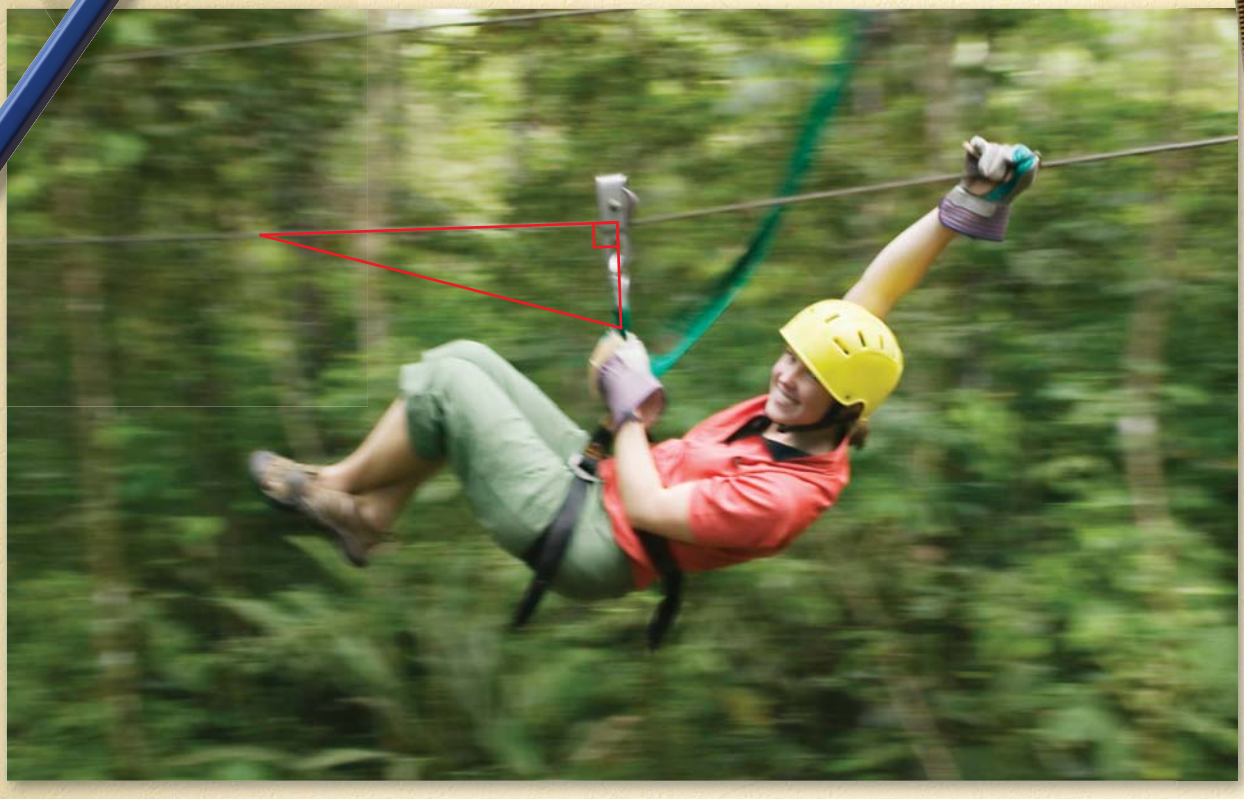
NL Math 10: Chapter 7

February 24, 2011, 16:25

7 Trigonometry



During the school break, Anna and her family went on an adventure tour. Anna rode a zipline down a mountainside and over a falls. Anna just got a summer job as a camp counsellor. She is one of a team working to develop a zipline for some young children at the camp. Anna has an idea and wants to develop a scale diagram.

1. What measurements does Anna need before she can begin drawing the scale diagram?
2. She decides to use a scale of 1 square represents 10 ft. Explain the meaning of this scale.
3. What skills might Anna need to learn in order to develop a zipline?

Key Words

- polygon
- similar figures
- corresponding sides
- opposite side
- adjacent side
- tangent ratio
- sine ratio
- cosine ratio
- primary trigonometric ratios

Career Link

Michael builds models of vintage trucks. Each model is a copy of a truck that is smaller than the actual truck. He uses math to make sure his models are built to scale.

7 Get Ready

Convert Fractions to Decimals

1. Convert each fraction to a decimal. Round the answer to two decimal places, if necessary.

a) $\frac{4}{5}$

b) $\frac{10}{12}$

c) $\frac{32}{21}$

d) $\frac{1.2}{4.5}$

e) $\frac{8}{9}$

f) $\frac{6.8}{2.5}$

Scale

2. Katie uses a scale of 1 cm represents 500 m to make a scale diagram. Some actual measurements are below. How long will she need to make each measurement on the scale diagram?

a) ■ cm represents 1 km

b) ■ cm represents 8 km

c) ■ cm represents 12 km

d) ■ cm represents 15 km

e) ■ cm represents 22 km

f) ■ cm represents 27 km

3. Draw a scale diagram of each rectangle. Use a scale of 1 in. represents 2 ft.

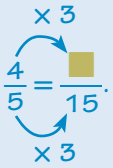
a)

b)

Proportions

4. Solve each equivalent fraction.

a) $\frac{4}{5} = \frac{\square}{15}$



To keep the proportion, multiply 4 × 3. What value belongs in the box?

b) $\frac{2}{3} = \frac{\square}{12}$

c) $\frac{8}{12} = \frac{\square}{6}$

d) $\frac{\square}{14} = \frac{1}{7}$

5. Solve for each unknown quantity.

a) $\frac{6}{5} = \frac{18}{x}$

b) $\frac{9}{x} = \frac{36}{24}$

c) $\frac{16}{22} = \frac{32}{x}$

d) $\frac{5.5}{x} = \frac{11}{3.6}$

6. Solve for each unknown quantity.

a) $\frac{1}{10} = \frac{x}{24}$

Isolate x.

$\frac{1}{10} = \frac{x}{24}$

$\frac{1}{10}(24) = \frac{x}{24}(24)$ Multiply both sides by 24.

$\frac{24}{10} = x$

What is the value of x?

b) $\frac{1}{5} = \frac{x}{12}$

c) $\frac{2}{3} = \frac{x}{16}$

d) $\frac{4}{3} = \frac{x}{9}$

e) $\frac{x}{12} = \frac{4}{30}$

f) $\frac{x}{9} = \frac{7}{10}$

Solve Equations

7. Solve for x.

a) $5 = \frac{15}{x}$

Isolate x.

$5 = \frac{15}{x}$

$5(x) = \frac{15}{x}(x)$ Multiply both sides by x.

$5x = 15$

What is the value of x?

b) $8 = \frac{32}{x}$

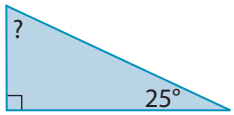
c) $1.5 = \frac{60}{x}$

d) $4.2 = \frac{63}{x}$

Right Triangles

8. Determine the measure of each unknown angle.

a)



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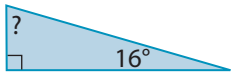

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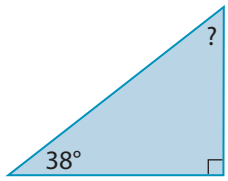
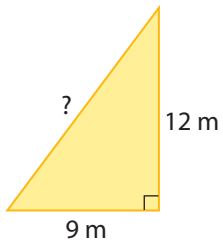


9. Determine each unknown side length. Express your answers to two decimal places, if necessary.

a)



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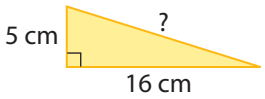

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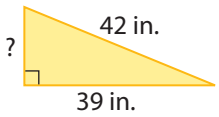
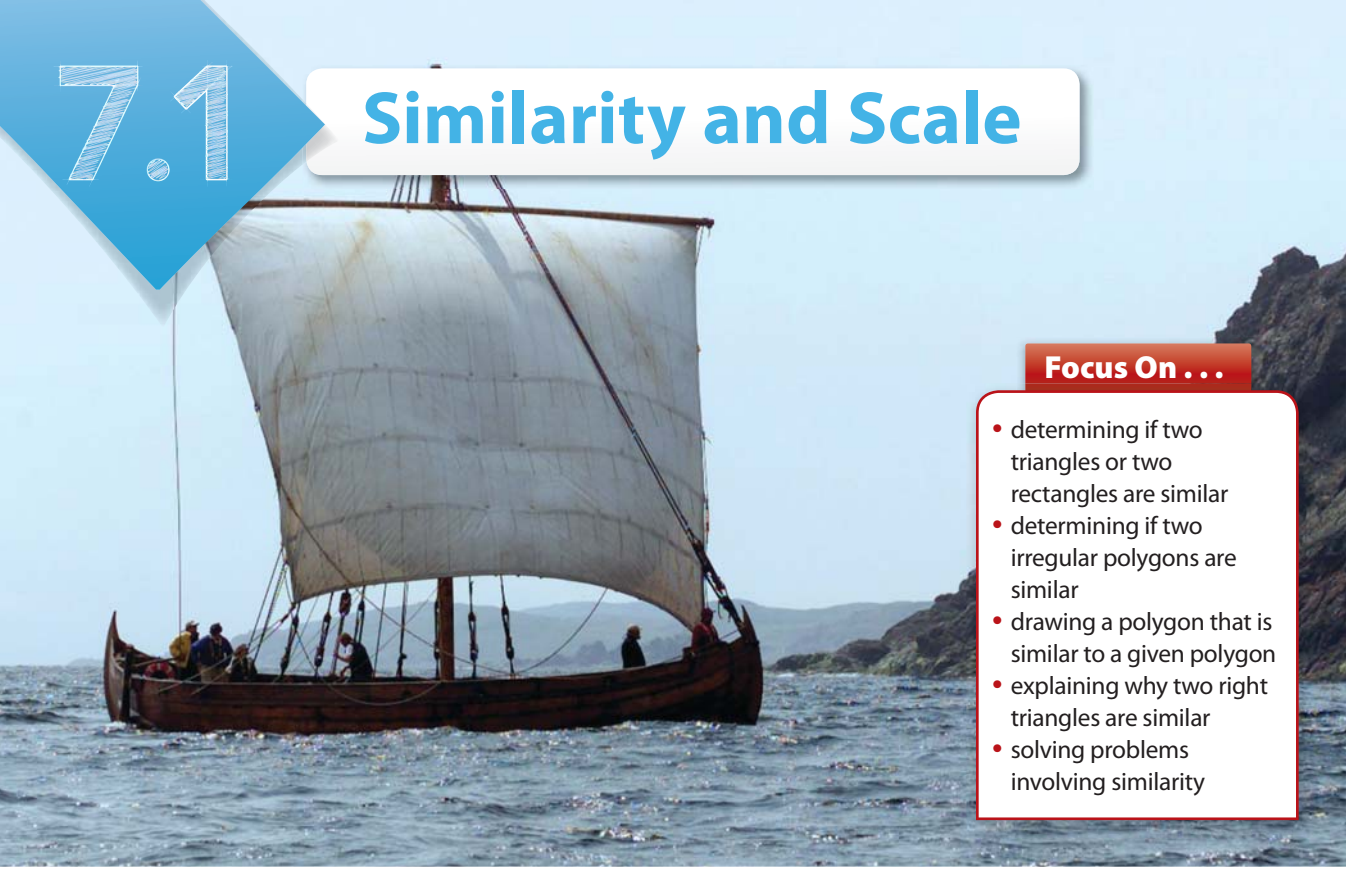


7.1 Similarity and Scale



Focus On . . .

- determining if two triangles or two rectangles are similar
- determining if two irregular polygons are similar
- drawing a polygon that is similar to a given polygon
- explaining why two right triangles are similar
- solving problems involving similarity

Materials

- protractor
- ruler
- grid paper
- Explore Similar Polygons worksheet

This Viking knarr is housed at Norstead, a reconstructed Viking village. A model ship builder wants to make a scale model of the ship. The scale model will be an exact copy of the actual ship, except that each linear dimension will be $\frac{1}{25}$ the size. The model builder will need to calculate the exact dimensions for the model from the actual ship.

Explore Similar Polygons

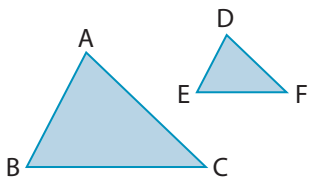
polygon

- a two-dimensional closed figure

similar figures

- have the same shape but different size
- have equal corresponding angles
- have proportional corresponding sides

A model ship builder wants to determine if two shapes are similar. Determine whether $\triangle ABC$ is **similar** to $\triangle DEF$ by answering the following questions.



1. Measure each angle in $\triangle ABC$ and $\triangle DEF$. Record the measurements in a table similar to this one.

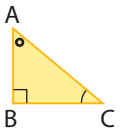
	$\triangle ABC$			$\triangle DEF$		
Angle	A	B	C	D	E	F
Angle measure						

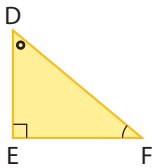
F.Y.I.

Corresponding angles have the same relative position. In the diagrams, the following angles are corresponding:
∠A and ∠D
∠B and ∠E
∠C and ∠F

corresponding sides

- sides that have the same relative position





- AB and DE, BC and EF, and CA and FD are corresponding sides

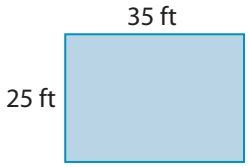
2. What do you notice about the corresponding angles of the triangles?

3. Measure each side length of △ABC and △DEF, in millimetres. Record the measurements in a table similar to this one.

	△ABC			△DEF		
Side	AB	BC	CA	DE	EF	FD
Side length						

4. What do you notice about the **corresponding sides** of the triangles?

5. a) A house foundation measures 35 ft by 25 ft. Using grid paper, create a scale diagram of the foundation. Choose an appropriate scale.



b) What do you know about the angles in any rectangle?

c) How many times longer are the sides of the foundation than the corresponding sides on your scale diagram?

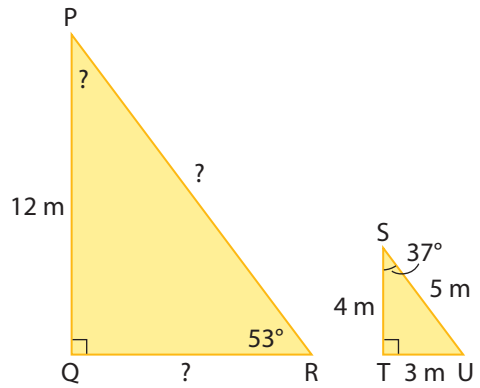
6. Reflect

a) What can you conclude about the corresponding angles of the two rectangles in step 5?

b) What can you conclude about the corresponding sides of the two rectangles?

c) What does this tell you about the two rectangles?

7. Extend Your Understanding △PQR and △STU are similar.



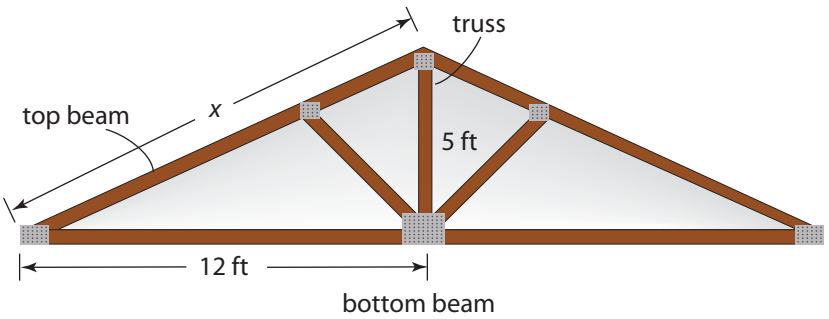
a) Determine the unknown side lengths.

b) Determine the unknown angle measures.

On the Job 1

Determine Similar Triangles

James is building a custom dollhouse for the Hudson family. The dollhouse will be an exact copy of their home. The roof of the home is supported by trusses like those shown below. Each truss measures 12 ft from the edge of the roof to the centre. The height of the truss is 5 ft.



