# Math at Work 11: Chapter 1

February 20, 2012, 16:17

# 1 Surface Area

Jasper, AB

Oil companies need to protect their storage tanks from rust and corrosion. Before painting the tanks, they need to know the surface area of each cylinder.

Three-dimensional (3-D) figures like these cylinders are often made up of two-dimensional (2-D) shapes.

**1.** What 2-D shapes make up a cylinder?

**2. a)** Name three other 3-D figures commonly used for storing items.

**b)** For each 3-D figure you named, list the 2-D shapes it is made up of.

## Key Words

rectangular prism
net
surface area
triangular prism
cylinder
diameter
radius
square-based pyramid
slant height
cone
sphere

## Career Link

Dan has applied for a job as an oil tank cleaner/painter. To be hired, he needs to know how to use pressure wash equipment. Physical fitness is also important, since he must be able to climb in and out of tanks and crawl through small spaces.

# 1 Get Ready

## SI and Imperial Units of Length

**1. a)** What are four SI units used to measure length? List them from shortest to longest.

**b)** Which SI units would you use to measure each item?

- the length of a salmon
- the thickness of a coin
- the height of an apartment building
- the distance from Hopedale, NL, to Montréal, QC


**2. a)** What are four imperial units used to measure length? List them from shortest to longest.

**b)** Which of these imperial units would you use to measure each item?

- the diameter of a car tire
- the distance from Labrador City, NL, to Brandon, MB
- the width of a football field
- the length of a cell phone

## Convert Units

**3.** Convert each SI length to the unit shown.

10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

**a)** 4.5 metres → ■ centimetres

**b)** 275 millimetres → ■ centimetres

**c)** 1500 metres → ■ kilometres

**d)** 1 m 80 cm → ■ cm → ■ m

**4.** Convert each imperial length to the unit shown.

**a)** 6 feet → ■ inches

There are 12 inches in 1 foot.
So, 6 feet is 6 × 12 = 72 inches.

**b)** 42 in. → ■ ft

**c)** 8 yards → ■ feet

**d)** 10′ 6″ → ■′ → ■″

**5.** Estimate each length in the unit shown.

**a)** 6 feet → ■ metres

One metre is a little longer than 3 feet.
So, 6 feet is close to 2 metres.

**b)** 10 miles → ■ kilometres

**c)** 20 cm → ■ in.

**d)** 10 metres → ■ yards → ■ feet

## Proportional Reasoning

**6.** Solve for $y$.

**a)** $\frac{3}{8} = \frac{y}{56}$

**b)** $\frac{5}{12} = \frac{y}{60}$

**c)** $\frac{3}{10} = \frac{y}{75}$

**d)** $\frac{7}{12} = \frac{y}{30}$

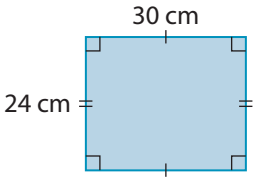
$8 \times 7 = 56$
So, $y$ must be 7 times greater than 3.
$7 \times 3 = 21$

## Area

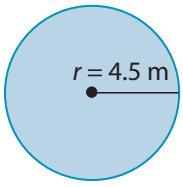
**7.** Determine the area of each shape.

Area of a rectangle = length × width
Area of a circle = $\pi r^2$
Area of a triangle = $\frac{1}{2}$base × height

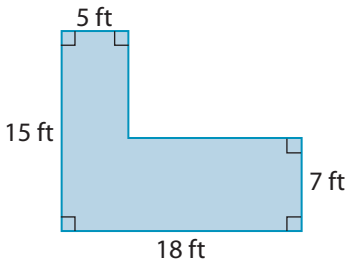
**a)**



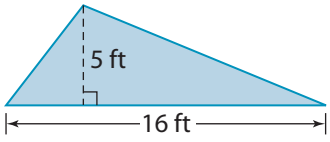

**b)**




**c)**




**d)**




**8.** Evaluate, without using a calculator.

**a)** $9 \times 5$

**b)** $7 \times 6$

**c)** $13 \times 10$

**d)** $6 \times 6$

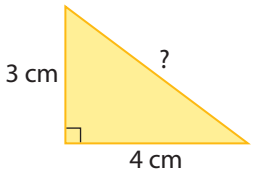
**e)** $8^2$

**f)** $12 \times 4\frac{1}{2}$

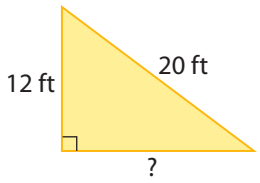
## The Pythagorean Relationship

**9.** Determine the unknown side lengths in each right triangle.

**a)**




**b)**




## Solving Equations

**10.** Solve.

**a)** $P = 2(l + w)$
$l = 5$, $w = 8$. Solve for $P$.

**b)** $V = lwh$
$l = 7$, $w = 4$, $h = 2.5$. Solve for $V$.

**c)** $A = \frac{1}{2}bh$
$A = 18$, $b = 3$. Solve for $h$.

**d)** $SA = 6s^2$
$s = 1.2$. Solve for $SA$.

# 1.1 Nets and Surface Area of 3-D Objects

**Focus On . . .**

- drawing nets of 3-D objects
- determining the area of the 2-D shapes that make up a 3-D object
- determining the surface area of 3-D objects

***Products come in packages of various shapes and sizes. A container starts out as a 2-D shape. Then, it is folded, rolled, or bent into a 3-D shape.***

**Materials**

- cardboard box
- ruler
- scissors
- grid paper
- tape

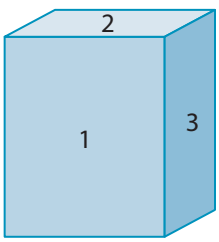
**rectangular prism**

- a 3-D figure with two rectangular bases that are the same size and shape

## Explore the Net and Surface Area of a Rectangular Prism

**1. a)** Use a box in the shape of a **rectangular prism**. Label the six faces of the box, using numbers 1 to 6.

**b)** Which pairs of faces are identical?



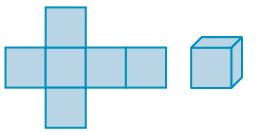
**2. a)** Measure the dimensions of each face of the box. For all of the measurements, use the same SI unit or imperial unit of your choice. Calculate the area of each face. Copy and fill in the table shown.

| First Area | # of Identical Faces | Total Area of Identical Faces |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

**b)** What is the total area of the six faces of the box?

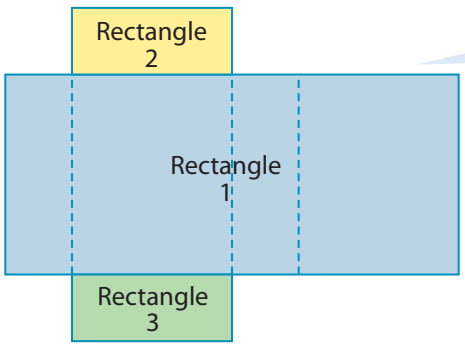
**net**

- a 2-D diagram that can be folded to create a 3-D object



net 3-D object

**3. a)** Create a **net** of the box. Cut the box along its edges so that you can lie the box flat. Do not worry about overlapping tabs that allow the box to be sealed.




Your net will look like a shape made up of three rectangles.

It may not look exactly like this drawing.

**b)** Measure and record the dimensions of the three rectangles in the net. Calculate the area of each rectangle.

**c)** What is the total area of the three rectangles in the net?

**4. Reflect** How does the total area you calculated in #2b) compare with your calculation in #3c)?

**5. Extend Your Understanding**

**a)** On centimetre grid paper, make a sketch of a net for a box that is 6 cm long, 4 cm wide, and 2 cm high.

**b)** Cut out the net and fold it into a box. Tape the box together.

**c)** Measure the length, width, and height of the box. Do these measurements match the dimensions given? Compare your results with those of a classmate.

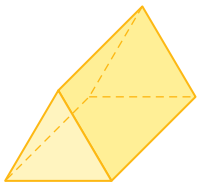
**d)** What product might come in a container this size?

## On the Job 1

### Calculate the Surface Area of a Triangular Prism

**triangular prism**

- a 3-D figure with only two triangular faces that are the same size and shape



**surface area**

- the sum of the areas of all the faces of a 3-D object
- measured in square units or units$^2$

**Strategy**

**Simplify the Original Problem**

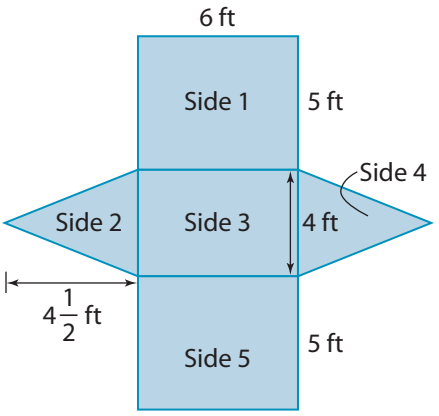
Many tents are in the shape of a **triangular prism**. Tent manufacturers need to know the **surface area** of a tent to determine the amount of material they need to make it.


**a)** Sketch a net of a tent. Label the dimensions.

**b)** Determine the amount of material needed to make the tent.

#### Solution

**a)** There are five sides to this tent. Number each side.




**b)** Calculate the area of each side.

Side 1: $A = l \times w$

$A = 6 \text{ ft} \times 5 \text{ ft}$

$A = 30 \text{ ft}^2$

Side 5 is identical to Side 1. So, it is also 30 ft$^2$.

Side 2: $A = \frac{1}{2}bh$

$A = 0.5 \times 4 \text{ ft} \times 4.5 \text{ ft}$

$A = 9 \text{ ft}^2$

Side 3: $A = l \times w$

$A = 6 \text{ ft} \times 4 \text{ ft}$

$A = 24 \text{ ft}^2$

Side 4 is identical to Side 2. So, it is also 9 ft$^2$.

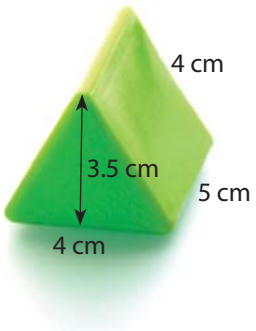
$SA = 30 \text{ ft}^2 + 30 \text{ ft}^2 + 9 \text{ ft}^2 + 9 \text{ ft}^2 + 24 \text{ ft}^2$

$SA = 102 \text{ ft}^2$

They will need 102 ft$^2$ of material to make the tent.

