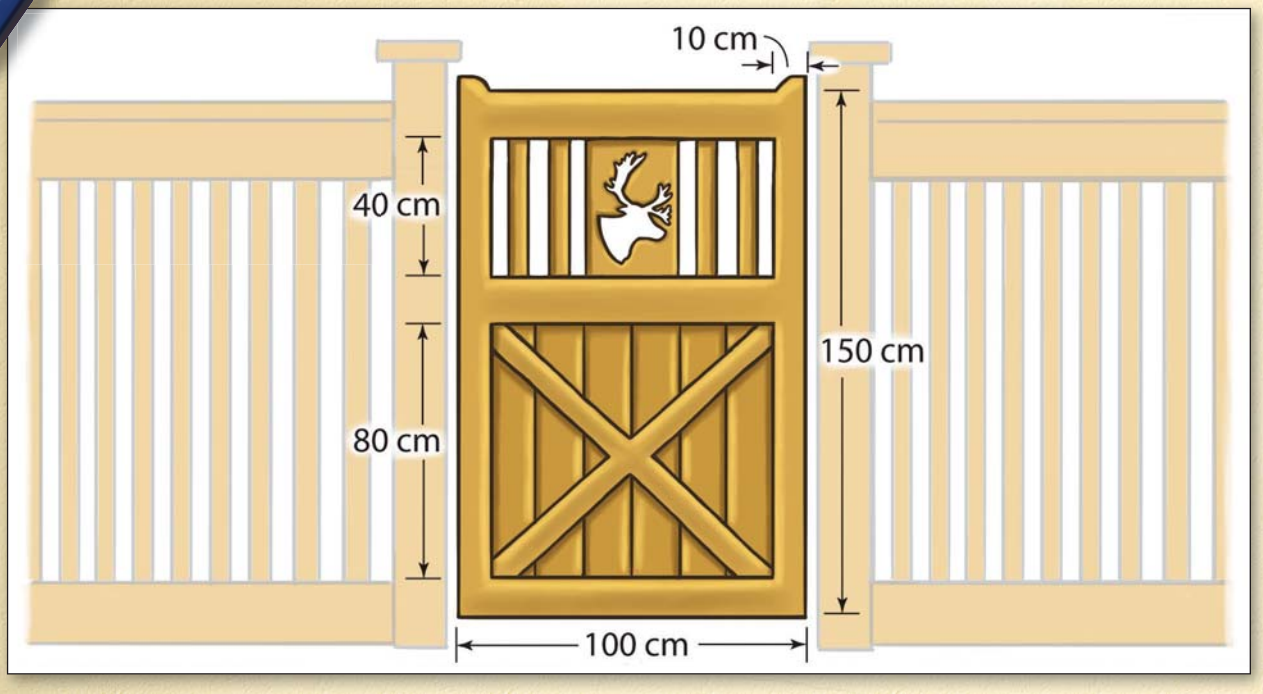
NL Math 10: Chapter 6

February 24, 2011, 16:33

6 Pythagorean Relationship



Brendan is building a gate based on a design he saw on the Internet. He changed the design to suit his family's garden.

1. a) What geometric shapes do you see on the gate? Sketch each shape.
b) Label every length that you know.

2. Put a question mark on any side lengths you cannot determine.

3. a) Identify the size of every angle you can.
b) Which shapes have the same angles?

4. For what jobs do you need these skills?

Key Words

right triangle
hypotenuse
leg
Pythagorean relationship
square

Career Link

Andrew is a landscape designer. He enjoys designing and building walkways and gardens. Andrew likes the creative design part of his job. He also likes the construction part that brings his ideas into reality.

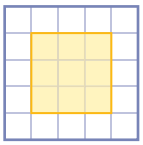
6 Get Ready

Squares

1. Determine the area of each square drawn on centimetre grid paper.

Remember to use the correct units for area.

a)



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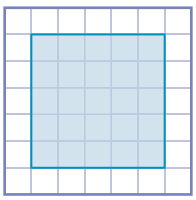

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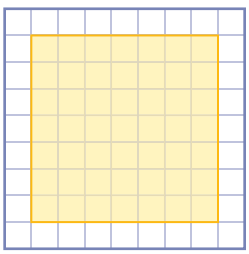


2. On centimetre grid paper, draw each square. Then, determine the area of each square.

a) 8 cm by 8 cm

b) 10 cm by 10 cm

c) 12 cm by 12 cm

3. Evaluate without using a calculator.

a) 2^2

b) 4^2

c) 9^2

d) 11^2

e) 40^2

f) 70^2

4. Estimate. Then, calculate.

a) 1.1^2

b) 5.9^2

c) 7.7^2

d) 10.2^2

e) 51^2

f) 100.5^2

How close were your estimates to your calculated answers?

Add and Subtract Squares

5. Evaluate.

a) $6^2 + 1^2$

b) $4^2 + 3^2$

c) $11^2 + 2^2$

d) $10^2 + 10^2$

e) $40^2 + 5^2$

f) $12^2 + 4^2$

6. Find each value.

a) $7^2 - 1^2$

b) $8^2 - 3^2$

c) $9^2 - 1^2$

d) $10^2 - 5^2$

e) $12^2 - 2^2$

f) $30^2 - 20^2$

Square Roots

A square root can be thought of as the length of each side of a given square. For example, a 6-cm by 6-cm square has an area of 36 cm^2. The square root of 36 is 6, or $\sqrt{36} = 6$.

7. Determine each square root. Do not use a calculator.

a) $\sqrt{4}$

b) $\sqrt{16}$

c) $\sqrt{36}$

d) $\sqrt{100}$

8. Evaluate without using a calculator. Then, evaluate using a calculator. Write your answer to one decimal place.

a) $\sqrt{5}$

b) $\sqrt{15}$

c) $\sqrt{34}$

d) $\sqrt{103}$

How close were your answers without a calculator to your answers with a calculator?

Solve Equations

9. Solve for the variable.

a) $a^2 = 16$

b) $x^2 = 25$

c) $b^2 = 49$

d) $y^2 = 100$

e) $s^2 = 121$

f) $c^2 = 10\,000$

10. Solve for the variable.

a) $c^2 = 3^2 + 4^2$

b) $f^2 = 5^2 + 12^2$

c) $t^2 = 12^2 + 9^2$

d) $p^2 = 8^2 + 6^2$

Angles

11. On grid paper, sketch each angle.

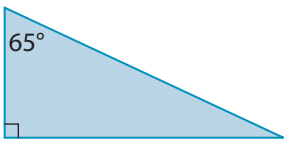
a) 90°

b) 45°

c) 120°

d) 60°

12. What is the measure of the third angle in the triangle?



6.1 Right Triangles

Focus On . . .

- exploring right triangles
- estimating the length of the hypotenuse of a right triangle

The photo shows buildings at the Norstead Viking Village at L'Anse aux Meadows. Can you see the right triangles? Take a look in your home, at school, and at the rest of the world around you. Where do you see right triangles?

right triangle

- a triangle with an angle of 90°

Materials

- centimetre grid paper
- ruler
- protractor

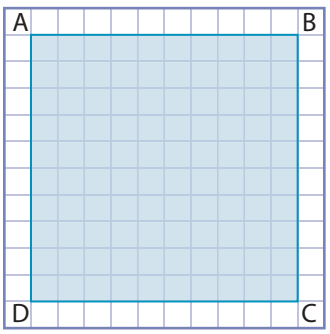
F.Y.I.

The plural form of "vertex" is "vertices." A vertex is the point where two sides of a figure meet.

Explore Right Triangles

1. a) On centimetre grid paper, draw a 10-cm by 10-cm square.

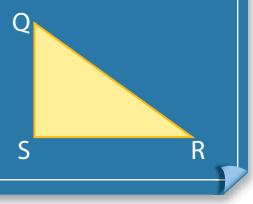
b) Label the vertices A, B, C, and D, as shown.



c) Draw a diagonal connecting A and C.

F.Y.I.

Triangles are named by their vertices, such as △QRS.



2. Name the two triangles.

3. ∠ADC is a right angle. Name the right angle in the other triangle.

4. a) Estimate the measure of ∠DAC.

b) List the other angles that appear to have the same measure.

5. a) Choose one triangle. Add the measure of the right angle and your estimates of the other two angles.

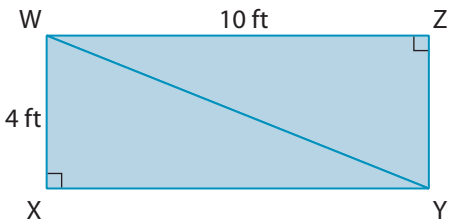
b) Predict the total measure of the three angles inside the other triangle. Explain your thinking.

6. Reflect

a) Estimate the length of the diagonal, AC.

b) Explain your reasoning.

7. Extend Your Understanding Below is rectangle WXYZ and the diagonal WY.



a) Estimate the measure of ∠XWY.

b) Name any other angles that appear to have the same measure as ∠XWY.

c) Estimate the measure of ∠XYW.

d) Name any other angles that appear to have the same measure as ∠XYW.

e) Measure the angles in parts a) and c).

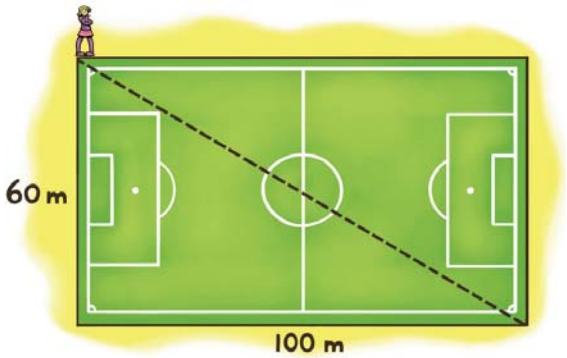
f) What is the total measure of the three angles that make up each triangle?

g) Estimate the length of WY.