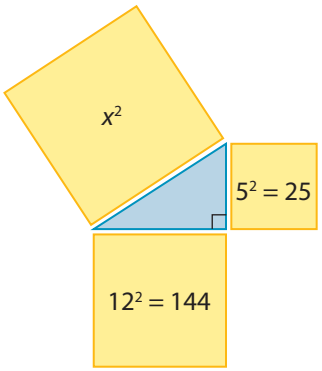
a) Before he can start his model, James needs to know the length of the top beam. Determine this length.

b) James plans to use a scale of 1 to 10, in which 1 ft represents 10 ft. Determine the lengths of the top beam, bottom beam, and centre of the truss, to the nearest appropriate unit.

Solution

a) Use the Pythagorean relationship to calculate the length of the top beam.

Sketch the situation.



Use x to represent the length of the hypotenuse.

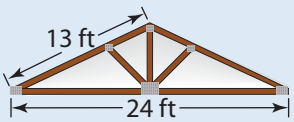
$$144 + 25 = x^2$$
$$169 = x^2$$
$$\sqrt{169} = x$$
$$13 = x$$

The top beam of the truss is 13 ft long.

b) The side lengths of similar triangles are proportional. Draw and label a diagram. Set up a proportion for each measurement.

Each truss has two top beams, one bottom beam, and one centre upright.

top beam = 13 ft
bottom beam = 24 ft



James set up a proportion for each measurement. He let x represent each unknown length. He used his scale of 1 ft represents 10 ft.

Length of bottom beam:

$$\frac{1}{10} = \frac{x}{24}$$

$$\frac{1(24)}{10} = \frac{x(24)}{24}$$

Isolate x.
Multiply each side by 24.

$$\frac{24}{10} = x$$

$$2.4 = x$$

Convert 0.4 feet to inches by multiplying by 12.

The bottom beam on the model will be about 2.4 ft, or $28\frac{4}{5}$ in., long.

Height of the centre upright:

$$\frac{1}{10} = \frac{x}{5}$$

$$\frac{5}{10} = x$$

$$\frac{1}{2} = x$$

The centre of the truss will be $\frac{1}{2}$ ft, or 6 in., long.

Length of top beam:

$$\frac{1}{10} = \frac{x}{13}$$

$$\frac{13}{10} = x$$

What did James multiply by to isolate the variable?

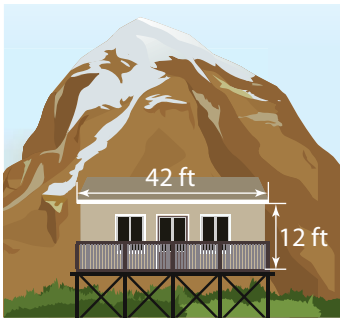
$$1.3 = x$$

Convert 0.3 ft to inches by multiplying by 12.

The top beam will be about 1.3 ft, or $15\frac{3}{5}$ in., long.

Your Turn

The front of the Hudsons' house is 42 ft long and 12 ft high. Apply your knowledge of similar rectangles to calculate these dimensions for a scale model of the house. Use a scale of 1 represents 24.



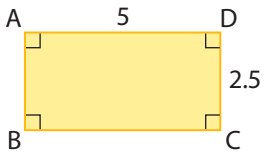
Check Your Understanding

Try It

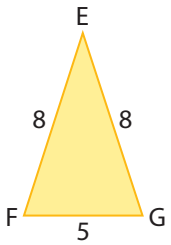
For #1 to #4, match the shape in column A with the similar shape in column B.

Column A

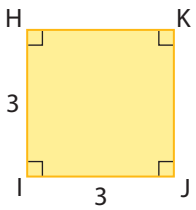
1.



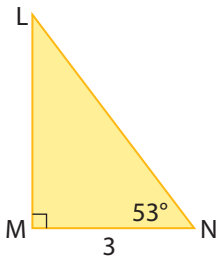
2.



3.

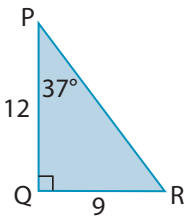


4.

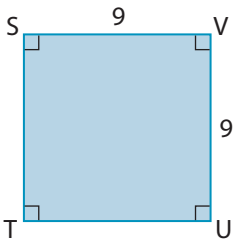


Column B

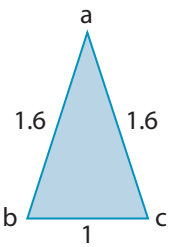
a)



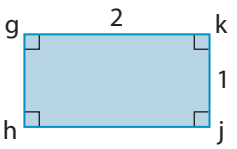
b)



c)

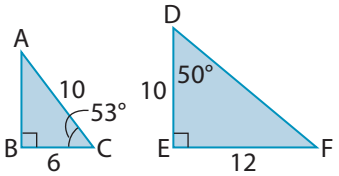


d)



5. Determine if each pair of shapes is similar. Explain your reasoning.

a)



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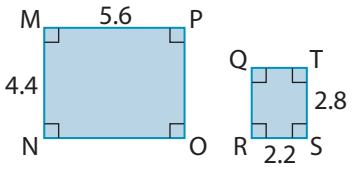
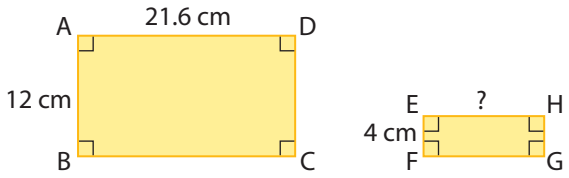


6. Each pair of polygons is similar. Determine the unknown side length.

a)



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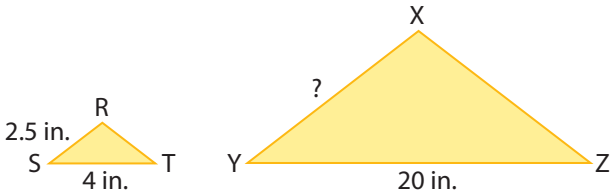
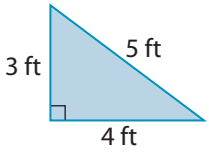
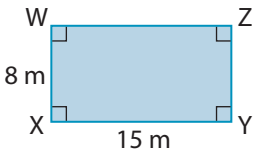


7. Draw a scale diagram of each shape. Use an appropriate scale.

a)

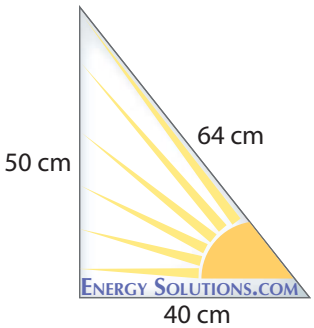


b)



Apply It

8. a) The sign for Sierra's company is a right triangle, as shown. Sierra wants to make miniature copies of the sign for keychains. She would like the side that is 64 cm to measure 8 cm on the key chain. How long should the other two sides be?



b) Draw a scale diagram of the sign using a different scale than the one used for the keychain.

9. The Marshalls fly a Canadian flag that measures 36 in. by 72 in. Draw a scale diagram of the outline of the flag. Include the scale.

On the Job 2

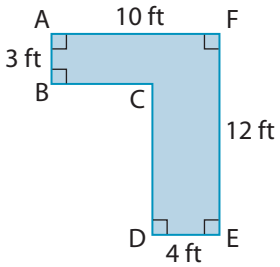
Determine Similar Irregular Polygons

Darlene is a landscaper. She is designing and planting a heritage garden that is an L-shape. Darlene needs a scale diagram of the garden layout. Once the client approves the design, she will be ready to plant.

F.Y.I.

Heritage plants were commonly grown in the past. Heritage plants are often tougher and resist diseases and insects better than some newer plants. The Memorial University Botanical Garden in St. John's makes heritage plants available to local plant nurseries.

a) The actual dimensions of the garden are shown. All the corners are square. How can Darlene draw a scale diagram using an appropriate scale?



b) How can Darlene determine if her scale diagram is similar to the original diagram?

Tools of the Trade

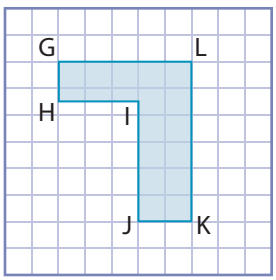
Landscapers plan and create outdoor spaces for homeowners and businesses. They plant and maintain trees, shrubs, lawns, and gardens. They build structures such as fences, walkways, decks, and patios. To learn more about landscapers, go to www.mhrmathatwork10.ca and follow the links.

Solution

a) Darlene uses quarter-inch grid paper.

She lets the side length of one square represent 2 ft.

The scale is 1 square represents 2 ft^2.



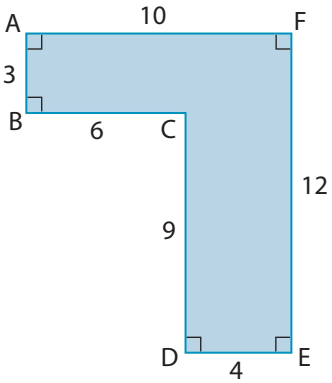
b) Darlene checks whether the corresponding angles are equal.

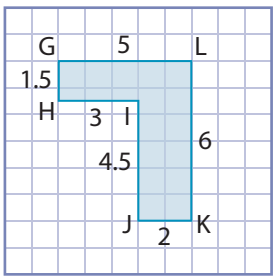
All of the angles in the garden measure 90°.

$\angle A = \angle G$
$\angle B = \angle H$
$\angle D = \angle J$
$\angle E = \angle K$
$\angle F = \angle L$

She knows that all of the angles in the garden measure 90°.

She uses proportional reasoning to compare corresponding sides.





$\frac{AB}{GH} = \frac{3}{1.5}$
$= 2$

$\frac{BC}{HI} = \frac{6}{3}$
$= 2$

$\frac{CD}{IJ} = \frac{9}{4.5}$
$= 2$

$\frac{DE}{JK} = \frac{4}{2}$
$= 2$

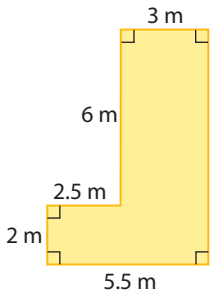
$\frac{EF}{KL} = \frac{12}{6}$
$= 2$

$\frac{AF}{GL} = \frac{10}{5}$
$= 2$

The values of the ratios are equal. All the corresponding sides are proportional.

Your Turn

A layout for kitchen flooring is shown. Using grid paper, create a scale diagram. Use an appropriate scale.



Check Your Understanding

Try It

1. The shapes shown are similar. Determine the unknown side lengths.

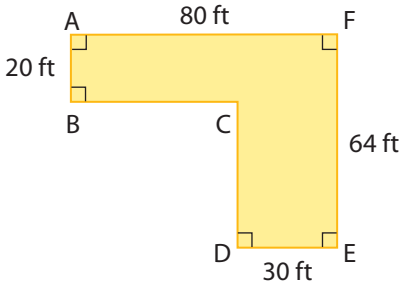


Figure A

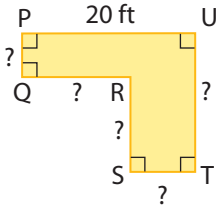


Figure B

2. Draw a scale model of each shape, using a scale of 1 represents 5.

a) How long will each side length be?

b) What will each angle measure be?

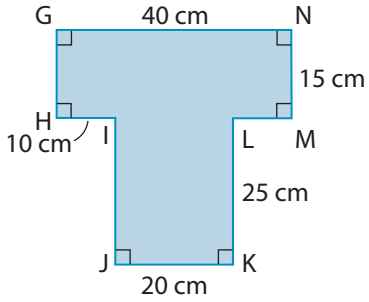


Figure A

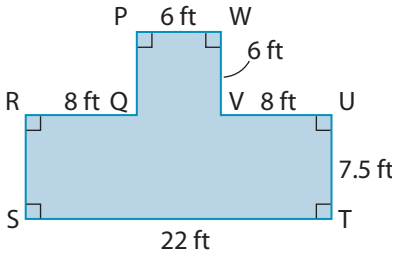
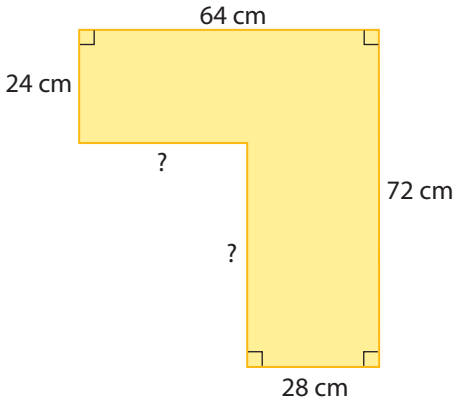


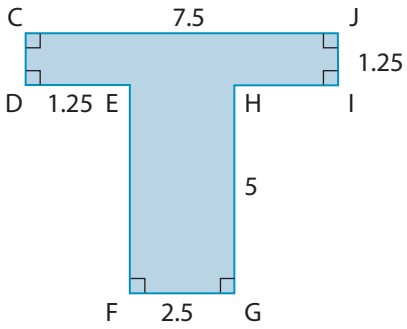
Figure B

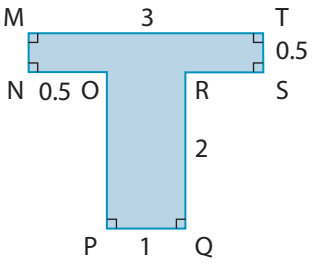
3. Brandon was asked to make a scale model of the shape below. He decided to use a scale of 1 represents 8. Determine the length of each side of Brandon's model. Show your answer on a sketch of the shape.



4. a) For the side lengths shown, express each pair of corresponding sides as a ratio.

b) Use the ratios to check that the corresponding sides are proportional.



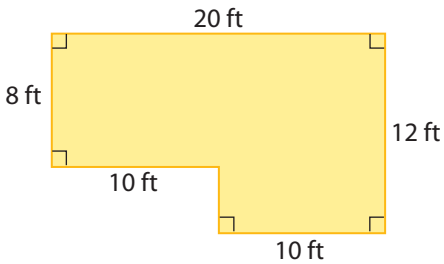


Tools of the Trade

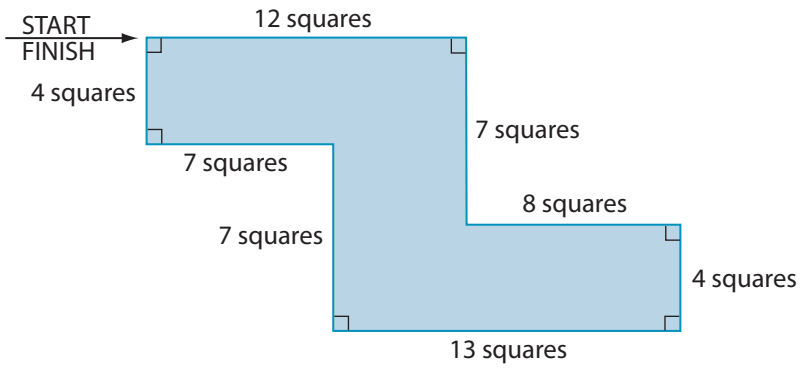
Interior designers make design plans and estimate costs for projects. They often use scale diagrams to help design indoor spaces. To learn more about interior designers, go to www.mhrmathatwork10.ca and follow the links.

Apply It

5. Ellen recently bought her first house. She wants to draw a scale diagram of her living room and dining room. Then, she can arrange templates of her furniture on the drawing before moving in. Below is a sketch of the area. Draw a scale diagram using an appropriate scale.

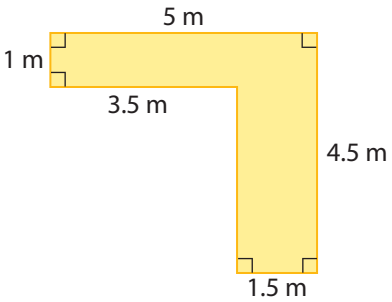


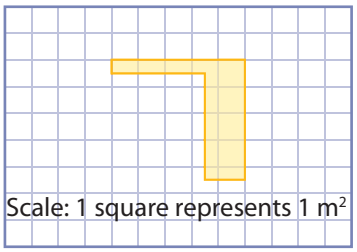
6. Kyle sketched the outline of the game board shown. Draw a scale diagram of the game board using an appropriate scale.