#### Your Turn

A children's sorting game includes a plastic triangular prism.




**a)** Sketch the triangular prism. Label the approximate dimensions.

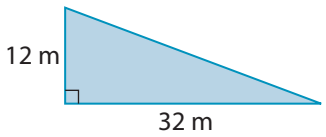
**b)** What is the total area of plastic needed to make the triangular prism?

## Check Your Understanding

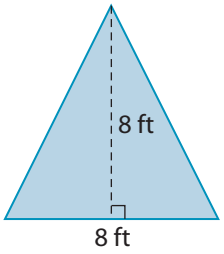
### Try It

**1.** Determine the area of each triangle.

**a)**




**b)**




**2.** Which imperial unit would be best to express the area of each item?

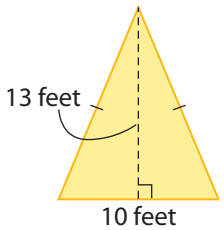
**a)** a computer screen

**b)** the surface of a lake

**c)** a backyard

**d)** the ceiling in a bedroom

**3.** What is the area of the triangle shown?




**4.** Which SI unit would be best to express the area of each item?

**a)** a gymnasium floor

**b)** a forest

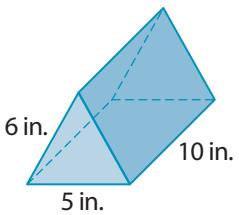
**c)** a wrapper for a mint

**d)** the lid for a tub of ice cream

**5.** The triangular prism shown has an approximate height of 5.5 in.

**a)** Sketch a net of the triangular prism.

**b)** Calculate the surface area of the prism.




### Apply It

**6.** Determine the total area of the material needed to create the wedge tent shown. Include the material needed for the floor. Express your answer to the nearest square foot.

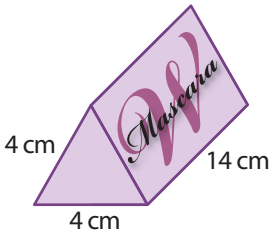
**F.Y.I.**

Some manufacturers use biodegradable packaging for their products. One snack manufacturer uses biodegradable plant material for its chip bags. This packaging is good for the environment because it decomposes completely. There is no waste to throw away.

**Web Link**

For more information about biodegradable packaging, go to www.mcgrawhill.ca/school/learningcentres and follow the links.

**7.** A brand of mascara comes in a cardboard box in the shape of a triangular prism. The height of the triangular face at each end of the prism is about 3.5 cm.

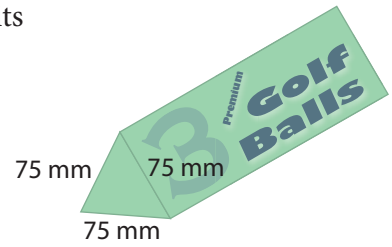



**a)** On grid paper, sketch a net of the box.

**b)** Calculate the area of the five faces of the box.

**c)** Why might the total amount of cardboard needed to make this box be slightly greater than your answer to part b)?

**8.** A company is thinking of changing its package for a set of three golf balls to a box in the shape of a triangular prism. The diameter of a golf ball is approximately 43 mm. The balls will be placed three in a row.




**a)** What is the minimum length of the longest side of the box?

**b)** Sketch a net of the box.

**c)** Calculate the total area of cardboard needed to make the five sides of the box. Round your answer to the nearest square millimetre. **Hint:** Use the Pythagorean relationship to determine the height of the triangular face at each end of the prism. Do not worry about the overlap needed to glue the box together.

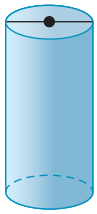
**d)** Calculate the total area of cardboard needed for a *rectangular* prism that would hold three golf balls.

**e)** Give one reason why the company might *not* use the triangular prism box for its golf balls.

**f)** Give one reason why the company might use the triangular prism box for its golf balls.

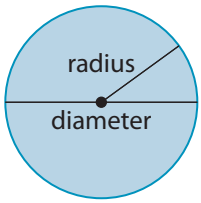
**cylinder**

- a 3-D object with two circular faces that are the same size and a curved surface



**diameter**

- the distance across a circle through its centre



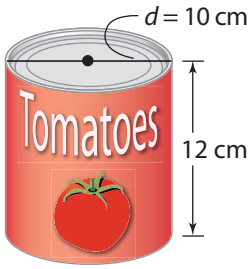

**radius**

- the distance from the centre of a circle to a point on the circumference

## On the Job 2

### Calculate the Surface Area of a Cylinder

Manufacturers package many foods in cans in the shape of a **cylinder**. A can of tomatoes has a **diameter** of 10 cm and a height of 12 cm.




**a)** Sketch a net of the can. Label all dimensions.

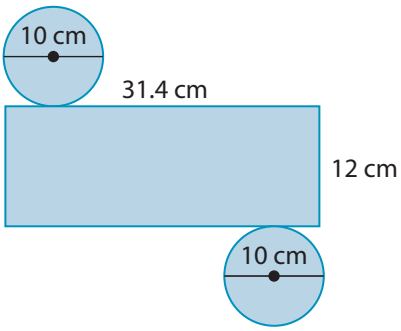
**b)** What amount of metal is needed to make one can, to the nearest square centimetre?

#### Solution

**a)** The top and bottom of the can are circles. Each circle has a diameter of 10 cm. The curved surface of the can is a rectangular sheet of metal that has been rolled into a tube.

$C = \pi d$
$C = \pi \times 10$
$C = 31.415\ldots$

The circumference of the circle is about 31.4 cm. This is the length of the rectangle that gets rolled into the tube.




**b)** Calculate the area of the rectangle.

$A = l \times h$
$A = 31.4\text{ cm} \times 12\text{ cm}$
$A = 376.8\text{ cm}^2$

The area of the rectangle is approximately 377 $\text{cm}^2$.

Calculate the area of the circle.

The **radius** is half the diameter. So, $r = 5$ cm.

$A = \pi r^2$
$A = \pi \times (5\text{ cm})^2$
$A = 78.539\ldots\text{ cm}^2$

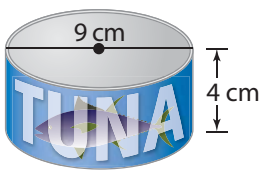
The area of each circle is approximately 78.5 $\text{cm}^2$.

$SA = 377\text{ cm}^2 + 78.5\text{ cm}^2 + 78.5\text{ cm}^2$
$SA = 534\text{ cm}^2$

It takes 534 $\text{cm}^2$ of metal to make one can.

#### Your Turn

Tuna is also packaged in cylindrical cans.




**a)** Sketch a net of the can. Label all dimensions.

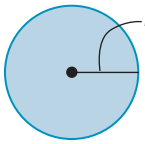
**b)** Find the area of metal needed to make one can.

## Check Your Understanding

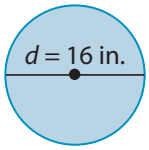
### Try It

**1.** Calculate the area of each circle, to the nearest square unit.

**a)**

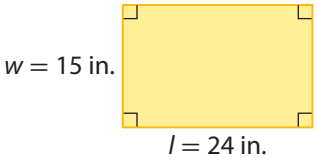



**b)**

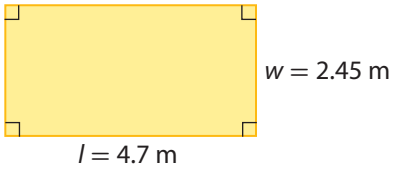



**2.** Calculate the area of each rectangle.

**a)**

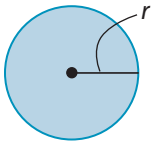



**b)**

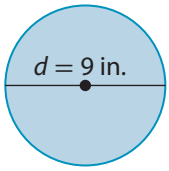



**3.** Calculate the circumference of each circle, to the nearest unit.

**a)**




**b)**




### Apply It

**4.** Salad dressing is sold with a paper label around the neck of the bottle. Determine the area of this label, to the nearest square millimetre.


**5.** Sea salt is shipped in cylindrical, plastic containers. The diameter of the container is $2\frac{1}{2}$ in.


**a)** Sketch a net of the salt container. Label all dimensions.

**b)** A graphic artist is creating a new design for the label that wraps around the container. Calculate the maximum area for the label, to the nearest square inch.

**c)** Calculate the total area of the plastic needed to make one container, to the nearest square inch.

**6. a)** Convert the answer to #2a) to square feet.

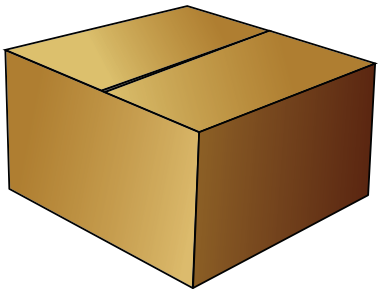
**b)** Convert the answer to #2b) to square centimetres.

**F.Y.I.**

1 square foot
= 144 square inches

## Work With It

**1.** Fresh seafood is sometimes shipped in insulated cardboard boxes. One type of box is approximately 18″ long, 15″ wide, and 12″ high.



**a)** What is the surface area of the box in square inches?

**b)** The length of the box is doubled to 36″. Does the surface area of the box double? Support your answer with calculations.

**2.** Oil storage tanks need to be painted to help prevent rust and corrosion. A tank has a height of 20 metres and a diameter of 15 metres.

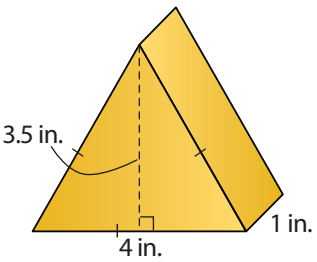
**a)** Sketch a net of an oil tank.

**b)** Determine the area that needs to be painted on one tank. **Note:** The bottom does not get painted. Round your answer to the nearest square metre.

**c)** There are 48 tanks that need to be painted. One way to determine the total area is to multiply the rounded answer in part b) by 48. Explain the possible problem with using this method. Support your answer with calculations.