On the Job 1

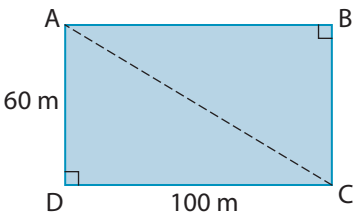
Estimate the Length of the Hypotenuse of a Right Triangle

Rita usually walks around the soccer field. She thinks that cutting across the rectangular soccer field might be a shorter walk. Estimate the distance saved by walking along the diagonal.



Solution

Draw a diagram of the soccer field. Label the vertices.



Rita is walking from A to C.

The diagonal, AC, creates two right triangles. AC is the **hypotenuse** of $\triangle ABC$ and $\triangle ACD$.

Method 1: Compare to the Legs of a Triangle

Estimate the length of AC.

AC looks longer than **leg** AB. So, AC must be greater than 100 metres.

AC looks shorter than the sum of legs AB and BC.

$$\begin{aligned} AB + BC &= 100 + 60 \\ &= 160 \end{aligned}$$

AC must be less than 160 metres.

A reasonable estimate for the length of the hypotenuse is 120 metres to 125 metres.

$160 - 120 = 40$

$160 - 125 = 35$

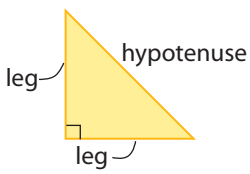
So, Rita saves about 35 metres to 40 metres by cutting across the field.

hypotenuse

- the longest side of a right triangle
- it is located opposite the right angle

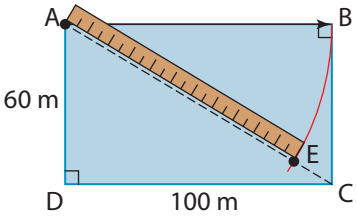
leg

- one of the two shorter sides that form the right angle



Method 2: Rotate the Longer Leg of a Triangle

Use point A as a fixed point. Imagine rotating leg AB.
AB would cross the hypotenuse, AC, at approximately E.



Use a ruler and your finger to trace the path of the rotating leg.

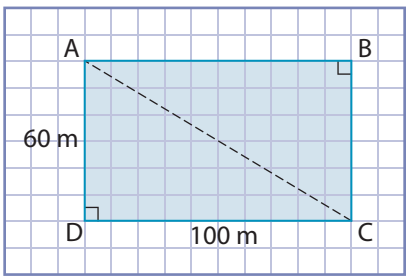
AE is approximately 100 metres. A reasonable estimate for AC is 115 metres to 120 metres.

$160 - 115 = 45$
$160 - 120 = 40$

So, Rita saves about 40 metres to 45 metres by cutting across the soccer field.

Method 3: Use a Scale Diagram

On centimetre grid paper, draw a scale diagram of the soccer field. Use a scale of 1 cm represents 10 m.



Measure the length of the diagonal, AC.
The diagonal measures about 11.6 cm or 11.7 cm.

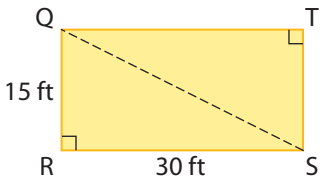
Apply the scale of the diagram.
1 cm represents 10 m.
So, 11.6 cm represents 116 m, and 11.7 cm represents 117 m.

The hypotenuse, AC, is about 116 metres to 117 metres.
$160 - 116 = 44$
$160 - 117 = 43$

So, Rita saved about 43 metres to 44 metres by cutting across the field.

Your Turn

Use two methods to estimate the length of the hypotenuse, QS.

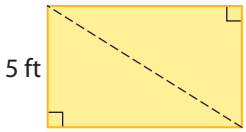


Check Your Understanding

Try It

1. Estimate the length of the hypotenuse by comparing it to the legs of the triangle.

a)



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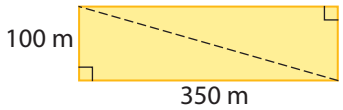

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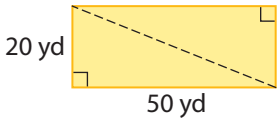

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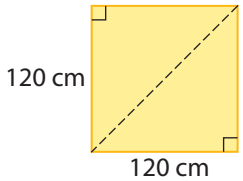
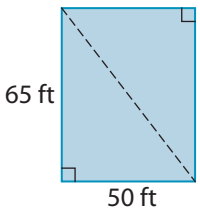


2. Estimate the length of each hypotenuse by rotating the longer leg of the triangle.

a)



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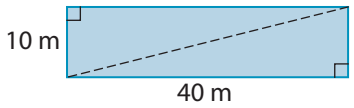

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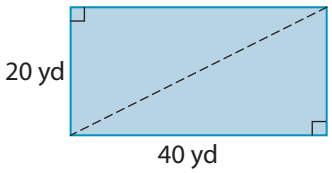

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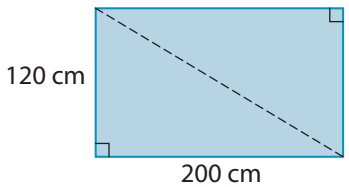
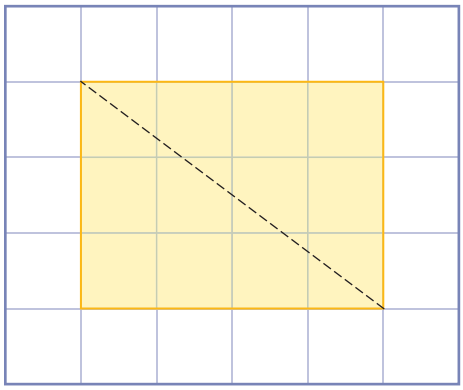


3. Measure the length of the hypotenuse on each diagram. Then, use the scale of 1 cm represents 1 m to estimate the length of each hypotenuse, in metres.

a)



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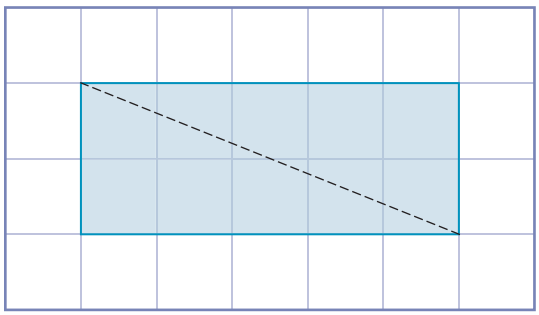
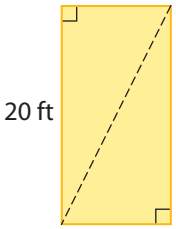


4. Make a scale diagram of each triangle below. Use the scale to estimate the length of each hypotenuse, in feet.

a)



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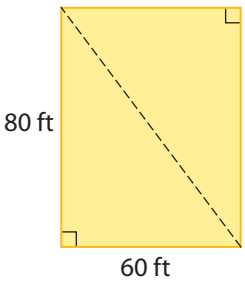
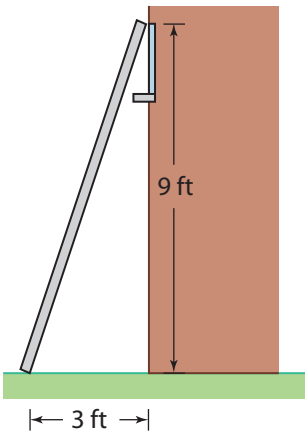
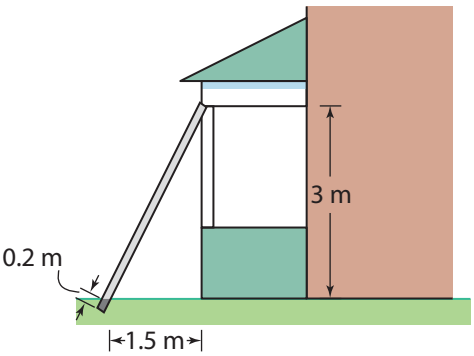


Apply It

5. Nadine wants to clean the windows of her house. She estimates that the tops of the windows are 9 feet above ground. She places the bottom of an extension ladder 3 feet from the base of the house. Estimate how long she needs to make the ladder.

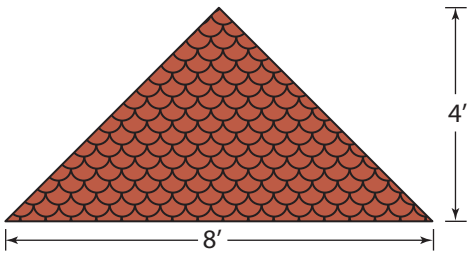


6. Peter is replacing the columns of a porch. First, he needs to brace the porch roof. The braces will run from the bottom of the porch roof to holes in the ground. The height from the ground to the bottom of the roof is 3 m. The holes will be 1.5 m from the house. An extra 0.2 m must be added to the braces so that they will reach the bottom of the holes. Estimate the length Peter will need to cut each brace.