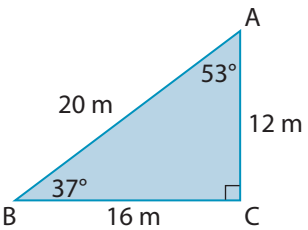
Work With It

1. The actual measurements of Sarah's garden are shown below. Sarah drew a scale diagram of her garden to help her decide what to plant. Explain where Sarah made an error in her scale diagram. Then, draw a correct scale diagram.

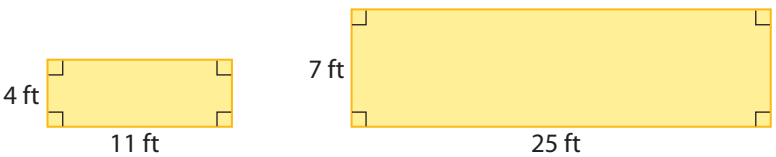




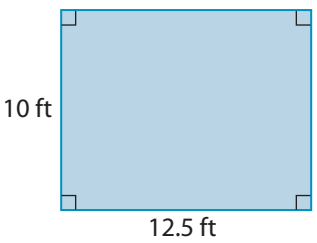
2. List the angle measures and side lengths of two triangles that are similar to $\triangle ABC$.



3. Explain how you know the rectangles shown are not similar.

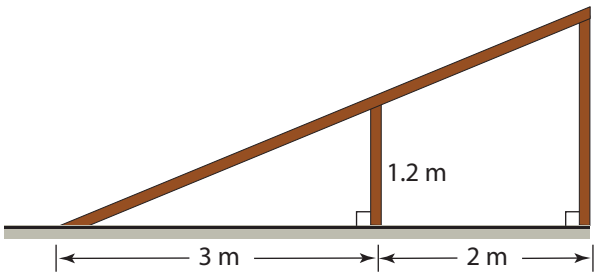


4. Kenny wants to draw a scale diagram of his bedroom. On his drawing, 1 square will represent 6 in. How long should the dimensions of the bedroom be in his drawing?



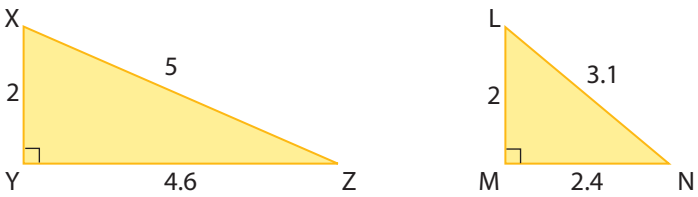
Discuss It

5. A ramp leads to a loading dock. The ramp has a vertical support 2 m from the base of the loading ramp and 3 m from the base of the ramp. The vertical support is 1.2 m in height. What is the height of the loading dock?



6. Two right triangles have different side lengths. Each has an angle that measures 30°. Are the triangles similar? Explain. Use an example to support your answer.

7. Mary Kate is not clear about similar triangles. She drew these two triangles and says they are similar. Is she correct? Explain.



8. Suppose that two polygons are similar.

a) What must be true about the side lengths of the two shapes?

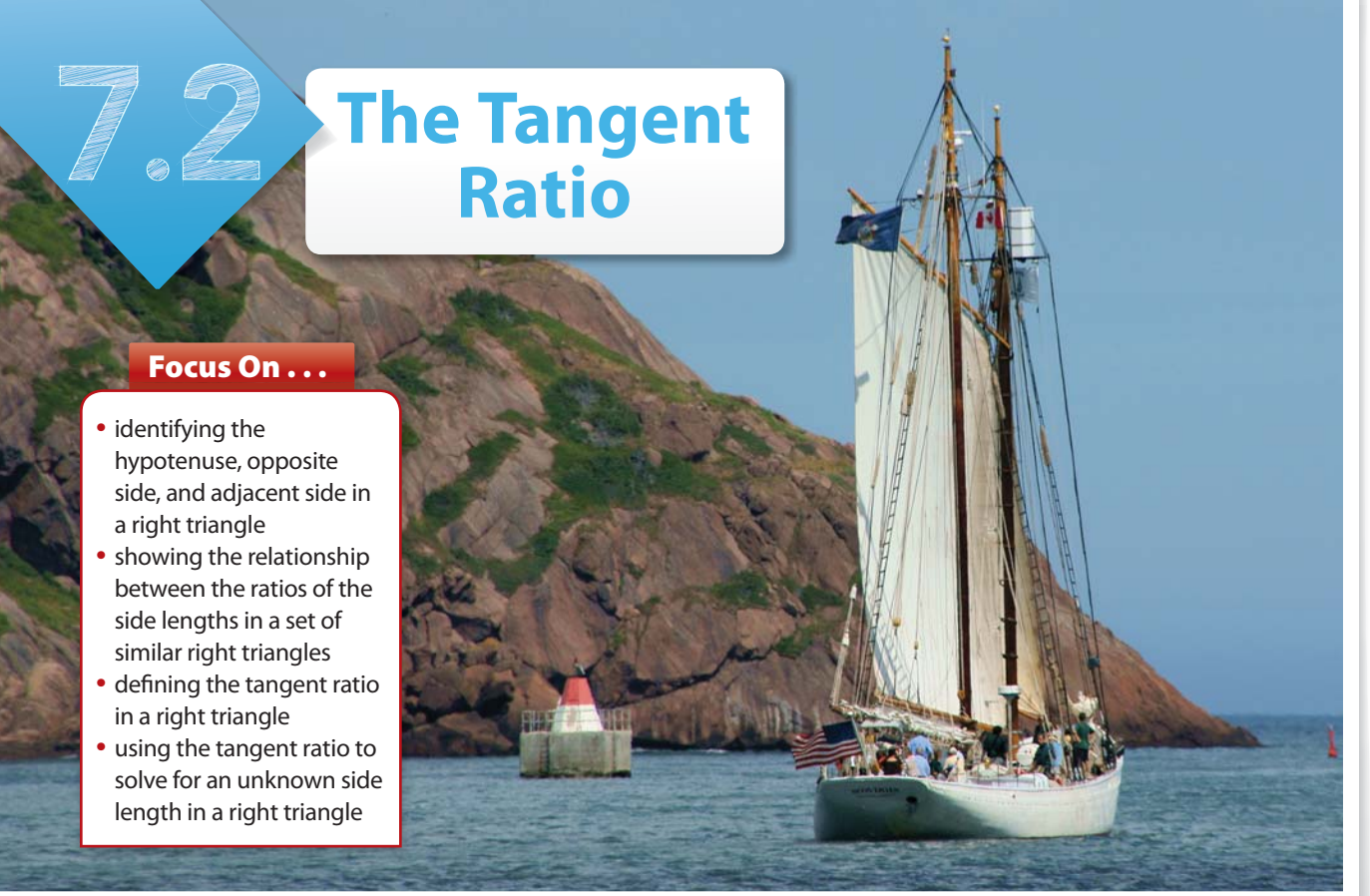
b) What must be true about the angles?

9. a) Write a problem that involves determining an unknown side length in similar shapes.

b) Solve your problem. Then, exchange problems with a classmate.

c) Solve your classmate's problem. Discuss the strategies you used.

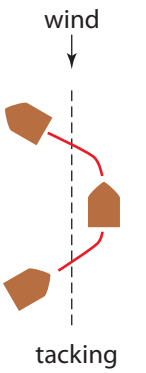
7.2 The Tangent Ratio



Focus On . . .

- identifying the hypotenuse, opposite side, and adjacent side in a right triangle
- showing the relationship between the ratios of the side lengths in a set of similar right triangles
- defining the tangent ratio in a right triangle
- using the tangent ratio to solve for an unknown side length in a right triangle

A sailboat cannot sail directly into the wind. It must zigzag back and forth against the wind. Sailors use right triangles and ratios to determine the distance of each tack.

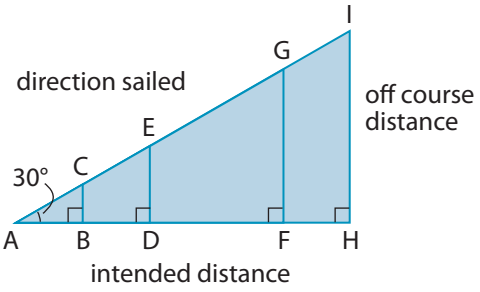


Materials

- ruler
- scientific calculator
- Explore Right Triangles worksheet
- Extended Explore Right Triangles worksheet

Explore Right Triangles

Suppose you are sailing at a tacking angle of 30° from A. Each triangle in the diagram shows a tacking angle of 30° and a different off course distance.

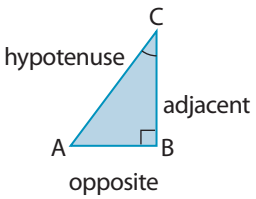


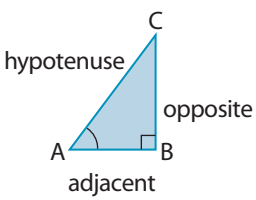
opposite side

- the side across from the angle in a right triangle that you are working with

adjacent side

- the side beside the angle in a right triangle that you are working with
- this is not the hypotenuse

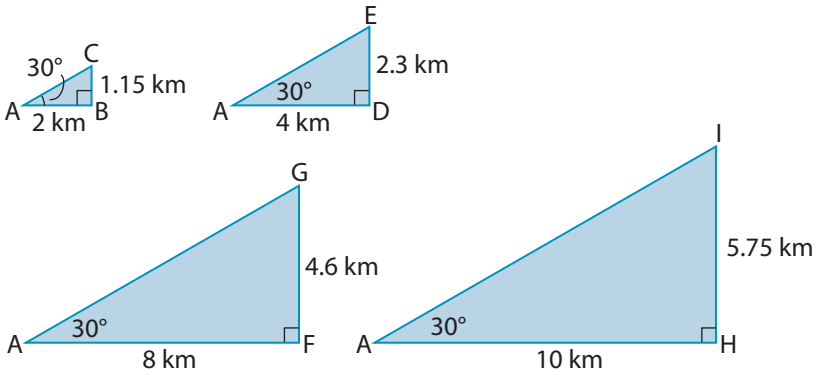




Web Link

Explore the tangent ratio using dynamic geometry software. Go to www.mhrmathatwork10.ca and follow the links.

1. The diagram shows the four triangles that are formed.



Compare the off course distance to the distance you intended to go. Complete a table similar to the one shown below. For each triangle,

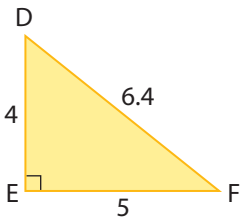
- record the off course distance and the intended distance
- calculate the ratio of $\frac{\text{off course distance}}{\text{intended distance}}$

Triangle	Off Course Distance	Intended Distance	$\frac{\text{Off Course Distance}}{\text{Intended Distance}}$
ABC			
ADE			
AFG			
AHI			

2. Describe any pattern you notice in the ratios. Compare the ratios with those of a classmate.

3. Reflect The ratio of $\frac{\text{off course distance}}{\text{intended distance}}$ is an example of a tangent ratio for angle A. Write a formula you could use to calculate the tangent ratio of any angle. Use the terms **opposite side** and **adjacent side** in your formula.

4. Extend Your Understanding Use your formula to state the tangent ratio for ∠F.

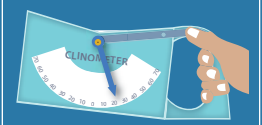


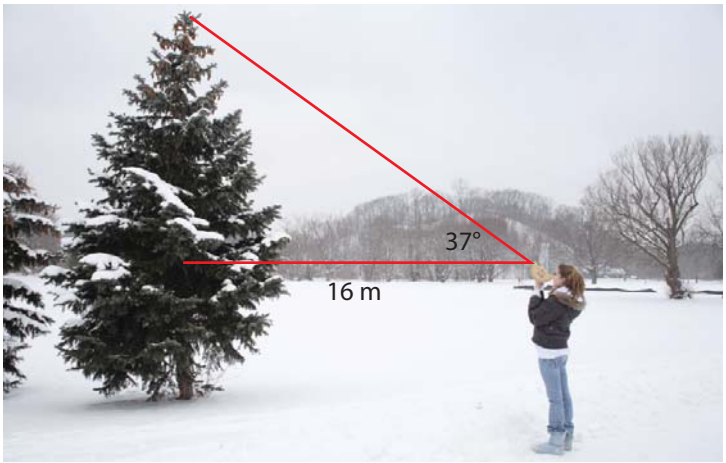
On the Job 1

Determine a Height Using the Tangent Ratio

F.Y.I.

A clinometer is used to help determine the height of tall objects, such as buildings, trees, and flagpoles. It can also be used to measure angles from the ground. To use the clinometer shown, you sight an object, and then read a scale.



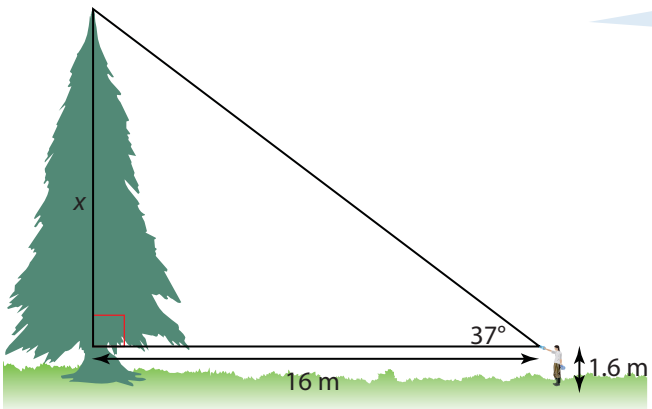


Chelsea is a forestry technician who collects data about the health of trees in boreal forests. She uses a tool called a "clinometer" to help determine the height of black spruce trees. Suppose she stands 16 m from the base of a black spruce tree. Using the clinometer, she determines that the angle to the top of the tree is 37°. The height at which she held the clinometer is 1.6 m. How can Chelsea determine the height of the tree, to the nearest metre?

Solution

Chelsea sketches the situation. She will need to add the height at which she held the clinometer to the height of the tree.

She uses the **tangent ratio** to determine the height of the tree.



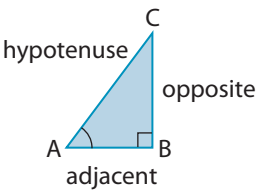
The adjacent side is known. So is the angle.

I can use the tangent ratio to determine the opposite side.

Since 37° is less than 45°, I know that x is less than 16 m. So, the tree will be less than 16 + 1.6 = 17.6 m tall.

tangent ratio

- for an angle in a right triangle, the ratio of the length of the opposite side to the length of the adjacent side
- the short form for the tangent ratio of ∠A is tan A
- $\tan A = \frac{\text{opposite}}{\text{adjacent}}$



Chelsea uses x to represent the height from the clinometer to the top of the tree.

Using the angle of 37°, she identifies the lengths of the sides of the triangle.

Length of opposite side: x
Length of adjacent side: 16 m

$$\tan 37° = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 37° = \frac{x}{16}$$

$$16(\tan 37°) = \frac{x}{16}(16)$$

$$16(\tan 37°) = x$$

$$12.056\ldots = x$$

C 16 TAN 37 =

12.0568648

Tan 37° is close to 0.75. This means that the opposite side is about $\frac{3}{4}$ the length of the adjacent side.

$\frac{3}{4} = \frac{?}{16}$

The tree should be about 12 metres tall.

Add the height at which the clinometer was held: 12 + 1.6

To the nearest metre, the black spruce tree is 14 m tall.

Tech Link

Make sure your calculator is in the degree mode.

Use the following key sequence to calculate the tan for 37°.

C TAN 37 =

Your calculator may have slightly different key strokes. Experiment or check with a classmate to find out which ones work.

F.Y.I.

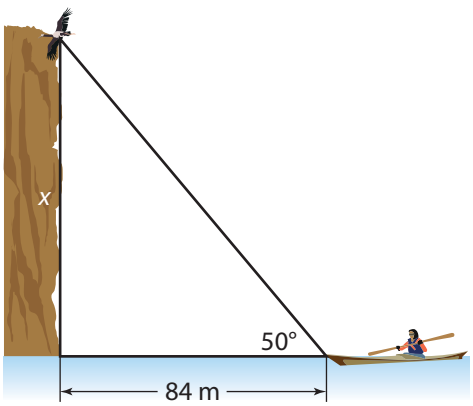
More than 24 000 northern gannets breed on Bird Rock. Bird Rock is the second largest gannet rookery in North America.

