McGraw-Hill Ryerson *MathLinks 7* Program Overview

The McGraw-Hill Ryerson *MathLinks* program has three components.

STUDENT RESOURCE

The student resource introduces topics in real-world contexts. In each section, **Explore the Math** and **Discover the Math** activities encourage students to develop their own understanding of new concepts. Worked **examples** present solutions in a clear, step-by-step manner, and then the **Key Ideas** and **Communicate the Ideas** summarize the new principles.

The student resource includes sections that can be used as assessment tools: **chapter review**, **practice test**, **Wrap It Up!**, and **cumulative review**. Technology is integrated throughout the program and includes the use of calculators, drawing software, spreadsheets, and the Internet.

TEACHER'S RESOURCE

The teaching and assessment suggestions that are provided in this Teacher's Resource include

- sample responses for the **Explore the Math** and **Discover the Math** questions
- sample responses for the **Communicate the Ideas** questions
- common student errors and suggested remedies
- suggestions for **Assessment *as* Learning**, **Assessment *for* Learning**, and **Assessment *of* Learning**

SOLUTIONS MANUAL

The solutions manual provides full worked solutions for all questions in the numbered sections of the student resource, as well as for questions in the **chapter review**, **practice test**, and **cumulative review** features.

An Introduction to *MathLinks 7* Teacher's Resource

The teaching notes for each chapter have the following structure:

Opening Matter and Charts

- These are provided on Roman numeral page numbers immediately after each chapter tab.
- These pages provide an overview of the chapter outcomes and the concepts, skills, and processes that will be assessed.
- The **Chapter Planning Chart** provides
  - suggested timing for the numbered sections, chapter review, practice test, Wrap It Up!, games, and challenge
  - suggested assignments for most students
  - a list of related blackline masters available on the CD-ROM
  - a list of materials and technology tools needed for each lesson
- The **Chapter Assessment Planner** identifies
  - the location of Assessment *as* Learning, Assessment *for* Learning, and Assessment *of* Learning opportunities in the chapter
  - the related assessment tools that are available on the CD-ROM or in the Teacher's Resource notes

Chapter Opener

The Chapter Opener includes

- a description of the math that will be covered in the chapter
- suggestions for introducing students to the chapter's topics
- an introduction to the chapter problem, called **Math Link**
- instructions for students to create a **Foldable™** organizer for the key terms in the chapter

Numbered Sections

The opening page lists

- **Specific Outcomes** that the section covers in whole or in part
- **Materials** needed for the section
- **Technology Tools** needed for the section
- **Blackline Masters** useful for extra practice, assessment, and adaptations

Teaching Notes

The key items include the following:

- **Warm-Up** exercises reinforce material in previous sections.
- Answers for the **Explore the Math** and **Discover the Math** questions let you know the expected outcome of these activities.
- **Activity Planning Notes** give insights or point out connections that might not be readily apparent on first reading of the worked **examples**.
- Sample responses for the **Communicate the Ideas** questions provide the type of answers students are expected to give.

- **Assessment** boxes give a variety of short assessment strategies and related supported learning for Assessment *as* Learning, Assessment *for* Learning, and Assessment *of* Learning. These boxes are provided for each of the activities in the student resource numbered sections.
- A **Question Planning Chart** specifies the questions to be assigned.
  - Essential: the minimum, usually knowledge and skill questions, that all students should be able to complete to address the outcomes
  - Typical: questions that most students should be fairly successful with
  - Extension/Enrichment: questions that extend the concepts horizontally and provide additional challenge
- A **Math Link** box describes what students will achieve with the Math Link activity and provides strategies for students to complete it successfully.

End of Chapter Items

- The chapter sections are followed by a **chapter review** and a **practice test**.
- The chapter problem is finalized in a **Wrap It Up!** Related notes provide ideas for handling this assessment opportunity. A rubric and suggested assessment notes are provided.
- **Math Games** appear after the practice test. These can be used for additional reinforcement, alternative assessment, or end-of-chapter review.
- The final page in each chapter is a **Challenge in Real Life**. This activity can be used as a holistic assessment tool, as an extra activity for gifted and enriched students, and/or by all students as a motivating activity related to real life. A rubric and suggested assessment notes are provided.
- **Cumulative reviews** reinforce the previous four chapters.
- The cumulative reviews are followed by a **Task**. This activity can be used as a holistic assessment tool for cross-strand work or as an extra activity for gifted and enriched students. A rubric and suggested assessment notes are provided.

The Teacher's Resource CD-ROM also provides editable masters:

- **Generic Masters** such as grid paper
- **Blackline Masters** related to each chapter:
  - a student self-assessment master for each chapter
  - an open-ended diagnostic assessment opportunity
  - extra practice questions for each section
  - how-to information for technology tools, where the curriculum suggests use of technology
  - scaffolding for each **Math Link** and **Wrap It Up!** for students who need supported learning
  - a **chapter test**
  - answers for ***MathLinks 7*** **student resource** and blackline master questions

Characteristics of McGraw-Hill Ryerson's *MathLinks* Program

McGraw-Hill Ryerson's *MathLinks* program is based on a view that all students can be successful in mathematics and should have the opportunity and support to learn mathematics with depth and understanding.[1] The program is built on principles of effective practice and on research about how early adolescents learn—prerequisites for achieving a balanced approach to instruction in mathematics.

Mathematics: Making Links

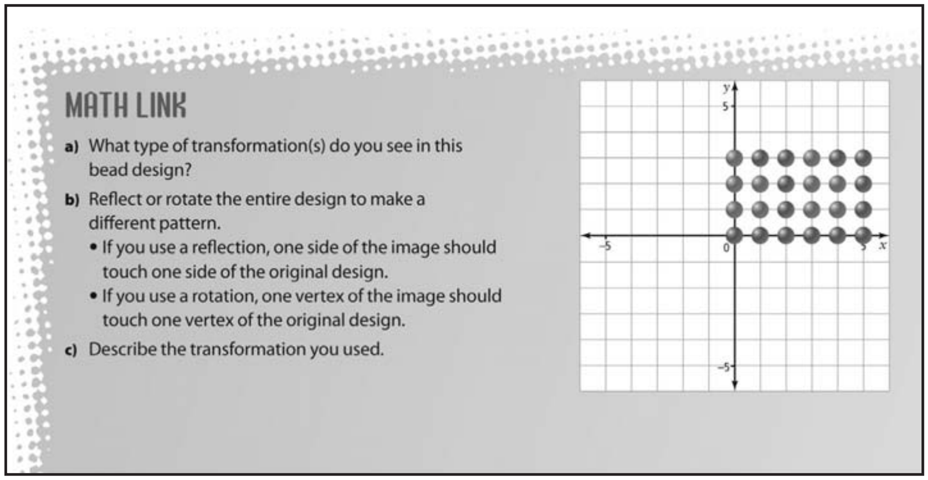
Throughout the student resource, students are given the opportunity to see the links between real life and mathematics.

- Every chapter is introduced with a **Math Link** problem that models mathematics in the real world, engages students' interest, and gives students a meaningful purpose for learning the mathematics presented in the chapter. The Math Link provides an important foundation for the concepts and skills developed throughout the chapter. The problem is designed to engage students by making links between the mathematics in the chapter and students' personal experiences, as well as between mathematics and the real world.

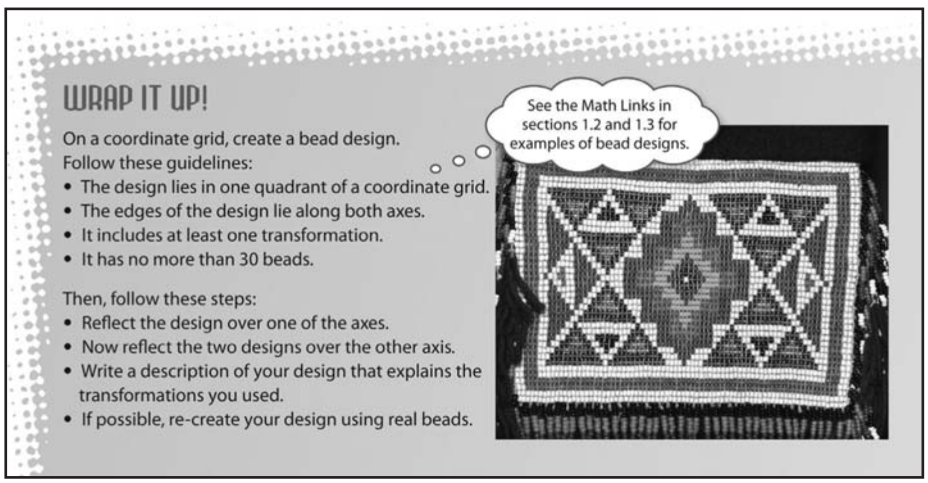
> **MATH LINK**
>
> A number of cultures use designs in their artwork. Many Aboriginal peoples use beads to decorate their ceremonial clothing or to create jewellery. How would you create a bead design of your own on a Cartesian plane?

[1]National Council of Teachers of Mathematics, *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 2000.

- The Math Link is revisited at the end of most lessons. This provides students with the opportunity to apply newly acquired concepts and skills in the context of the original problem.



- At the end of the chapter the **Wrap It Up!** offers an open-ended assessment opportunity for students to demonstrate their understanding by solving the problem introduced at the beginning of the chapter.