**3.** The outer casing of a battery advertises the name of the company that made the battery. The height of a C battery is 50 mm and the diameter is 26.2 mm. The casing of a battery extends 2 mm past the top and bottom of the tube. This allows it to wrap slightly around the circles at the ends. Determine the area of the battery casing, to the nearest tenth of a square millimetre.

**4.** A hotel wants to provide its customers with a welcome gift of handmade chocolates. The chocolates will come in a box in the shape of a triangular prism. The rectangular faces of the box will be 1 inch by 4 inches. The side lengths of the triangular faces will be 4 inches. The height of the triangular faces will be about 3.5 inches.




**a)** Create a net of the box. Use the exact dimensions given.

**b)** What is the surface area of the box?

**c)** Cut out your net and construct the box.

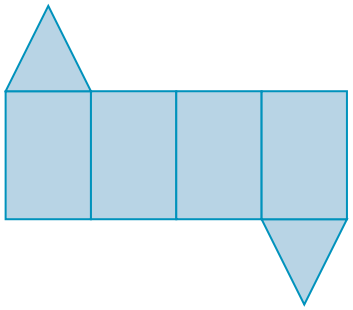
**d)** Check the dimensions by measuring. Do the measurements match the given dimensions?

## Discuss It

**5.** Explain how knowing how to calculate the area of a 2-D shape allows you to determine the surface area of a 3-D object.

**6.** Why do you think a manufacturer might use a box in the shape of a triangular prism to hold the mascara?

**7. a)** With a partner, discuss why this cannot be a diagram of the net of a triangular prism.



**b)** What changes need to be made for this net to be correct?

**8. a)** On centimetre grid paper, draw a net of a cube with edge lengths of 2 cm.

**b)** What is the surface area of the cube?

**c)** Without calculating, discuss with a partner what the surface area of a 3-cm cube might be.

# 1.2 Estimating Surface Area

**Focus On . . .**

- using length references to estimate the dimensions and the surface area of an object
- using area references to estimate the surface area of an object

***Some tasks require exact measurements. Other tasks do not. Calculating the wall area for painting a room does not require exact measurements. You can estimate the total surface area by using personal references.***

**Materials**

- ruler or measuring tape

## Explore Length and Area References

height = about 2 m

In earlier grades, you probably created personal references for imperial and SI lengths. Use a ruler or tape measure and collect some new personal references. Here are two methods for doing this.

1. ***Method 1: Measure the Reference***

   **a)** Think of a part of your body or a common item around you. Do you know the length of it? If so, you can use it as an estimate for a certain length.

   For example, the height of any door where you live is likely about 2 metres. You can now use a door as a reference for 2 metres. What is another example?

**b)** Copy the table shown. Record as many references, in both SI units and imperial units, as you can think of.

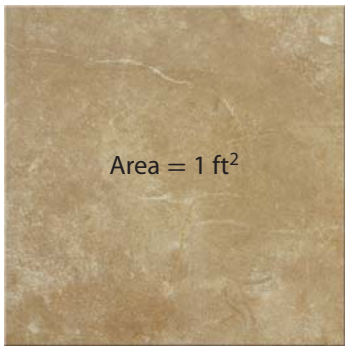
| Reference | SI Length | Imperial Length |
|---|---|---|
| My hand width | | |
| My arm span | | |
| My desk height | | |
| | | |

**Strategy**

**Develop Alternative Approaches**

**2.** ***Method 2: Reference the Measure***

**a)** You can collect references for standard units of length. For example, some floor tiles are exactly 12 inches by 12 inches. Use this kind of tile as a reference for a length of 1 foot and for an area of 1 square foot. What is another example?

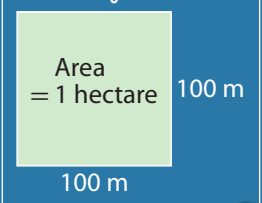



**b)** Copy the table shown. Record as many references, in both SI units and imperial units, as you can think of.

| SI Length | Reference | Imperial Length | Reference |
|---|---|---|---|
| 1 cm | | 1 in. | |
| 50 cm | | 1 ft | |
| 1 m | | 1 yd | |

**F.Y.I.**

Land can be measured in hectares. A hectare is an area of 100 m by 100 m.




**3. Reflect**

**a)** What is your reference for 1 centimetre? Use this to create a reference for 1 square centimetre.

**b)** Work with a partner. Create area references for 1 square foot and 1 square metre.

**4. Extend Your Understanding** Go outside and create references for three larger areas. For example, round the length and width of a parking space to the nearest metre or foot. What is the area of that parking space? You now have a visual reference for that area.

## On the Job 1

### Use Length References to Estimate Surface Area

Dylan is preparing to paint his daughter's bedroom closet. How can he estimate the surface area of the closet's three interior walls?

**Solution**

Width:

The combined length of Dylan's feet is about the same as the distance from the front of the closet to the back. He estimates the width of the closet as 2 ft.

Length:

When Dylan stands outside the closet with his arms outstretched, his arms are slightly longer than the length of the closet. Dylan is about 6 feet tall. He knows that his arm span is approximately the same as his height. He estimates the length of the closet to be $5\frac{1}{2}$ ft.

Height:

Dylan knows that the ceiling height in most houses is 8 feet. He does not need to use any reference.

Total:

Approximate area of back wall $= l \times h$

$= 5\frac{1}{2}$ ft $\times$ 8 ft

$= 44$ ft$^2$

$5 \times 8 = 40$

$\frac{1}{2} \times 8 = 4$

The area of the back wall is about 44 ft$^2$.

Approximate area of each side wall $= w \times h$

$= 2$ ft $\times$ 8 ft

$= 16$ ft$^2$

The area of each side wall is about 16 ft$^2$.

$SA = 44 \text{ ft}^2 + 16 \text{ ft}^2 + 16 \text{ ft}^2$

$SA = 76 \text{ ft}^2$

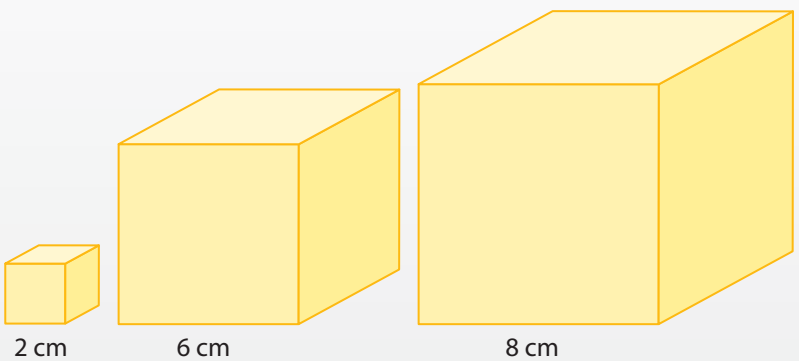
Dylan estimates the total surface area of the three interior walls to be about 76 square feet.

### Your Turn

Estimate the area of the four walls of a room in your school using your personal references for length.

**Puzzler**

You have three cubes. One cube has edges measuring 2 cm, one has edges of 6 cm, and one edges of 8 cm. You glue the cubes together so that you have the least possible exposed surface area. What is that surface area?



## Check Your Understanding

### Try It

**1. a)** Use your imperial length references to estimate the area of this page.

**b)** Measure the dimensions of this page in imperial units. Calculate the area of the page.

**c)** Compare your estimate and calculation. Are they close? If not, how can you improve your estimate?

An estimate is not a guess. Choose numbers that are easy to work with and close to the actual values. Use mental mathematics to estimate close to the actual measurements.

**2.** Repeat #1 using SI length references.

**3.** Look at the inside front cover and the inside back cover of the *Math at Work 11* student resource. Notice how the coloured paper just wraps around the hard cardboard cover. Open the book and turn it over so that you can see the front and back covers and the spine.

**a)** Estimate the area of the coloured paper covering the book.

**b)** Measure the dimensions of the coloured paper. Calculate the area.

**c)** Compare your estimate and your calculation.

### Apply It

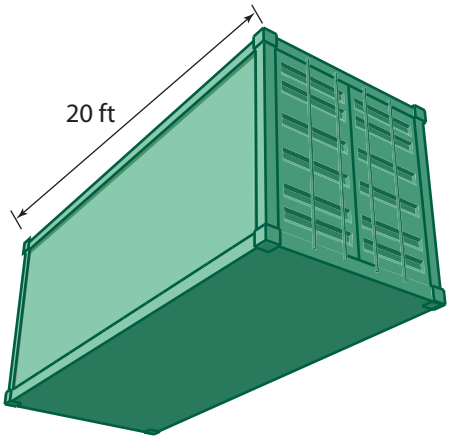
**4. a)** Estimate the surface area of the four walls of your classroom in square metres and in square feet.

**b)** One litre of paint covers about 10 $m^2$. Estimate the number of litres of paint needed to apply two coats to the walls of your classroom.

**c)** One gallon of paint covers about 400 $ft^2$. Estimate the number of gallons of paint needed to apply two coats.

**d)** Measure the dimensions of your classroom in metres and in feet. Calculate the total wall area in square metres and square feet. How close to the actual area are your estimates in part a)?

**5.** A moving company uses portable storage containers. The containers are dropped off at the old residence and then moved to the new residence. As part of their maintenance, the company needs to repaint the containers.

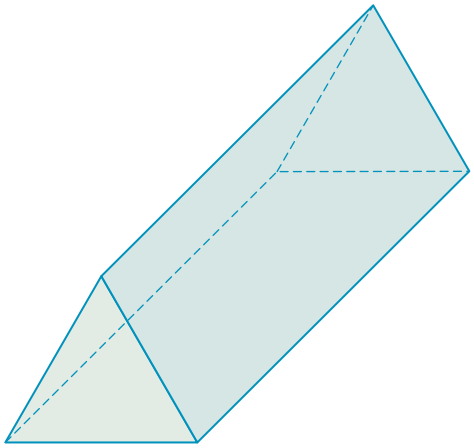



**a)** Estimate the surface area of the container shown.

**b)** Explain to a partner how you determined your estimate.

**6.** The picture shows a 6′ wooden picnic table. Brenna plans to coat the entire picnic table with wood sealer. Estimate the total surface area that she needs to coat. Show your work.