Your Turn

Andrew is kayaking at Cape St. Marys Ecological Reserve. He sights the northern gannet rookery on Bird Rock.

a) Estimate the height of the rookery.

b) How high is Bird Rock above sea level? Express your answer to the nearest metre.

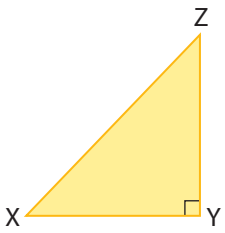


Check Your Understanding

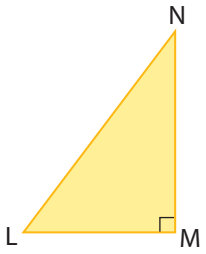
Try It

1. Sketch each triangle. Label the hypotenuse, adjacent side, and opposite side for each specified angle.

a) $\angle X$



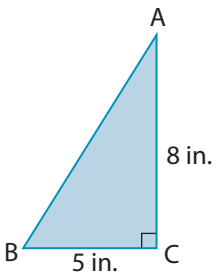
b) $\angle N$



2. Write each tangent ratio.

a) tan A

b) tan B



3. Using a calculator, determine each tangent ratio. Round to two decimal places.

a) tan 32°

b) tan 58°

4. **a)** What do you notice about your answers to #2 and #3?

b) Explain why this is so, using the tangent ratio.

5. Using a scientific calculator, evaluate each tangent ratio to three decimal places.

a) tan 17°

b) tan 86°

c) tan 29°

d) tan 56°

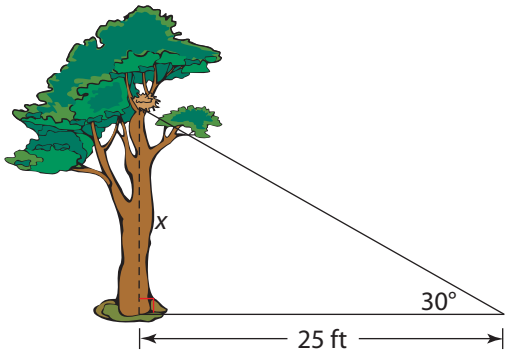
6. Sketch and label a right triangle to show each tangent ratio.

a) $\tan C = \frac{5}{2}$

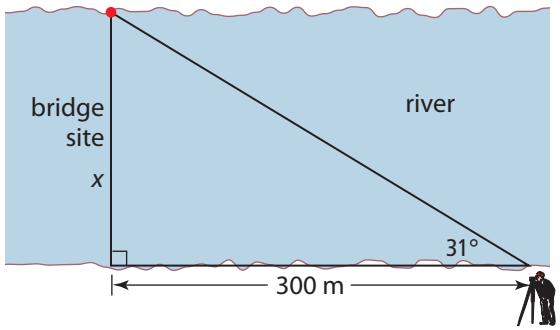
b) $\tan D = \frac{2}{3}$

Apply It

7. Suppose you see the bird nest shown. Estimate how far off the ground the bird nest is. **Hint**: How can you use the tangent ratio to estimate?

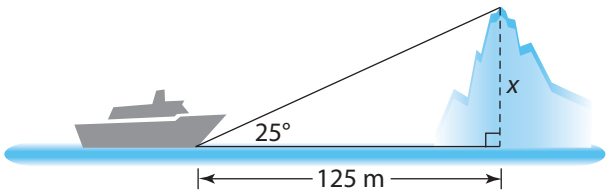


8. A surveyor needs to determine the width of a river in order to build a bridge. The distance from the surveyor to the bridge site is 300 m. The surveyor measures a 31° angle to the bridge site across the river. How wide is the river, to the nearest metre?



9. Cody measures icebergs to help understand the effects of climate change. He used a sextant to measure the angle between the ocean and the top of an iceberg. The ship's radar was used to determine the distance from the ship to the iceberg. What is the height of the iceberg, to the nearest metre?

A sextant is a tool used to measure the angle between two objects.



Tools of the Trade

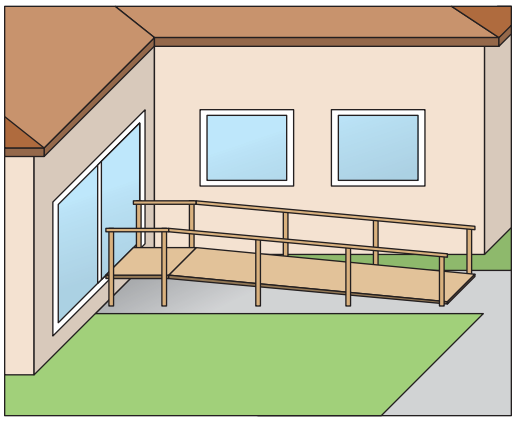
Surveyors use computers and tools to measure distances and angles. They mark boundaries, make maps, and write reports. Surveyors plan building lots in subdivisions and determine air space for airports. To learn more about surveying, go to www.mhrmathatwork10.ca and follow the links.

F.Y.I.

When sailing past an iceberg, it is difficult to know how large the iceberg is. You can see about 10% of an iceberg above the water. Suppose an iceberg has a height of 30 m above water. What is the depth of the bottom of the iceberg?

On the Job 2

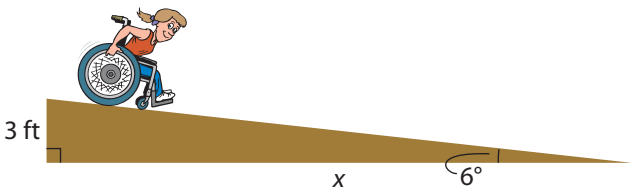
Calculate a Distance Using the Tangent Ratio



Laura plans to have a wheelchair ramp built for her mother. The building code states the angle that the ramp makes with the ground cannot be greater than 6°. The landing is 3 feet off the ground. The walkway along the ground is 30 ft long. The ramp cannot exceed this length. Laura needs to find out if the ramp will fit between the edge of the landing and the far end of the walkway. If it does, she will hire a contractor. How can Laura determine how far along the walkway the ramp will reach? Express the answer to the nearest half foot.

Solution

Laura draws and labels a diagram.



She uses x to represent the length of the ramp along the walkway.

Using the angle of 6°, she identifies the lengths of the sides of the triangle.

Length of opposite side: 3 ft
Length of adjacent side: x

$$\tan 6° = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 6° = \frac{3}{x}$$

$$x = \frac{3}{\tan 6°}$$

$$x = 28.543\ldots$$

C 3 ÷ TAN 6 =
28.54309336

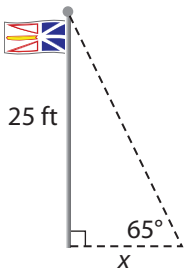
Tan 6° is about 0.1 or $\frac{1}{10}$.
So, 3 is about one-tenth the length of the adjacent side.
The value of x will be close to 30.

Isolate x.
Multiply both sides by x.
Divide both sides by tan 6°.

The ramp will reach about $28\frac{1}{2}$ feet down the walkway. This is less than 30 feet, which is the length of the walkway. Laura calls the contractor.

Your Turn

A wire supports a flagpole. The wire reaches 25 ft up the flagpole. The wire forms an angle of 65° with the ground.



a) How far away is the wire from the base of the flagpole? Express your answer to the nearest foot.

b) Check the reasonableness of your answer. Calculate the tan of 65° and use that to estimate the distance from the base of the flagpole to the end of the wire.

Check Your Understanding

Try It

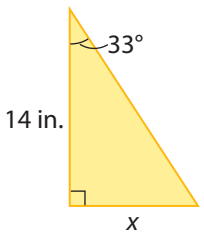
1. a) Draw a right triangle. Label the angles EFG. Make ∠F the right angle.

b) Label the opposite side and the adjacent side to ∠G.

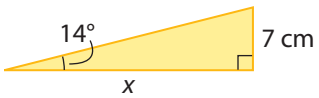
c) State the tangent ratio of ∠G.

2. Determine the value of x, to the nearest unit.

a)



b)



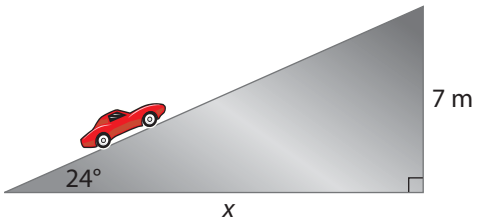
3. a) Sketch a right triangle. Label the angles LMN. Make ∠M the right angle. The measure of ∠N is 54°. The side opposite ∠N is 98 cm long.

b) How long is the side adjacent to ∠N? Express the answer to the nearest centimetre.

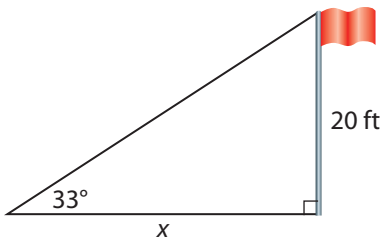
Apply It

4. Determine the value of each variable. Round each answer to the nearest unit.

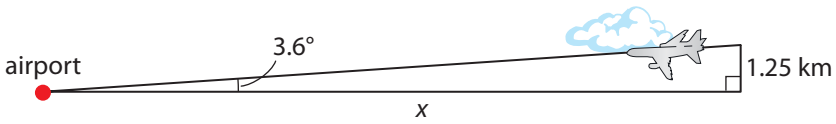
a)



b)



5. When a plane is landing at Gander Airport, the pilot approaches the runway at a constant angle. If the plane is 1.25 km above ground, how far away is the airport runway? Round the answer to the nearest tenth of a kilometre.



F.Y.I.

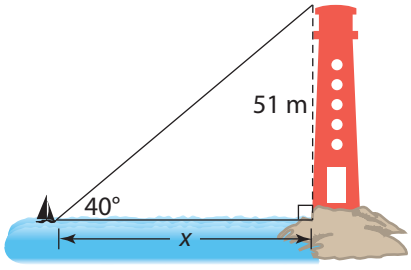
The concentration of bald eagles around Placentia Bay is among the highest in eastern North America.

6. Liam is collecting data about how far away bald eagles live from the nearest place they can catch fish. He is standing at the water's edge and sights a bald eagle's nest in a tree. The nest is 27 m above ground. He measures a 19° angle to the bald eagle nest.

a) Sketch the situation.

b) How far is the nest from the water's edge? Round your answer to the nearest metre.

7. A lighthouse is located 51 m above sea level. The lighthouse warns boats of the danger of rocks close to shore. The safe distance for boats from shore is 75 m. The angle that a boat makes with the top of the lighthouse is 40°.



a) Is the boat a safe distance from shore? Justify your answer to the nearest tenth of a metre.

b) Check the reasonableness of your answer. Calculate the tan of 40°. Use the result to estimate the distance from the boat to shore.

Work With It

F.Y.I.

The Hibernia is an offshore oil rig. It is located in the Jeanne d'Arc Basin off the coast of Newfoundland. The Hibernia platform is the largest oil rig in the world.

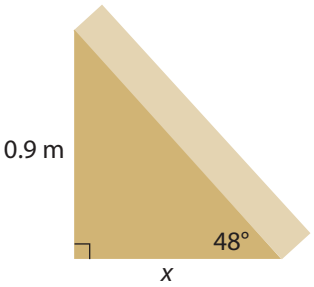
1. April needs to determine the height of an oil rig. She is on a ship that is 75 m from the base of the platform. April measures a 37° angle to the top of the oil rig.

a) Sketch a diagram.

b) Estimate the height of the oil rig.

c) How tall is the oil rig, to the nearest metre?

2. a) The diagram shows a ramp. Determine the length, x, of the base of the ramp. Round your answer to the nearest tenth of a metre.

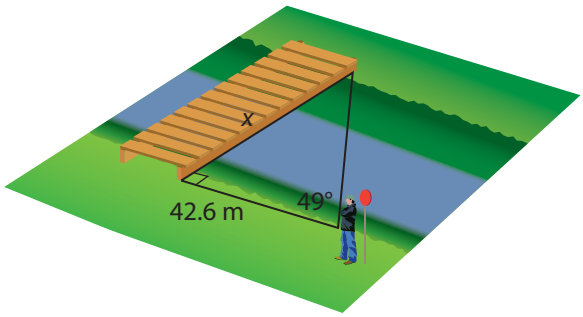


b) Check your answer using a different strategy.

F.Y.I.

The T'Railway is a rail trail. It runs almost 900 km from St. John's to Port aux Basques, along the former railway system. T'Railway Provincial Park is open to hikers, cyclists, skiers, horseback riders, snowmobilers, and ATV riders.

3. The members of an ATV club plan to replace a bridge on the T'Railway. Before ordering materials, they need to determine the length of the bridge. The diagram shows the measurements. To the nearest metre, how long will the bridge be?



4. Jake needs to cut down a dead yellow birch tree that is close to his garage. He is planning where to make it fall so that it will miss the building and a nearby fence. To help him, he needs to know the height of the tree. Jake stands 14 m from the base of the tree. Using a clinometer, he determines the angle to the top of the tree is 31°.

a) Draw a diagram to show the situation.

b) Determine the height of the tree, to the nearest tenth of a metre.

Discuss It

5. David says that if you know all of the angle measures in a right triangle, you can use the tangent ratio to determine the side lengths. Is he correct? Explain.

6. Can two different-sized triangles have the same tangent ratio for one corresponding angle? Explain.

7. Suppose you have a right triangle but do not know the size of the other angles. Can you use the tangent ratio to solve for an unknown side length? Explain.

8. With a partner, discuss how the tangent ratio might be used in a trade or a job.

9. Alice was asked to determine the height of the tree shown, using the tangent ratio.

Her work is shown.

$$\tan 21^\circ = \frac{\text{adjacent}}{\text{opposite}}$$

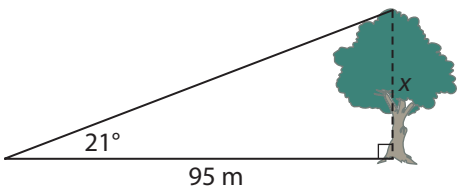
$$\tan 21^\circ = \frac{95}{x}$$

$$x(\tan 21^\circ) = 95$$

$$x = \frac{95}{\tan 21^\circ}$$

$$x = 247.483\ldots$$

The tree is about 247.5 m tall.



a) Identify Alice's error.

b) Correct the error.

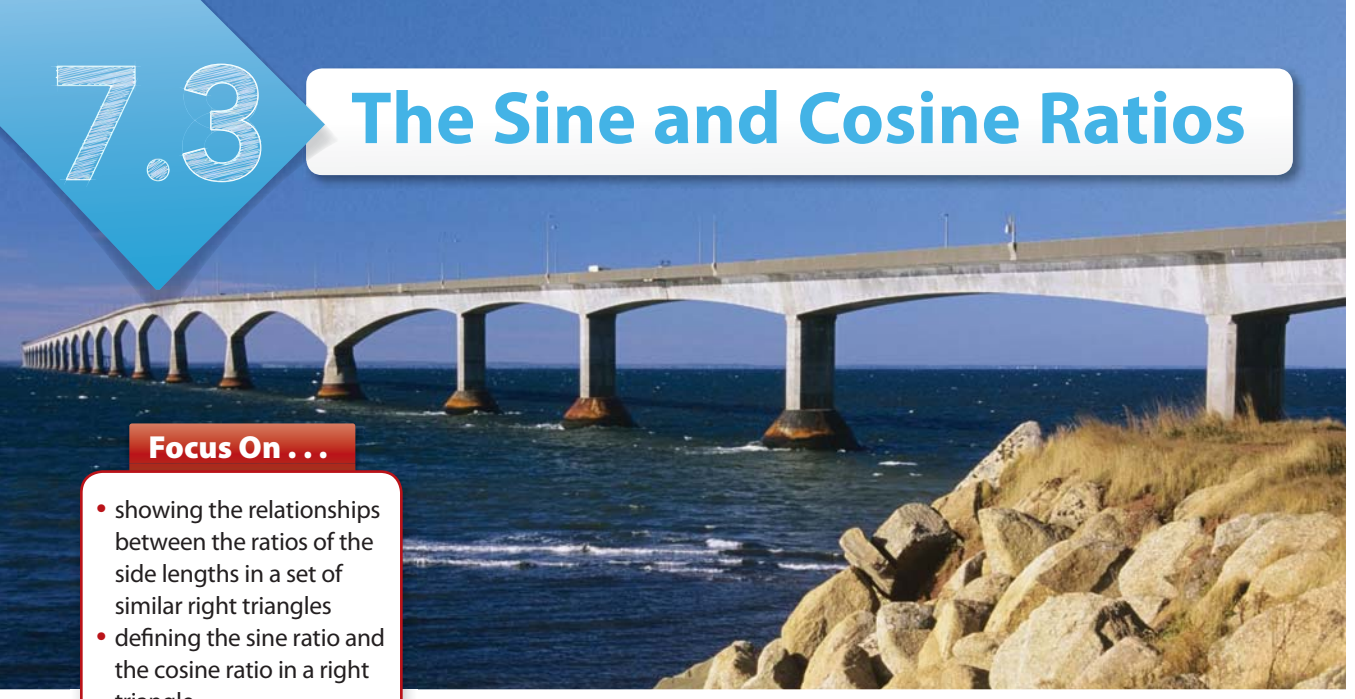
10. Brad was asked to estimate the height of the tree in #9. His work is shown.