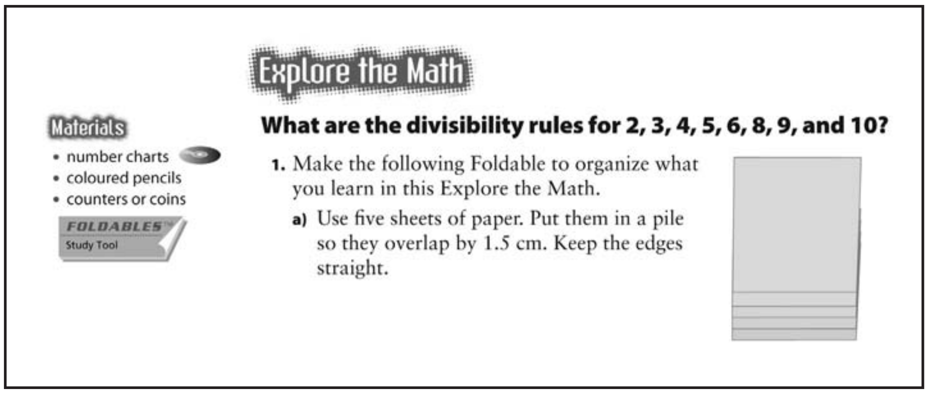
- Most concepts or procedures in the chapter are introduced in a real-life context.
- **What You Will Learn** at the beginning of each chapter lists in student-friendly language the outcomes of the mathematics curriculum that are covered in the chapter. These outcomes may be from different strands that naturally fit together and further illustrate how the program makes important links among concepts within the discipline and with the real world.
- Connections with other curriculum areas, such as science, geography, and art, are evident in a number of lessons in many of the chapters. The Teacher's Resource also identifies the various ways that concepts developed in the chapter are linked to concepts in the different strands.

Procedural Fluency and Conceptual Understanding

The three-part lesson structure in McGraw-Hill Ryerson's *MathLinks* program is designed so that students engage in learning that develops both conceptual understanding and procedural fluency. The three parts are described below.

Explore the Math or Discuss the Math

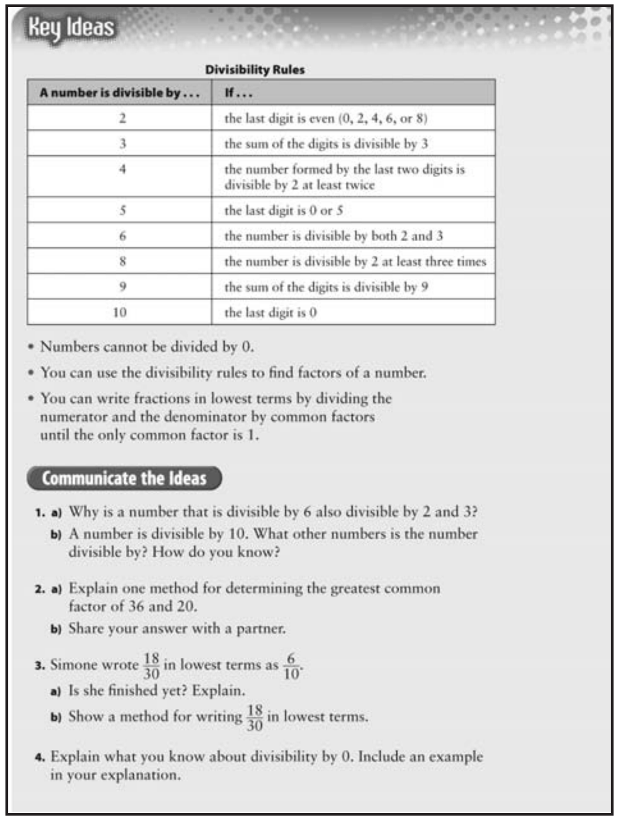
- begins with a focus question that identifies the learning objective of the lesson
- provides a hands-on or minds-on activity to generalize learning about the key concepts and to answer the original focus question in the **Reflect on Your Findings**



- provides worked **examples** of the mathematics being modelled
- makes use of commonly available concrete materials and mathematics manipulatives
- provides opportunities for students to check their understanding of concepts, through **Show You Know** questions, before proceeding to the next example

Key Ideas and Communicate the Ideas

- summarize the key concepts or big ideas of the lesson
- consolidate student learning through questions that include explaining or comparing concepts, identifying and correcting errors, and discussing as a group



Key Ideas

Divisibility Rules

A number is divisible by . . .	If . . .
2	the last digit is even (0, 2, 4, 6, or 8)
3	the sum of the digits is divisible by 3
4	the number formed by the last two digits is divisible by 2 at least twice
5	the last digit is 0 or 5
6	the number is divisible by both 2 and 3
8	the number is divisible by 2 at least three times
9	the sum of the digits is divisible by 9
10	the last digit is 0

- Numbers cannot be divided by 0.
- You can use the divisibility rules to find factors of a number.
- You can write fractions in lowest terms by dividing the numerator and the denominator by common factors until the only common factor is 1.

Communicate the Ideas

1. a) Why is a number that is divisible by 6 also divisible by 2 and 3?
   b) A number is divisible by 10. What other numbers is the number divisible by? How do you know?
2. a) Explain one method for determining the greatest common factor of 36 and 20.
   b) Share your answer with a partner.
3. Simone wrote $\frac{18}{30}$ in lowest terms as $\frac{6}{10}$.
   a) Is she finished yet? Explain.
   b) Show a method for writing $\frac{18}{30}$ in lowest terms.
4. Explain what you know about divisibility by 0. Include an example in your explanation.

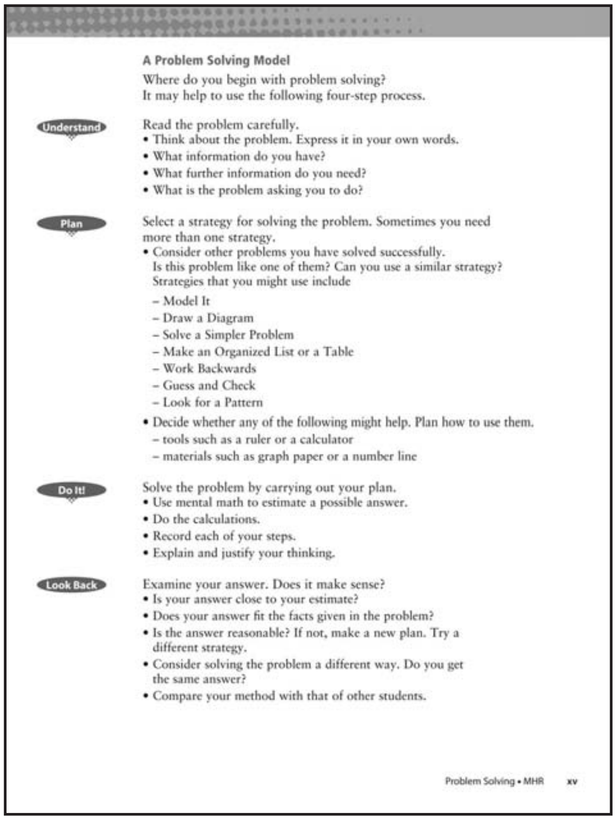
Exercises (Practise/Apply/Extend)

- allow practice of new skills and application of learning to different situations
- provide opportunities for solving problems in a variety of contexts
- provide opportunities for students to extend their thinking (synthesizing, analysing, evaluating, etc.) by using what was discussed in the chapter in a different context or a different way

Problem Solving

Problem solving is central to the McGraw-Hill Ryerson *MathLinks* program. For this reason, a variety of problem solving experiences are provided throughout the lessons:

- A four-step **problem solving model** is outlined at the beginning of the student resource: Understand, Plan, Do It!, Look Back.
- **Problem solving strategies** are reinforced at the beginning of the student resource. These pages serve as a reference for students as they solve problems within the chapters.



A Problem Solving Model

Where do you begin with problem solving?
It may help to use the following four-step process.

Understand

Read the problem carefully.
- Think about the problem. Express it in your own words.
- What information do you have?
- What further information do you need?
- What is the problem asking you to do?

Plan

Select a strategy for solving the problem. Sometimes you need more than one strategy.
- Consider other problems you have solved successfully. Is this problem like one of them? Can you use a similar strategy? Strategies that you might use include
  - – Model It
  - – Draw a Diagram
  - – Solve a Simpler Problem
  - – Make an Organized List or a Table
  - – Work Backwards
  - – Guess and Check
  - – Look for a Pattern
- Decide whether any of the following might help. Plan how to use them.
  - – tools such as a ruler or a calculator
  - – materials such as graph paper or a number line

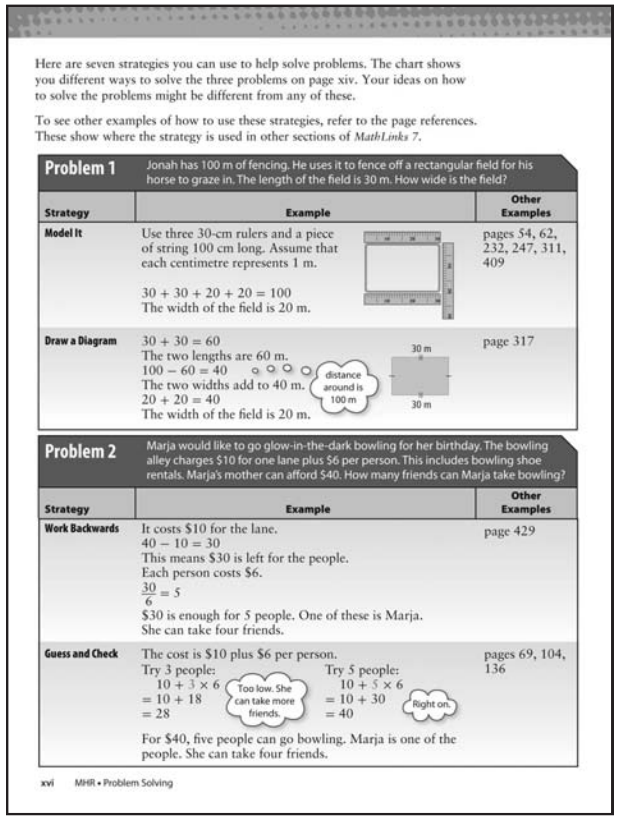
Do It!

Solve the problem by carrying out your plan.
- Use mental math to estimate a possible answer.
- Do the calculations.
- Record each of your steps.
- Explain and justify your thinking.

Look Back

Examine your answer. Does it make sense?
- Is your answer close to your estimate?
- Does your answer fit the facts given in the problem?
- Is the answer reasonable? If not, make a new plan. Try a different strategy.
- Consider solving the problem a different way. Do you get the same answer?
- Compare your method with that of other students.

Problem Solving • MHR xv



Here are seven strategies you can use to help solve problems. The chart shows you different ways to solve the three problems on page xiv. Your ideas on how to solve the problems might be different from any of these.

To see other examples of how to use these strategies, refer to the page references. These show where the strategy is used in other sections of *MathLinks 7*.

