a) What is the range?
  b) What are the mode, median, and mean?
  c) Identify any possible outlier(s). Should the outlier(s) be removed from the data set? Explain why or why not.
  d) Which measure of central tendency changes the most when the outlier is removed? By how much does it change?

13. For every soccer practice, Cam keeps track of the number of goals he makes in ten attempts. In the last ten practices, he made the following numbers of goals:

2, 6, 8, 7, 8, 9, 7, 8, 6, 8

  a) What is the range?
  b) What is the median as a percent?
  c) What is the mean as a percent?
  d) What is a possible explanation for Cam's low score on the first day?
  e) How would the mean and median change if Cam did not include his low score of 2?
  f) If Cam wishes to use the mean to describe his performance, should he include his low score? Explain.

WRAP IT UP!

Collect at least ten pieces of numerical data about a topic of your choice. Analyse the data to show what you know about working with data. Your report should include:
  • a description of why you collected this data set and how you collected it
  • the range, mean, median, and mode
  • an explanation of which measure of central tendency best represents the data
  • a description of any possible outliers
  • an explanation for whether any outliers should be used in determining the measures of central tendency

ESL

• Some English language learners may need to have the term *sledding* explained to them before they work on #10.

ESL, Language, and Memory

• Consider allowing students to use their Foldables during the practice test.

Activity Planning Notes

Have students start the practice test the same way they started the Chapter 12 Review. Have them write numbers 1 to 13 in their notebooks and circle each question using the colours they used on **BLM 12–1 Chapter 12 Self-Assessment** to identify which questions they need help with or do not yet understand.

This practice test can be given as an in-class or take-home assignment. These are the minimum questions that will meet the related curriculum outcomes: #1–#6, #9, #10, and #12.

Answers to the Chapter 12 Practice Test are provided on **BLM 12–17 Chapter 12 *MathLinks 7* Student Resource Answers**.

Assessment *of* Learning	Supported Learning
Chapter 12 Test After students complete the practice test, you may wish to use **BLM 12–13 Chapter 12 Test** as a summative assessment.	• Consider allowing students to use their chapter Foldable and/or a calculator. • Consider using the Math Games on page 456 or the Challenge in Real Life on page 457 to assess the knowledge and skills of students who have difficulty with tests.

Wrap It Up!

WRAP IT UP!

Collect at least ten pieces of numerical data about a topic of your choice.
Analyse the data to show what you know about working with data. Your
report should include:
• a description of why you collected this data set and how you collected it
• the range, mean, median, and mode
• an explanation of which measure of central tendency best represents the data
• a description of any possible outliers
• an explanation for whether any outliers should be used in determining the
  measures of central tendency

Practice Test • MHR   455

Suggested Timing
60–75 minutes

Materials
• encyclopedia
• computer with Internet access
• poster board

Blackline Masters
Master 1 Project Rubric

BLM 12–4 Section 12.1 Math Link

BLM 12–6 Section 12.2 Math Link

BLM 12–8 Section 12.3 Math Link

BLM 12–10 Section 12.4 Math Link

BLM 12–12 Section 12.5 Math Link

BLM 12–14 Chapter 12 Wrap It Up!

Supported Learning

Learning Style
• To students who need help getting
  started, suggest that they connect
  their project proposal to one of
  their interests or hobbies. If they
  love hockey, suggest that they
  analyse hockey statistics. If they are
  fond of horses, suggest they collect
  scientific data on speeds of horses
  compared with other animals.
• Allow students to choose among a
  written report, an oral report, and a
  visual display for the Wrap It Up!
• This chapter problem can be scaled
  up or down to meet the needs of
  your class. Decide whether students
  should work alone or in pairs.

Common Errors

• Some students may want to collect
  more than ten pieces of data.
R$_x$ Monitor students and set limits
  for the amount of data they are
  allowed to include.
• Some students may struggle
  due to poor choices and poor
  organization of their results.
R$_x$ Build in opportunities for
  students to check with you that
  they are on track.

Specific Outcomes

SP1 Demonstrate an understanding of central tendency
and range by:
• determining the measures of central tendency (mean,
  median, mode) and range
• determining the most appropriate measures of central
  tendency to report findings.
SP2 Determine the effect on the mean, median and mode
when an outlier is included in a data set.