Tan 21°is close to 0.4. This means that the opposite side is almost twice the length of the adjacent side. The tree should be about 180 m tall.

a) Identify the error.

b) Correct the error.

7.3 The Sine and Cosine Ratios



Focus On . . .

- showing the relationships between the ratios of the side lengths in a set of similar right triangles
- defining the sine ratio and the cosine ratio in a right triangle
- using the sine ratio and the cosine ratio to solve for an unknown side length in a right triangle
- determining which trigonometric ratio is most appropriate to solve a problem

You can use trigonometry to calculate distances that cannot be measured directly. An example is the distance that the Confederation Bridge spans across the Northumberland Strait. The Confederation Bridge connects Prince Edward Island and New Brunswick. It is the longest bridge in the world that crosses ice-covered water. That makes this bridge an engineering marvel.

Explore the Sine and Cosine Ratios

Materials

- grid paper
- ruler
- scientific calculator
- Explore the Sine and Cosine Ratios worksheet

In section 7.2 you learned about the tangent ratio. This ratio compares the opposite and adjacent sides of a right triangle. There are two other ratios that compare the side lengths of a right triangle. These ratios involve the hypotenuse. They are called the sine ratio and the cosine ratio.

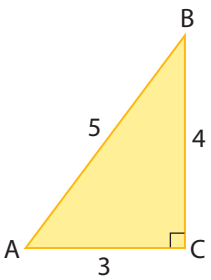
F.Y.I.

The vertex of a triangle is the point where two sides meet. The plural is "vertices."

1. a) On grid paper, draw two right triangles. Make the side lengths of each triangle 3 cm, 4 cm, and 5 cm.

b) Label the vertices of $\triangle ABC$. Make $\angle C$ the right angle.

c) In the first triangle, use $\angle A$ as your reference angle. Label the hypotenuse, and opposite and adjacent sides.

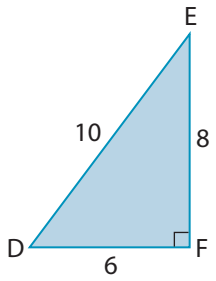


d) In the second triangle, use $\angle B$ as your reference angle. Label the hypotenuse, and opposite and adjacent sides.

2. a) Draw two right triangles with side lengths of 6 cm, 8 cm, and 10 cm.

b) Label the vertices of $\triangle DEF$. Make $\angle F$ the right angle.

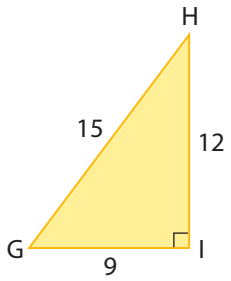
c) Repeat step 1c) and step 1d). Use $\angle D$ and $\angle E$ as your reference angles.



3. a) Draw two right triangles with side lengths of 9 cm, 12 cm, and 15 cm.

b) Label the vertices of $\triangle GHI$. Make $\angle I$ the right angle.

c) Repeat step 1c) and step 1d). Use $\angle G$ and $\angle H$ as your reference angles.



4. a) Record the side lengths of each triangle in a table similar to this one.

Reference Angle	Opposite Side	Adjacent Side	Hypotenuse	$\frac{\text{Opposite Side}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent Side}}{\text{Hypotenuse}}$
$\angle A$					
$\angle D$					
$\angle G$					
$\angle B$					
$\angle E$					
$\angle H$					

b) Compare your ratios with those of a classmate. Describe any patterns that you notice.

5. Reflect

a) What relationships do you notice among the ratios?

b) What does this tell you about the different triangles you drew?

c) What conclusion can you make about how the ratios relate to each reference angle?

6. Extend Your Understanding

a) Predict the side lengths of two more triangles that have the same ratios as those in the table.

b) Sketch the triangles. Then, check your prediction by calculating the ratios $\frac{\text{opposite side}}{\text{hypotenuse}}$ and $\frac{\text{adjacent side}}{\text{hypotenuse}}$.

Web Link

Explore the sine ratio and the cosine ratio using dynamic geometry software. Go to www.mhrmathatwork10.ca and follow the links.

On the Job 1

Determine a Length Using the Sine Ratio

Charles is framing the outside walls of a horse barn. He nails one wall section together and raises it upright. Each wall stud is 12 ft high.

a) He uses supports to brace the wall and keep the wall from swaying. Each support brace needs to be placed at a 45° angle. How long should the support brace be? Express the answer to the nearest foot.

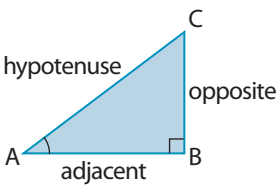
b) Suppose a support brace for a different wall is 15 feet long. It is placed at a 45° angle. What is the height of the stud where the support brace is nailed? Express the answer to the nearest foot.

Tools of the Trade

A framer is a carpenter who builds the frame of a building. Framers build walls using studs and headers, floors using beams and joists, and roofs using trusses. To learn more about framers, go to www.mhrmathatwork10.ca and follow the links.

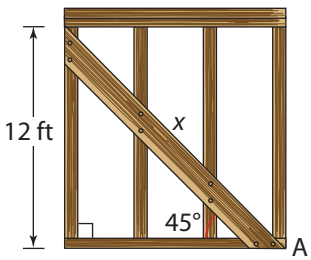
sine ratio

- for an angle in a right triangle, the ratio of the length of the opposite side to the length of the hypotenuse
- the short form for the sine ratio of ∠A is sin A
- $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$



Solution

a) Charles sketches the situation.



Use the **sine ratio** to determine the length of the support brace.

Charles uses x to represent the length of the support brace.

∠A is 45°.

Length of opposite side: 12 ft

Length of hypotenuse: x

$$\sin 45° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 45° = \frac{12}{x}$$

$$x = \frac{12}{\sin 45°}$$

C 12 ÷ SIN 45 =

16.97056275

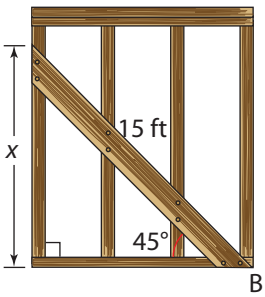
$$x = 16.970\ldots$$

The support brace needs to be approximately 17 feet long.

Sin 45° is close to 0.7. This means that the opposite side is a little less than $\frac{3}{4}$ the length of the hypotenuse. $\frac{3}{4} = \frac{12}{?}$ The length should be about 16 ft. To get this answer, divide 12 by the sine ratio.

b) Sketch the situation.

Use the sine ratio to determine the height of the stud where the brace is nailed.



The length of the hypotenuse is known.

The length of the opposite side is unknown.

Charles used x to represent the distance.

Using $\angle B$, he identified the lengths of the sides of the triangle.

Length of opposite side: x

Length of hypotenuse: 15 ft

$$\sin 45° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 45° = \frac{x}{15}$$

$$15(\sin 45°) = x$$

$$10.606\ldots = x$$

C 15 SIN 45 =

10.60660172

The stud is nailed at approximately 11 ft up. This is close to the estimate.

Sin 45° is close to 0.7. This means that the opposite side is almost $\frac{3}{4}$ the length of the hypotenuse. $\frac{3}{4}$ of 16 is 12. The stud will be nailed to the brace less than 12 ft up.

Tech Link

Make sure your calculator is in the degree mode.

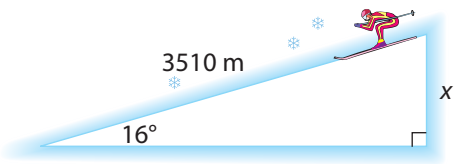
Use the following key sequence to calculate the sin for 45°.

C SIN 45 =

Your calculator may have slightly different key strokes. Experiment or check with a classmate to find out which ones work.

Your Turn

A ski run at Marble Mountain is 3510 m long. The average angle of the ski course run is 16°.



a) What is the height of the ski run? Round the answer to the nearest metre.

b) Suppose a different ski run has a height of 800 m. The average angle of the ski run is 16°. How long is the ski run? Express the answer to the nearest metre.

Check Your Understanding

Try It

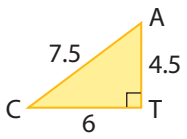
1. Write each sine ratio. Express each ratio in lowest terms.

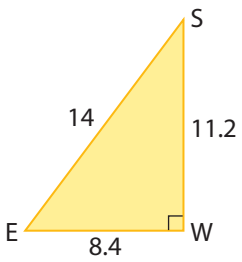
a) sin A

b) sin C

c) sin S

d) sin E





2. Using a scientific calculator, determine each sine ratio to three decimal places.

a) sin 17°

b) sin 86°

c) sin 29°

d) sin 56°

3. Sketch and label a right triangle to show each sine ratio.

a) $\sin C = \frac{3}{8}$

b) $\sin D = \frac{2}{5}$

4. Solve for x. Express the answers to one decimal place.

a) $\sin 27° = \frac{x}{15}$

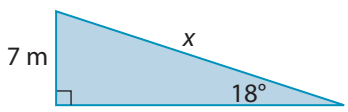
b) $\sin 38° = \frac{22}{x}$

c) $\sin 45° = \frac{x}{35}$

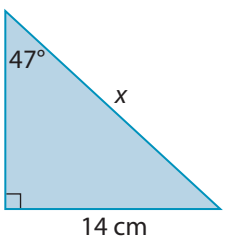
d) $\sin 76° = \frac{2.5}{x}$

5. Estimate and then determine each length of x. Round each calculation to the nearest tenth of a unit.

a)



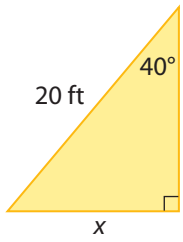
b)



6. Determine each length of x. Round each answer to the nearest tenth of a unit.

a)

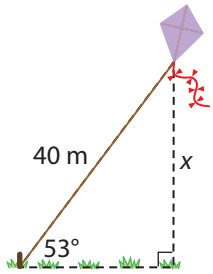
b)



Apply It

7. a) The string attached to a kite is 40 m long. The string is staked to the ground and makes an angle of 53° with the ground. How high is the kite flying? Round the answer to the nearest tenth of a metre.

b) Show how you know that your answer is reasonable.



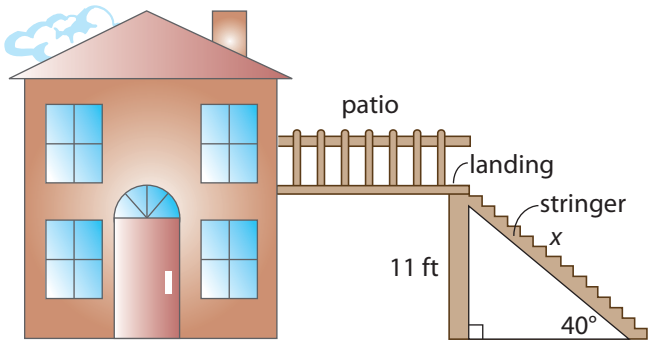
F.Y.I.

A 2 by 8 board is about 2 in. deep by 8 in. wide. It is sold in different lengths.

8. Clare and Kevin are building a patio on the second storey of their house. They plan to build stairs from the patio to the ground. The stairs need to make a 40° angle with the ground. The height of the stairs to the landing will be 11 ft.

a) How long will the stringer be, to the nearest tenth of a foot?

b) Clare and Kevin need to buy 2 by 8 boards for the stringer. They are available in 12-ft lengths. How many 2 by 8 boards will they need? Explain.



9. Emily's class installed a slide in the local park. The slide is 3.5 m long and makes a 46° angle with the ground.

a) Sketch a right triangle to model the situation. Label the measurements you know. Use a variable to represent the height.

b) Determine how high off the ground the top of the slide is. Express the answer to the nearest tenth of a metre.

On the Job 2

Determine a Length Using the Cosine Ratio

Erin owns a moving company. The loading ramp was too steep for moving appliances so she decided to buy a longer ramp.

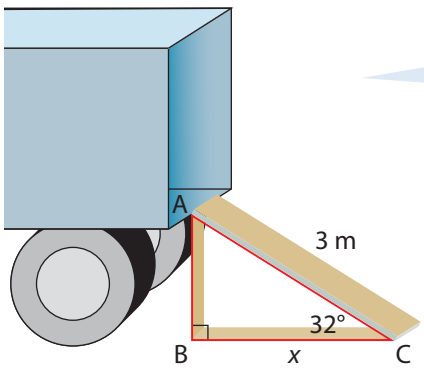
a) The new ramp is 3 m long. It forms an angle of 32° with the ground. How far out from the truck does it extend? Express the answer to the nearest tenth of a metre.

b) Suppose Erin buys a different ramp. It is placed at an angle of 32°. The ramp extends 3.5 m from the truck. How long is the ramp, to the nearest tenth of a metre?

Solution

a) Sketch the situation.

Use the **cosine ratio** to determine the distance the ramp extends from the truck.



The length of the hypotenuse is known.
The length of the adjacent side is unknown.

Erin uses x to represent the distance.

Using $\angle C$, she identifies the lengths of the sides of the triangle.

Length of adjacent side: x
Length of hypotenuse: 3 m

$$\cos 32° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 32° = \frac{x}{3}$$

$$3(\cos 32°) = x$$

$$2.544\ldots = x$$

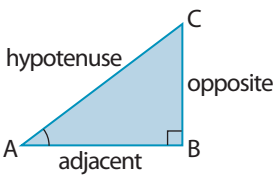
C 3 COS 32 =
2.544144288

Cos 32° is close to 0.8. This means that the adjacent side is more than $\frac{4}{5}$ the length of the hypotenuse.
$\frac{4}{5} = \frac{3}{x}$
Solve for x to get the estimate.

The horizontal length of the ramp is about 2.5 m. This is close to the estimate.

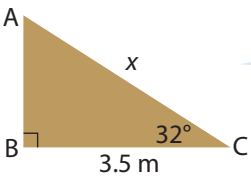
cosine ratio

- for an angle in a right triangle, the ratio of the length of the adjacent side to the length of the hypotenuse
- the short form for the cosine ratio of $\angle A$ is cos A
- $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$



b) Erin sketches the situation.

She uses the cosine ratio to determine the length of the ramp.



The length of the adjacent side is known.
The length of the hypotenuse is unknown.

Erin uses x to represent the length of the ramp.

Using $\angle C$, she identifies the lengths of the sides of the triangle.

Length of adjacent side: 3.5 m

Length of hypotenuse: x

$$\cos 32° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 32° = \frac{3.5}{x}$$

$$x = \frac{3.5}{\cos 32°}$$

$$x = 4.127\ldots$$

C 3.5 ÷ COS 32 =
4.127124412

Cos 32° is close to 0.8. This means that the adjacent side is a little more than $\frac{4}{5}$ the length of the hypotenuse. The ramp should be a little more than 4 m long.

The ramp is approximately 4.1 m long.
This is close to the estimate.

Tech Link

Make sure your calculator is in the degree mode.