**7.** A manufacturer makes glass triangular prisms that refract light into the colours of the rainbow. The manufacturer protects each triangular prism with bubble wrap. The picture shows the actual size of a glass triangular prism.



**a)** Without measuring, estimate the surface area of the prism. Use imperial or SI units, whichever you prefer.

**b)** Measure the dimensions of the prism. Calculate its surface area. Compare your answer with your estimate in part a). How close are your estimate and your calculation?

**Web Link**

To view a demonstration of how light refracts through a prism, go to www.mcgrawhill.ca/school/learningcentres and follow the links.

## On the Job 2

### Use Area References to Estimate Surface Area

Ria wants to lay laminate floor tiles in her basement. One case of tiles covers about 15 square feet. How can Ria estimate how many cases she needs?

**Strategy**

**Develop a Strategy**

What if the basement did not have ceiling tiles? How could you use a personal reference to estimate the area of the floor?

### Solution

The floor and ceiling have the same area. So, Ria uses the ceiling tiles to estimate the area of the floor.

Each ceiling tile is 2 ft by 4 ft. So, each tile has an area of 8 $ft^2$.

Ria counts

- 38 full ceiling tiles
- 12 tiles that look like they are half tiles

$A = (38 \times 8\ ft^2) + (12 \times 4\ ft^2)$
$A = 304\ ft^2 + 48\ ft^2$
$A = 352\ ft^2$

A tile cut in half will cover about 4 $ft^2$.

The area of the ceiling is approximately 352 $ft^2$. So, the area of the floor is also about 352 $ft^2$.

$352 \div 15 = 23.466\ldots$

Each case of laminate tiles covers about 15 $ft^2$.

Ria needs between 23 and 24 cases of laminate floor tiles. She will get 24 cases to be sure she has enough.

**Tools of the Trade**

Many contractors buy 10% more floor tiles than they need for the estimated area. This is in case of errors in measuring or cutting.

### Your Turn

Use your area references to estimate the floor area of a room in your school.

## Check Your Understanding

### Try It

**1.** Use your reference for 1 square inch to estimate each area.

**a)** a Canadian $5 bill

**b)** a computer screen

**c)** the screen on a cell phone

**d)** a loonie

**e)** the bottom of a glass

**f)** the sole of your shoe

**2.** One and a half standard $8\frac{1}{2}''$ by 11″ sheets of paper can be a reference for about 1 $ft^2$. Use this reference to estimate the area of each item.

**a)** your desktop

**b)** the door to the classroom

**c)** a display board in your school

**d)** the seat of your chair

**3.** Solve.

**a)** 1 ft = ■ in.

**b)** 12 in. × 12 in. = ■ $in.^2$

**c)** $\frac{1}{2}$ ft = ■ in.

**d)** 6 in. × 6 in. = ■ $in.^2$

**e)** $\frac{1}{2}$ of 144 in. = ■ in.

**f)** $\frac{1}{4}$ of 144 ft = ■ ft

**4.** Solve.

**a)** 1 m = ■ cm

**b)** 100 cm × 100 cm = ■ $cm^2$

**c)** $\frac{1}{2}$ m = ■ cm

**d)** 50 cm × 50 cm = ■ $cm^2$

**e)** $\frac{1}{2}$ of 10 000 cm = ■ cm

**f)** $\frac{1}{4}$ of 10 000 m = ■ m

**5.** Use your reference for 1 square centimetre to estimate each area.

**a)** a sales receipt

**b)** a keyboard

**c)** a postage stamp

**d)** a nickel

**e)** a DVD

**f)** a key

**6.** Lie four metre sticks on the floor of your classroom to make a square. The area inside the square is 1 $m^2$. Use this reference to estimate the area of each item.

**a)** the door to the classroom

**b)** the ceiling of your classroom

**c)** a display board in your school

**7. a)** What fraction of 1 $ft^2$ is a 6″-by-6″ ceramic tile?

**b)** What fraction of 1 $m^2$ is a mirror that is 25 cm by 25 cm?

## Apply It

**8.** A standard hockey puck is 1″ thick with a 3″ diameter. Estimate the surface area of one puck.


**9.** Rolls of tape are sometimes packaged in a stack covered with plastic wrap. Each roll of tape is about 1 inch thick and about 3 inches in diameter.

**a)** Estimate the surface area of the plastic wrap needed for a stack of 10 rolls of tape.

**b)** Explain why your estimate must be less than 10 times your estimate for #8.

## Work With It

1. You have been hired to paint a room in your school. Use a reference to estimate the surface area of the room's four walls.

2. Estimate the area of bubble wrap needed to protect a plate that has a diameter of 10 inches.

3. Maggie has a 20-cm cube-shaped box that she wants to gift wrap. Sheets of wrapping paper come in three sizes. Which size should Maggie buy? Explain your choice.

   Size A: 30 cm by 30 cm

   Size B: 60 cm by 40 cm

   Size C: 90 cm by 50 cm

4. An upholsterer needs to re-cover the three cushions on the couch shown. Estimate how much fabric will be needed.


## Discuss It

5. Describe how you would use a reference to estimate the area of plastic wrap needed to cover a DVD case.

6. A tube is 12 in. high and has a diameter of 4 in. The area of the wrapper for this tube is about 1 square foot. Explain why.

7. Ria estimates that she needs about 24 cases of laminate flooring for her basement. Explain why Ria does not need to buy an exact amount of flooring.

# 1.3 Using Formulas for Surface Area of 3-D Objects

**Focus On . . .**

- using formulas to determine the surface area of rectangular and triangular prisms, pyramids, and cylinders

***Walk down a grocery store aisle. You will see that packaging comes in all shapes and sizes. While the dimensions of 3-D objects may differ, the methods of calculating surface area remain the same.***

## Explore Formulas for the Surface Area of Rectangular Prisms

**Materials**

- box
- ruler
- grid paper

**1. a)** Use a box in the shape of a rectangular prism. How many sides does the box have?

**b)** How many pairs of identical sides are there?

**2.** Label the longest edge of the box the length. Label the shorter edge the width. The height is the vertical edge.

**3.** The area of the bottom of the box is length × width.
The area of the top of the box is also length × width.

**a)** Identify one face of the box. Which two dimensions do you multiply to determine the area of this face?

**b)** Identify the other face with the same area.

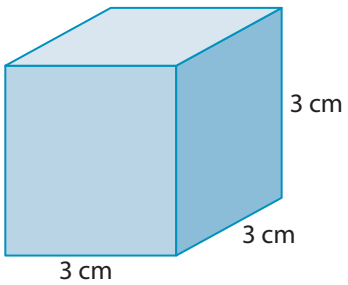
**4.** Which two dimensions do you multiply to determine the area of each of the two remaining faces?

**5. Reflect** Write a formula for the surface area of the rectangular prism.

**6. Extend Your Understanding**

**a)** Work with a partner. Create a formula for determining the surface area of a cube.

**b)** Use your formula to determine the surface area of a 3-cm cube.



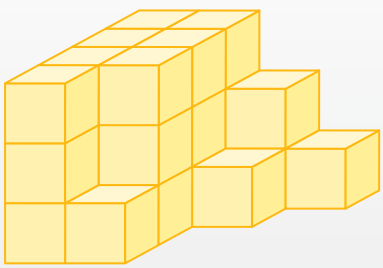

**c)** Draw a net of a 3-cm cube.

**d)** Determine the area of your net.

**e)** Does your formula work?

**Puzzler**

How many cubes are in the stack? Assume that there are no cubes missing from the back of the stack.



## On the Job 1

### Use a Formula to Calculate the Surface Area of a Rectangular Prism

For big paint jobs, the surface area measurement must be more exact than for a small job such as painting a closet. Calculate the surface area of the exterior of an industrial storage container that is 45 feet long, 9 feet 6 inches wide, and 8 feet high.

#### Solution

The total surface area of a rectangular prism equals the area of all six sides.

$SA = 2(\text{length} \times \text{width}) + 2(\text{length} \times \text{height}) + 2(\text{width} \times \text{height})$

Calculate the surface area using the formula
$SA = 2(lw + lh + wh)$, where $l$ = length, $w$ = width, and $h$ = height
$SA = 2[(45 \text{ ft} \times 9.5 \text{ ft}) + (45 \text{ ft} \times 8 \text{ ft}) + (9.5 \text{ ft} \times 8 \text{ ft})]$
$SA = 2(427.5 \text{ ft}^2 + 360 \text{ ft}^2 + 76 \text{ ft}^2)$
$SA = 2 \times 863.5 \text{ ft}^2$
$SA = 1727 \text{ ft}^2$

6 in. $= \frac{1}{2}$ ft
$= 0.5$ ft

The surface area of the storage container is 1727 square feet.

#### Your Turn

Calculate the approximate surface area of the exterior of a freight train boxcar with dimensions 20 m by 3.2 m by 3 m.

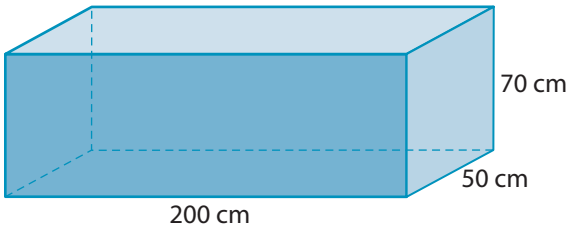
It may help you to sketch a net of the boxcar.

## Check Your Understanding

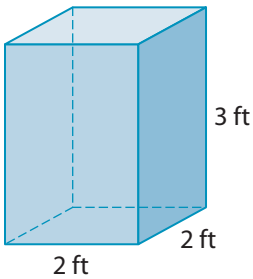
### Try It

**1.** Calculate the surface area of each rectangular prism.

**a)**




**b)**

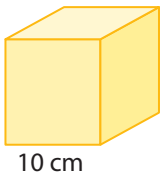



**2.** **a)** Calculate the area of one face of this cube.

**b)** Calculate the surface area of the entire cube.

**c)** Suppose the side length of the cube doubles. What is the surface area of a cube with a side length of 20 cm?

**d)** Does the surface area double?



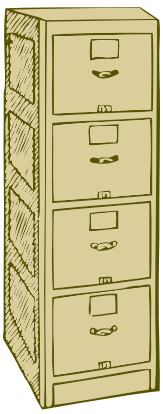

### Apply It

**3.** Kevin purchased a filing cabinet for his home office. He wants to paint the exterior a different colour. The dimensions of the cabinet are 140 cm high by 40 cm wide by 70 cm deep.