Activity Planning Notes

Discuss the following points with students:
• some possible ways of collecting numerical data (survey, research, etc.)
• how they will present their data (tables, charts, etc.) and their findings

To give students ideas, remind them of some topics that have been covered
in the chapter: earthquake magnitude, sports scores, and heart rate. Have
students design a proposal and get your approval before starting.

Assessment *of* Learning	Supported Learning
Wrap It Up! The Wrap It Up! is intended to allow students to apply their knowledge of the measures of central tendency to a topic that interests them. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this Wrap It Up! Page 455a provides notes on how to use this rubric for the Wrap It Up!	• Having students brainstorm questions associated with specific topics may help to prompt students who are experiencing difficulty in deciding what questions to ask. • It is important for students to include justification in their analysis. • Some students may need to review the Math Links before they begin. • Students who have not done the earlier Math Links and are having difficulty with this Wrap It Up! may need to go back and do the Math Links. The following worksheets may help them: **BLM 12–4 Section 12.1 Math Link, BLM 12–6 Section 12.2 Math Link, BLM 12–8 Section 12.3 Math Link, BLM 12–10 Section 12.4 Math Link,** and **BLM 12–12 Section 12.5 Math Link.** • Some students might benefit from using **BLM 12–14 Chapter 12 Wrap It Up!**, which provides scaffolding. • Observe how accurately students design, explain, and justify their plans; how clearly they organize the data collected; and how well they understand what they are attempting to measure.

The chart below shows the **Master 1 Project Rubric** for tasks such as the Wrap It Up! and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❏ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❏ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❏ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• correctly completes the question with all supporting explanations and work
4 **(Above Acceptable)**	❏ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❏ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❏ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• correctly completes bullets 1, 2, 3, and 4 but has a weak or omitted solution in bullet 5
3 **(Meets Acceptable)**	❏ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❏ Procedures are **basic** and may contain a **major error or omission** ❏ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• correctly completes bullets 1, 2, and 3 *or* • correctly completes bullets 2, 3, and 4
2 **(Below Acceptable)**	❏ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❏ Procedures are **basic** and may contain **several major mathematical errors** ❏ Communication is **weak**	• correctly completes bullets 1 and 2 *or* • correctly completes bullets 2 and 3 *or* • correctly completes bullets 3 and 4 based on incorrect bullets 2 and 3
1 **(Beginning)**	❏ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❏ Communication is **weak or absent**	• correctly completes either bullet 1 or 2 or 3

Math Games

Suggested Timing
40–50 minutes

Materials
• 4 six-sided dice
• paper clip

Blackline Masters
BLM 12–15 Data Duel Spinner

Assessment *for* Learning	Supported Learning
Data Duel Observe how students calculate mean, median, mode, and range.	• Provide students with **BLM 12–15 Data Duel Spinner** for making the spinner. • Monitor students to check that they are calculating the mean, median, mode, and range correctly. • Have students record the numbers and type of data requested so that they can check each other's answers.

Specific Outcomes

SP1 Demonstrate an understanding of central tendency and range by:
• determining the measures of central tendency (mean, median, mode) and range
• determining the most appropriate measures of central tendency to report findings.

Common Errors

• Some students may have trouble determining the median.

R_x Make sure students understand the following ideas:
  – If the number of values in the set is even, the median is the average of the two middle values.
  – If the number of values is odd, the median is the middle value.

Activity Planning Notes

Read through the game with students. Use the following data to review the definitions of mean, median, mode, and range: 8, 5, 8, 4, 5, 5, 12, 9, 7, 7. Focus on these points:
• To determine the mean, find the sum of the set of values and then divide by the number of values:
  mean $= (8 + 5 + 8 + 4 + 5 + 5 + 12 + 9 + 7 + 7) \div 10 = 7$
• The median is the middle number when the set is written in order:
  4, 5, 5, 5, 7, 7, 8, 8, 9, 12
  In this case, the median is the mean of these two numbers: $(7 + 7) \div 2 = 7$
• The mode is the most frequently occurring value, which is 5 for this set.
• The range is the difference between the highest value in the set and the lowest: range $= 12 - 4 = 8$

Challenge in Real Life

Mathematical Processes

✓	Communication
✓	Connections
☐	Mental Mathematics and Estimation
✓	Problem Solving
✓	Reasoning
✓	Technology
☐	Visualization

Specific Outcomes

SP1 Demonstrate an understanding of central tendency and range by:
• determining the measures of central tendency (mean, median, mode) and range
• determining the most appropriate measures of central tendency to report findings.