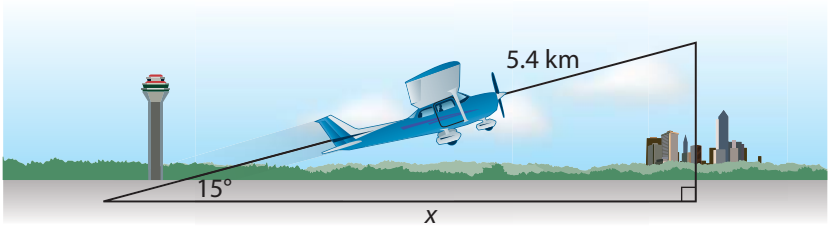
Use the following key sequence to calculate the cos for 32°.

C COS 32 =

Your calculator may have slightly different key strokes. Experiment or check with a classmate to find out which ones work.

Your Turn

a) During a test flight, a water bomber took off and climbed steadily at an angle of 15°. It travelled 5.4 km to get to the forest fire nearby. How far is the forest fire from the takeoff point? Round the answer to the nearest tenth of a kilometre.



b) Suppose a water bomber took off and climbed steadily at an angle of 15°. It travelled 4.5 km from the takeoff point. How far has it travelled toward the forest fire? Round the answer to the nearest tenth of a kilometre.

Check Your Understanding

Try It

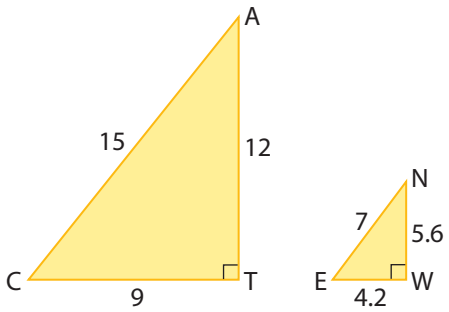
1. For the triangles shown, write each cosine ratio. Express each ratio in lowest terms.

a) cos A

b) cos C

c) cos N

d) cos E



2. Using a scientific calculator, evaluate each cosine ratio. Round the answers to three decimal places.

a) cos 17°

b) cos 86°

c) cos 29°

d) cos 56°

3. Solve for x. Express the answers to one decimal place.

a) $\cos 58° = \frac{3}{x}$

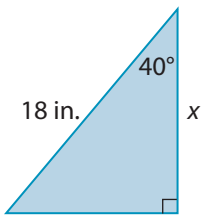
b) $\cos 6° = \frac{x}{12}$

c) $\cos 14° = \frac{1.9}{x}$

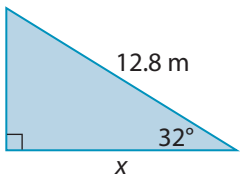
d) $\cos 63° = \frac{x}{14}$

4. Estimate and then calculate the length of x in each of the following. Round each answer to the nearest tenth of a unit.

a)

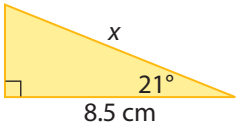


b)

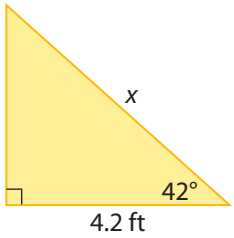


5. What is the length of x in each of the following? Round each answer to the nearest tenth of a unit.

a)

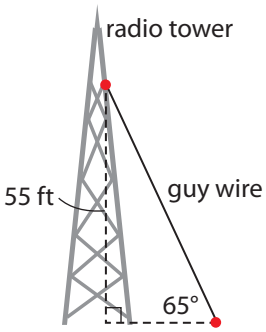


b)



Apply It

6. A guy wire supports a radio transmission tower. The wire reaches 55 ft up the tower. The wire forms an angle of 65° with the ground. How long is the guy wire? Express your answer to the nearest half foot.



7. A wheelchair ramp is going to be installed outside the front doors of a school. The plans call for an angle of 5° and a ramp that is 8 m long. The horizontal distance of the ramp cannot be longer than 8.2 m.

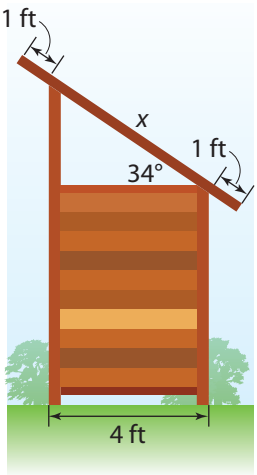
a) Sketch the situation.

b) Will the ramp fit? Justify your answer.

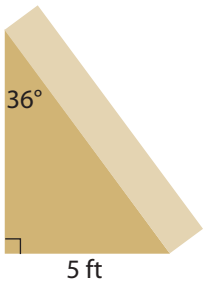
F.Y.I.

One cord of wood is 4 ft high by 8 ft long by 4 ft wide.

8. Randy is designing a lean-to for storing firewood. The lean-to will hold two full cords of wood. Randy decides on a roof angle of 34° so snow will slide off easily. He wants to add 1 ft to each end of each rafter to allow an overhang at the front and back of the lean-to. How long will the roof rafters need to be? Round your answer to the nearest tenth of a foot.



9. Determine the length of the two unmarked sides in this ramp. Express the answers to the nearest tenth of a foot.



On the Job 3

Solve a Problem Using Trigonometry

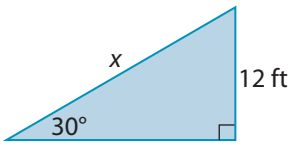
There are plans to install a new escalator in St. John's International Airport. The building code requires that the angle of an escalator to the floor be 30°. Two students decide to determine the length of the escalator.

a) Megan knows that the escalator will have an overall height of 12 ft. How long is the escalator?

b) Chris knows that the escalator will have a horizontal distance of 20.8 ft. How long is the escalator?

Solution

a) Megan drew a diagram of the situation.



The opposite side is known.
The hypotenuse is unknown.

Let x represent the length.

Using the reference angle of 30°, identify the lengths of the sides of the triangle.

Length of opposite side: 12 ft
Length of hypotenuse: x

$$\sin 30° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 30° = \frac{12}{x}$$

$$x = \frac{12}{\sin 30°}$$

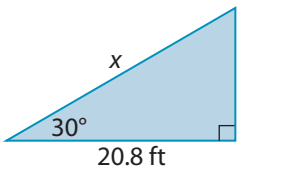
$$x = 24$$

C 12 ÷ SIN 30 =
24

Sin 30° is 0.5. This means that the opposite side is $\frac{1}{2}$ the length of the hypotenuse. The escalator should be 24 feet long.

According to Megan, the escalator is 24 ft long.

b) Chris sketched the situation.



The adjacent side is known.
The hypotenuse is unknown.

Method 1: Use the Cosine Ratio

Let x represent the length.

Using the reference angle of 30°, identify the lengths of the sides of the triangle.

Length of adjacent side: 20.8 ft
Length of hypotenuse: x

$$\cos 30° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 30° = \frac{20.8}{x}$$

$$x = \frac{20.8}{\cos 30°}$$

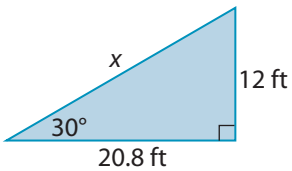
$$x = 24.017\ldots$$

Cos 30° is about 0.9. This means that the adjacent side is close to $\frac{9}{10}$ the length of the hypotenuse. The escalator should be about 24 feet long.

According to Chris, the escalator is approximately 24 ft long.

Method 2: Use the Pythagorean Relationship

Chris knows two side lengths.



$$12^2 + 20.8^2 = x^2$$

$$144 + 432.64 = x^2$$

$$576.64 = x^2$$

$$\sqrt{576.64} = x$$

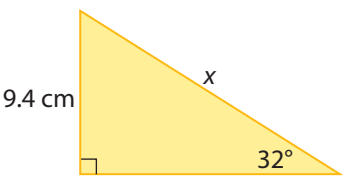
$$24.013\ldots = x$$

The escalator is approximately 24 ft long.

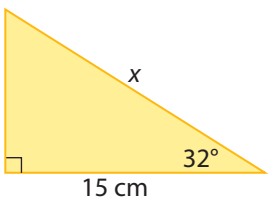
Your Turn

Determine the unknown side length in each triangle. Round each answer to the nearest tenth of a centimetre.

a)



b)

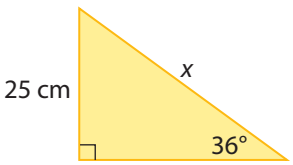


Check Your Understanding

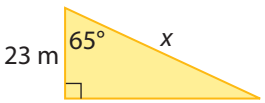
Try It

1. For each triangle, decide whether to use the sine ratio or the cosine ratio. Then, determine the unknown side length. Round your answers to the nearest tenth of a unit.

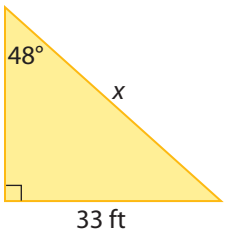
a)



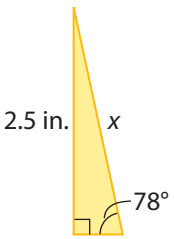
b)



c)

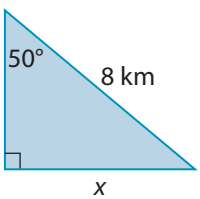


d)

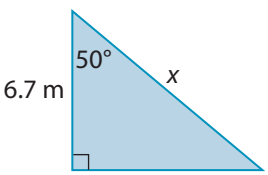


2. Determine the unknown side length in each triangle. Express each answer to the nearest tenth of a unit.

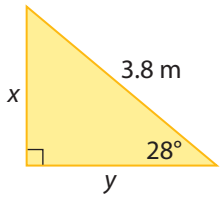
a)



b)



3. a) Determine the unknown sides of the triangle. Express the answers to the nearest tenth of a metre.



b) Solve for one unknown side using a different method than you used in part a).

Apply It

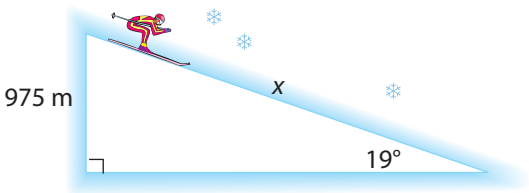
4. A firefighter is on a ladder against a wall. The ladder makes a 75° angle with the ground and is 16 feet away from the wall.

a) Sketch the situation.

b) Estimate the length of the ladder.

c) How long is the ladder? Round your answer to the nearest foot.

5. Volunteers are preparing a ski course for a downhill ski competition. They plan to line the edge of the course with straw bales. Snow will be piled on to cover the straw. The volunteers need to know the length of the course so they can order enough straw bales. Estimate and then calculate the length of the ski course. Express your calculation to the nearest metre.



6. Jessica is flying a kite and has reeled out 200 ft of string. The string of the kite makes a 70° angle with the ground.

a) Sketch the situation.

b) How high is the kite? Round your answer to the nearest foot.

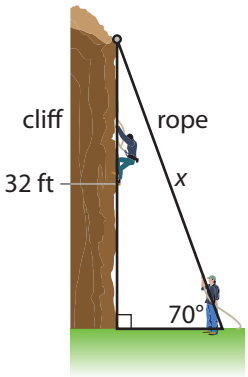
c) Check that your answer is reasonable using a different strategy.

Work With It

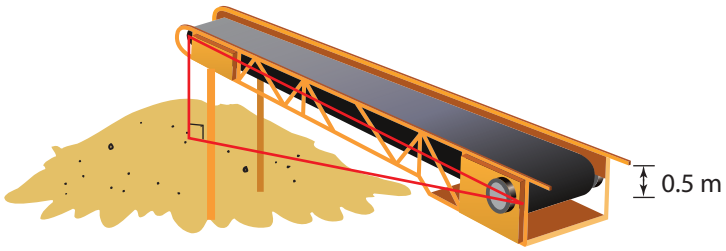
F.Y.I.

"Rappel" means to descend a cliff. A belayer is the person on the ground who secures a rock climber. The belayer and the rock climber each wear a harness that is attached to a rope. The belayer controls how much slack is in the rope.

1. Kelsey wants to rappel down a cliff that is 32 ft high. Philip is the belayer and stands near the base of the cliff. He pulls the rope taut to the ground. How long is the rope from the ground to the top of the cliff, to the nearest foot?



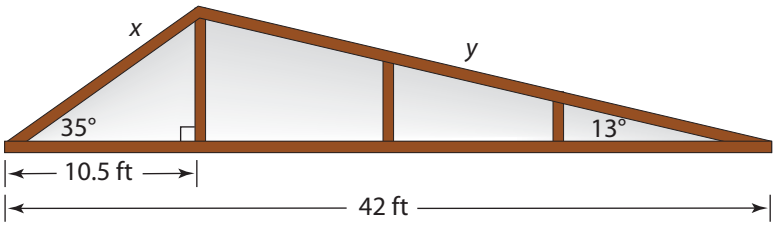
2. A mining company uses a conveyor to move ore to a stockpile. The bottom of the conveyor is 8.5 m from the centre of the stockpile. The conveyor makes a 30° angle with the stockpile. The belt on the conveyor needs to be replaced. It measures twice the length of the conveyor plus 0.5 m for each end.



a) Draw and label a diagram of the situation.

b) How long is the conveyor belt? Express the answer to the nearest tenth of a metre.

3. Earl is building a cabin. He has drawn a diagram to design his roof. Determine the lengths of x and y. Express the answers to the nearest tenth of a foot.



Discuss It

4. **a)** Develop a problem that involves using the sine or cosine ratio to solve it. Then, solve the problem.

b) Explain how you decided which ratio to use.

5. Diane listed the strategies she uses when asked to solve a problem involving the sine or cosine ratio.

Study the picture, if given.
Sketch the situation and label the given measures.
Analyze the information and choose a strategy.
Estimate the unknown side length.

a) Why is estimating the answer important? Discuss with a classmate.

b) What strategy do you recommend after determining the solution?

6. Ben and Jerry both evaluated $\sin 17° = \frac{8}{x}$.

Ben:

$\sin 17^\circ = \frac{8}{x}$

$x = \frac{8}{\sin 17^\circ}$

$x = 27.362...$

Jerry:

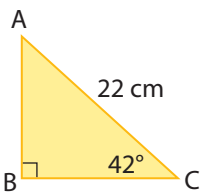
$\sin 17^\circ = \frac{8}{x}$

$x = 8(\sin 17^\circ)$

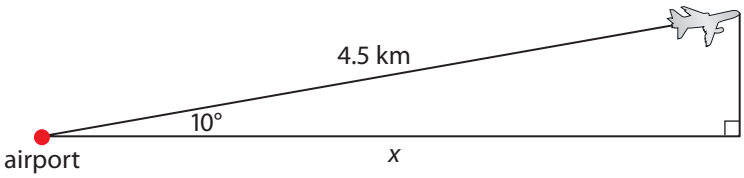
$x = 2.338...$

Who was correct? Explain.

7. Boyd and Maria were asked to solve for side AB in the triangle shown. Boyd used the Pythagorean relationship while Maria used the sine ratio. What method would you use? Explain why.



8. A student was asked to estimate the horizontal distance that a plane has travelled when it has climbed 4.5 km. Assume the pilot climbs steadily at an angle of 10°.



Her work is shown.

Cos 10° is about 0.9. This means that the adjacent side is a little more than the length of the hypotenuse. The horizontal distance should be about 4.6 km.

a) Identify the error. **b)** Correct the error.

7.4 Determining Unknown Angles

Focus On . . .

- solving problems that involve an unknown angle in a right triangle
- using trigonometric ratios to solve problems

F.Y.I.

The Louisbourg lighthouse was built between 1731 and 1734. The original stone lighthouse was the first lighthouse in Canada. It is 55 ft high.

You can use trigonometry to determine the measure of an unknown angle if you know two side lengths in a right triangle. An example is the angle from a rock to the top of the lighthouse at Louisbourg, NS.