**a)** Calculate the surface area of the filing cabinet.

**b)** Kevin will not paint the bottom of the cabinet, since it sits on the floor. What is the total area that he will paint?

**c)** Express your answer to part b) in square metres.

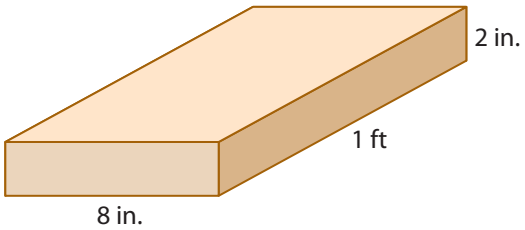


**4.** Jake is going to put plastic lining inside a rectangular pond. The pond has a width of 6 ft, length of 9 ft, and depth of 3 ft. Jake incorrectly calculates the surface area of the inside of the pond. What is the correct answer?

$SA = 2(lw + lh + wh)$
$SA = 2[(9\text{ ft} \times 6\text{ ft}) + (9\text{ ft} \times 3\text{ ft}) + (6\text{ ft} \times 3\text{ ft})]$
$SA = 2[(54\text{ ft}^2) + (27\text{ ft}^2) + (18\text{ ft}^2)]$
$SA = 198\text{ ft}^2$
The surface area of the inside of the pond is $198\text{ ft}^2$.

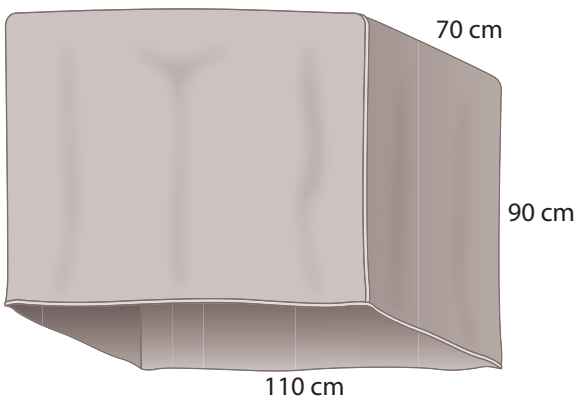
**5.** Lori creates fabric-covered gift boxes that she sells at craft shows.

**a)** The lid of a cardboard box is shown below. Calculate the surface area of the five sides of the lid that are visible when the box is closed.




**b)** What is the minimum area of the material she needs for the lid?

**6.** One style of barbecue cover is in the shape of an open-ended rectangular prism. Determine the area of the material needed for the barbecue cover shown. **Note:** The cover has no bottom.



## On the Job 2

### Use a Formula to Calculate the Surface Area of a Triangular Prism

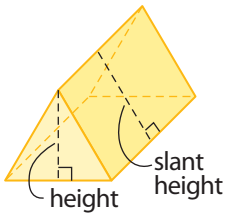
Meghan is building a greenhouse in the shape of a triangular prism. It will be 3 metres high, 4 metres wide, and 6 metres long. The **slant height** of the greenhouse will be 3.6 metres. Meghan needs to buy glass to start the job.

**a)** What is the surface area of the triangular prism?

**b)** What is the exposed surface area of the greenhouse?

**slant height**

- the shortest distance from the edge of the base of a 3-D figure to its highest point




**F.Y.I.**

Another formula for calculating the surface area of a triangular prism is $SA = lw + 2\left(\frac{1}{2}wh\right) + 2ls$.

#### Solution

**a)** The surface area of a triangular prism equals the area of all five sides:

area of base + area of 2 triangular ends + area of 2 slanted walls

You can calculate the surface area using the formula

$SA = lw + wh + 2ls$,
where $l$ = length,
$w$ = width,
$h$ = height
$s$ = slant height

The area of one triangular end is $\frac{1}{2} \times w \times h$.

So, the area of two triangular ends is $2 \times \frac{1}{2} \times w \times h = wh$.

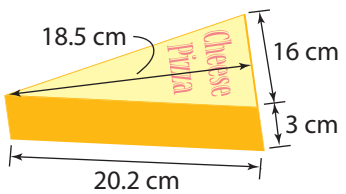
$SA = (6\text{ m} \times 4\text{ m}) + (4\text{ m} \times 3\text{ m}) + (2 \times 6\text{ m} \times 3.6\text{ m})$
$SA = 24\text{ m}^2 + 12\text{ m}^2 + 43.2\text{ m}^2$
$SA = 79.2\text{ m}^2$

The total surface area of the triangular prism is $79.2\text{ m}^2$.

**b)** The floor of the greenhouse is not exposed. So, the surface area of the exposed part of the greenhouse is about $79.2\text{ m}^2 - 24\text{ m}^2 = 55.2\text{ m}^2$.

#### Your Turn

Packaging suppliers sell pizza slice boxes in the shape of a triangular prism. Calculate the exterior surface area of the box shown.

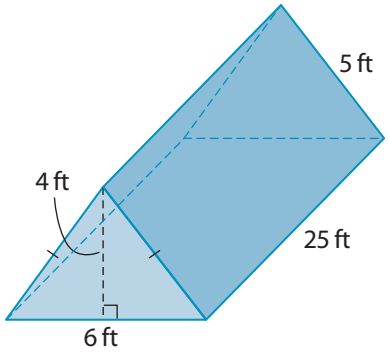


## Check Your Understanding

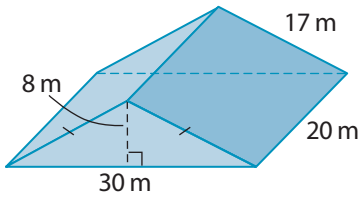
### Try It

**1.** Calculate the surface area of each triangular prism.

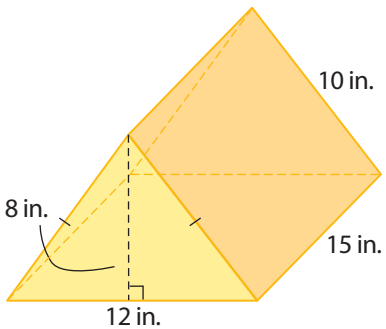
**a)**




**b)**




**2. a)** Calculate the total surface area of the triangular prism.

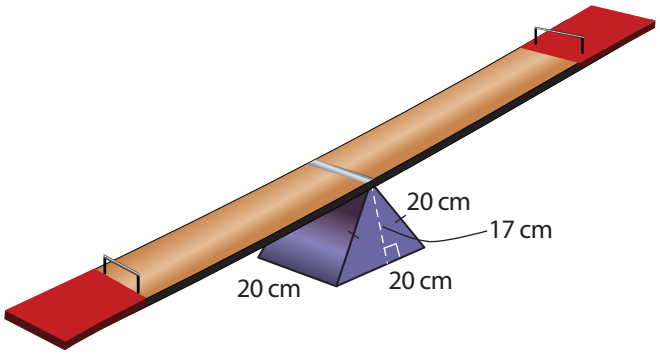



**b)** Suppose the length of the original triangular prism doubles from 15 in. to 30 in. What is the surface area now? Does the surface area double?

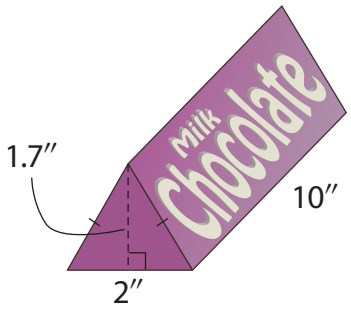
**c)** Suppose the height of the original triangular prism doubles from 8 in. to 16 in. What is the surface area now? Does the surface area double?

## Apply It

**3.** The fulcrum, or base, of a seesaw is a triangular prism. Determine its surface area.




**4.** A chocolate bar comes packaged in the shape of a triangular prism.




**a)** Determine the slant height of the package.

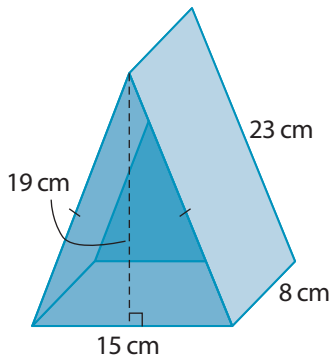
**b)** Calculate the exterior surface area of the package.

Do not take into account any tabs that are needed to glue the package together.

**F.Y.I.**

The slant height makes a right triangle.


**5.** Bruno and Lena are giving pieces of their wedding cake as gifts. Each piece of cake is packaged in the shape of an open triangular prism with the approximate dimensions shown. Calculate the exterior surface area of the box.

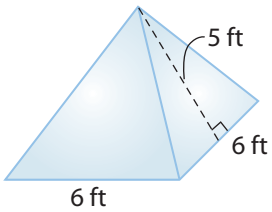


## On the Job 3

### Use a Formula to Calculate the Surface Area of a Square-Based Pyramid

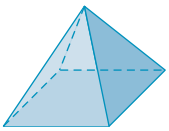
The Muttart Conservatory is a botanical garden located in Edmonton, AB.

Pyramid shapes are used in architecture, landscaping, advertising, and packaging to create a unique look. Calculate the surface area of a **square-based pyramid** with a length of 6 feet and a slant height of 5 feet.




**square-based pyramid**

- a 3-D figure with a square base and four triangular sides that connect at one point



#### Solution

The base of the pyramid is a square. All four faces of the pyramid are identical triangles.

The total surface area of a square-based pyramid equals the area of all five sides.

$SA$ = area of square base + 4(area of each triangle)

Calculate the surface area using the formula

$SA = l^2 + 2ls$, where $l$ = length and $s$ = slant height

$SA = (6\text{ ft})^2 + 2(6\text{ ft} \times 5\text{ ft})$

$SA = 36\text{ ft}^2 + 60\text{ ft}^2$

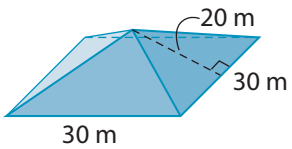
$SA = 96\text{ ft}^2$

$\left(4\frac{1}{2}lh\right) = 2lh$

The surface area of the pyramid is 96 square feet.