Activity Planning Notes

You may wish to use the following steps to introduce and complete this challenge:

1. Read through Managing a Business as a class. Ask a couple of students to share their experiences with shopping in a sports shoe store. Have them talk about the variety of sizes, styles, and prices they saw. Show examples of the various styles of shoes by pointing them out on the feet of students in class or by providing pictures.

2. Lead a class discussion about managing a shoe store. Talk about why it is necessary to know customer needs. Discuss what would happen if the store had too many shoes with styles, sizes, and prices that did not meet customers' needs and too few that did. You may wish to expand the discussion to other businesses that would use this type of data collection and analysis. If possible, invite the manager of a local business to come in and speak to students about this topic.

Learning Style, Language, and Memory

• Some students may need additional reinforcement to process the information and the instructions.

ESL and Language

• Some students may need help with the vocabulary: *central tendency, mean, median,* and *mode.*

Gifted and Enrichment

• Encourage students to research how store managers collect their sales data and what their data is for youth shoe sizes, styles, and prices.

• Students could make a business plan for a youths' sports shoe store. They could name their store and use their survey results to decide what sizes, styles, and prices of shoes they will sell.

3. Talk about how students could create their surveys and what survey questions to ask. Discuss bias and good sampling practices.

4. Clarify that the task is to

• create a survey on sports shoe sizes, style preferences, and the amount of money students will spend (Note: Suggest that students limit the shoe sizes to whole sizes and round prices to the nearest $10 so that the results will give a clear central tendency.)

• survey the students in their class and then organize the data

• analyse the data, including measures of central tendency

• choose which measure of central tendency is best for each set of results

5. Review the **Master 1 Project Rubric** with students so that they will know what is expected.

This Challenge in Real Life can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment *for* Learning	Supported Learning
Managing a Business Discuss the Challenge in Real Life with the class, including how the survey could be developed. Then have students individually do the survey and analyse the data.	• You may wish to provide to students **BLM 12–16 Sports Shoe Survey** to help them organize the results from their survey. • Some students may need coaching as they decide which measure of central tendency is best for each set of results. • For a second challenge, complete with teaching notes and student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

Assessment *of* Learning	Supported Learning
Managing a Business Discuss the Challenge in Real Life with the class, including how the survey could be developed. Then have students individually do the survey and analyse the data.	• Use **Master 1 Project Rubric** to assist you in assessing student work. Page 457a provides notes on how to use the rubric for this challenge. • To view student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as the Challenge in Real Life and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 (Standard of Excellence)	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution with justification present
4 (Above Acceptable)	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete response to parts a) and b), and the justification in part c) is incorrect or missing *or* • provides a complete response to parts a), b), and c) using a measure of central tendency that is not substantiated by the analysis, though the rationale is correct
3 (Meets Acceptable)	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides correct parts a) and b) with most or all measures of central tendency justified mathematically
2 (Below Acceptable)	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides correct part a) with data organized to reflect the three categories • has started some analysis but it does not move beyond initial steps
1 (Beginning)	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• has collected some data but there is no formal organization for further analysis

Chapters 9-12 Review

Suggested Timing
60–75 minutes

Supported Learning

Learning Style
• Allow students to complete the Chapters 9–12 Review using any combination of oral description, diagrams, and written answers.

ESL
• Some English language learners may need the term *cockroaches* explained to them.

Chapter 9 Add and Subtract Integers

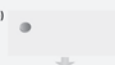1. What addition statement does each diagram represent?
   a)
   b)

2. What subtraction statement does each diagram represent?
   a)   b)

3. What is each sum or difference?
   a) $(+5) + (-5)$ b) $(+7) + (-4)$
   c) $(-9) - (-7)$ d) $(+8) - (-8)$
   e) $(+3) - (+7)$ f) $(-2) - (-9)$

4. One October morning in Prince Edward, Saskatchewan, the temperature was $-3°C$. In the late afternoon, the temperature was $12°C$ higher. What was the temperature in the late afternoon?

5. The surface of Great Bear Lake is 156 m above sea level. The bottom of the lake is 257 m below sea level. Estimate and calculate the depth of the lake.