Problem 1 Jonah has 100 m of fencing. He uses it to fence off a rectangular field for his horse to graze in. The length of the field is 30 m. How wide is the field?

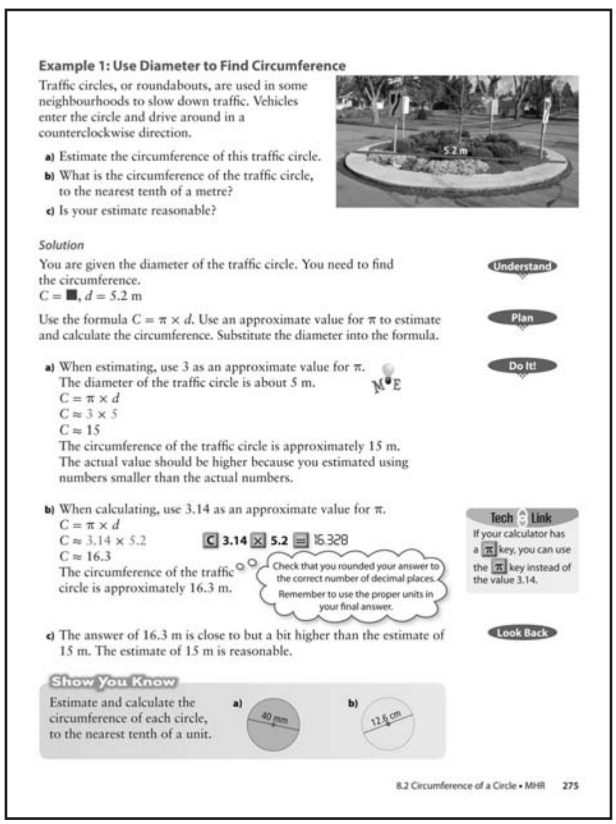
Strategy	Example	Other Examples
Model It	Use three 30-cm rulers and a piece of string 100 cm long. Assume that each centimetre represents 1 m. $30 + 30 + 20 + 20 = 100$ The width of the field is 20 m.	pages 54, 62, 232, 247, 311, 409
Draw a Diagram	$30 + 30 = 60$ The two lengths are 60 m. $100 - 60 = 40$ (distance around is 100 m) The two widths add to 40 m. $20 + 20 = 40$ The width of the field is 20 m. (30 m, 30 m)	page 317

Problem 2 Marja would like to go glow-in-the-dark bowling for her birthday. The bowling alley charges $10 for one lane plus $6 per person. This includes bowling shoe rentals. Marja's mother can afford $40. How many friends can Marja take bowling?

Strategy	Example	Other Examples
Work Backwards	It costs $10 for the lane. $40 - 10 = 30$ This means $30 is left for the people. Each person costs $6. $\frac{30}{6} = 5$ $30 is enough for 5 people. One of these is Marja. She can take four friends.	page 429
Guess and Check	The cost is $10 plus $6 per person. Try 3 people: $10 + 3 \times 6 = 10 + 18 = 28$ (Too low. She can take more friends.) Try 5 people: $10 + 5 \times 6 = 10 + 30 = 40$ (Right on.) For $40, five people can go bowling. Marja is one of the people. She can take four friends.	pages 69, 104, 136

xvi MHR • Problem Solving

- Examples throughout *MathLinks 7* show the problem solving model and strategies being used in context.



Example 1: Use Diameter to Find Circumference

Traffic circles, or roundabouts, are used in some neighbourhoods to slow down traffic. Vehicles enter the circle and drive around in a counterclockwise direction.

5.2 m

a) Estimate the circumference of this traffic circle.
b) What is the circumference of the traffic circle, to the nearest tenth of a metre?
c) Is your estimate reasonable?

Solution

Understand

You are given the diameter of the traffic circle. You need to find the circumference.
C = ■, $d = 5.2$ m

Plan

Use the formula $C = \pi \times d$. Use an approximate value for π to estimate and calculate the circumference. Substitute the diameter into the formula.

Do It!

a) When estimating, use 3 as an approximate value for π.
The diameter of the traffic circle is about 5 m.
$C = \pi \times d$
$C \approx 3 \times 5$
$C \approx 15$
The circumference of the traffic circle is approximately 15 m.
The actual value should be higher because you estimated using numbers smaller than the actual numbers.

b) When calculating, use 3.14 as an approximate value for π.
$C = \pi \times d$
$C \approx 3.14 \times 5.2$
$C \approx 16.3$
C 3.14 × 5.2 = 16.328
The circumference of the traffic circle is approximately 16.3 m.

Check that you rounded your answer to the correct number of decimal places. Remember to use the proper units in your final answer.

Tech Link
If your calculator has a π key, you can use the π key instead of the value 3.14.

Look Back

c) The answer of 16.3 m is close to but a bit higher than the estimate of 15 m. The estimate of 15 m is reasonable.

Show You Know

Estimate and calculate the circumference of each circle, to the nearest tenth of a unit.

a) 40 mm
b) 12.6 cm

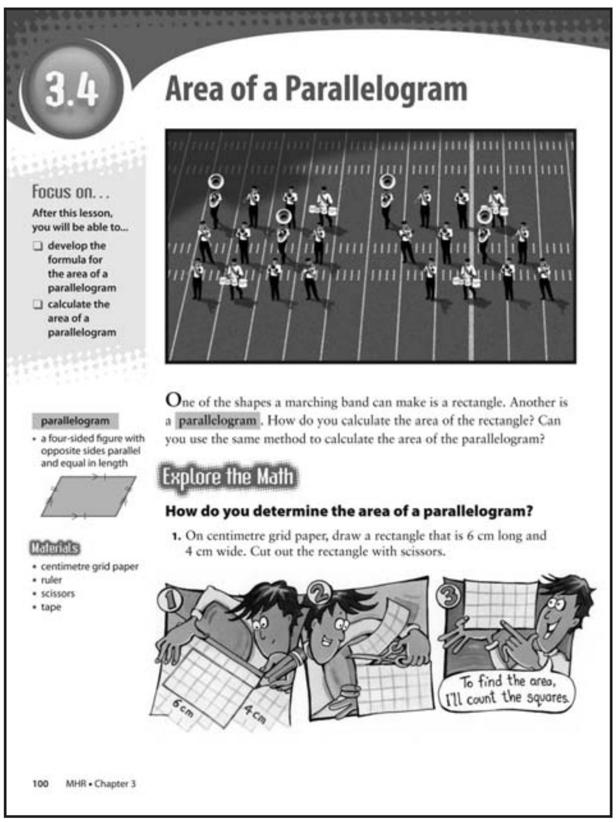
8.2 Circumference of a Circle • MHR 275

- Students are frequently asked to discuss their methods for solving problems. Doing so reinforces thinking and helps students realize that there may be multiple methods for solving a problem.
- Each chapter begins with a **Math Link** problem that often models mathematics in the real world. This problem is wrapped up at the end of the chapter in the form of a performance task.
- A problem provides the focus for learning in **Explore the Math** or **Discuss the Math**, often making use of concrete materials.
- Students are challenged to higher levels of thinking and to extend their thinking in the **Extend** section of exercises, the **Extended Response** section in the practice test, the **Math Games**, and the **Challenge in Real Life**.

Differentiating Instruction

Care has been taken in the McGraw-Hill Ryerson *MathLinks* program to ensure that all students—including special needs students (with learning disabilities or gifted), students at risk, English language learners, and students from various cultures—can access the mathematics and experience success with the program. The Teacher's Resource includes strategies for supporting these students.

- Visuals that illustrate how to carry out explorations accompany the instructions. These visuals help the student to "see" the process. They also aid in the acquisition of mathematical language.


3.4 Area of a Parallelogram

Focus on...

After this lesson, you will be able to...

- develop the formula for the area of a parallelogram
- calculate the area of a parallelogram

One of the shapes a marching band can make is a rectangle. Another is a parallelogram. How do you calculate the area of the rectangle? Can you use the same method to calculate the area of the parallelogram?

parallelogram
- a four-sided figure with opposite sides parallel and equal in length

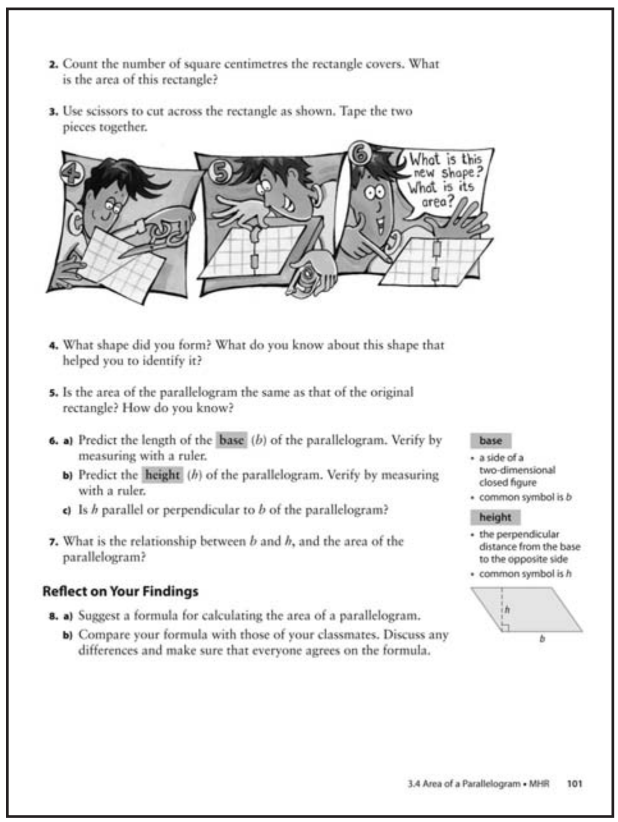
Explore the Math

How do you determine the area of a parallelogram?