Materials

- grid paper
- ruler
- protractor
- scientific calculator
- Explore Angle Measures worksheet

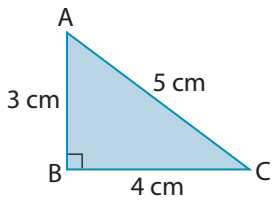
Explore Angle Measures

1. a) On grid paper, draw a right triangle with sides 3 cm, 4 cm, and 5 cm.

b) Using ∠C, label the opposite and adjacent sides, and the hypotenuse.

c) Estimate the measure of ∠C. Record your estimate.

d) Use a protractor to measure ∠C. Record your measurement.



primary trigonometric ratios

- the three ratios sine, cosine, and tangent, defined in a right triangle

2. Create a table similar to this one. Complete the table.

∠C	
Length of opposite side	
Length of adjacent side	
Length of hypotenuse	
Ratio for sin C	
Ratio for cos C	
Ratio for tan C	

3. Record your answer for each part.

a) Determine the measure of ∠C using the sine ratio. Work backward. Use the inverse function on your calculator. It allows you to apply a **primary trigonometric ratio** in reverse.

$$\angle C = \sin^{-1}\left(\frac{3}{5}\right)$$

C SIN^{-1} 3 ÷ 5 =

36.86989765

Round the calculator display to the nearest degree.

b) Determine the measure of ∠C using the cosine ratio.

c) Determine the measure of ∠C using the tangent ratio.

4. How close was your estimate to the calculated measure of ∠C?

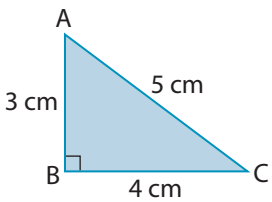
5. Compare the three values you calculated in #3.

6. a) What is the sum of the measures of the angles in a triangle?

b) What is the measure of a right angle?

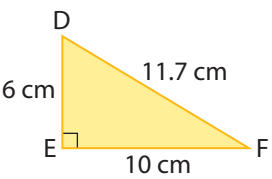
c) How can you determine the measure of ∠A now that you know the measure of ∠C?

d) How could you have used trigonometry to determine the measure of ∠A?



7. Reflect How can you decide which trigonometric ratio to use for calculating the measure of an unknown angle?

8. Extend Your Understanding Determine the measure of ∠D and ∠F.



Tech Link

Make sure your calculator is in degree mode.

Use the following key sequence to calculate $\sin^{-1}\left(\frac{3}{5}\right)$.

C 2nd SIN 3 ÷ 5 =

Your calculator may have slightly different key strokes. Experiment or check with a classmate to find out which ones work.

On the Job 1

Determine an Unknown Angle Using the Tangent Ratio

F.Y.I.

A dead-end pole is a pole placed where the line ends or goes in another direction. Dead-end poles have guy wires to support them.

After Hurricane Igor, many power lines were damaged. Matthew is an apprentice lineman. He is installing a dead-end pole to replace a damaged one. He attaches a guy wire 7.5 m up the pole. The guy wire is anchored to the ground 3.4 m from the base of the pole.

a) What angle does the guy wire make with the ground? Round the answer to the nearest degree.

b) What angle does the guy wire make with the pole?

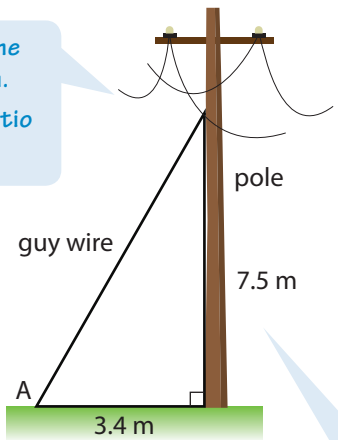
Tools of the Trade

Linemen provide, maintain, and restore power to homes and businesses. They build power line poles and towers. Linemen also remove trees and shrubs that are too close to power lines. To learn more about linemen, go to www.mhrmathatwork10.ca and follow the links.

Solution

a) Sketch the situation.

The adjacent side and the opposite side are known.
I can use the tangent ratio to determine the angle.



Let A represent the angle.

Using ∠A, identify the lengths of the sides of the triangle.

Length of opposite side: 7.5 m
Length of adjacent side: 3.4 m

The opposite side is a little more than twice the length of the adjacent side. The angles at each end of the guy wire add to 90°. The size of ∠A must be about two-thirds of that, or about 60°.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{7.5}{3.4}$$

$$\angle A = \tan^{-1}\left(\frac{7.5}{3.4}\right)$$

$$\angle A = 65.613\ldots$$

The guy wire forms an angle of approximately 66° with the ground.

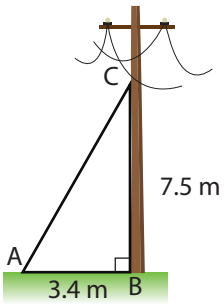
b) The angles in a triangle should have a sum of 180°.

Method 1: Subtract the Known Angles

$180° - 90° - 66° = 24°$

The guy wire forms an angle of 24° with the pole.

Method 2: Use a Formula



$$\angle A + \angle B + \angle C = 180°$$
$$66° + 90° + \angle C = 180°$$
$$156° + \angle C = 180°$$
$$\angle C = 180° - 156°$$
$$\angle C = 24°$$

$\angle C$ is 24°.

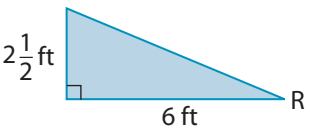
Check:

$66° + 90° + 24° = 180°$

Your Turn

A bike ramp extends 6 ft along the ground. It is $2\frac{1}{2}$ ft off the ground.

a) What is the angle of the ramp with the ground? Express the answer to the nearest degree.

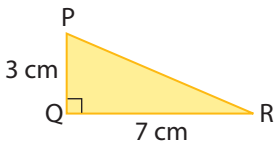


b) What is the size of the angle at the top of the ramp?

Check Your Understanding

Try It

1. **a)** On grid paper, sketch a right triangle with sides 3 cm and 7 cm.



b) Estimate the measure of ∠R.

c) Calculate ∠R, to the nearest degree.

d) How close was your estimate to the actual value of ∠R?

e) What does the ratio of tan R represent?

f) What does $\tan^{-1}\left(\frac{3}{7}\right)$ represent?

F.Y.I.

The measures of angles are often rounded to the nearest degree or nearest tenth of a degree.

2. Determine each angle measure to the nearest tenth of a degree.

a) $\tan^{-1}(0.6523)$

b) $\tan^{-1}(1.253)$

c) $\tan^{-1}(0.3976)$

d) $\tan^{-1}(2.689)$

3. Calculate the measure of each angle to the nearest degree.

a) $\tan A = 0.1123$

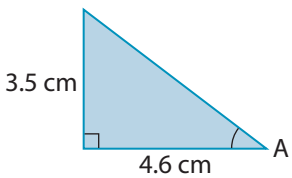
b) $\tan B = 1.3625$

c) $\tan C = 16.432$

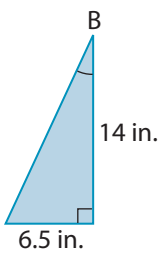
d) $\tan D = 3.5554$

4. What is the size of the angle marked on each diagram? Express each answer to the nearest degree.

a)



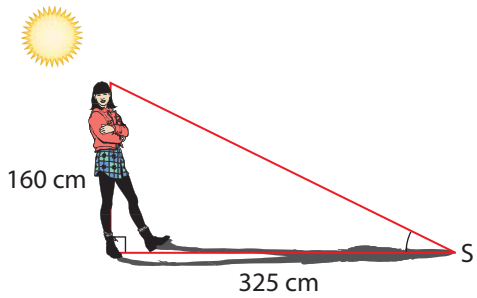
b)



5. Determine the size of the third angle in each part of #4.

Apply It

6. Bonnie is 160 cm tall. She casts a shadow 325 cm long. What is the measure of the angle to the Sun, to the nearest degree?



7. Terri Lynn, a student painter, props a ladder against a house. The ladder reaches 7.5 m up the side of the house. It is a distance of 3 m from the wall. For ladder safety, the ladder needs to form an angle of 75° with the ground.

a) Sketch the situation.

b) What angle does the ladder form with the ground?

c) Has Terri Lynn placed the ladder safely? Explain.

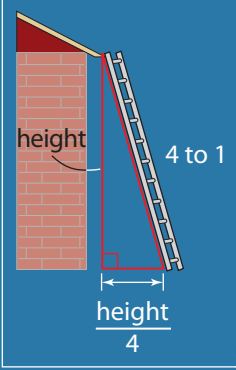
8. Kyle is shopping for a snowmobile ramp. He wants a ramp that is 8 ft long. He wants it to extend $7\frac{1}{2}$ ft from the back of the truck. The height of the truck's tailgate is $2\frac{1}{2}$ ft.

a) Estimate and then determine the angle of the ramp with the ground.

b) For safety reasons, it is important to keep the ramp angle shallow. What could Kyle do to lower the ramp angle when he is loading his snowmobile?

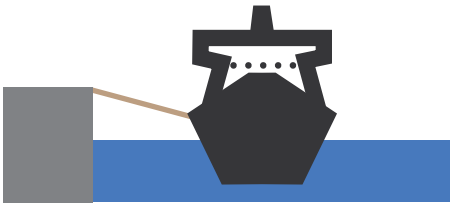
F.Y.I.

The correct angle to set up a ladder safely is 75° with the ground. If the angle is too large, the ladder could fall backward. If the angle is too small, the bottom of the ladder could slide out. You can judge the angle by using the 1 in 4 rule. For 1 unit away from a wall, go 4 units up the wall.



On the Job 2

Solve a Problem Involving an Angle Measure Using the Sine or Cosine Ratio



Katrina is a tour boat operator located in St. John's Harbour. Tour boat operators have to comply with a new safety standard for gangplanks. Gangplanks are not allowed to slope more than 15°. Katrina has a 4.3-m gangplank from the wharf to her boat. The gangplank lies parallel to the wharf when the tide is out and rises with the tide. High tide is 0.9 m.

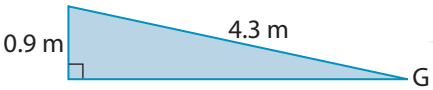
a) Calculate the angle of the gangplank at high tide.

b) Does the gangplank meet the safety standard? Justify your answer.

Solution

a) Sketch the situation.

Decide which trigonometric ratio to use.



The opposite side and the hypotenuse are known.

Use the sine ratio to determine the angle.

Let G represent the angle.

Using ∠G, identify the lengths of the sides of the triangle.

Length of opposite side: 0.9 m
Length of hypotenuse: 4.3 m

$$\sin G = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin G = \frac{0.9}{4.3}$$

$$\angle G = \sin^{-1}\left(\frac{0.9}{4.3}\right)$$

$$\angle G = 12.081\ldots$$

At high tide, the gangplank forms an angle of 12° with the wharf.

b) At high tide the gangplank does not slope more than 15°. So, the gangplank allows for an acceptable slope at high tide.

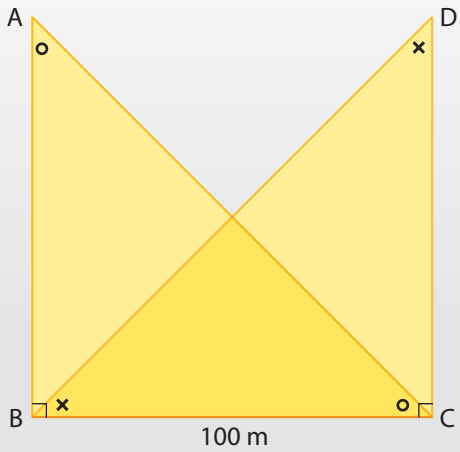
Your Turn

Suppose another boat operator has a gangplank that is 3.2 m long. Does this gangplank meet the requirements for an acceptable slope at high tide? Justify your answer.

Puzzler

a) What is the length of AB? CD? AC? BD?

b) How many different strategies can you use to solve this puzzle? Show your work.



Check Your Understanding

Try It

1. Determine each value to the nearest degree.

a) $\sin^{-1}(0.6523)$ **b)** $\sin^{-1}(0.253)$

c) $\sin^{-1}(0.3976)$ **d)** $\sin^{-1}(0.689)$

2. Determine each value to the nearest tenth of a degree.

a) $\cos^{-1}(0.5238)$ **b)** $\cos^{-1}(0.2167)$

c) $\cos^{-1}(0.9176)$ **d)** $\cos^{-1}(0.2689)$

3. Calculate the measure of each $\angle A$ to the nearest degree.

a) $\sin A = 0.1123$ **b)** $\sin A = 0.3625$

c) $\sin A = 0.432$ **d)** $\sin A = 0.5554$

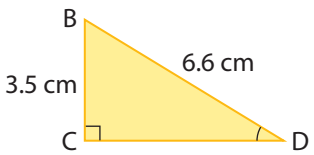
4. Determine the measure of each $\angle A$ to the nearest degree.

a) $\cos A = 0.1023$ **b)** $\cos A = 0.6235$

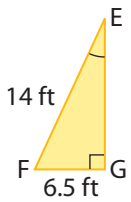
c) $\cos A = 0.3432$ **d)** $\cos A = 0.5013$

5. Determine the size of the indicated angle in each triangle. Express your answers to the nearest degree.

a)

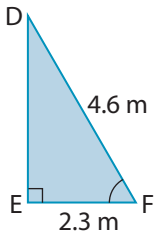


b)

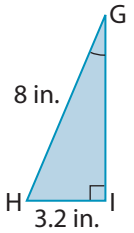


6. Determine the indicated angle in each triangle. Express your answers to the nearest degree.

a)

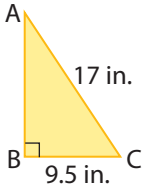


b)

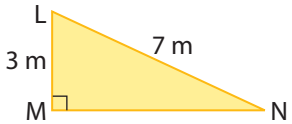


7. Solve for the unknown angles in each triangle. Express your answers to the nearest degree.

a)



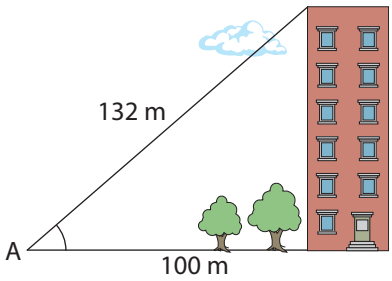
b)



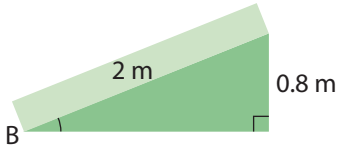
Apply It

8. Estimate and then determine the measure of each indicated angle. Express each calculation to the nearest degree.

a)

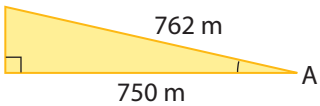


b)



9. Determine the measure of the third angle for each triangle in #8.

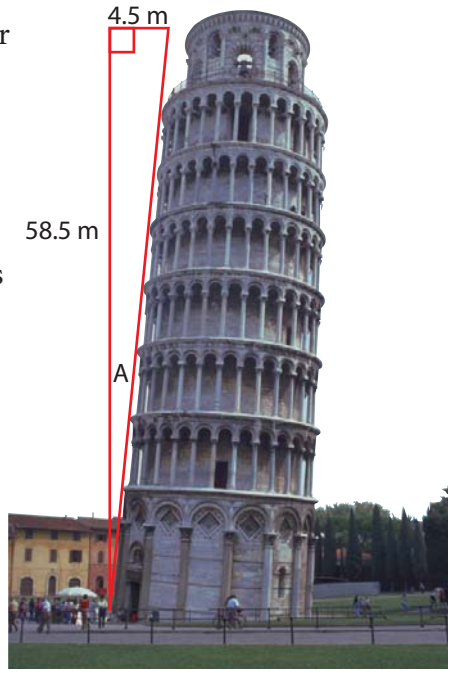
10. A mining engineer is planning to build a haul road to an open pit. The horizontal distance of the road is 750 m. The road will be 762 m long. For road safety, the grade of the road must not be more than 4.6°. Will the proposed road meet the requirements? Justify your answer.



11. Anthony saw the Leaning Tower of Pisa online. He knows the tower is 58.5 m on the high side. The edge of the top leans 4.5 m outside the edge of the foundation. Anthony wants to know the angle that the tower is leaning at.

a) Estimate the measure of the angle.

b) Calculate the measure of the angle, to the nearest degree.



F.Y.I.

The Leaning Tower of Pisa in Italy is famous for its tilt. It took from 1173 until 1370 to complete the bell tower. Even before it was completed, the tower began to lean to one side. This happened because it was built on a marsh.

Work With It

Materials
- clinometer
- measuring tape

1. **MINI LAB** Work with a partner or small group. Choose five tall items located around your school. Some good choices might include the school building, the flagpole, or a very tall tree. Make sure that you can get close to the bottom of everything you choose. Do the following steps for each item.