Work With It

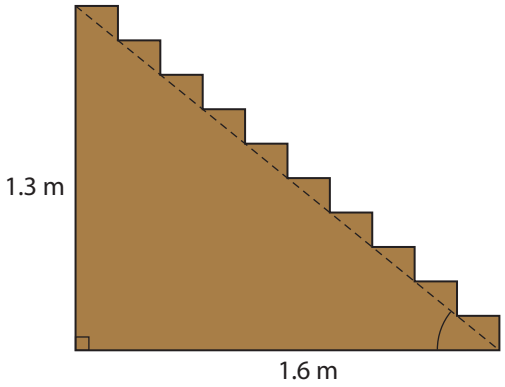
1. Jody plans to put up a set of solar panels to run outdoor lights. She will lay them flat on one slope of her shed roof. Do you think that a 6-foot by 6-foot set of solar panels would fit on the shed roof? Explain.



F.Y.I.

A solar panel must be placed at the best angle to catch the sun's rays. The angle depends on the latitude of the location and the time of year. The angle of a solar panel in St. John's is greater than in a location further south.

2. Han read that the angle for a set of stairs should be 35.5° to 42.5° from the ground.



a) Measure the angle marked by the arc in the diagram. Are the stairs within the guideline?

b) Estimate the length of the hypotenuse.

c) If the rise is changed to 1.6 m, what is an estimate for the length of the hypotenuse?

d) Draw a diagram to represent the stairs in part c). Measure the angle of the stairs. Are they within the guideline?

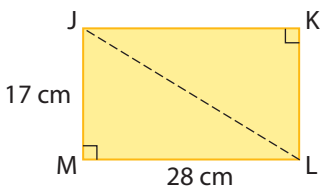
How is it possible to know the angle of the stairs in part d) without drawing a diagram?

Discuss It

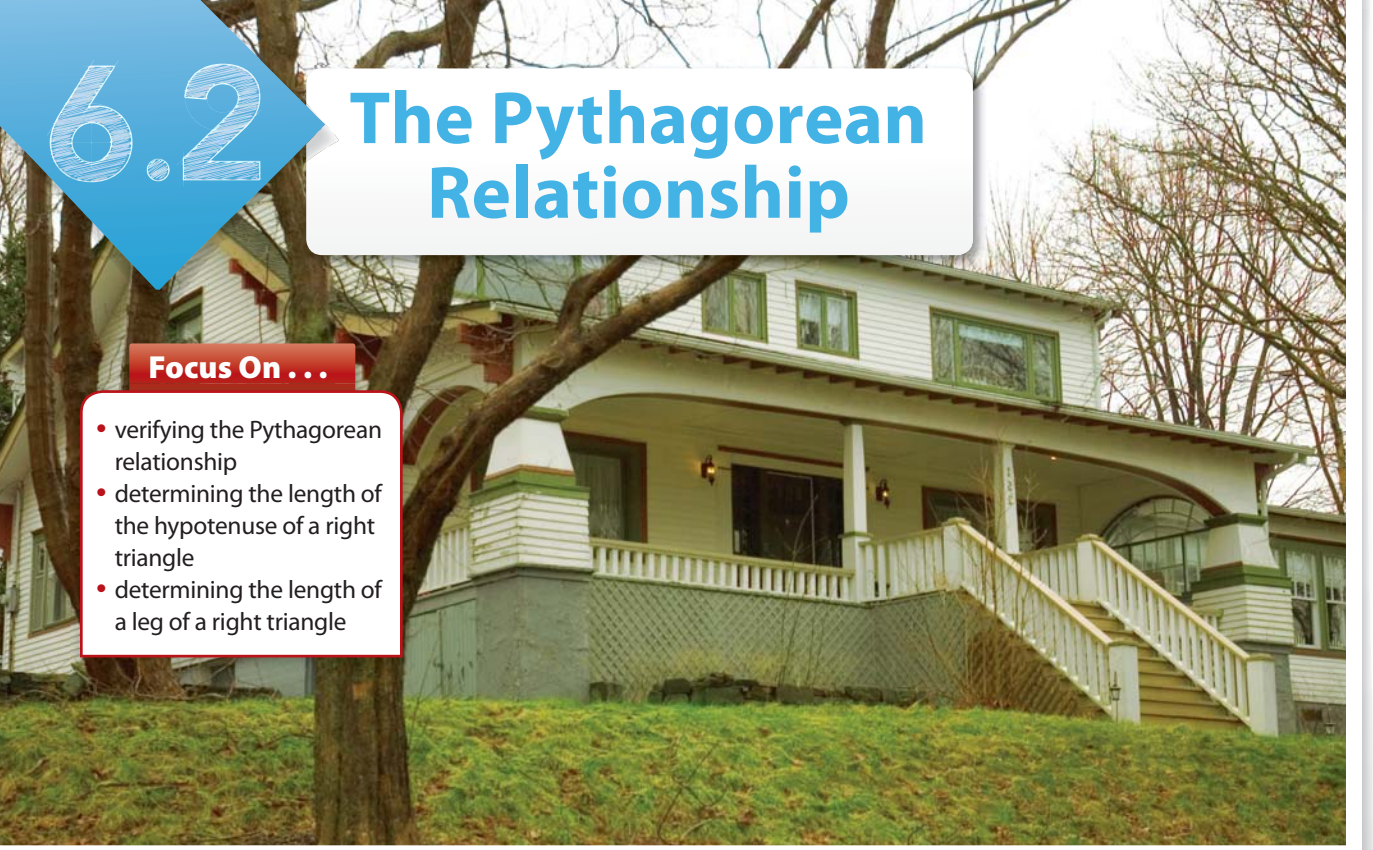
3. If you know the length of the two legs of a right triangle, you know what range of values the length of the hypotenuse must lie within. Explain this statement.

4. When the length of a hypotenuse is measured with a ruler, it is usually called an "approximate measurement." Why?

5. Kayla and Ariel each estimated the length of the hypotenuse, JL. Ariel said it was between 17 cm and 28 cm. Kayla said that it was less than 45 cm but more than 28 cm. Who is correct? How do you know?



6.2 The Pythagorean Relationship



Focus On . . .

- verifying the Pythagorean relationship
- determining the length of the hypotenuse of a right triangle
- determining the length of a leg of a right triangle

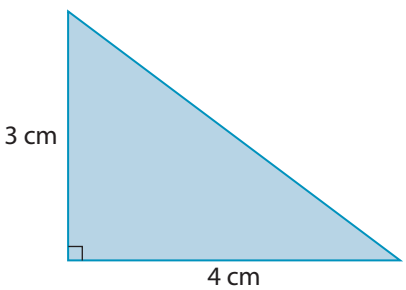
It was 2500 years ago that Greek mathematician Pythagoras developed a theorem describing the relationship among the three sides of a right triangle. Today, the relationship helps people determine measurements and angles when they are working on tasks on the job and at home. The photo shows Merchant's Manor, a historical building in Carbonear. Do you see the right triangle used in part of its construction?

Explore the Pythagorean Relationship

Materials

- centimetre grid paper
- ruler
- scissors
- tape or glue

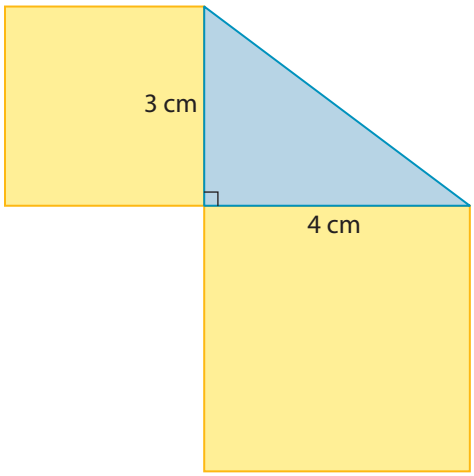
1. In the centre of a piece of centimetre grid paper, draw a right triangle with legs 3 cm and 4 cm.



2. a) Measure the length of the hypotenuse.

b) Label the lengths of all three sides of the triangle.

3. a) Sketch a 3-cm by 3-cm square. Use the 3-cm leg of the triangle as one side of the square.



4. a) Sketch a 4-cm by 4-cm square. Use the 4-cm leg of the triangle as one side of the square.

b) From the corner of the grid paper, cut out a square with sides that are the same length as the length of the hypotenuse. Place the square so that the hypotenuse is one of the sides.

5. a) What is the area of the 3-cm by 3-cm square? Write that value inside the square.

b) What is the area of the 4-cm by 4-cm square? Write that value inside the square.

c) What is the area of the square attached to the hypotenuse? Write that value inside the square.

6. Reflect How are these three areas related?

7. Extend Your Understanding

a) Repeat the steps with a right triangle that has legs that are 5 cm and 12 cm. Do you get the same relationship as in step 6?

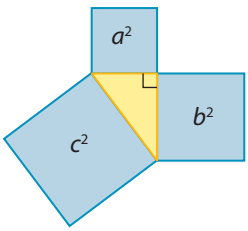
b) What do you think is the relationship among the sizes of the squares on each side of a right triangle?