Materials
- centimetre grid paper
- ruler
- scissors
- tape

1. On centimetre grid paper, draw a rectangle that is 6 cm long and 4 cm wide. Cut out the rectangle with scissors.

100 MHR • Chapter 3


2. Count the number of square centimetres the rectangle covers. What is the area of this rectangle?

3. Use scissors to cut across the rectangle as shown. Tape the two pieces together.

4. What shape did you form? What do you know about this shape that helped you to identify it?

5. Is the area of the parallelogram the same as that of the original rectangle? How do you know?

6. a) Predict the length of the base (b) of the parallelogram. Verify by measuring with a ruler.
   b) Predict the height (h) of the parallelogram. Verify by measuring with a ruler.
   c) Is h parallel or perpendicular to b of the parallelogram?

7. What is the relationship between b and h, and the area of the parallelogram?

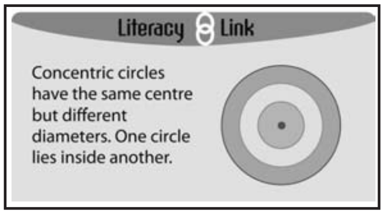
base
- a side of a two-dimensional closed figure
- common symbol is b

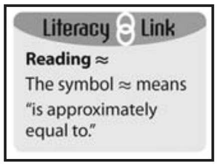
height
- the perpendicular distance from the base to the opposite side
- common symbol is h

Reflect on Your Findings

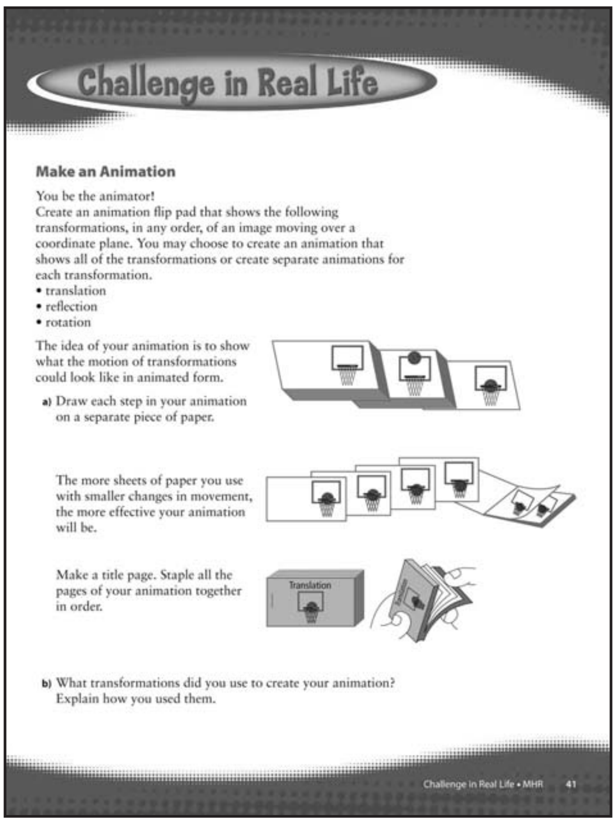
8. a) Suggest a formula for calculating the area of a parallelogram.
   b) Compare your formula with those of your classmates. Discuss any differences and make sure that everyone agrees on the formula.

3.4 Area of a Parallelogram • MHR 101





- Visuals and graphics are paired with questions and content in other strategic locations in the student resource.
- **Literacy Links** provide students with strategies for how to read and understand mathematical language.
- The Teacher's Resource provides strategies and blackline master support for accommodating different learning styles, special needs, English language learners, and at-risk students.
- Support for combined grades situations appears on the *MathLinks 7* book site at **www.mathlinks7.ca**.
- The Teacher's Resource and *MathLinks 7* book site offer further support in the form of concrete activities, additional practice, and diagnostic strategies to support students who may have gaps in their learning.
- The open-ended nature of many of the **Math Link**, **Wrap It Up!**, **Challenge in Real Life**, and **Task** activities accommodates the needs of all students by allowing for multiple entry points.



Challenge in Real Life

Make an Animation

You be the animator!
Create an animation flip pad that shows the following transformations, in any order, of an image moving over a coordinate plane. You may choose to create an animation that shows all of the transformations or create separate animations for each transformation.
- translation
- reflection
- rotation

The idea of your animation is to show what the motion of transformations could look like in animated form.

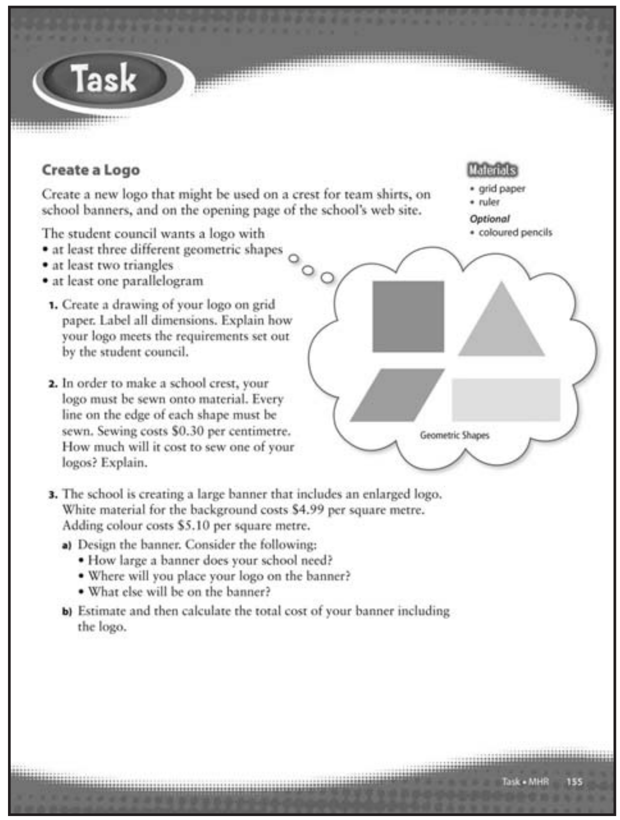
a) Draw each step in your animation on a separate piece of paper.

The more sheets of paper you use with smaller changes in movement, the more effective your animation will be.

Make a title page. Staple all the pages of your animation together in order.

b) What transformations did you use to create your animation? Explain how you used them.

Challenge in Real Life • MHR 41



Task

Create a Logo

Create a new logo that might be used on a crest for team shirts, on school banners, and on the opening page of the school's web site.

Materials
- grid paper
- ruler

Optional
- coloured pencils

The student council wants a logo with
- at least three different geometric shapes
- at least two triangles
- at least one parallelogram

1. Create a drawing of your logo on grid paper. Label all dimensions. Explain how your logo meets the requirements set out by the student council.
2. In order to make a school crest, your logo must be sewn onto material. Every line on the edge of each shape must be sewn. Sewing costs $0.30 per centimetre. How much will it cost to sew one of your logos? Explain.
3. The school is creating a large banner that includes an enlarged logo. White material for the background costs $4.99 per square metre. Adding colour costs $5.10 per square metre.
   a) Design the banner. Consider the following:
      - How large a banner does your school need?
      - Where will you place your logo on the banner?
      - What else will be on the banner?
   b) Estimate and then calculate the total cost of your banner including the logo.

Task • MHR 155

- Features titled **Did You Know?** present interesting information related to the math of the lesson.

Adaptations

MathLinks 7 considers a broad range of needs and learning styles, including those of students requiring adaptations, students with limited proficiency in English, and gifted learners.

- Visuals and multiple representations of concepts and instructions support **visual learners**, **English language learners**, and **struggling readers**.
- **Literacy Links** boxes and **Key Words** bolded, highlighted, and defined in the margin support struggling readers and promote mathematics literacy for all learners.

- Relevant contexts including **multicultural examples** engage students and provide a purpose for the mathematics being learned.
- **Extend** questions, **Math Games**, **Challenges in Real Life**, and **Tasks** provide additional motivation for **gifted learners**.
- **Explore the Math** and **Discuss the Math** activities provide additional opportunities for hands-on and minds-on learning.

This Teacher's Resource and the *MathLinks 7* student resource provide support in addressing multiple intelligences and learning styles, through additional activities, adaptation suggestions, English language learner support, and supported learning strategies.

Reaching All Students

Students may experience difficulty meeting WNCP standards for a variety of reasons. General cognitive delays, social–emotional issues, behavioural difficulties, health-related factors, and extended or sporadic absences from instruction underlie the math difficulties experienced by some students. However, these factors do not explain the challenges other students encounter. For these students, math difficulties are usually related to three key areas.

Three Key Areas Underlying Math Difficulties

Language

Students with language learning difficulties demonstrate difficulty reading and understanding math vocabulary and math story problems, and determining saliency (picking out the most important details from irrelevant information). Processing information that is presented using oral or written language is often difficult for these students, who may be more efficient learners when information is presented in a nonverbal, visual format. Diagrams and pictorial representations of math concepts are usually more meaningful to these students than lengthy oral or written descriptions.

English language learners (ELL) enter Canadian classrooms with varied background knowledge. Teachers struggle to include them in lessons. ELL students used to flourish in math classes when the focus was on numbers and reproducing formulas. Now the focus has shifted to a more investigative type of learning, with more emphasis on oral and written communication. Many ELL students can do the computation but do not understand the questions because of the increased language intensity. Simple activities inserted in lessons can help these students with little time and effort on the part of the teacher. Consider the following:

- Encourage students to create a translation dictionary at the back of their math notebook. In this dictionary, they could translate the new words they are learning in their math class into their own language. Some students may be unfamiliar with the English alphabet and need help setting up this dictionary.
- Ask students in the class to tell you the way to say the word for *perimeter* in their language. Then take a minute to have students write the word in their language on the board.
- Use math word walls that include the translations for words into the different languages of students in the class. Also, have students use drawings and symbols to define the math terms.
- Have students understand the meaning of the words through kinesthetic learning: "Show me the perimeter of your desk. Show me the area of your desk."

Visual/Perceptual/Spatial/Motor

Some students demonstrate difficulties understanding and processing information that is presented visually and in a nonverbal format. Language support to supplement and make sense of visually presented information is often beneficial (e.g., verbal explanation of a visual chart). Visual, perceptual, spatial, and motor difficulties may be evident in students' written output, as well as in their ability to process information that has been input visually. Difficulties with near- and far-point copying, accurately aligning numbers in columns, properly sequencing numbers, and illegible handwriting are examples of output difficulties in this area.