#### Your Turn

Calculate the surface area of a square-based pyramid with a length of 30 metres and a slant height of 20 metres.

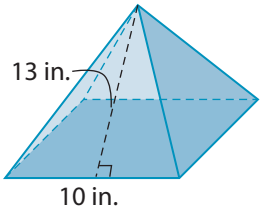


## Check Your Understanding

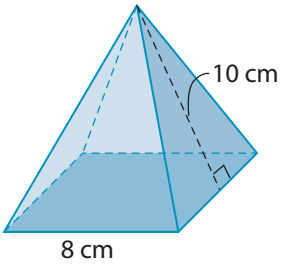
### Try It

**1.** Calculate the surface area of each square-based pyramid.

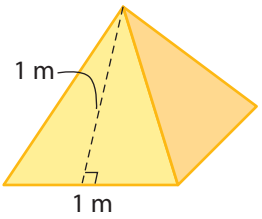
**a)**




**b)**

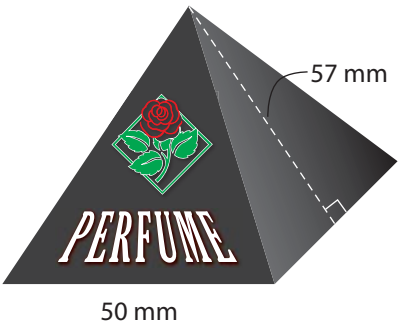



**2.** A square-based pyramid sitting on the ground was built with only four triangular panels. The base length is 1 metre and the slant height is 1 metre. What is the surface area of the pyramid, not including the square area of ground that the pyramid sits on?




### Apply It

**3.** A new brand of perfume has a box in the shape of a square-based pyramid. The box has a length of 50 mm and a slant height of 57 mm.




**a)** Calculate the amount of cardboard needed for the pyramid.

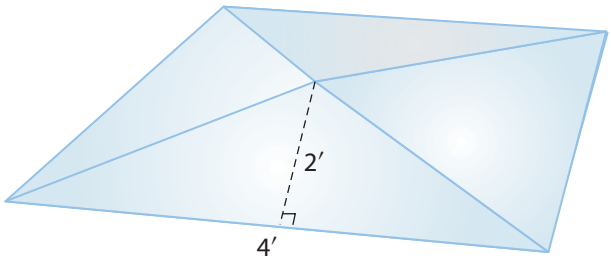
**b)** The manufacturer wants to connect the name of the perfume to the shape of the package. Suggest a suitable name for this perfume.

**4.** Sales of the perfume in #3 have been strong. The manufacturer is making a larger bottle. The base of the new box remains 50 mm in length, but the slant height doubles to 114 mm.

**a)** Without calculating, predict the surface area of the new box.

**b)** Calculate the surface area of the new box. Compare your calculation with your prediction in part a).

**5.** A company that installs windows and doors makes pyramid-shaped skylights for the roofs of houses. The skylight has no base. Calculate the surface area of the acrylic needed for a square-based skylight that is 4′ wide and has a slant height of 2′.



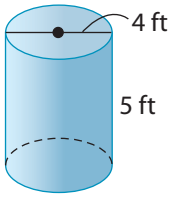

**Puzzler**

How could the cubes shown be cut to create a set of cubes with twice the surface area? Show your reasoning.

## On the Job 4

### Use a Formula to Calculate the Surface Area of a Cylinder

A cylindrical water tank needs to have a protective coating applied to its surface. Calculate the outer surface area of a tank that is 5 ft tall and has a diameter of 4 ft. Express your answer in square feet.

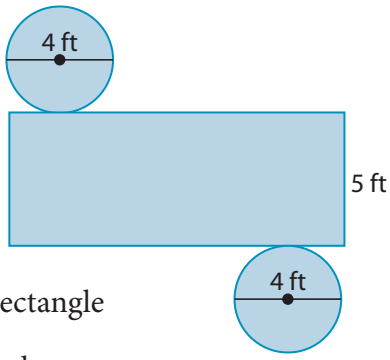



**F.Y.I.**

The measurement of 5 ft can be referred to as the *height* or the *length* of the cylinder.

### Solution

The total surface area of a cylinder equals the area of the two circles at each end plus the area of the rectangle that wraps around the cylinder.




**Strategy**

**Develop a Strategy**

How could you determine the surface area of the cylindrical tank without changing all measurements to feet?

$SA = 2(\text{area of circular ends}) + \text{area of rectangle}$

Calculate the surface area using the formula

$SA = (2 \times \pi r^2) + (\pi d \times h)$, where $r$ = radius, $d$ = diameter, and $h$ = height

$SA = [2 \times \pi \times (2 \text{ ft})^2] + (\pi \times 4 \text{ ft} \times 5 \text{ ft})$

$SA = 25.132\ldots \text{ ft}^2 + 62.831\ldots \text{ ft}^2$

$SA = 87.964\ldots \text{ ft}^2$

The length of the rectangle is equal to the circumference of the circular ends: $\pi d$.

The surface area of the tank is approximately 88 $\text{ft}^2$.

### Your Turn

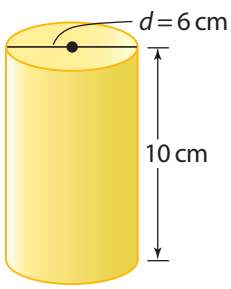
A plastic industrial drum has a diameter of 16 in. and a height of 23 in. What is the surface area of the drum? Express your answer to the nearest square inch.

## Check Your Understanding

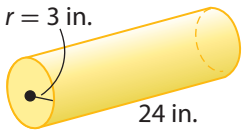
### Try It

**1.** Calculate the surface area of each cylinder. Round each answer to the nearest square unit.

**a)**



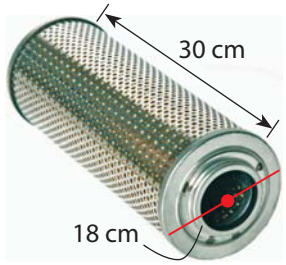

**b)**




**2.** A tube is an open-ended cylinder. Calculate the surface area of a 3-foot-long cardboard tube that has a 2-inch diameter. Express your answer to the nearest square inch.

### Apply It

**3.** The air filter in some vehicles is cylindrical in shape. Determine the surface area of the mesh around the outside of this air filter. Express your answer to the nearest tenth of a square centimetre.




**4.** A cylindrical tank has an above-ground height of 40 ft and a diameter of 60 ft. Determine the exterior surface area of the above-ground part of the tank, to the nearest square foot.

**5.** A cardboard tube is used as a mould for concrete. What is the exterior surface area of cardboard needed for a 12-foot-long tube with an 18-inch diameter, to the nearest square foot?

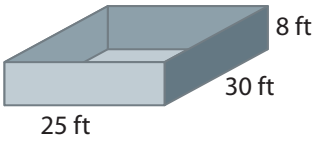
**6.** A stage set will include six cylindrical posts. Each post is 1 metre high, with a diameter of 0.6 m. Calculate the total area of wallpaper needed to cover the height of six posts. Express your answer to the nearest tenth of a square metre.

**Tools of the Trade**

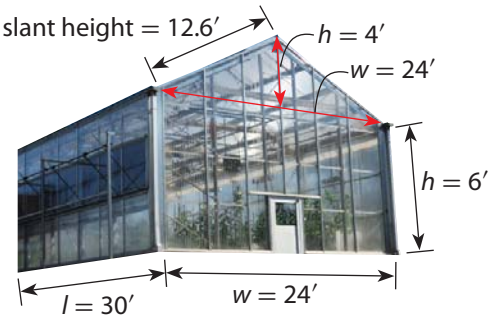
Builders use water-resistant cardboard tubes to make concrete posts. The tubes can also be used as moulds for simple columns. For more information on using cardboard tubes for concrete, go to www.mcgrawhill.ca/school/learningcentres and follow the links.

## Work With It

**1.** A contractor is spray-sealing the concrete floor and walls of the basement of a building. The dimensions of the floor are 30 ft by 25 ft. The walls are 8 ft high. Determine the total surface area that needs to be sealed.

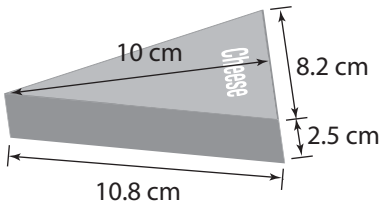



**2.** Mario is building a greenhouse in the shape of a rectangular prism with a triangular prism on top. Mario needs to buy glass to start building. What is the surface area of the greenhouse?




**3.** Pink fibreglass insulation sometimes comes rolled up in a cylinder of plastic wrap. The ends of the cylinder are not wrapped. Calculate the surface area of the plastic wrap needed for a roll with a diameter of 12 inches and a height of 15 inches. Express your answer in square inches.

**4.** A brand of cheese is packaged with a protective foil wrap.




**a)** Calculate the approximate surface area of one piece of cheese. Express your answer in square centimetres.

**b)** Explain why the answer to part a) would be too small for the area of each foil wrap.

**5.** Alexis wants to repaint the upper rectangular cabinet doors in her kitchen. Each cabinet door is 16″ by 20″ by $\frac{1}{2}$″. There are eight doors. She is painting the entire door of each cabinet. Calculate the total surface area she will paint.

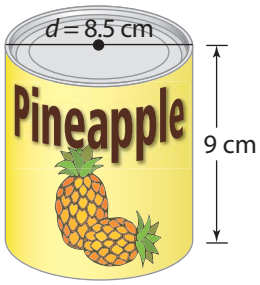
**6.** John makes novelty gift boxes. A chocolate store has ordered 150 boxes in the shape of square-based pyramids. The length of the base of each box is 3 inches. The slant height is also 3 inches.

**a)** What is the surface area of one box?

**b)** A sticker with the chocolate company's logo will cover one of the triangular faces of the box. What is the area of the sticker?