On the Job 1

Determine the Length of the Hypotenuse of a Right Triangle

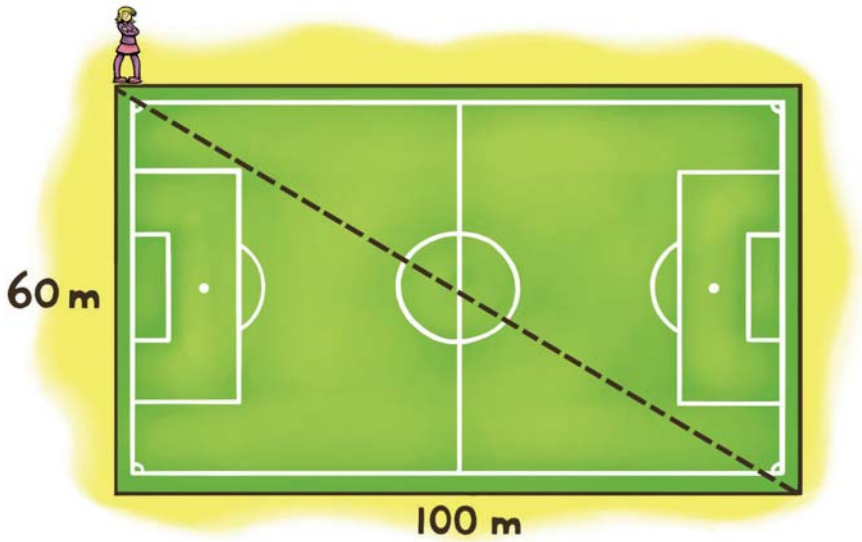
Pythagorean relationship

- the relationship between the lengths of the sides of a right triangle
- the sum of the areas of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse



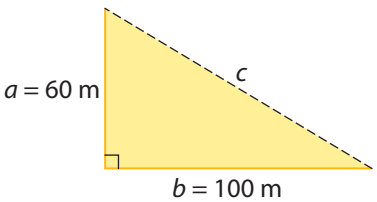
$a^2 + b^2 = c^2$

In the previous section, you estimated the distance Rita saved by cutting across the diagonal of a soccer field. Use the **Pythagorean relationship** to calculate the distance saved by walking along the diagonal. Give your answer to the nearest hundredth of a metre.



Solution

Draw a sketch. Label the sides of the triangle, as shown.



Sketch the squares attached to the three sides of the triangle.

Calculate the area of the two smaller squares.

$a^2 = 60^2$

$a^2 = 3600$

The area of the square attached to leg a is 3600 m^2.

$b^2 = 100^2$

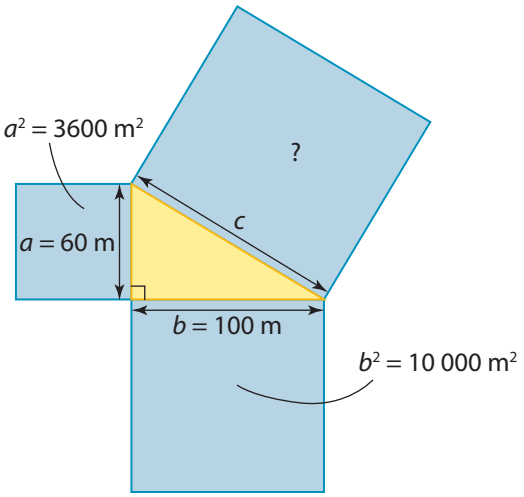
$b^2 = 10\,000$

The area of the square attached to leg b is 10 000 m^2.

Web Link

To explore the Pythagorean relationship using an interactive activity, go to www.mhrmathatwork10.ca and follow the links.

Label the area of each smaller square.



Using the Pythagorean relationship, you know that the area of the square attached to the hypotenuse is the sum of the areas of the two smaller squares.

$c^2 = a^2 + b^2$

$c^2 = 3600 + 10\ 000$

$c^2 = 13\ 600$

The area of the big square is 13 600 m^2.

The length of the hypotenuse is the square root of 13 600.

$c^2 = 13\ 600$

$c = \sqrt{13\ 600}$

$c = 116.619\ldots$

The diagonal of the soccer field is about 116.62 metres.

Tech Link

Follow the steps to calculate the square root of 13 600.

Press **C** **2nd** **√** **13 600** **=**

Your calculator may have slightly different key strokes. Experiment or check with a classmate.

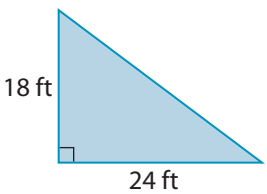
The distance around the edge of the soccer field is 160 metres.

$160 - 116.62 = 43.38$

Rita saved a distance of 43.38 m by walking along the diagonal.

Your Turn

Make a sketch of a right triangle with legs of length 18 feet and 24 feet. Use the Pythagorean relationship to determine the length of the hypotenuse.

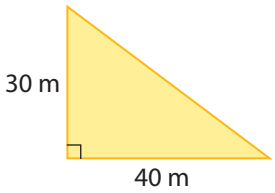


Check Your Understanding

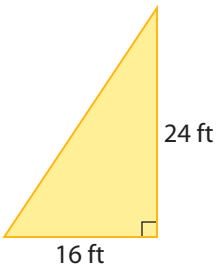
Try It

1. State two values that the length of each hypotenuse must lie between.

a)

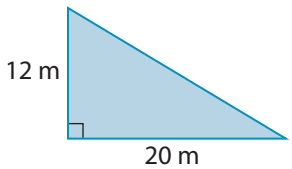


b)

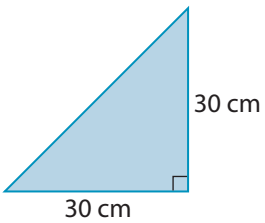


2. Use the Pythagorean relationship to determine the length of the hypotenuse of each right triangle. Round your answers to the nearest tenth of a unit.

a)

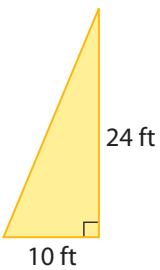


b)

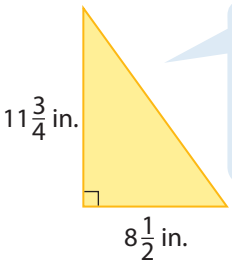


3. Estimate the length of the hypotenuse of each right triangle. Then, use the Pythagorean relationship to determine the length of the hypotenuse. Where necessary, round your answer to the nearest whole unit.

a)



b)



Change the mixed fractions to decimal numbers.

4. **a)** Convert your answer in #3a) to inches.

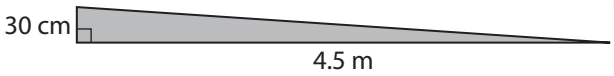
b) Convert your answer in #3b) to feet and inches.

How many inches are there in a foot?

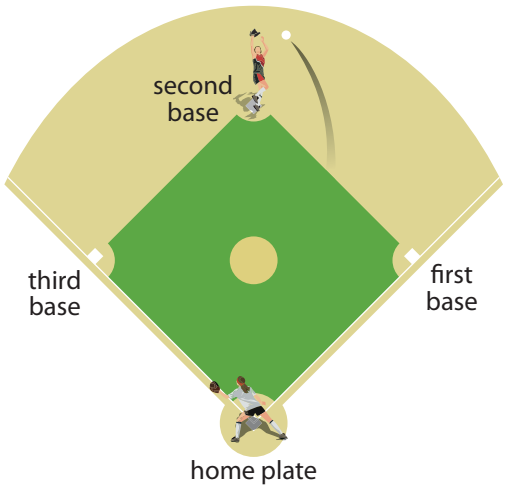
Apply It

5. A ramp from a walkway to a doorway has a height of 30 cm. The length along the ground is 4.5 m. Calculate the distance a wheelchair travels along the ramp. Write your answer to the nearest hundredth of a metre.

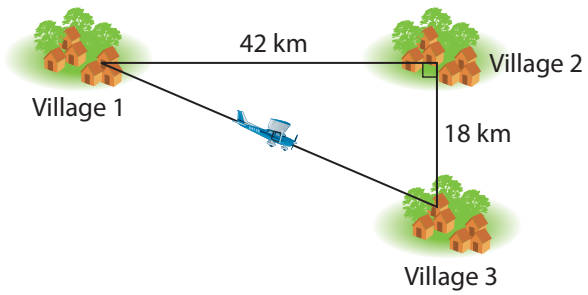
First, change 30 cm to metres.



6. In softball, the bases are often set 65 feet apart in the shape of a square. The catcher throws the ball from home plate to second base. How far is the catcher's throw? Round your answer to the nearest foot.



7. Some small villages can be accessed by plane only.



a) Estimate the distance of the flight from Village 1 to Village 3.

b) Determine the distance of the flight from Village 1 to Village 3. Round your answer to the nearest kilometre.

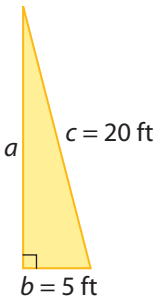
On the Job 2

Determine the Length of One Leg of a Right Triangle

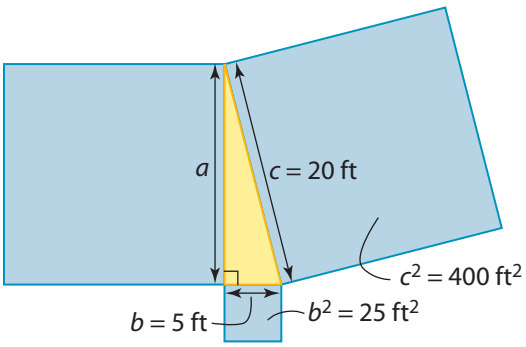
Jeanette needs to clean her eavestroughs. She owns a 20-foot ladder. She wants to know how high up the side of her house the ladder will reach if she puts the bottom of the ladder 5 feet from the house. Write your answer to the nearest foot.

Solution

Sketch a diagram.



Draw squares attached to each of the three sides of the triangle. Label the known areas of the squares.



Use the Pythagorean relationship.

$a^2 + b^2 = c^2$

$a^2 + 25 = 400$

$a^2 = 375$

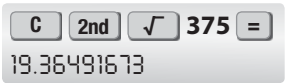
The area of the square attached to leg a is 375 ft^2.