Memory (Short-Term Memory, Working Memory, and Long-Term Memory)

Students with auditory short-term memory or visual short-term memory difficulties find it hard to remember what they have just heard or seen. A weak working or active memory makes it difficult for students to hold information in their short-term memory and manipulate it (e.g., hold what they have just heard and then perform a mathematical operation with that information). For others, the retrieval of information from long-term memory (e.g., remembering number facts and previously taught formulas) is difficult. Students with long-term memory difficulties may have problems storing information in their long-term memory, as well as retrieving it.

At-Risk Students

"At-risk" students are in danger of completing their schooling without adequate skill development to function effectively in society. Risk factors include low achievement, retention difficulties, behaviour problems, poor attendance, low socioeconomic status, and attendance at schools with large numbers of poor students.

Neither failing such students nor putting them in pullout programs has produced much gain in achievement, but there are certain approaches that do help.

- Allow students to proceed at their own pace through a well-defined series of instructional objectives.
- Involve students in learning about something that is relevant to them, such as money management or wise shopping.
- Get parents/guardians involved in their child's learning as much as possible.

Meeting the Needs of All Learners

Teachers may wish to use the following suggestions to help meet the needs of all students:

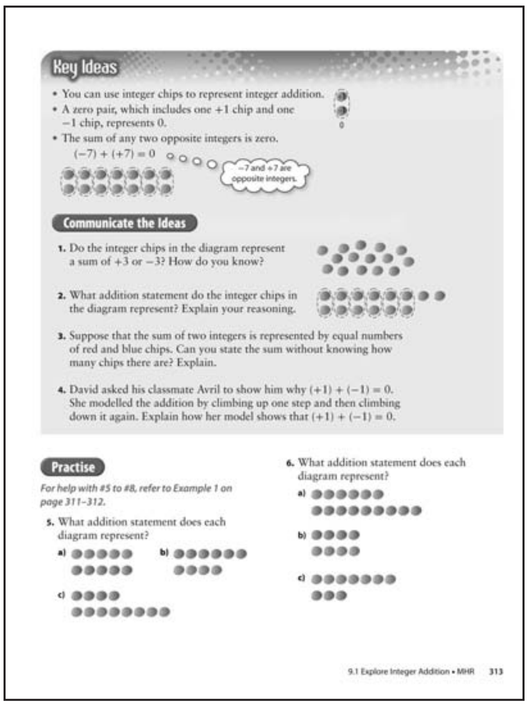
- Plan to cover the basic material really well and only use the extension-type practice questions for the top students. Where possible, use additional questions when working through the Explore the Math sections. Do not assume students will get the concept by going over the material only once.
- Consider the resources for multiple learning styles and needs.
- Include a lot of hands-on, concrete practice to explain the math concepts.
- Monitor the amount of math terminology and simplify it if necessary and possible.

Concrete Materials

The McGraw-Hill Ryerson *MathLinks* program engages students in a variety of worthwhile mathematical tasks that span the continuum from concrete to abstract.

Concept development in the program generally begins with students working with concrete materials. Most explorations in **Explore the Math** have students using commonplace materials and conventional mathematical manipulatives in a hands-on approach. Pictorial images of the materials support the text and accommodate the stages of the investigations in the absence of concrete materials. After an appropriate number of hands-on opportunities, students move from the pictorial to the symbolic in the **examples**, **Show You Know**, and **Practise/Apply/Extend** exercises.

An example of how students move through the continuum of learning can be seen in the development of the concept of addition of integers. Students begin the exploration using positive and negative integer chips. Pictorial images of addition with the chips are paired with text in the initial stages of the exploration. Eventually, the pictorial images are removed and the student is presented only with the symbolic.



Key Ideas

- You can use integer chips to represent integer addition.
- A zero pair, which includes one +1 chip and one −1 chip, represents 0.
- The sum of any two opposite integers is zero.

$(-7) + (+7) = 0$

Communicate the Ideas

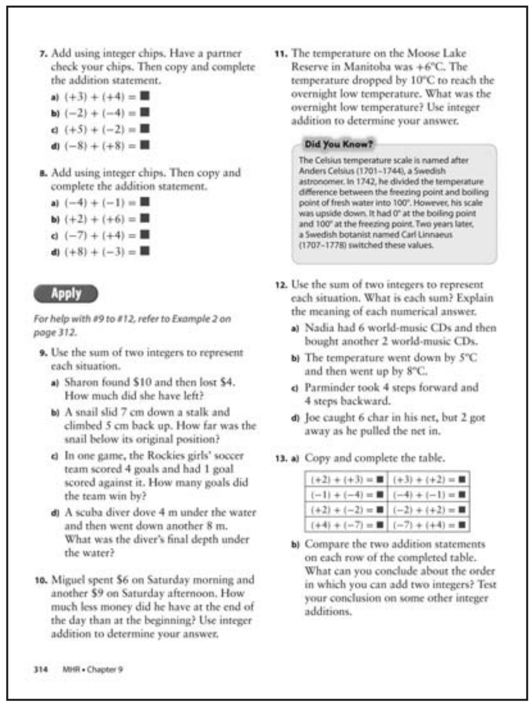
1. Do the integer chips in the diagram represent a sum of +3 or −3? How do you know?
2. What addition statement do the integer chips in the diagram represent? Explain your reasoning.
3. Suppose that the sum of two integers is represented by equal numbers of red and blue chips. Can you state the sum without knowing how many chips there are? Explain.
4. David asked his classmate Avril to show him why $(+1) + (-1) = 0$. She modelled the addition by climbing up one step and then climbing down it again. Explain how her model shows that $(+1) + (-1) = 0$.

Practise

For help with #5 to #8, refer to Example 1 on page 311–312.

5. What addition statement does each diagram represent?
   a)
   b)
   c)
6. What addition statement does each diagram represent?
   a)
   b)
   c)

9.1 Explore Integer Addition • MHR 313



7. Add using integer chips. Have a partner check your chips. Then copy and complete the addition statement.
   a) $(+3) + (+4) = ■$
   b) $(-2) + (-4) = ■$
   c) $(+5) + (-2) = ■$
   d) $(-8) + (+8) = ■$
8. Add using integer chips. Then copy and complete the addition statement.
   a) $(-4) + (-1) = ■$
   b) $(+2) + (+6) = ■$
   c) $(-7) + (+4) = ■$
   d) $(+8) + (-3) = ■$

Apply

For help with #9 to #12, refer to Example 2 on page 312.

9. Use the sum of two integers to represent each situation.
   a) Sharon found $10 and then lost $4. How much did she have left?
   b) A snail slid 7 cm down a stalk and climbed 5 cm back up. How far was the snail below its original position?
   c) In one game, the Rockies girls' soccer team scored 4 goals and had 1 goal scored against it. How many goals did the team win by?
   d) A scuba diver dove 4 m under the water and then went down another 8 m. What was the diver's final depth under the water?
10. Miguel spent $6 on Saturday morning and another $9 on Saturday afternoon. How much less money did he have at the end of the day than at the beginning? Use integer addition to determine your answer.
11. The temperature on the Moose Lake Reserve in Manitoba was +6°C. The temperature dropped by 10°C to reach the overnight low temperature. What was the overnight low temperature? Use integer addition to determine your answer.

Did You Know?

The Celsius temperature scale is named after Anders Celsius (1701–1744), a Swedish astronomer. In 1742, he divided the temperature difference between the freezing point and boiling point of fresh water into 100°. However, his scale was upside down. It had 0° at the boiling point and 100° at the freezing point. Two years later, a Swedish botanist named Carl Linnaeus (1707–1778) switched these values.

12. Use the sum of two integers to represent each situation. What is each sum? Explain the meaning of each numerical answer.
    a) Nadia had 6 world-music CDs and then bought another 2 world-music CDs.
    b) The temperature went down by 5°C and then went up by 8°C.
    c) Parminder took 4 steps forward and 4 steps backward.
    d) Joe caught 6 char in his net, but 2 got away as he pulled the net in.
13. a) Copy and complete the table.

$(+2) + (+3) = ■$	$(+3) + (+2) = ■$
$(-1) + (-4) = ■$	$(-4) + (-1) = ■$
$(+2) + (-2) = ■$	$(-2) + (+2) = ■$
$(+4) + (-7) = ■$	$(-7) + (+4) = ■$

    b) Compare the two addition statements on each row of the completed table. What can you conclude about the order in which you can add two integers? Test your conclusion on some other integer additions.

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Technology

Lessons are designed to provide students with the opportunity to develop their skills in the use of calculators and spreadsheets, but not to rely on this technology to think mathematically. Students are also asked to research on the Internet for information related to problems they are required to solve.

The student resource provides technology learning that matches technology requirements for curriculum expectations.

Blackline masters of text-based technology activities that can easily be used in a computer laboratory are also included in the Teacher's Resource. The masters include directions for using a number of different software applications common in many classrooms.

> "Assessment should not merely be done to students; rather, it should also be done for students to guide and enhance their learning." [2]

ASSESSMENT

The primary purpose of assessment is to improve student learning. Assessment data help teachers determine the instructional needs of students throughout the learning process. Some assessment data are used to evaluate students for reporting purposes.

Assessment must be purposeful and include all students. It should be varied appropriately to reflect student learning styles and clearly communicated to students and parents/guardians. Teachers can use diagnostic assessment to determine prior knowledge, formative assessment to inform instructional planning, and summative assessment to determine how well students have achieved the outcomes at the end of a learning cycle.

There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. The table below describes each of these assessment types and lists some of the opportunities for these types of assessment outlined in the Teacher's Resource. Please note that many of these assessment opportunities could be used for a number of purposes, depending on how and why the teacher decides to use them.

	Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Description	• generally occurs before instruction • is used to determine what students know and can do • informs teachers about students' prior knowledge and skills • facilitates metacognition by encouraging students to be active and critical assessors of their own learning	• occurs during the learning process • provides information, direction, and feedback to teachers and students that support adjustment and improvement	• allows students to synthesize their knowledge at the end of a unit of learning • provides students with feedback on a broader range of learning goals • helps teachers identify whether students have met the curriculum outcomes
Examples in the *MathLinks* Program	• chapter self-assessment BLM • Get Ready (see reference after assessment planner) • Reflect on Your Findings • Communicate the Ideas • What I Need to Work On part of the chapter Foldable • Math Learning Log	• Show You Know • Practise • Math Links • chapter review • practice test • Math Games • Challenge in Real Life	• Wrap It Up! • Challenge in Real Life • chapter test • Task • End-of-Year Exam

[2] National Council of Teachers of Mathematics, *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 2000.