STEP 1

Estimate how tall you think the item is. Record your estimate.

STEP 2

a) Person with clinometer: Stand far enough away from the item that you can see the top of it. Use the clinometer to site the top of the item. Measure and record the angle to the top. Record this angle measure.

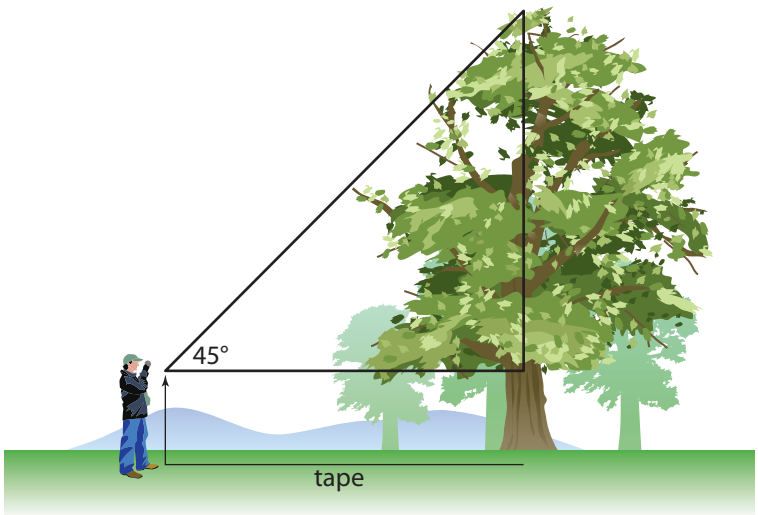
b) Person with the measuring tape: Measure and record the distance between the person with the clinometer and the base of the item. Measure and record the height at which the clinometer is held.

STEP 3

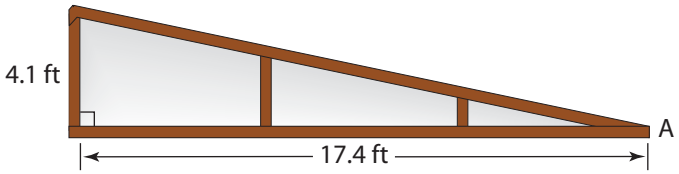
Draw and label a diagram to show what you have recorded. Use trigonometry to determine the height of the item. Add the height at which the clinometer was held.

STEP 4

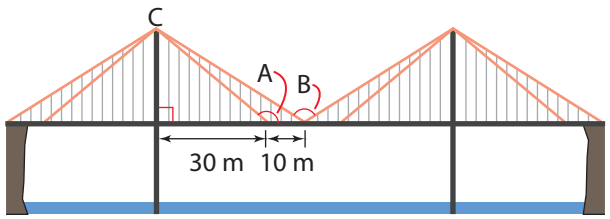
How close is your estimate to the actual height of the item?



2. A roof truss is shown. What is the measure of ∠A to the nearest degree?



3. Steel cables are used to support a suspension bridge. In the diagram, two cables are shown.



a) Cable AC is 52 m long. Determine the measure of the angle that cable AC forms with the ground. Express the answer to the nearest degree.

b) The tower on the bridge is 40 m in height. What angle does cable BC make with the ground?

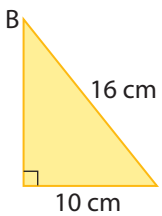
Discuss It

4. Draw a right triangle in which the tangent ratio of one angle is equal to 1. Describe the triangle.

5. Nikita wants to use the sine ratio to determine the measure of an angle. She says she needs to know only two sides of a right triangle. Is she correct? Explain.

6. With a partner, discuss how the primary trigonometric ratios might be used in one of the following jobs: fire tower ranger, surveyor, builder, ski ramp designer.

7. Adrian was asked to estimate the size of ∠B in the diagram shown. He says, "I know the opposite side and the hypotenuse. I can use the cosine ratio to determine the angle. The opposite side is a little more than half the length of the hypotenuse. The two unknown angles add to 90°. So, the size of ∠B must be about half of that, or 45°.



a) Explain Adrian's error.

b) Correct the error.

7 Skill Check

What You Need to Know

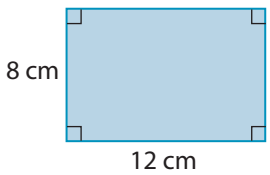
Section	After this section, I know how to . . .
7.1	■ determine if two triangles, two rectangles, or two irregular polygons are similar ■ draw a polygon that is similar to a given polygon ■ solve problems involving similarity
7.2	■ define the tangent ratio in a right triangle ■ use the tangent ratio to solve for an unknown side length in a right triangle
7.3	■ define the sine ratio and the cosine ratio in a right triangle ■ use the sine ratio and the cosine ratio to solve for an unknown side length in a right triangle
7.4	■ solve problems that involve an unknown angle in a right triangle ■ use trigonometric ratios to solve problems

If you are unsure about any of these questions, review the appropriate section or sections of this chapter.

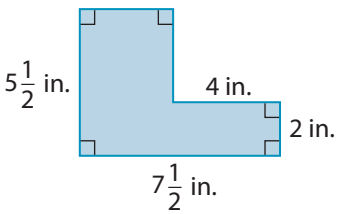
7.1 Similarity and Scale, pages 324–335

1. Using grid paper, create a scale diagram for each shape shown. Use an appropriate scale.

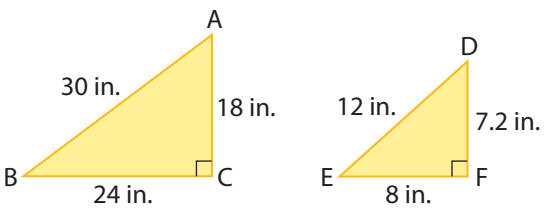
a)



b)



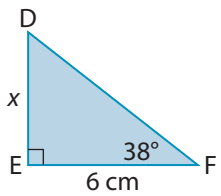
2. Determine whether the triangles shown are similar. Explain your reasoning.



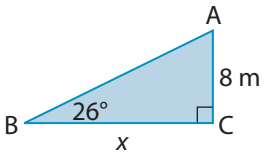
7.2 The Tangent Ratio, pages 336–347

3. Determine the value of x, to the nearest tenth of a unit.

a)



b)

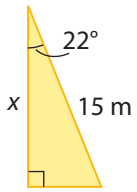


4. A wire is attached to the top of a flagpole. The wire reaches a stake in the ground 7 m from the foot of the pole. The angle formed between the wire and the ground is 58°. Determine the height of the flagpole to the nearest tenth of a metre.

7.3 The Sine and Cosine Ratios, pages 348–363

5. Estimate and then determine the length x in each triangle. Express each answer to the nearest tenth of a metre.

a)

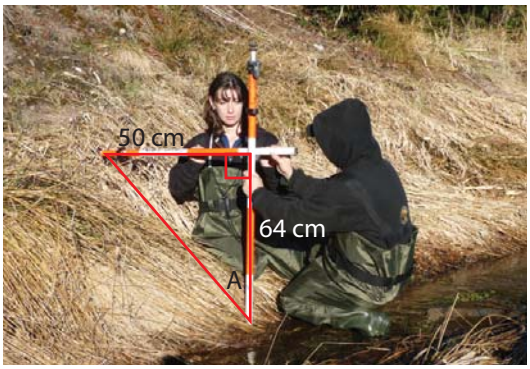


b)

6. A ramp inclines at an angle of 25°. The height of the ramp is 1.2 m. What is the horizontal length of the ramp to the nearest tenth of a metre?

7.4 Determine Unknown Angles, pages 364–375

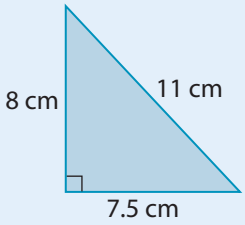
7. A conservation group is checking the risk of flooding along the Churchill River. The group needs to calculate the angle of the river bank. They set up two measuring poles as shown. They measure the vertical height to be 64 cm and horizontal distance to be 50 cm. What is the angle of the river bank, to the nearest degree?



7 Test Yourself

For #1 to #5, select the best answer.

1. Devon wants to draw a triangle similar to the one shown. Which set of dimensions could he use for the similar triangle?



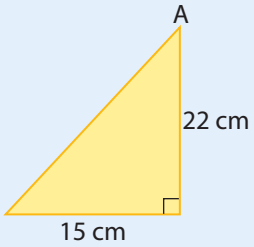
A 4 cm, 3.5 cm, 5 cm

B 12 cm, 11.25 cm, 17 cm

C 16 cm, 14 cm, 22 cm

D 24 cm, 22.5 cm, 33 cm

2. What is the measure of $\angle A$?



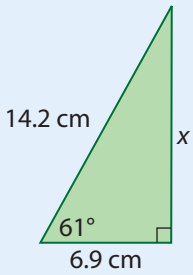
A 34°

B 43°

C 47°

D 56°

3. Which equation could be used to determine the value of x?



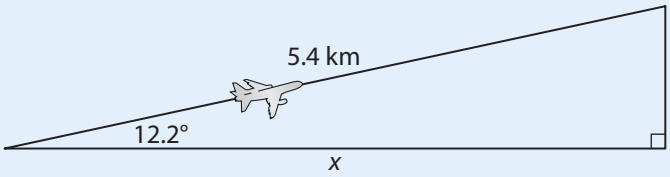
A $\cos 61° = \frac{x}{6.9}$

B $\cos 61° = \frac{x}{14.2}$

C $\sin 61° = \frac{6.9}{14.2}$

D $\tan 61° = \frac{x}{6.9}$

4. Cindi was asked to solve the following problem. A pilot takes off and climbs at a steady rate at an angle of 12.2°. Determine the horizontal distance the plane has gone when it has travelled 5.4 km. Her solution is shown.



Step 1 $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$

Step 2 $\cos 12.2° = \frac{x}{5.4}$

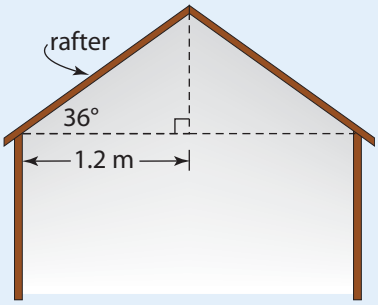
Step 3 $\frac{\cos 12.2°}{5.4} = x$

Step 4 $0.999... = x$

Where did Cindi make her first error?

A Step 1 **B** Step 2 **C** Step 3 **D** Step 4

5. Paul is designing a storage shed to build beside the house. He wants an overhang of 30 cm on each side of his roof. How long should each rafter be?



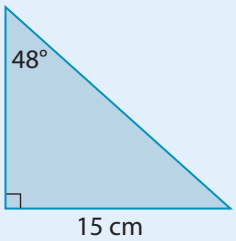
A 1.8 m

B 2.3 m

C 3.8 m

D 4.4 m

6. Determine the unknown side lengths and angle measure in the triangle. Give each side length to the nearest tenth of a unit and the angle to the nearest degree.



7. Gerianne leans a ladder that is 30 ft long against a two-storey house. The feet of the ladder are 7 ft from the base of the house.

a) Sketch a diagram of the situation.

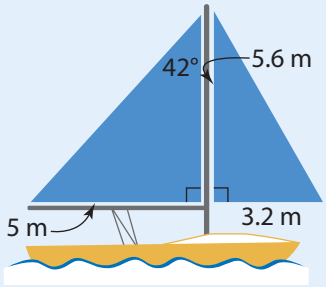
b) How high up the side of her house does the ladder reach? Express your answer to the nearest foot.

c) What angle does the ladder make with the ground? Express your answer to the nearest degree.

8. A sailboat has two sails shaped like right triangles.

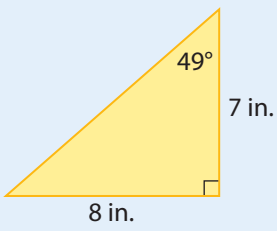
a) The mainsail is 5 m long. The top of the sail makes an angle of 42° with the top of the mast. Determine the unknown side lengths and angle measures of the mainsail. Express lengths to the nearest tenth of a metre and angles to the nearest degree.

b) The jib is 3.2 m long and 5.6 m tall. Determine the unknown side length and angle measures of the jib. Express length to the nearest tenth of a metre and angles to the nearest degree.



9. a) What two different methods could you use to determine the hypotenuse in the triangle shown?

b) Use one method to determine the length of the hypotenuse. Use a different method to check your answer. Show all your calculations.



7 Chapter Project

Design a Zipline for a Children's Camp

Plan a zipline for a children's camp.

1. Draw the plan, and then create a scale model that includes the following:
- ✓ the height and length of the zipline
- ✓ where the zipline is connected at each end
- ✓ safety features
- ✓ a clear scale

2. a) Decide on the height and length of your zipline.

b) Sketch your plan. Use trigonometry and the Pythagorean relationship to help you determine all lengths and angles in your drawing.

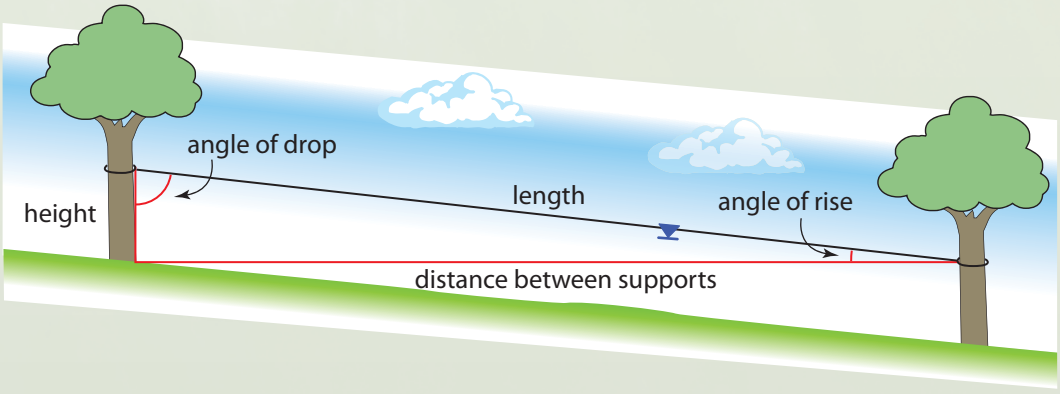
3. a) Decide on the type of scale model you will make. For example, you might create a 3-D model using string, small tree branches, and a cardboard base. List what you need to create the model.

b) Choose an appropriate scale for your model.

c) Create the model to scale.

Web Link

For information on building ziplines, go to www.mhrmathatwork10.ca and follow the links.



7 GAMES AND PUZZLES

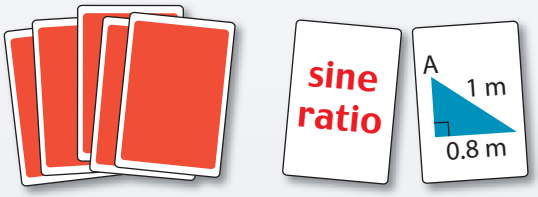
Trig Solitaire

Materials
- Trig's a Snap cards
- coin

1. Organize the set of cards into three categories. Start with the sine ratio, cosine ratio, and tangent ratio cards.
2. Beside each ratio, place the correct formula.
3. Decide which group each triangle card belongs in. Some may belong in more than one group, if there is more than one way to solve the problem.

Trig's a Snap

Play a card game. Take turns flipping cards until the last two cards flipped are a match. There are several ways to make a match:
✓ a trigonometry ratio and a formula
✓ a trigonometry ratio and a triangle
✓ a formula and a triangle



SNAP! The opposite side and hypotenuse are known. That's the sine ratio. So, these are a pair.

1. Play the game with a partner. Flip a coin to decide who shuffles and deals. The other person plays first.
2. The dealer deals all the cards, face down.
3. Players take turns flipping the top card from their pile.
4. If the cards on the top of each pile match, the first player to say "Snap!" wins all the flipped cards. If the player calls "Snap!" when the cards on top of each pile do not match, the opposing player wins all the flipped cards.
5. The first player to run out of cards loses the round.
6. Shuffle the cards and play again.
7. Describe a strategy you can use that might help you win the game. Try your strategy.