Leg a of the triangle equals the square root of 375.

$a^2 = 375$

$a = \sqrt{375}$

$a = 19.364\ldots$

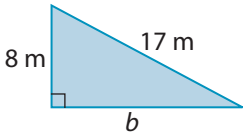


Leg a of the triangle is approximately 19 ft.

Jeanette's ladder reaches about 19 feet up the side of her house.

Your Turn

Use the Pythagorean relationship to determine the length of leg b of the right triangle.



Web Link

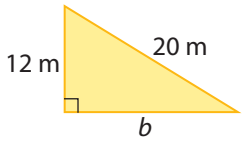
To try an interactive activity that explores the Pythagorean relationship, go to www.mhrmathatwork10.ca and follow the links.

Check Your Understanding

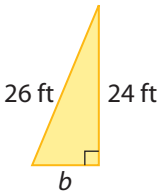
Try It

1. Use the Pythagorean relationship to determine the length of leg b in each right triangle.

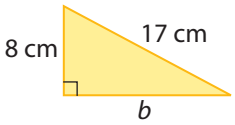
a)



b)

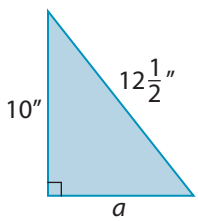


c)

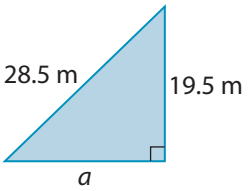


2. Estimate the length of leg a in each right triangle. Then, use the Pythagorean relationship to determine the length of leg a. Write your answers to the nearest whole unit.

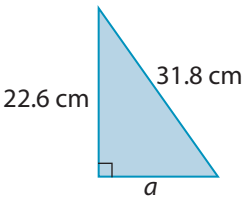
a)



b)

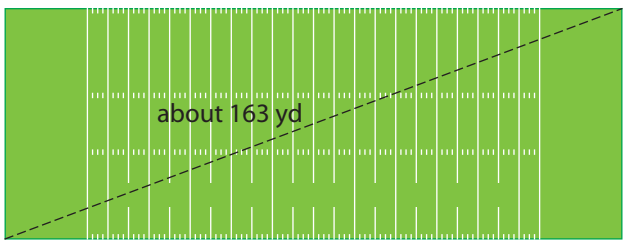


c)



Apply It

3. Ed and Bill cut across the diagonal of a rectangular football field that is 150 yards long. They estimate the length of the diagonal to be 163 yards.



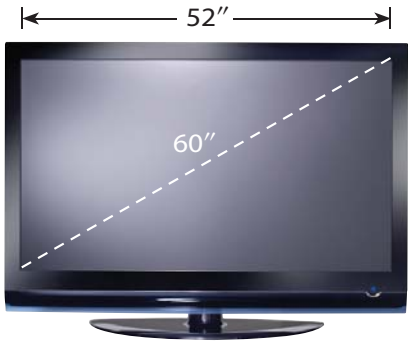
a) Use their estimate to determine an approximate measurement for the width of the football field. Express your answer to the nearest yard.

b) About how much distance did Ed and Bill save compared to walking along the edge of the field?

F.Y.I.

The size of a TV is determined by the length of its diagonal. For example, a 60-in. TV measures 60 in. along the diagonal of the rectangular screen.

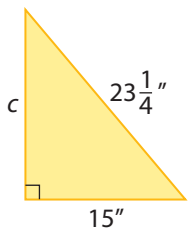
4. A 60-inch TV measures 52 inches along the top of the screen. What is the height of the screen? Round your answer to the nearest inch.



5. The loading ramp of a moving van is 10 feet long. The floor of the storage compartment of the van is 27 inches off the ground. What is the distance along the ground from the back of the van to the point where the ramp touches the ground? Write your answer to the nearest inch.

6. When asked to calculate the length of c, a student wrote the following.

$$a^2 + b^2 = c^2$$
$$\left(23\frac{1}{4}\right)^2 + 15^2 = c^2$$
$$540.5625 + 225 = c^2$$
$$765.5625 = c^2$$
$$c = \sqrt{765.5625}$$
$$c = 27.67$$



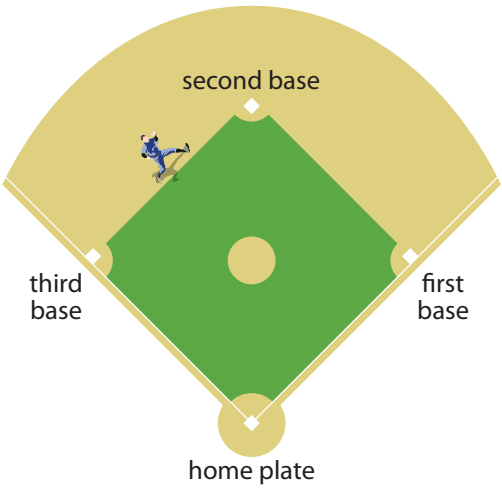
a) Do you agree that c is the longest side in this triangle? If not, explain the error in the student's thinking.

b) Explain why you cannot always write the Pythagorean relationship as $a^2 + b^2 = c^2$.

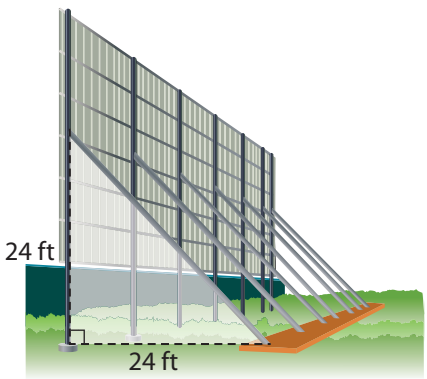
Work With It

1. A vehicle's hatch opening measures 95 centimetres high and 110 centimetres wide at its greatest width. Do you think a round table with a diameter of 150 centimetres will fit through the opening? Show your work.

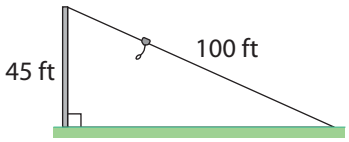
2. The shortstop normally plays halfway between second base and third base. On a square baseball diamond where the bases are 90 feet apart, how far is the throw from shortstop to first base? Round your answer to the nearest foot.



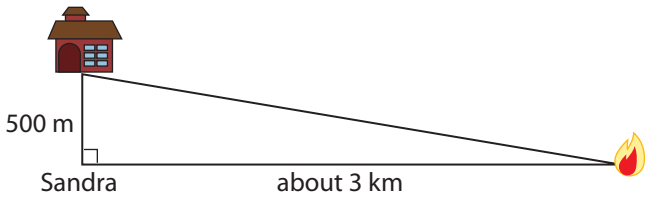
3. The diagonal pipe braces are set in concrete 24 feet away from the pipe columns. They are attached to the columns 24 feet up. Determine the length of the diagonal braces. Round your answer to the nearest foot.



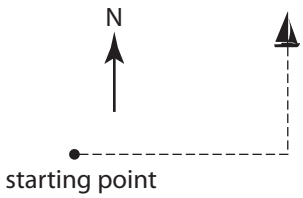
4. A 100-foot zip line is anchored to the top of a pole that is 45 feet high. How far from the base of the pole will the zip line connect to the ground? Round your answer to the nearest foot.



5. From an observation point, Sandra estimates the distance to a forest fire to be 3 km. She contacts the fire station, which is 500 m away. Estimate the distance between the fire station and the forest fire.



6. a) Joanna sails 8 miles east from Trinity Bay, and then travels 4 miles north. How far is Joanna from her starting point? Round your answer to the nearest mile.



b) Joanna continues to sail north. She reaches a point that is 15 miles on the diagonal from her starting point. How far north has she sailed in total? Round your answer to the nearest mile.

Discuss It

7. a) Evan says, "Solving for the hypotenuse of a right triangle is an addition problem." What does he mean?

b) Based on the answer to part a), what kind of problem is solving for one of the legs of a right triangle? Why?

8. Toby says that for #5, "You don't need a calculator. The fire is 3500 metres away."

a) Why might Toby think this?

b) Describe how to do the question correctly.

9. Claire is working out a triangle problem. The squares of the legs are 64 cm^2 and 36 cm^2. The square of the longest side of the triangle is 101 cm^2. Based on the Pythagorean relationship, what can Claire conclude about the triangle?

10. Research and write about practical uses of the Pythagorean relationship in the past and in the present.

6.3 Using the Pythagorean Relationship

Focus On . . .

- showing that a 3-4-5 triangle is a right triangle
- determining if a triangle is a right triangle
- determining if a corner is square using the Pythagorean relationship
- solving problems involving right triangles

square

- a corner that is exactly 90°

When creating a rectangular garden, you can check that the corners are square by using a right triangle that has sides in a ratio of 3 : 4 : 5. People often refer to the "3-4-5 triangle" or the "3-4-5 method" when doing construction or other types of work. Any triangle with side lengths in a ratio of 3 : 4 : 5 is a right triangle. This is true because $3^2 + 4^2 = 5^2$.

Explore the 3-4-5 Triangle

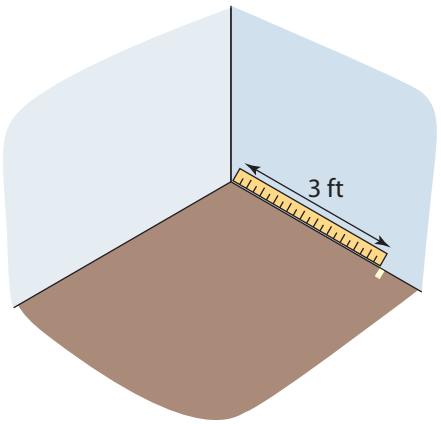
Materials

- measuring tape
- masking tape

Determine whether a corner of your classroom is square.