An **assessment planner** at the beginning of each chapter shows the location of each of these assessment types and lists the related assessment tool(s).

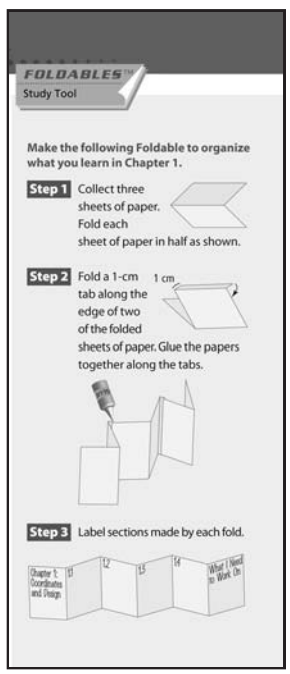
Chapter 1 Assessment Planner

Assessment Options	Type of Assessment	Assessment Tool
Chapter Opener	Assessment *as* Learning (TR pages i, 3)	BLM 1–2 Chapter 1 Self-Assessment Chapter 1 Foldable
1.1 The Cartesian Plane	Assessment *as* Learning (TR pages 6, 8, 10) Assessment *for* Learning (TR pages 10, 11, 13, 15)	Math Learning Log (TR page 10) Master 2 Two Stars and One Wish BLM 1–2 Chapter 1 Self-Assessment
1.2 Create Designs	Assessment *as* Learning (TR pages 13, 15, 17) Assessment *for* Learning (TR pages 14, 16, 17)	Math Learning Log (TR page 17) BLM 1–2 Chapter 1 Self-Assessment
1.3 Transformations	Assessment *as* Learning (TR pages 20, 24, 28) Assessment *for* Learning (TR pages 21, 22, 26, 29)	Math Learning Log (TR page 28) BLM 1–2 Chapter 1 Self-Assessment
1.4 Horizontal and Vertical Distances	Assessment *as* Learning (TR pages 31, 33, 35) Assessment *for* Learning (TR pages 32, 34)	Math Learning Log (TR page 35) BLM 1–2 Chapter 1 Self-Assessment
Chapter 1 Review	Assessment *for* Learning (TR page 36) Assessment *as* Learning (TR page 37)	Math Learning Log (TR page 37) BLM 1–2 Chapter 1 Self-Assessment
Chapter 1 Practice Test	Assessment *as* Learning (TR page 38) Assessment *of* Learning (TR page 39)	BLM 1–2 Chapter 1 Self-Assessment BLM 1–12 Chapter 1 Test
Chapter 1 Wrap It Up!	Assessment *of* Learning (TR page 39a)	Master 1 Project Rubric
Chapter 1 Math Games	Assessment *for* Learning (TR page 40)	
Chapter 1 Challenge in Real Life	Assessment *for* Learning (TR page 40a) Assessment *of* Learning (TR page 40a)	Master 1 Project Rubric

Assessment *as* Learning

The *MathLinks 7* student resource and this Teacher's Resource have the following components to assist the teacher in programming by identifying student weaknesses and gaps:

- The chapter opener, which features a topic introduction and a **Foldables** activity. The Foldable gives students a way to organize their learning and provides them with opportunities to express their understanding in their own words. The last part of each chapter Foldable asks students to keep track of what they need to work on, allowing them to be self-directed learners.



- The **Reflect on Your Findings** questions at the end of each Explore the Math and Discuss the Math provide early opportunities for students to construct knowledge about the section content.
- The **Communicate the Ideas** questions allow students to explore their initial understandings of a concept.
- The **Warm-Up** exercises, journal questions, and **Math Learning Log** suggestions in this Teacher's Resource provide additional support in identifying and facilitating student learning.
- The suggested assignments, questions, and activities in the **Supported Learning** boxes in this Teacher's Resource meet the needs of all learners by addressing language, memory, motor, learning styles, gifted and enrichment, and English language learner needs.
- Introductory questions designed to open discussion in the classroom and exploration activities are provided in this Teacher's Resource where appropriate.
- The **self-assessment blackline masters** at the beginning of each chapter allow students to consider how well they understand and can work with various concepts, skills, and processes. By observing and monitoring their own progress, students become self-directed learners.

Assessment *for* Learning

Assessment *for* Learning tools are provided throughout the *MathLinks 7* student resource and this Teacher's Resource.

- Ideas for assessing student readiness for each chapter are provided in the Roman numeral pages after the assessment planner.
- Additional support in this Teacher's Resource and on the *MathLinks 7* book site provides assistance for identifying and supporting weaknesses in students' learning.
- The Teacher's Resource provides **blackline masters** for students that complement the student resource in areas where assessment indicates students may need further support.
- The **Reflect on Your Findings** and **Communicate the Ideas** questions provide an opportunity to determine students' understanding of concepts through conversations and/or written work.
- The **Show You Know** questions target key skills of a section.
- Students can use the **Practise** assignments in each section to check their understanding.
- The **Math Links** at the end of most sections allow students to apply the lesson's concepts to a problem that is linked to the Wrap It Up! at the end of each chapter.
- The **chapter reviews** and **cumulative reviews** provide opportunities to assess knowledge/understanding, applications, communications, mental math, and problem solving.

Assessment *of* Learning

Assessment *of* Learning is provided in the following ways:

- **Practice tests** are provided at the end of the chapters in the student resource, and this Teacher's Resource provides **chapter tests**.
- The **Wrap It Up!** at the end of each chapter provides teachers with an opportunity to check whether students have synthesized the concepts and procedures. Rubrics for the **Wrap It Up!** are included in this Teacher's Resource. Student exemplars are on the *MathLinks 7* book site.
- **Math Games** at the end of each chapter give students and teachers another opportunity for assessment. These games are linked to concepts studied in the chapter. Some games also review outcomes in previous chapters.
- A **Challenge in Real Life** is provided at the end of every chapter and is accompanied by a rubric and suggested scoring in this Teacher's Resource. Student exemplars are on the *MathLinks 7* book site.
- **Cumulative reviews** at the end of Chapters 4, 8, and 12 provide ongoing reinforcement of basic skills, processes, and concepts.
- A **Task** is included after every four chapters. An accompanying rubric and suggested scoring can be found in this Teacher's Resource. Student exemplars are provided on the *MathLinks 7* book site.
- A **Computer Assessment Bank (CAB)** offers a database of additional questions.

Portfolio Assessment

Student-selected portfolios provide a powerful platform for assessing students' mathematical thinking. Portfolios provide the following benefits:

- help teachers assess students' growth and mathematical understanding
- give insight into students' self-awareness about their own progress
- help parents/guardians understand their child's growth

MathLinks 7 has many components that provide ideal portfolio items. Including any or all of the following chapter items is a non-threatening, formative way to gain insight into students' progress:

- student responses to the **chapter opener**
- answers to the **Reflect on Your Findings** questions, which give students early opportunities to construct knowledge about the section content
- answers to the **Communicate the Ideas** questions, which allow students to explore their initial understanding of concepts
- journal and **Math Learning Log** responses, which show student understanding of the chapter skills and processes
- student responses to the **Wrap It Up!** assignments
- **Task** and **Challenge in Real Life** assignments, which show student understanding, usually across several chapters and strands

Master 1 Project Rubric

The **Master 1 Project Rubric** may be used for all assessments of **Wrap It Up!** assignments, **Challenge in Real Life** activities, and **Tasks**. This unique rubric includes

- a Score/Level grade ranging from 1 to 5 (Beginning to Standard of Excellence)
- a Holistic Descriptor for each grade range, describing the level of understanding and communication skills
- Specific Question Notes, which provide suggested solutions typical of each grade range. These notes are meant to represent what the majority of students display. They are by no means exhaustive of all possible solutions. Teachers are encouraged to continually refer to both the specific and holistic pieces of the rubric.

Score/Level	Holistic Descriptor	Specific Question Notes
5 (Standard of Excellence)	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete solution with one or more initial transformations • includes reflections that may contain a minor point error that does not affect the understanding • provides an explanation that is clear and complete
4 (Above Acceptable)	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete design with at least one initial transformation • plots points with some errors present as a result of the transformation and/or reflection (two at most) • includes a clear explanation that addresses most of the requirements
3 (Meets Acceptable)	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• creates a basic design with only one initial transformation • makes some errors in the transformations • includes a minimal explanation
2 (Below Acceptable)	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• creates a design that is missing several of the basic requirements for the initial design • fails to include the transformation, or the transformation has an error and/or the reflection is in one axis only • provides little or no explanation
1 (Beginning)	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• begins design in one quadrant *or* • draws a design that does not contain a transformation attempt, and a reflection may be attempted but many errors are present

Teachers are encouraged to share the rubric with students early in the year. This will help them become active participants in their own assessment and program planning. Discussing and building the Specific Question Notes with students allows them to engage actively in their learning.

CAPITALIZING ON DIVERSITY AND REAL LIFE

Throughout the student resource, students are given opportunities to see how mathematics connects to real life by being engaged in meaningful problem solving situations. Most chapters are introduced with a problem that models real life. Visual images used to introduce lessons, as well as those in the **Explore the Math** or **Discuss the Math** and exercise sets, depict the cultural diversity within classrooms. Examples of mathematics from other cultures are evident throughout the student resource. Names used in the lessons and exercises also reflect the diversity of Canadian society.

Cultural Sensitivity

Dice are used in many ways to support the outcomes addressed in the new WNCP curriculum. In many cultures, these items are used for a variety of purposes, including non-competitive games and learning. It is important, however, to be sensitive to religious and cultural groups that do not support the use of dice for any reason. If students from these groups are in your classroom, you may wish to replace dice with number cubes.