**c)** What is the height of the sticker?

**7.** In most grocery stores, you can buy a can of pineapple pieces.




**a)** Calculate the area of metal needed to produce the can. Round your answer to the nearest tenth of a square centimetre.

**b)** Calculate the area of the label. Allow for an overlap of 1 cm so the ends of the label can be glued together. Round your answer to the nearest tenth of a square centimetre.

**c)** A larger can of pineapple pieces has twice the height. Does the surface area of the can double? Does the surface area of the label double?

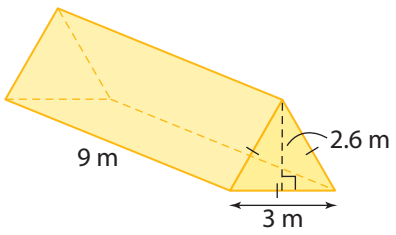
## Discuss It

**8.** How would you explain to someone what *surface area* means?

**9.** Vicki says that the surface area of a box is equal to how much it will hold. Do you agree or disagree?

**10.** The surface area of a 2-D shape is measured in square units. Why is the surface area of a 3-D shape not measured in cubic units?

**11.** Alex used the following formula to calculate the surface area of the triangular prism shown: $SA = 3(lw) + 2\left(\frac{1}{2}wh\right)$. Is he correct? Explain.

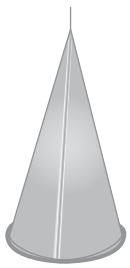


# 1.4 Surface Area of Cones and Spheres

**Focus On . . .**

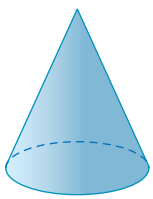
- using formulas to determine the surface area of cones and spheres

***Photographers use a light tent to soften lighting and reduce shadows. Light tents come in various shapes. One type of tent is shaped like a cone.***



**cone**

- a 3-D figure with a circular base and a curved surface that runs from the base to the highest point



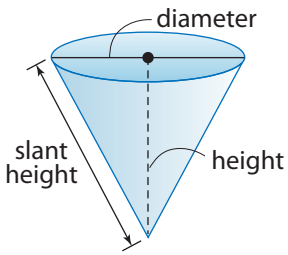
**Materials**

- cone-shaped paper cup
- ruler
- grid paper
- scissors
- tennis ball or other small ball

## Explore the Surface Area of Cones and Spheres

### Part A: Surface Area of Cones

**1. a)** Measure the diameter and the height of a cone paper cup to the nearest millimetre. Record the measurements.



**b)** What is the radius of the circle at the base of the cone?

**c)** The radius, the height, and the slanted side of the cone form a right triangle. Use the Pythagorean relationship to calculate the slant height.

**d)** Measure the slant height of the cone, to the nearest millimetre. Compare this measurement to your answer in part c).

**Strategy**

**Draw or Model**

**2. a)** On grid paper, trace the circle that forms the base of the paper cup.

**b)** Estimate the circumference of the circle.

**c)** Calculate the circumference of the circle. Label it on your diagram.

**d)** Estimate the area of the circle.

**e)** Calculate the area of the circle. Label it on your diagram.

**3. a)** Cut the cup in a straight line from the base to the tip. Uncurl the paper to make it lie as flat as possible.

**b)** On grid paper, trace the flattened shape of the paper cup.

**c)** Identify the slant height of the cone and the circumference of the circle. Label them on your diagram.

**4. Reflect**

**a)** Which surface do you think has the greater area: the base of the cone or the curved surface of the cone?

**b)** What if the dimensions of the cone changed? Would your answer to part a) still be true?

**c)** Estimate the area of the curved surface of the cone. Do not include the area of the base.

**Strategy**

**Guess and Check**

**5. Extend Your Understanding**

**a)** Calculate the area of the curved surface this way:
$\pi \times$ radius $\times$ slant height

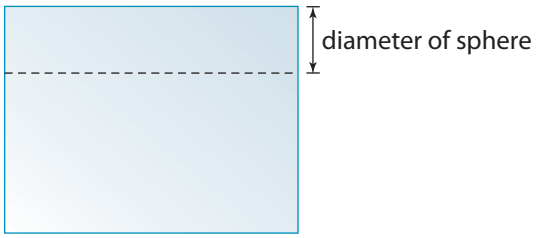
**b)** Compare your answer to step 5a) to your estimate in step 4c).

**c)** Write a formula for calculating the surface area of a cone that includes the circular base plus the curved surface.

## Part B: Surface Area of a Sphere

**6. a)** Estimate the diameter of a tennis ball or other small ball.

**b)** Cut a strip from a sheet of $8\frac{1}{2}''$ by $11''$ paper lengthwise, so that the width of the strip has the same measurement as the diameter of the sphere.




**7. a)** Wrap the strip of paper around the circumference of the sphere. Cut the length of the strip so that the two ends just meet when wrapped around the sphere.

**b)** Measure the length of the rectangle you cut out.

**8.** Copy and fill in Columns 1 and 2 of the table.

| | Column 1 | Column 2 | Column 3 |
|---|---|---|---|
| Rectangle | width = | length = | Area = |
| Sphere | diameter = | circumference = | Surface Area = |

**9. a)** What is the area of the rectangle? Record this value in Column 3.

**b)** The area of the rectangle is equal to the surface area of the sphere. What is the surface area of the sphere? Record this value in Column 3.

**10. Reflect**

**a)** Write the formula for the area of a rectangle.

**b)** Use *diameter* and *circumference* to write a formula for the surface area of a sphere.

**11. Extend Your Understanding**

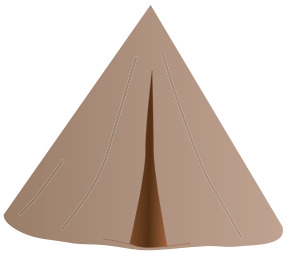
**a)** What is the formula for the circumference of a circle?

**b)** Use the formula in part a) to write another formula for the surface area of a sphere. Remember to simplify your formula as much as possible.

$d = 2r$

## On the Job 1

### Calculate the Surface Area of a Cone



Kara is designing a tent that approximates the shape of a cone. The tent is made of two pieces of nylon. One piece is the circular base. The other piece is the curved upper surface. The tent design has a base diameter of 2.4 metres and a height of 2 metres. Express your answers to the following questions to the nearest hundredth of a unit.

**a)** Sketch a net of the tent.

**b)** What is the slant height, $s$, of the tent?

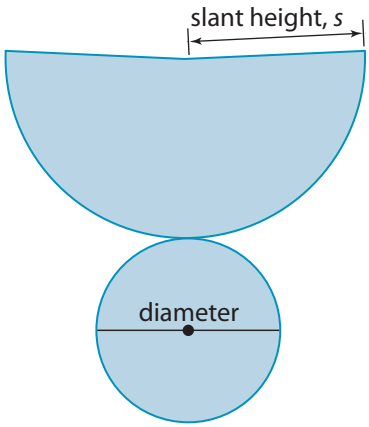
**c)** What is the surface area of the curved surface?

**d)** What is the surface area of the base?

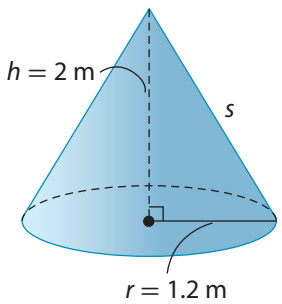
**e)** What is the total area of nylon needed to construct the tent?

### Solution

**a)** For the net, the base of the tent is in the shape of a circle. The curved part of the tent is in the shape of wedge.




**b)** Use the Pythagorean relationship to determine the slant height, $s$.




The diameter is 2.4 m.
So, the radius is 1.2 m.

$s^2 = h^2 + r^2$
$s^2 = 2^2 + 1.2^2$
$s^2 = 4 + 1.44$
$s^2 = 5.44$
$s = \sqrt{5.44}$
$s = 2.332\ldots$

The slant height of the tent is about 2.33 m.

**c)** The formula for the surface area of the curved surface of a cone is $SA = \pi rs$, where $r$ is the radius of the base of the cone and $s$ is the slant height of the cone

The formula can also be written as $SA = \pi \times r \times s$.

$SA = \pi \times 1.2 \times 2.33$
$SA = 8.783\ldots$


The surface area of the curved surface of the tent is about 8.78 $m^2$.

**d)** The surface area of the base of a cone is a circle.

The formula for the area of a circle is $SA = \pi r^2$, where $r$ is the radius of the circle

$SA = \pi \times 1.2^2$
$SA = 4.523\ldots$


The surface area of the base of the tent is about 4.52 $m^2$.

**e)** The total surface area of a cone equals the total area of the two surfaces.

$SA = 4.52 + 8.78$
$SA = 13.30$

You can calculate the total surface area using the formula $SA = \pi r^2 + \pi rs$, where $r$ is the radius of the base of the cone and $s$ is the slant height of the cone

The total area of nylon needed to construct the tent is about 13.3 $m^2$.

## Your Turn

A larger model of the conical tent has a radius of 4 feet and a slant height of 8 feet. Express your answers to the following questions to the nearest square foot.

*Conical* is another way of saying "cone shaped."

**a)** Sketch a net of the tent.

**b)** What is the surface area of the curved surface of the tent?

**c)** What is the surface area of the base of the tent?

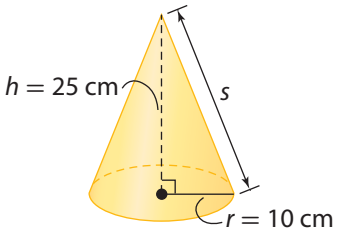
**d)** What is the total area of nylon needed to make the tent?

## Check Your Understanding

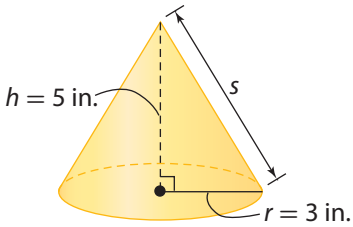
### Try It

**1.** What is the slant height of each cone? Round your answers to the nearest tenth of a unit.

**a)**




**b)**




**2.** Sketch a net of the cone in #1b). Label the radius and the slant height.

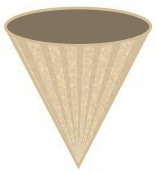
**3.** What is the area of the base of each cone in #1? Round your answers to the nearest tenth of a square unit.