1. Choose one corner of your classroom.

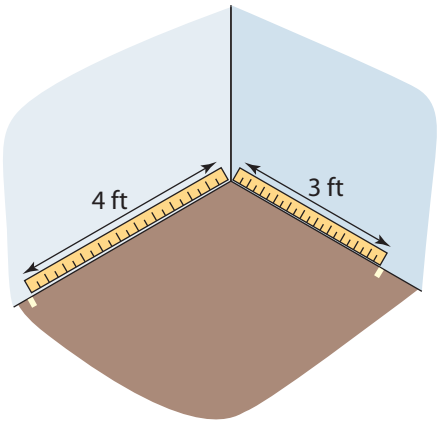
2. Measure a spot on the floor that is 3 feet along the wall. Mark this spot with a piece of masking tape.



F.Y.I.

"Adjacent" means near or next to.

3. Return to the corner. On the adjacent wall, measure and mark a spot on the floor 4 feet from the corner.



4. Measure the diagonal distance from one mark to the other.

5. Reflect

a) If the distance between the two points is 5 feet, what does that mean about the corner of the classroom?

b) If the distance between the two points is less than 5 feet, what does that mean about the corner of the classroom?

c) If the distance between the two points is more than 5 feet, what does that mean about the corner of the classroom?

6. Extend Your Understanding Use your knowledge of 3-4-5 triangles to determine whether the other corners of your classroom are square.

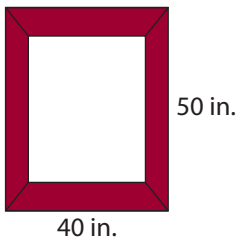
Web Link

The 3-4-5 triangle is based on the concept of the Pythagorean triple. To learn more about the Pythagorean triple, go to www.mhrmathatwork10.ca and follow the links.

On the Job 1

Check for a Right Angle Using the Pythagorean Relationship

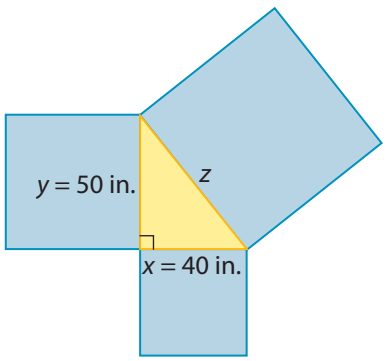
George is making a rectangular picture frame. He wants to check that the frame has square corners. Calculate what the length of the diagonal must be for the frame to be square. Round your answer to the nearest inch.



Solution

Sketch a diagram.

Use the Pythagorean relationship to determine the length of the hypotenuse of a right triangle with legs that are 40 inches and 50 inches.



$z^2 = x^2 + y^2$
$z^2 = 40^2 + 50^2$
$z^2 = 1600 + 2500$
$z^2 = 4100$

The area of the triangle attached to the hypotenuse is 4100 square inches.

The length of the hypotenuse is the square root of 4100.

$z = \sqrt{4100}$
$z = 64.031\ldots$

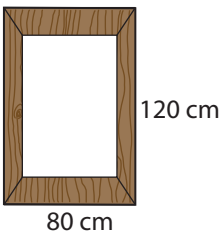
The diagonal distance across the picture frame must be approximately 64 inches for the frame to be square.

F.Y.I.

Any variables can be used for the Pythagorean relationship. Just make sure you use the correct values for the hypotenuse and the two legs when writing the relationship.

Your Turn

A gate frame has dimensions 80 cm by 120 cm. What must the diagonal distance from corner to corner be for the frame to be square? Show your answer to the nearest centimetre.

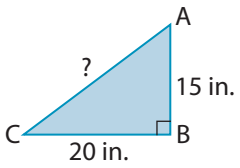


Check Your Understanding

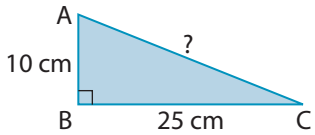
Try It

1. Determine what the length of AC must be for ∠B to be a right angle. Where necessary, round your answer to the nearest tenth of a unit.

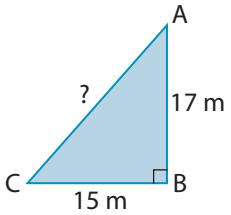
a)



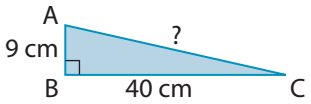
b)



c)

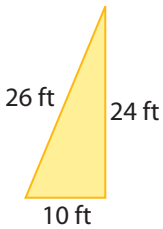


d)



2. Is each triangle a right triangle? How do you know?

a)



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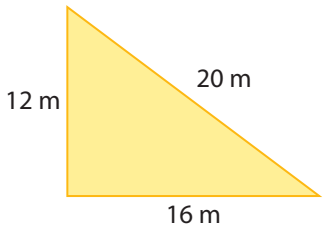
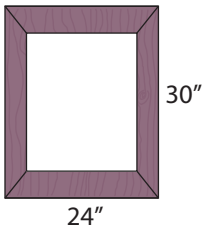


Apply It

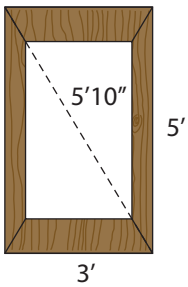
3. Use what you know about 3-4-5 triangles to determine whether the corner of a window frame in your school is square. Justify your answer.

4. Bob has a piece of matting for a picture frame. Its dimensions are 24″ by 30″. He wants to know whether the matting has been cut properly. What length should the diagonal measurement of the matting be for Bob to know that it is square? Round your answer to the nearest inch.

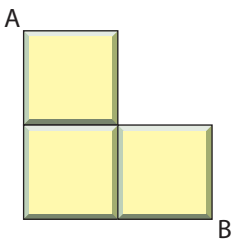


5. A landscaper is staking the outline of a rectangular garden. The width of the garden is 14 feet, and the length is 20 feet. The landscaper measures the distance from one corner to the opposite corner to be 23 feet. Will the new garden have 90° corners? Show your work.

6. Rick wonders if the frame of his gate is square. The horizontal pieces of the frame are 3 ft long. The vertical pieces are 5 ft long. He measures the diagonal to be 5 ft 10 in. Is the frame square? Explain why or why not.



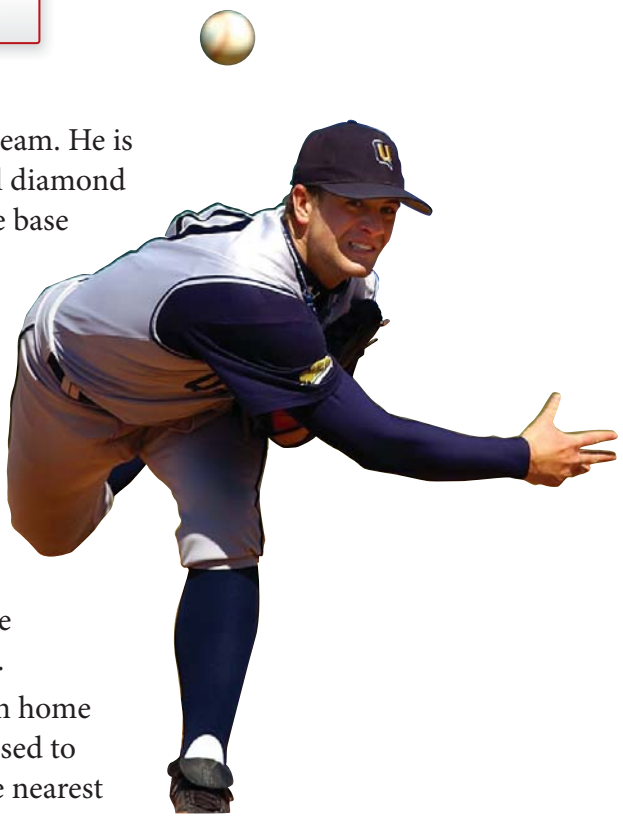
7. Sam has laid the first few tiles on her bathroom floor. Each tile is 10 cm by 10 cm. What does the diagonal measurement from A to B need to be if she is laying the tiles square? Write your answer to the nearest tenth of a centimetre.



8. Andrea draws a triangle with sides that measure 15 cm, 36 cm, and 39 cm. Max says it cannot be a right triangle because it is not a 3-4-5 triangle. Is Max correct? Explain.

On the Job 2

Correct a Right Angle



Mick is the coach of a baseball team. He is measuring out a square baseball diamond in a field. The distance from one base to the next is supposed to be 20 metres. Mick has measured the distance between home plate and first base, and the distance between first and second base. They are both 20 metres. However, Mick thinks that something does not look right. He measures the distance between home plate and second base. It is 30 metres.

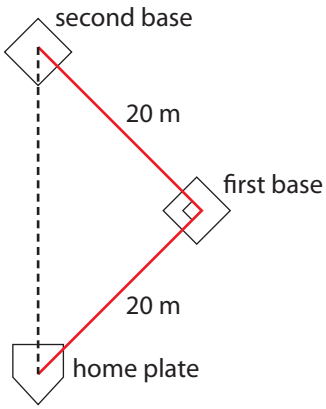
a) What is the distance between home plate and second base supposed to be? Write your answer to the nearest hundredth of a metre.

b) What is the problem?

c) How can Mick fix the problem?