Home Connections

The design of the McGraw-Hill Ryerson *MathLinks* program recognizes that students' learning in mathematics also takes place outside of the classroom as they complete their homework, work with parents/guardians, and employ their mathematical skills in everyday life. The following features support learning outside of the classroom:

- **Key Ideas** and worked **examples** serve as references for students and parents when doing homework.
- Visuals and **Key Ideas** allow investigations to be easily followed independently of the teacher.
- Opportunities for bringing mathematics activities home are provided through **Practise/Apply/Extend**, **Math Links**, **Math Games**, **Challenges in Real Life**, and **Tasks**.

COOPERATIVE LEARNING

There are multiple opportunities throughout the program for teachers to use different types of classroom groupings. The Explore the Math explorations lend themselves to being completed in groups, but teachers are free to choose class groupings that meet the needs of their students. Additional suggestions are also provided in this Teacher's Resource.

Students learn effectively when they are actively engaged in the process of learning. Most sections of *MathLinks 7* begin with a hands-on activity that fosters this approach. These activities are best done through cooperative learning during which students work together—either with a partner or in a small group of three or four—to complete the activity and develop generalizations about the topic or process.

Group learning such as this is an important aspect of a constructivist educational approach. It encourages interactions and increases chances for students to communicate and learn from each other.[3]

Teachers' Role—In classrooms where students are adept at cooperative learning, the teacher becomes the facilitator, guide, and progress monitor. Until students have reached that level of group cooperation, however, you as the teacher will need to coach them in how to learn cooperatively. This may include

- making sure that the materials are at hand and directions perfectly clear, so that students know what they are doing before starting group work
- carefully structuring activities so that students can work together
- coaching how to provide peer feedback in a way that allows the listener to hear and attend
- constantly monitoring student progress and providing assistance to groups having problems with either group cooperation or the math at hand

Group Composition—The size of group you use may vary from activity to activity. Small-group settings allow students to take risks that they might not take in a whole class.[4] Research suggests that small groups are fertile environments for developing mathematical reasoning.[5]

Results of international studies suggest that groups of mixed ability work well in mathematics classrooms.[6] If your class is new to cooperative learning, you may wish to assign students to groups according to the specific skills of each individual. For example, you might pair a student who is talkative but weak in number sense and numeration with a quiet student who is strong in those areas. You might pair a student who is weak in many parts of mathematics but has excellent spatial sense with a stronger mathematics student who has poor spatial sense. In this way, student strengths and weaknesses complement each other, and peers have a better chance of recognizing the value of working together.

Cooperative Learning Skills—When coaching students about cooperative learning, consider task skills and working relationship skills.

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[3] Sternberg, R.J., and W.M. Williams, *Educational Psychology* (Boston, MA: Allyn & Bacon, 2002).

[4] Van De Walle, J., *Elementary and Middle School Mathematics: Teaching Developmentally*, 4th ed. (Boston, MA: Addison Wesley Longman, 2000).

[5] Artzt, A.F., and S. Yaloz-Femia, "Mathematical Reasoning During Small-Group Problem Solving," in L. Stiff and F. Curcio (eds.), *Developing Mathematical Reasoning in Grades K–12* (Reston, VA: National Council of Teachers of Mathematics, 1999), 115–26.

[6] Kilpatrick, J., J. Swafford, and B. Findell, *Adding It Up: Helping Children Learn Mathematics* (Washington, DC: National Academy Press, 2001).

Task Skills	Working Relationship Skills
• following directions • communicating information and ideas • seeking clarification • ensuring that others understand • actively listening to others • staying on task	• encouraging others to contribute • acknowledging and responding to the contributions of others • checking for agreement • disagreeing in an agreeable way • mediating disagreements within the group • sharing • showing appreciation for the efforts of others

Use class discussions, modelling, role-plays, and drama to provide positive task skills. For example, you might role-play different ways to provide feedback and have a class discussion on which ones students like and why. You might discuss common group roles and how group members can use them. Make sure students understand that the same person can play more than one role.

Role	Job	Sample Comment
Leader	• makes sure the group is on task and everyone is participating • pushes group to come to a decision	Let's do this. Can we decide ... ? This is what I think we should do ...
Recorder	• manages materials • writes down data collected or measurements made	This is what I wrote down. Is that what you mean?
Presenter	• presents the group's results and conclusions	This is what the group thinks ...
Organizer	• watches time • keeps on topic • encourages getting the job done	Let's get started. Where should we start? So far we've done the following ... Are we on topic? What else do we need to do?
Clarifier	• checks that members understand and agree	Does everyone understand? So, what I hear you saying is ... Do you mean that ... ?

Types of Groups

Three group types are commonly used in the mathematics classroom.

Think/Pair/Share—This consists of having students individually think about a concept and then pick a partner to share their ideas. For example, students might work on the **Communicate the Ideas** questions and then choose a partner to discuss the concepts with. Working together, the partners could expand on what they understood individually. In this way, they learn from each other, learn to respect each other's ideas, and learn to listen.

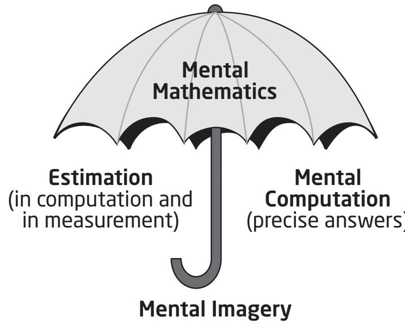
Cooperative Task Group—Task groups of two to four students can work on activities in the **Explore the Math** and **Discover the Math** sections. As a group, students can share their understanding of what is happening during the activity and how that relates to the mathematics topic, at the same time as they develop group cooperation skills.

Jigsaw—Another common cooperative learning group is called a jigsaw. In this technique, individual group members are responsible for researching and understanding a specific area of information for a project. Individual students then share what they have learned so that the entire group gets information about all areas being studied. For example, during data management, this type of group might have "experts" in using various types of software to make a circle graph. Group members could then coach each other in using the particular software.

Another way of using the jigsaw method is to assign "home" and "expert" groups during a large project. For example, students researching the scoring methods for various sports competitions might have a home group of four in which each member is responsible for researching one of diving, figure skating, snowboarding, and gymnastics. Individual members could then move to expert groups. Expert groups would include all of the students responsible for researching each of the sports. Each of the expert groups would research their particular sport. Once the information had been gathered and prepared for presentation, individual members of the expert group would return to their home group and teach other members about their sport.

MENTAL MATHEMATICS

A major goal of mathematics instruction for the twenty-first century is for students to make sense of the mathematics in their lives. The development of all areas of mental mathematics is a major contributor to this comfort and understanding. Mental mathematics is the mental manipulation of knowledge dealing with numbers, shapes, and patterns to solve problems.



The diagram above shows the various components under the umbrella of Mental Mathematics. All three are considered mental activities and interact with each other to make the connections required for mathematics understanding. Estimation and mental math are not topics that can be isolated as a unit of instruction; they must be integrated throughout the study of mathematics.

Estimation

Estimation refers to the approximate answers for calculations, a very practical skill in today's world. The development of estimation skills helps refine mental computation skills, enhances number sense, and fosters confidence in math abilities, all of which are key in problem solving. Over 80% of out-of-school problem solving situations involve mental computation and estimation.[7]

Estimation does not mean guessing at answers. Rather, it involves a host of computational strategies that are selected to suit the numbers involved. The goal is to refine these strategies over time with regular practice, so that estimates become more precise. The ultimate goal is for students to estimate automatically and quickly when faced with a calculation. These estimations allow for recognition of errors on calculator displays, provide learners with a strategy for checking the reasonableness of their calculations, and give students a strategy for finding an answer when only an approximation is necessary.

Mental Imagery

Mental imagery in mathematics refers to the images in the mind when one is doing mathematics. It is this mental representation, or conceptual knowledge, that needs to be developed in all areas of mathematics. Capable math students "see" the math and are able to perform mental manoeuvres in order to make connections and solve problems. These images are formed

[7]Reys, B. J., and R.E. Reys, "One Point of View: Mental Computation and Computational Estimation—Their Time Has Come," *Arithmetic Teacher* (Vol. 33, No. 7, 1986), 4–5.

when students manipulate objects, explore numbers and their meanings, and talk about their learning. Students must be encouraged to look into their mind's eye and "think about their thinking."

Asking, "What do you see in your mind's eye?" when asked to visualize, as in the exercises below, forces students to think about the images they are using to help them solve problems. Students are often surprised when fellow students share their personal images; the discussion generated is very worthwhile.

Try these mental imaging exercises with your students.

Example 1: Draw the mental image you have for each of the following: • $\frac{2}{3}$ • 75% of the questions on the page • a 175° angle	**Example 2:** Use mental imagery to answer the following: **1.** How many edges does a cube have? **2.** If I am facing east, what direction is to my left? **3.** What is the perimeter of a 90 cm × 30 cm shelf?

Mental Computation

Mental computation refers to an operation used to obtain the precise answer for a calculation. Unlike traditional algorithms, which involve one method of calculation for each operation, mental computations include a number of strategies—often in combination with each other—for finding the exact answer. As with estimation, strategies for mental computation develop in quantity and quality over time. A thorough understanding of, and facility with, mental computation allows students to solve complicated multi-step problems without spending needless time figuring out calculations and is a valuable prerequisite for proficiency with algebra. Students need regular practice in these strategies.

Some Points Regarding Mental Mathematics

- Students must have a knowledge of the basic facts (addition and multiplication) in order to estimate and calculate mentally. They learn the many strategies for fact learning in elementary school. With practice, they eventually commit these facts to memory. Without knowing the basic facts, it is unlikely that students will ever attempt to employ any estimation or mental math strategies, as these will be too tedious.
- The various estimation and mental calculation strategies must be taught and are best developed in context; opportunities must be provided for regular practice of these strategies. Having students share their various strategies is vital, as it provides possible options for classmates to add to their repertoire.
- Unlike the traditional paper-and-pencil algorithms, there are many mental algorithms to learn. With the learning, however, comes a greater facility with numbers. Key to the development of skills in mental math is the understanding of place value (number sense) and the number operations. This understanding is enhanced when students make mental math a focus as they calculate.