Chapter 10 Patterns and Expressions

6. Describe each pattern. What are the next two numbers in each pattern?
   a) 1, 4, 7, 10, …
   b) 8, 13, 18, 23, 26 …
   c) 17, 14, 11, 8, …

7. Look at the following number pattern.
   $\frac{2}{30} = 0.0666\ldots$, or $0.0\overline{6}$
   $\frac{5}{30} = 0.1666\ldots$, or $0.1\overline{6}$
   $\frac{8}{30} = 0.2666\ldots$, or $0.2\overline{6}$
   a) Describe the pattern.
   b) What is the next fraction in the pattern?
   c) What is $\frac{17}{30}$ written as a repeating decimal using bar notation?

8. The tile pattern shown is being used to cover a bathroom floor.
   Figure 1   Figure 2   Figure 3
   a) Make a table showing the number of white and blue tiles in the first five designs.
   b) Describe the pattern for the number of blue tiles in relation to the number of white tiles.

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Study Guide

Question(s)	Section(s)	Refer to	I can …
1	9.1 9.2	Example 1 Example 1	✓ add integers using integer chips ✓ add integers using a number line
2	9.3	Example 1	✓ subtract integers using integer chips
3	9.2 9.4	Example 2 Example 1	✓ add integers ✓ use addition to subtract integers
4, 5	9.5	Example	✓ decide when to add and subtract integers in solving problems
6, 7	10.1	Example 2	✓ describe patterns ✓ use patterns with repeating decimal numbers
8, 9	10.1 10.2 10.3	Example 2 Example 1 Example 2	✓ describe patterns using words, tables, or diagrams ✓ write an expression to represent a pattern ✓ make a table of values for an expression ✓ evaluate an expression
10	10.4	Example 2	✓ describe the relationship shown on a graph
11	11.1	Example 2	✓ describe the difference between an expression and an equation

c) Choose a variable and tell what it represents. Then use the variable to write an algebraic expression for the number of blue tiles.

d) How many blue tiles will there be if there are 24 white tiles?

9. The expression $2n + 4$ represents the perimeter of any figure in this pattern, where n is the figure number.
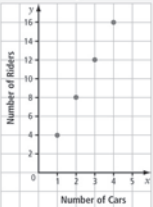
Figure 1   Figure 2   Figure 3

a) What would be the perimeter of Figure 12?

b) Make a table of values for the first six figures in the pattern.

10. The graph shows the number of roller coaster riders allowed on a roller coaster train, depending on the number of cars that make up the train.

a) Make a table of values for the first five values of x starting at $x = 1$.

b) What is an algebraic expression for the number of riders in relation to the number of cars?

c) Describe the pattern of points on the graph in two different ways.

d) If there are 10 cars in the roller coaster train, how many riders are allowed on the train?

Chapter 11 Solving Equations

11. The diagram represents an equation.

a) What are the two expressions that make up this equation?

b) What is the equation?

12. Solve by inspection. Verify your answer.

a) $k - 7 = 19$ b) $p + 12 = 12$

c) $2n = 18$ d) $\frac{c}{8} = 3$

13. Solve the equation modelled by each diagram. Check your solution.

a)

b)

14. The formula for the perimeter of an equilateral triangle is $P = 3s$. What side length is needed to make an equilateral triangle with a perimeter of 48 cm?

15. An adventure company charges $95 per day for canoeing equipment plus $10 per student for food. The total cost for one day can be modelled using the equation $C = 10n + 95$.

a) What do the variables C and n represent?

b) Students in one class raised $345 for a one-day trip. How many students can go?

Question(s)	Section(s)	Refer to	I can …
12, 13	11.2	Example 1 Example 2	✓ solve equations and record the process
14, 15	11.4	Example 2	✓ solve two-step equations and record the process
16	12.1 12.2	Example 1 Example 1	✓ determine mean, median, and mode for a given set of data
17, 18	12.1 12.2 12.3 12.4	Example 1 Example 1 Examples 1, 2 Example 2	✓ determine median and mode of a data set ✓ determine mean for a set of data ✓ determine the range and identify the outlier in a data set ✓ determine the effects of outliers
19, 20	12.2	Example 2	✓ solve problems by finding the mean
21	12.4	Example 2	✓ justify whether outliers should be included when determining measures of central tendency ✓ determine the effects of outliers
22	12.5	Examples 1, 2, 3	✓ determine when it is best to use the mean, median, or mode to describe a set of data

Chapter 12 Working With Data

16. The following numbers of haircuts were given at a salon in the last five days: 25, 29, 25, 26, 35.

a) What is the mode?

b) What is the median?

c) What is the mean?