Solution

a) Make a sketch of what the distance between home plate and second base should look like.



Determine the distance between home plate and second base. This is the length of the hypotenuse of the right triangle.

The sum of the areas of the squares attached to the legs equals the area of the square attached to the hypotenuse.

$$20^2 + 20^2 = 400 + 400$$
$$= 800$$

The area of the square attached to the hypotenuse should be 800 m^2.

The length of the hypotenuse is the square root of 800.

$\sqrt{800} = 28.284\ldots$

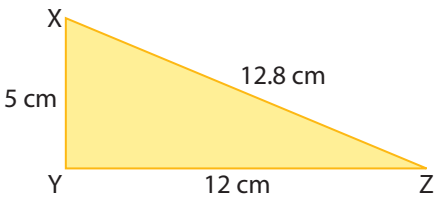
The distance from home plate to second base should be 28.28 m.

b) The problem is that Mick has a distance from home plate to second base that is 30 metres. This is almost 2 metres greater than 28.28 metres. This means that the angle at first base is not 90°.

c) Mick needs to bring second base closer to home plate. When the triangle has lengths of 20 m, 20 m, and 28.28 m, he will know he has a right angle at first base.

Your Turn

A triangle has side lengths of 5 cm, 12 cm, and 12.8 cm.



a) If the angle at Y is exactly 90°, what should the distance between X and Z be?

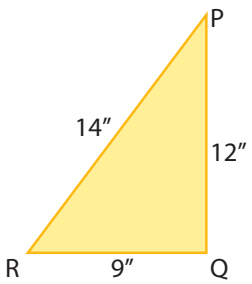
b) Do you think the angle at Y is greater than, less than, or equal to 90°? Explain your reasoning.

Check Your Understanding

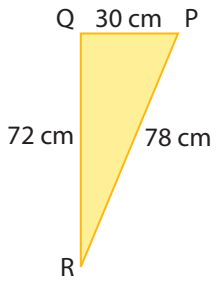
Try It

1. Is ∠Q in each triangle a right angle? Show your work.

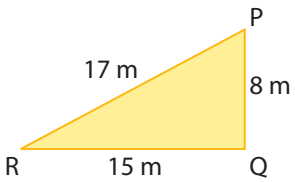
a)



b)



c)



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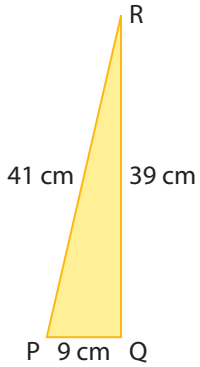


2. **a)** Which triangles in #1 are not right triangles?

b) What should be the length of the hypotenuse so that they become right triangles? Do not change the length of the legs. When necessary, write your answer to the nearest hundredth of a unit.

3. **a)** On grid paper, draw a right triangle of any size.

b) Measure and label the legs.

c) Use the Pythagorean relationship to determine the length of the hypotenuse.

d) Label the hypotenuse incorrectly by at least one whole unit.

e) Exchange your sketch with a classmate.

f) Estimate whether the labelled hypotenuse on your classmate's sketch is too long or too short.

g) Calculate the correct length of the hypotenuse.

h) Check your classmate's solution to your problem.

Tools of the Trade

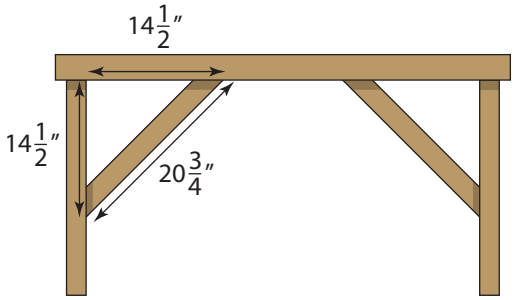
You can make a simple workbench from 2-by-4s, plywood, and some screws. To find out how to build one yourself, go to www.mhrmathatwork10.ca and follow the links.

Apply It

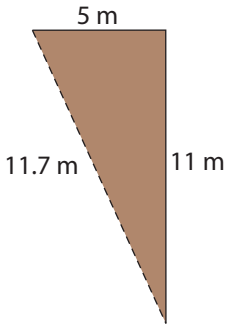
4. Graham is building a workbench. He adds a diagonal brace between the tabletop and one of the legs, as shown. He measures the length of the brace to check that the tabletop and leg are square. The brace is $20\frac{3}{4}$ in. long.

a) What should the length of the diagonal brace be so that the tabletop and leg are square?

b) What can Graham conclude about the angle made by the tabletop and the leg?



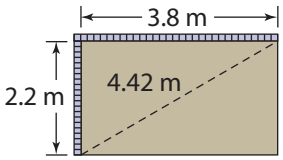
5. Molly is building a shed. The floor will be 5 m by 11 m. She begins to clear out an area where she will pour the concrete base of the shed. However, the corner does not appear to be at a right angle. When she measures the diagonal, it is 11.7 m.



a) What should the length of the diagonal be? Write your answer to the nearest tenth of a metre.

b) How can Molly fix the problem?

6. Martin is converting his garage into an apartment. He is building two walls that will frame a bathroom. He wants the corners to be perfectly square. The new walls measure 2.2 metres and 3.8 metres long. He measures the corner-to-corner distance to be 4.42 metres.



a) Can Martin be reasonably sure that the corner is a right angle? Explain.

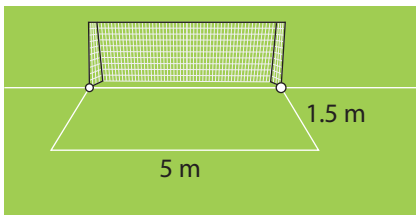
b) If the corner of the room is not square, suggest what Martin needs to do.

Work With It

1. **a)** Reena needs to mail a DVD. She has a box with a rectangular opening that measures 11 cm by 4 cm. Will a DVD with a diameter of 12 cm fit through the opening? Show your work.

b) To the nearest tenth of a centimetre, what is the largest diameter that will fit in the box?

2. Hank is marking off the goal creases for a children's soccer game. Each rectangular crease should measure 5 metres by 1.5 metres.



a) Calculate the correct measure of the diagonal for each goal crease. Write your answer to the nearest hundredth of a metre.

b) Hank measures one of the diagonals to be 5.3 m. What should Hank do to make the corner square?

3. Dan wants to pour a concrete patio. He has built a wooden frame that he will pour the concrete into. The dimensions of the frame are 9 ft by 12 ft.

a) On centimetre grid paper, draw a rectangle that is 9 squares by 12 squares.

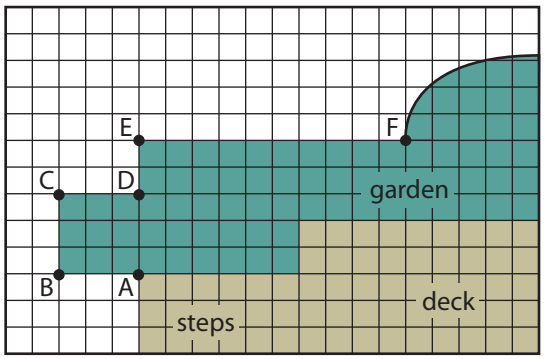
b) Draw the diagonals.

c) Use any method from the chapter to estimate the length of each diagonal.

d) Use the Pythagorean relationship to determine the length of the diagonal of the wooden frame.

e) If Dan's measurement of the diagonals matches your answer to part d), what can he conclude?

4. Fred and Marnie had a landscaper draw plans for their backyard. The design uses a combination of curved garden spaces and rectangular spaces. The design of one of the spaces is shown. The scale of the drawing is 1 square represents 1 metre.



a) The angle at B is a right angle. Calculate the distance between A and C. Write your answer to the nearest hundredth of a metre.

b) The angle at D is a right angle. Calculate the distance between C and E. Write your answer to the nearest hundredth of a metre.

c) Estimate the distance between D and F.

d) The angle at E should be 90°. In the completed garden, Marnie and Fred measure the distance between D and F to be 10.04 m. Is the angle at E the size it is supposed to be? Show your thinking.

e) Fred and Marnie want to fix the edge of the garden between D, E, and F to ensure a right angle at E. What should they do?

Discuss It

5. You have drawn two right triangles.

a) The hypotenuse of one triangle is larger than it should be. What can you conclude about the measure of the angle opposite the hypotenuse?

b) The hypotenuse of the other triangle is shorter than it should be. What can you conclude about the measure of the angle opposite the hypotenuse?

6. How can knowing about the Pythagorean relationship help you to determine whether an angle is *not* 90°?

7. Discuss why a 3-4-6 triangle cannot be a right triangle.

6 Skill Check

What You Need to Know

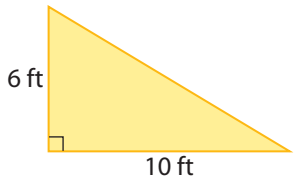
Section	After this section, I know how to . . .
6.1	■ explore right triangles ■ estimate the length of the hypotenuse of a right triangle
6.2	■ verify the Pythagorean relationship ■ determine the length of the hypotenuse or leg of a right triangle
6.3	■ determine if a triangle is a right triangle ■ determine if a corner is square using the Pythagorean relationship

If you are unsure about any of these questions, review the appropriate section or sections of this chapter.

6.1 Right Triangles, pages 284–291

1. **a)** Determine a reasonable range of values for the length of the hypotenuse.

b) Estimate the length of the hypotenuse. Imagine rotating the longer leg of the triangle to the hypotenuse.