- Mental math strategies are flexible; the student needs to select one that is appropriate for the numbers in the computation. Practice should be in the form of practising the strategy itself, selecting appropriate strategies for a variety of computation examples, and using the strategies in problem solving situations.
- Although students should not be pressured with time constraints when first learning a mental math strategy, it is beneficial to provide timed tests once they have some facility with mental computation. If too much time is provided, many students will resort to the traditional algorithm and will not use mental strategies.
- Mental math algorithms are used with whole numbers, fractions, and decimal numbers.
- Sometimes mental math strategies are used in conjunction with paper-and-pencil tasks. The questions are rewritten to make the calculation easier.
- The ultimate goal of mental mathematics is for students to estimate for reasonableness and to look for opportunities to calculate mentally.
- Encourage students to refer to the strategies by their name (e.g., front-end strategy). Once the strategies have been taught, post them around the room. Have students write problems in which a mental strategy would be the appropriate computation. Share these problems with the class.
- Students need to identify why particular procedures work; they should not be taught computation "tricks" without understanding.
- Those who are skilled in using mental mathematics will be able to transfer, relate, and apply mental strategies to paper-and-pencil tasks.

Keep in Mind

Practice in classrooms has traditionally been in the form of asking students to write the answers to questions presented orally. This is particularly challenging for students who are primarily visual learners. Although we are sometimes faced with computations of numbers we cannot see, most often the numbers are written down. This makes it easier to select a strategy. In daily life, we see the numbers when solving written problems (e.g., when checking calculations on a bill or invoice, when determining what to leave for tips, when calculating discounted prices from a price tag). Provide students with mental math practice that is sometimes oral and sometimes visual.

TIME LINES FOR *MATHLINKS 7*

The chart below shows estimated times for covering the material in *MathLinks 7*. Please note that times will vary depending on your particular class and its individual students. Field-testing shows that many classes can do some of this material in much less time than is outlined here, while it takes others more time. The chart shows an average. In most cases, the full course can be handled in 160 classes.

Also note that there are alternative ways to cover and assess many outcomes. For example, student achievement of chapter outcomes can be checked by having students do the **chapter review**, **practice test**, and **chapter test**, *or*, more holistically, by having students complete a related **Challenge in Real Life**, *or* by doing a combination of these things. Similarly, some of the exercise questions can be replaced by a **Math Games** activity, which provides a motivating way for students to do extra practice.

In a similar manner, you may wish to have some advanced students do the **Challenge in Real Life** for a particular chapter while other students work on the sections. In other chapters, the **Challenge in Real Life** may provide additional motivation for all students. Questions from the **cumulative review** could be used for extra practice for students who need it.

Chapter	1	2	3	4	5	6	7	8	9	10	11	12
Chapter Opener	20–30	20–30	20–30	20–30	20–30	20–30	20–30	20–30	20–30	20–30	20–30	20–30
Section 1	80–100	80–100	80–100	120–150	80–100	120–150	40–50	40–50	60–75	80–100	80–100	80–100
Section 2	80–100	80–100	80–100	80–100	80–100	80–100	80–100	80–100	60–75	80–100	80–100	80–100
Section 3	120–150	80–100	80–100	80–100	80–100	80–100	80–100	80–100	60–75	80–100	80–100	80–100
Section 4	80–100	80–100	80–100		80–100		80–100	40–50	40–50	100–125	80–100	80–100
Section 5			80–100		80–100			80–100	60–75			80–100
Chapter Review	40–50	80–100	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50
Practice Test	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50
Wrap It Up!	80–100	60–75	40–50	40–50	40–50	40–50	60–75	60–75	40–50	40–50	40–50	60–75
Math Games	40–50	40–50	40–50	40–50	40–50	40–50	40–50	60–75	40–50	60–75	40–50	40–50
Challenge in Real Life	40–50	60–75	40–50	60–75	60–75	60–75	60–75	60–75	60–75	60–75	60–75	80–100
Cumulative Review				60–75				60–75				60–75
Task				60–75				60–75				60–75

CURRICULUM CORRELATION

Strand/Outcome	Chapter/Section	Pages
Strand: Number		
General Outcome *Develop number sense.*		
Specific Outcomes		
1. Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0. [C, R]	Chapter 6: 6.1 Math Games	pp. 198–209 p. 226
2. Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems. [ME, PS, T]	Chapter 2: 2.1–2.4 Math Games Challenge in Real Life Task Math Games	pp. 44–73 p. 78 p. 79 p. 155 p. 302
3. Solve problems involving percents from 1% to 100%. [C, CN, PS, R, T]	Chapter 4: 4.1–4.3 Math Games Challenge in Real Life Task Challenge in Real Life	pp. 124–144 p. 150 p. 151 p. 155 p. 419
4. Demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions. [C, CN, R, T]	Chapter 4: 4.1–4.2 Chapter 10: 10.1 Challenge in Real Life	pp. 124–139 pp. 350–357 p. 151
5. Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences). [C, CN, ME, PS, R, V]	Chapter 6: 6.2–6.3 Challenge in Real Life Chapter 7: 7.1–7.4 Math Games Challenge in Real Life Math Games Task	pp. 210–221 p. 227 pp. 230–259 p. 264 p. 265 p. 302 p. 307
6. Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically. [C, CN, PS, R, V]	Chapter 9: 9.1–9.5 Math Games Challenge in Real Life	pp. 310–341 p. 346 p. 347
7. Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using: • benchmarks • place value • equivalent fractions and/or decimals. [CN, R, V]	Chapter 4: 4.1–4.2 Chapter 6: 6.2–6.3 Chapter 7: 7.1	pp. 124–139 pp. 210–221 pp. 230–236
Strand: Patterns and Relations (Patterns)		
General Outcome *Use patterns to describe the world and solve problems.*		
Specific Outcomes		
1. Demonstrate an understanding of oral and written patterns and their equivalent linear relations. [C, CN, R]	Chapter 10: 10.1–10.2, 10.4 Task	pp. 350–364, pp. 372–381 p. 461
2. Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. [C, CN, R, V]	Chapter 10: 10.3–10.4 Challenge in Real Life Task	pp. 365–381 p. 387 p. 461

Strand/Outcome	Chapter/Section	Pages
Strand: Patterns and Relations (Variables and Equations)		
General Outcome *Represent algebraic expressions in multiple ways.*		
Specific Outcomes		
3. Demonstrate an understanding of preservation of equality by: • modelling preservation of equality, concretely, pictorially and symbolically • applying preservation of equality to solve equations. [C, CN, PS, R, V]	Chapter 11: 11.2–11.4 Math Games	pp. 395–413 p. 418
4. Explain the difference between an expression and an equation. [C, CN]	Chapter 10: 10.2 (partial) Chapter 11: 11.1	pp. 358–364 pp. 390–394
5. Evaluate an expression given the value of the variable(s). [CN, R]	Chapter 10: 10.3 Math Games Task	pp. 365–381 p. 386 p. 461
6. Model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially and symbolically, where a and b are integers. [CN, PS, R, V]	Chapter 11: 11.2 Math Games	pp. 395–401 p. 418
7. Model and solve problems that can be represented by linear equations of the form: • $ax + b = c$ • $ax = b$ • $\frac{x}{a} = b, a \neq 0$ concretely, pictorially and symbolically, where a, b and c are whole numbers. [CN, PS, R, V]	Chapter 11: 11.3–11.4 Math Games Challenge in Real Life Task	pp. 402–413 p. 418 p. 419 p. 461
Strand: Shape and Space (Measurement)		
General Outcome *Use direct or indirect measurement to solve problems.*		
Specific Outcomes		
1. Demonstrate an understanding of circles by: • describing the relationships among radius, diameter and circumference of circles • relating circumference to pi • determining the sum of the central angles • constructing circles with a given radius or diameter • solving problems involving the radii, diameters and circumferences of circles. [C, CN, R, V]	Chapter 8: 8.1–8.2, 8.5 Challenge in Real Life Task	pp. 268–279, pp. 292–297 p. 303 p. 307
2. Develop and apply a formula for determining the area of: • triangles • parallelograms • circles. [CN, PS, R, V]	Chapter 3: 3.4–3.5 Task Chapter 8: 8.3	pp. 100–115 p. 155 pp. 280–286

Strand/Outcome	Chapter/Section	Pages
Strand: Shape and Space (3-D Objects and 2-D Shapes)		
General Outcome *Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.*		
Specific Outcomes		
3. Perform geometric constructions, including: • perpendicular line segments • parallel line segments • perpendicular bisectors • angle bisectors. [CN, R, V]	Chapter 3: 3.1–3.3 Math Games Challenge in Real Life	pp. 82–99 p. 120 p. 121
General Outcome *Describe and analyze position and motion of objects and shapes.*		
Specific Outcomes		
4. Identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs. [C, CN, V]	Chapter 1: 1.1–1.2 Math Games Challenge in Real Life	pp. 4–17 p. 40 p. 347
5. Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices). [C, CN, PS, T, V]	Chapter 1: 1.3–1.4 Challenge in Real Life	pp. 18–35 p. 41
Strand: Statistics and Probability (Data Analysis)		
General Outcome *Collect, display and analyze data to solve problems.*		
Specific Outcomes		
1. Demonstrate an understanding of central tendency and range by: • determining the measures of central tendency (mean, median, mode) and range • determining the most appropriate measures of central tendency to report findings. [C, PS, R, T]	Chapter 12: 12.1–12.3, 12.5 Math Games Challenge in Real Life Task	pp. 422–439, pp. 446–451 p. 456 p. 457 p. 461
2. Determine the effect on the mean, median and mode when an outlier is included in a data set. [C, CN, PS, R]	Chapter 12: 12.3–12.4	pp. 434–445
3. Construct, label and interpret circle graphs to solve problems. [C, CN, PS, R, T, V]	Chapter 8: 8.4–8.5 Math Games	pp. 287–297 p. 302
Strand: Statistics and Probability (Chance and Uncertainty)		
General Outcome *Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.*		
Specific Outcomes		
4. Express probabilities as ratios, fractions and percents. [C, CN, R, T, V]	Chapter 5: 5.1–5.2 Challenge in Real Life Math Games Task	pp. 158–170 p. 195 p. 302 p. 307
5. Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events. [C, ME, PS]	Chapter 5: 5.2–5.4 Challenge in Real Life Task	pp. 165–182 p. 195 p. 307
6. Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events. [C, PS, R, T]	Chapter 5: 5.3–5.5 Math Games	pp. 171–189 p. 194