17. The mode is 6 for the set of numbers 3, 4, 4, 5, 6, 6, 7, 8, 9, 10, x, and y. What are possible whole number values for x and y?

18. Robert takes his dog for a walk six days a week. The following times indicate how long they walked last week:

54 min, 56 min, 60 min, 58 min, 55 min, 28 min

a) What is the range?

b) Which time may be an outlier?

c) Why might this value be so different from the others?

d) If you remove the outlier, what is the new range?

19. Steven scored 338 points in 26 basketball games this fall. Micha played in 8 fewer games than Steven. Her mean score per game was 0.5 points higher than Steven's. How many total points did Micha score in her basketball season?

20. Thirty students wrote a science test. Twenty-five students achieved a mean mark of 74%. The other five students had a mean mark of 45%. What was the class mean mark? Express your answer as a percent.

21. Melissa found the following prices for five different brands of orange juice in the refrigerated section at the grocery store: $3.29, $2.99, $3.49, $6.98, and $3.79.

a) What is the range?

b) What are the median and the mean?

c) Which is the best measure of central tendency for the data?

d) Identify any possible outlier(s). Should the outlier(s) be removed from the data set? Explain why or why not.

e) How would removing the outlier(s) affect the median and the mean?

22. Roach-Away developed a new chemical to destroy cockroaches. Ten tests were performed. The percent of roaches destroyed were:

60, 99, 90, 99, 70, 91, 88, 71, 69, 99

If you were the owner of the company, which measure of central tendency would you use for advertising? Explain your choice.

Activity Planning Notes

Students might work independently to complete the questions, and then in pairs to compare solutions. The minimum questions that will meet the curriculum requirements are #1–#3, #5–#8, #11–#13, #15, #16, #18, and #21.

Assessment *for* Learning	Supported Learning
Chapters 9–12 Review This cumulative review provides an opportunity for students to assess themselves by completing selected questions in each chapter and checking their answers against the answers in the back of the student resource.	• Have students review the tests from each chapter and any challenges related to those chapters, identify the items that they had problems with, and do the questions related to those items. Have students do at least one question that tests skills from each chapter. • Have students revisit any chapter section they are having difficulty with.

Assessment *as* Learning	Supported Learning
Math Learning Log Once students have completed the Chapters 9–12 Review, have them reflect on their progress and complete a journal entry for each statement: – I continue to have difficulty with … – Here's how I plan to address what I am having difficulty with …	• Encourage students to try to clear up any problems they have had during the past four chapters. Work with them to provide the necessary coaching.

Task

The Birthday Party

1. Students in two classes were asked how many friends were invited to their last birthday party.
   Class A: 5, 10, 10, 6, 8, 9, 6, 5, 4, 6, 8
   Class B: 11, 9, 13, 11, 18, 12, 14, 18, 16, 15, 11, 14
   a) Find the mean, median, and mode for each class.
   b) What is the range for each class?

2. You are planning a birthday party. You can choose from the following three activities. Costs include all taxes.

   A Bowling: Cost is $12 per person. This includes the cost of food, drinks, shoe rental, and 1 h of bowling.

   B Swimming: Cost is $8 per person plus $55 for renting the pool.

   C Movie: Cost is $10 per person plus $30 for the party room.

   In all cases, the birthday person is free.
   You want to invite at least 8 friends. The total cost cannot be more than $200.
   Model the cost of each activity using an algebraic equation.

3. Which party would you choose? How many friends can you take? What is the total cost?

4. Explain the decisions you made in #3. Justify your response using at least two of a table of values, equations, or graphs.