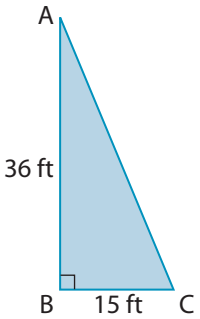
Use a ruler and your finger to trace the arc of the longer leg to the hypotenuse.

2. △ABC is a right triangle

a) On grid paper, make a scale diagram of △ABC.

b) Measure the length of the hypotenuse on the diagram.

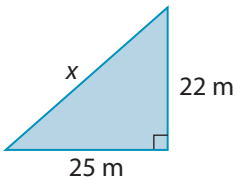
c) Use the scale of your diagram to determine the length, in feet, of the hypotenuse.



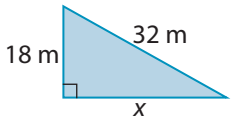
6.2 The Pythagorean Relationship, pages 292–302

3. Determine the length of x. Round your answers to the nearest hundredth of a metre.

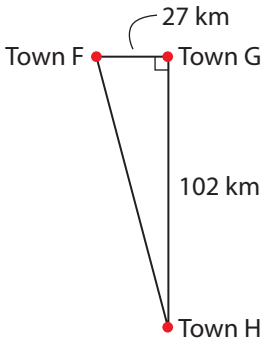
a)



b)



4. The distances between three towns are shown.



a) State a reasonable range of values for the distance between Town F and Town H.

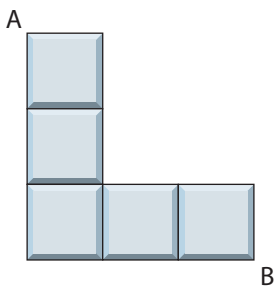
b) Estimate the distance between Town F and Town H.

c) Calculate the distance between these two towns. Round your answer to the nearest kilometre.

6.3 Using the Pythagorean Relationship, pages 303–313

5. Karen wants to put a round table into her car. The opening in the car measures 90 cm by 70 cm. Determine the largest diameter that could fit into the rectangular opening. Write your answer to the nearest centimetre.

6. Luka is tiling the walls of a shower stall. Each tile is 5 cm by 5 cm. Luka is concerned that his tiles are not set at exactly 90° angles. He measures the distance from the top left corner of tile A to the bottom right corner of tile B as 21 cm.



a) How far out of square are the tiles, to the nearest hundredth of a centimetre? Show your work.

b) What will be the effect if Luka does not correct this error?

Test Yourself

For #1 to #6, select the best answer.

1. Which of the following statements is always true about the hypotenuse of a right triangle?

A The hypotenuse is opposite the right angle.

B The hypotenuse is at the top of a right triangle.

C The hypotenuse is to the left of the right angle.

D The hypotenuse is to the right of the right angle.

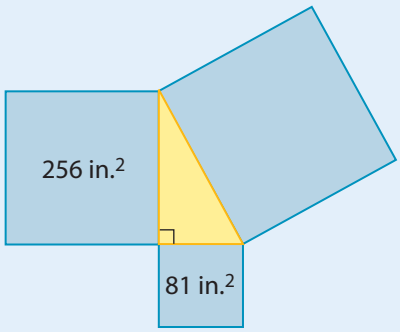
2. What is the area of the square attached to the hypotenuse?

A $256 - 81$

B $256 + 81$

C between 81 and 256

D $\sqrt{256} + \sqrt{81}$



3. What is the best estimate for the length of the hypotenuse in #2?

A greater than 9

B between 9 and 16

C greater than 16

D between 16 and 25

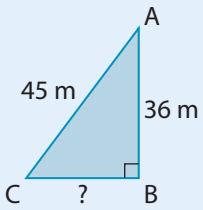
4. What is the length of BC?

A 3 m

B 9 m

C 27 m

D 32 m



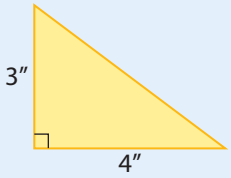
5. Without using a calculator, what is the best estimate for the length of the hypotenuse?

A about 5″

B $3'' + 4''$

C between 3″ and 4″

D exactly 5″



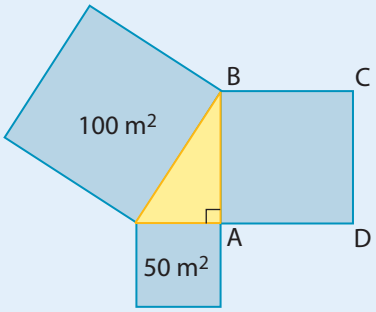
6. If the angle at A is exactly 90°, what is the area of square ABCD?

A exactly 50 m^2

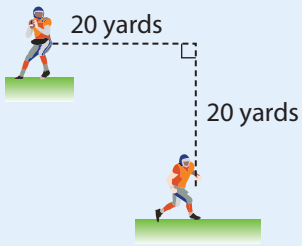
B exactly 150 m^2

C between 0 m^2 and 50 m^2

D between 50 m^2 and 100 m^2



7. Drew is the quarterback for his high school football team. He can throw the ball about 30 yards. A receiver lines up beside Drew. The receiver runs 20 yards down the field, turns right, and runs 20 more yards. Can Drew's pass reach the receiver? Explain your reasoning.



8. Bailey owns a 25-foot extension ladder. She puts the bottom of the ladder 5 feet from the house. Calculate how high up the side of her house the ladder will reach. State your answer in feet and inches.

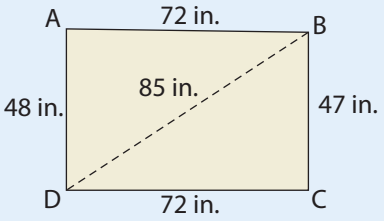
9. Janice lives in an old house. She is preparing to install new linoleum on the floor of her bathroom. She wonders if the corners of the room are square.

a) Is the angle at A exactly 90°? Show your work.

b) Is the angle at C exactly 90°? Show your work.

c) Do you think that ∠ABC and ∠ADC are greater than or less than 90°? Explain.

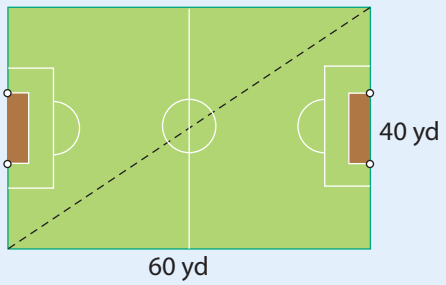
d) What values might Janice wish to know before having the linoleum cut to fit the room?



10. The dimensions of a rectangular soccer pitch for 10-year-olds are often 60 yards by 40 yards.

a) What must be the length of the diagonal of the soccer pitch? Round your answer to the nearest yard.

b) Write the length, width, and diagonal of the pitch in feet.



6 Chapter Project

Build a Gate or Picture Frame

Use craft sticks to construct a scale model of a gate or to construct a picture frame.

1. Follow these steps to complete a gate.

- Build a square or rectangular gate frame that will support the vertical slats attached to it.
- Ensure that all four corners of the frame are square.
- Install one diagonal support.
- Complete the gate using a design of your choice.

OR

Follow these steps to complete a picture frame.

- Build a square or rectangular picture frame.
- Ensure that all four corners of the frame are square.
- Attach craft sticks to the back side of the frame to make it strong.
- Optional: Complete the design by decorating your picture frame.

2. Include the following items as part of your project:

✓ a diagram of your gate or picture frame, with labels showing measurements and angles

✓ an explanation of how you know that the corners of your gate or picture frame are square

Remember to include your calculations.

6 GAMES AND PUZZLES

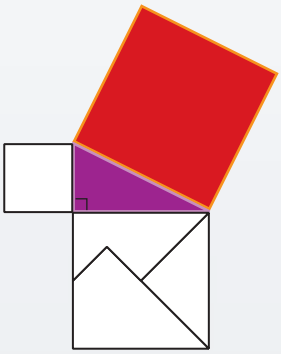
Pythagorean Puzzles

The Pythagorean relationship states that the sum of the areas of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse. The squares attached to the legs of two right triangles have been divided into puzzle pieces.

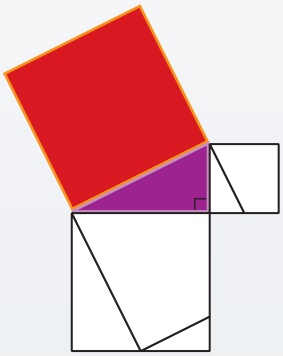
Materials

- Pythagorean Puzzles 1 and 2
- scissors
- timer

Puzzle 1



Puzzle 2



1. Player 1 receives Pythagorean Puzzle 1. Player 2 receives Pythagorean Puzzle 2.
2. Each player cuts out the unshaded puzzle pieces from their sheet. Then, each player cuts out the shaded right triangle and the square attached to the hypotenuse *as one piece*. Make sure that the two shaded shapes stay attached to each other.
3. Player 1 receives the pieces for both puzzles. Do not mix together the pieces for the two puzzles!
4. Player 2 says, "Go!" and starts the timer.
5. As quickly as possible, Player 1 rearranges each set of unshaded pieces to fill in the shaded square attached to the hypotenuse of each right triangle.
6. When Player 1 completes both puzzles, he or she says, "Done!" Player 2 immediately stops the timer.
7. The players switch roles. This time, the challenge is a bit different. Player 1 times while Player 2 uses the puzzle pieces to reassemble the squares on the *legs* of each right triangle.
8. The player with the faster time wins.