Suggested Timing
60–75 minutes

Materials
- ruler (optional)
- calculator (optional)
- cups and counters (optional)

Blackline Masters
Master 1 Project Rubric

Master 9 0.5 Centimetre Grid Paper

Mathematical Processes

- ✓ Communication
- ✓ Connections
- ✓ Mental Mathematics and Estimation
- ✓ Problem Solving
- ✓ Reasoning
- ✓ Technology
- ✓ Visualization

Specific Outcomes

SP1 Demonstrate an understanding of central tendency and range by:
- determining the measures of central tendency (mean, median, mode) and range
- determining the most appropriate measures of central tendency to report findings.

PR1 Demonstrate an understanding of oral or written patterns and their equivalent linear relations.

PR2 Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.

PR5 Evaluate an expression given the value of the variable(s).

PR7 Model and solve problems that can be represented by linear equations of the form:

- $ax + b = c$
- $ax = b$
- $\frac{x}{a} = b, a = 0$

concretely, pictorially and symbolically, where a, b and c are whole numbers.

1. **a)** Class A: mean: 7; median: 6; mode: 6
   Class B: mean: 13.5; median: 13.5; mode: 11
   b) Class A: range: 6; Class B: range: 9

2. Answers may vary. For example:
   Bowling: $b = 12p$ where b is the cost of bowling and p is the number of people
   Swimming: $s = 8p + 55$ where s is the cost of swimming and p is the number of people
   Movie: $m = 10p + 30$ where m is the cost of the movie and p is the number of people

3. Answers will vary.

4. Answers will vary.

Supported Learning

Learning Style

- You may wish to allow students to use calculators.
- Kinesthetic and concrete learners may want to model the cost of each party using cups and counters. They could model the bowling party as 12 cups and make a pattern to show what could be in each cup and still keep the answer below $200. The cost of swimming and the movie could be modelled in a similar manner.

Meeting the Needs of All Learners

- Invite the class to share different ways that they celebrate their birthdays. If students refer to playing games, have them demonstrate how to play the games.
- You may wish to change the party options to coincide with activities available in and near the local community. The costs could be adjusted accordingly.

Gifted and Enrichment

- Students might graph the cost of each activity and then use the graph to identify the number of guests who could attend each activity for under $200. They will need **Master 9 0.5 Centimetre Grid Paper**.
- Students might research the cost of bowling, swimming, and going to a movie at various locations, and then calculate the mean and median costs.

Activity Planning Notes

You may wish to use the following steps to introduce and complete this task:

1. Consider collecting data from the class on attendance at an event. Then, calculate the measures of central tendency and range.
2. Have students consider the birthday party options and the cost of each. Work together to model one of the options, using an algebraic equation. Have students model the other options on their own and decide which type of party they would prefer. Emphasize that their party preference does not have to be related to the math, but that they have to show how much the party they choose would cost compared to the cost of the other parties.
3. As a class, discuss reasons why someone might prefer one party over the other.
4. Clarify that the task is to
   - determine the mean, median, mode, and range for each class in #1
   - model the cost of all three parties using algebraic equations
   - calculate the maximum number of people who could attend each party, and still keep the price under $200
   - choose one party and justify your decision
   - explain all calculations
5. Review **Master 1 Project Rubric** with students so that they are familiar with the assessment criteria.

Assessment *of* Learning	Supported Learning
Discuss the Task with the class. Have students work together to model the cost for bowling in #2, and then have students do #1, the balance of #2, and then do #3 and #4 on their own. They may wish to discuss their ideas in a group.	• Use **Master 1 Project Rubric** to assist you in assessing students' work. Page 461a provides notes on how to use the rubric for this Task. • To view student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links. • For a second task, complete with teaching notes and student exemplars, go to **www.mathlinks7.ca**, access the Teachers' Site, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as this one and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 (Standard of Excellence)	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete response to the question but with a minor error (e.g., an incorrect graph or minor calculation error in a table or graph)
4 (Above Acceptable)	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a correct response to the question but with a more significant calculation error, or a weak justification, or no labelling or activity identification on a graph
3 (Meets Acceptable)	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides a correct response to parts 1, 2, and 3 *or* • provides a correct response to parts 2, 3, and 4 *or* • provides a correct response to part 1 and makes a significant start to the remaining parts
2 (Below Acceptable)	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a correct response to both parts of 1 *or* • provides a correct response to parts 3 and 4 based on an incorrect part 2 *or* • provides correct parts 1 and 2
1 (Beginning)	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides a correct response to any part of a question *or* • begins a process that could lead to a correct answer