**4.** What is the area of the curved surfaces of each cone in #1? Round your answers to the nearest tenth of a square unit.

**5.** What is the surface area of each cone in #1?

### Apply It

**6.** Max has designed a reusable coffee filter in the shape of a cone. It has a diameter of 19 cm and a slant height of 19.6 cm. What is the surface area of the filter? Round your answer to the nearest tenth of a square centimetre.



**7.** Michelle sells crafts every year at the Christmas arts and crafts show. This year, she is making Christmas trees made of cones covered in green paper. Each tree will have a diameter of 3 inches and a height of 8 inches.

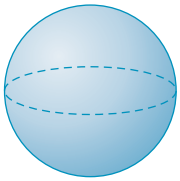
**a)** Determine the slant height of the tree. Round your answer to the nearest inch.

**b)** Determine the minimum amount of paper needed to make one tree. Michelle does not cover the bottom of the cone. Round your answer to the nearest square inch.

## On the Job 2

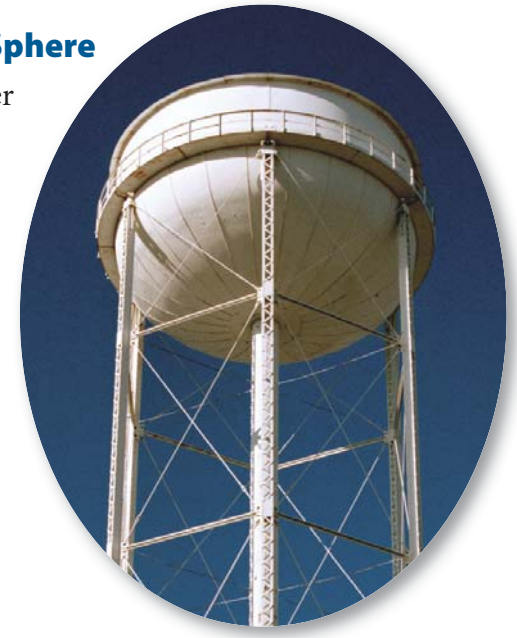
**sphere**

- a round, ball-shaped figure



### Calculate the Surface Area of a Sphere

Painters have been hired to paint a water storage tank that is in the shape of a **sphere**. The tank has a diameter of 22 metres. What is the outer surface area of the sphere? Express your answer to the nearest tenth of a square metre.



**F.Y.I.**

Most spherical storage tanks have a diameter of 10 m to 22 m. They can be used to hold liquids or gases.

#### Solution

You can calculate the surface area of a sphere using the formula $SA = 4\pi r^2$, where $r$ is the radius of the sphere

$SA = 4 \times \pi \times 11^2$

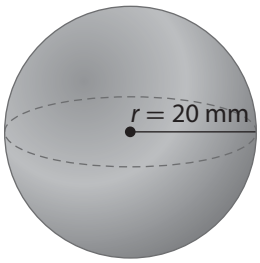
$SA = 1520.530\ldots$


The radius is half the diameter.

The surface area of the water tank is about 1520.5 $m^2$.

#### Your Turn

A manufacturer of industrial supplies makes rubber-coated steel balls. If a steel ball has a radius of 20 mm, what is the surface area that the rubber must cover? Round your answer to the nearest tenth of a square millimetre.




**F.Y.I.**

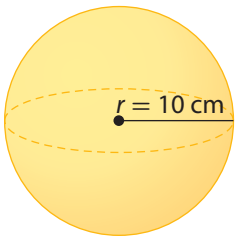
Rubber-coated steel balls are used in valves to ensure a leak-proof seal. These types of valves have many uses, including spray bottles. The rubber coating prevents any gas or liquid from leaking.

## Check Your Understanding

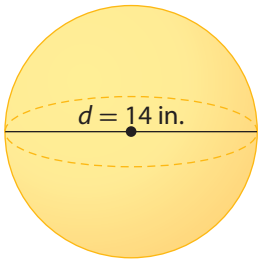
### Try It

**1.** What is the surface area of each sphere? Round each answer to the nearest tenth of a square unit.

**a)**




**b)**




**2.** For the following questions, round each answer to the nearest tenth of a square unit.

**a)** The radius of the sphere in #1a) doubles from 10 cm to 20 cm. What is the new surface area? Did the surface area double?

**b)** The diameter of the sphere in #1b) doubles from 14 cm to 28 cm. What is the new surface area? Did the surface area double?

### Apply It

**3.** Jennifer is a fitness instructor. The size of exercise ball she should use depends on her height. For the following questions, round each answer to the nearest tenth of a square centimetre.

| Ball Diameter | Height |
|---|---|
| 45 cm | up to 5′ 0″ (1.5 m) |
| 55 cm | 5′ 1″ to 5′ 6″ (1.5 m to 1.7 m) |
| 65 cm | 5′ 7″ to 6′ 0″ (1.7 m to 1.8 m) |
| 75 cm | 6′ 1″ and over |

**a)** Jennifer is 5′ tall. What is the surface area of the ball she should use?

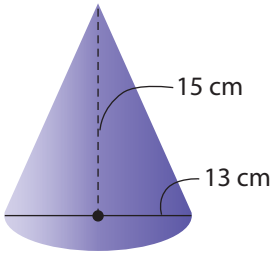
**b)** What is the surface area of the ball that someone your height should use?

**4.** Earth is almost a perfect sphere. The diameter of Earth at the equator is a little less than 8000 miles. Approximately 71% of Earth's surface is covered with water. What is the surface area of the water covering Earth, to the nearest square mile?

**5.** A manufacturer makes golf balls that have a diameter of about 43 mm. Each ball has an outer cover made of ionomer resin. What is the outer surface area of the ionomer resin on one ball? Express your answer to the nearest square millimetre.

## Work With It

**1.** Ferdinand is making paper party hats for his daughter's birthday party.

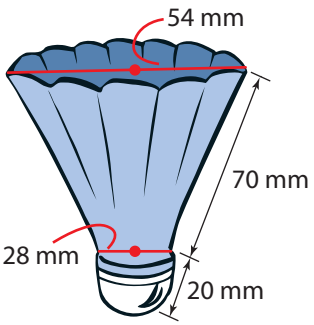



**a)** What is the minimum surface area of the paper needed to make one hat? Express your answer to the nearest tenth of a square centimetre.

**b)** Fifteen children will attend the party. What is the total surface area of paper needed to make 15 hats? Express your answer to the nearest square centimetre.

**2.** A manufacturer of educational items makes globes. Each globe is made of a plastic sphere with a paper covering. What is the surface area of a plastic sphere that has a diameter of 1 foot? Round your answer to the nearest tenth of a square inch.

**3.** A manufacturer of sports equipment makes badminton birdies. Calculate the approximate exterior surface area of the blue plastic used to make the birdie. Express your answer to the nearest tenth of a square millimetre.




## Discuss It

**4.** A cone and a cylinder have the same base diameter and the same height. Which one has the greater surface area? Use diagrams or calculations to explain your thinking.

**5.** A sphere and a cylinder have the same diameter and the same height. Which one has the greater surface area?

**6. a)** Explain when to use the formula $SA = \pi rs$.

**b)** Explain when to use $SA = \pi r^2 + \pi rs$.

# 1 Skill Check

## What You Need to Know

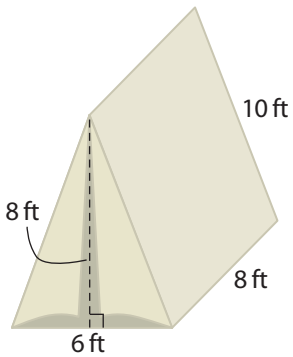
| Section | After this section, I know how to . . . |
|---|---|
| 1.1 | ■ draw nets of 3-D objects<br>■ determine the area of the 2-D shapes that make up a 3-D object<br>■ determine the surface area of 3-D objects |
| 1.2 | ■ use length references to estimate the dimensions and the surface area of an object<br>■ use area references to estimate the surface area of an object |
| 1.3 | ■ use formulas to determine the surface area of rectangular and triangular prisms, pyramids, and cylinders |
| 1.4 | ■ use formulas to determine the surface area of cones and spheres |

*If you are unsure about any of these questions, review the appropriate section or sections of this chapter.*

### 1.1 Nets and Surface Area of 3-D Objects, pages 6–14

**1.** **a)** Sketch and label a net of a tent with the dimensions shown.

**b)** Determine the amount of material needed to make the tent.




**2.** A soup can has a diameter of 8 cm and a height of 12 cm.

**a)** Sketch and label a net of the soup can.

**b)** Determine the total area of metal needed to make the can. Express your answer to the nearest tenth of a centimetre.

**c)** Explain why a can with a height of 24 cm will not have double the surface area of a can with a height of 12 cm.

### 1.2 Estimating Surface Area, pages 15–24

**3.** Use references for SI lengths to estimate the surface area of a bookshelf or another piece of furniture in your classroom.

**4.** Repeat #3 using references for imperial lengths.

## 1.3 Using Formulas for Surface Area of 3-D Objects, pages 25–39

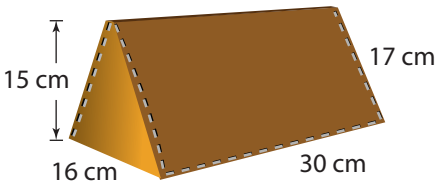
**5.** A stack of $8\frac{1}{2}''$-by-11″ sheets of printer paper is 2″ thick.

**a)** Use a formula to calculate the surface area of a wrapper for the printer paper.

**b)** Explain why the actual wrapper would likely have a surface area slightly greater than your answer to part a).

**6.** A set of candles is stacked and then packaged in plastic. Each candle is 8 cm in diameter and 5 cm high. Use a formula to calculate the surface area of a stack of three candles. Express your answer to the nearest tenth of a square centimetre.


**7.** Carly makes canvas handbags in the shape of a triangular prism. Each bag has a length of 30 cm, width of 16 cm, height of 15 cm, and slant height of 17 cm. Determine the minimum surface area of the canvas needed to make one bag.



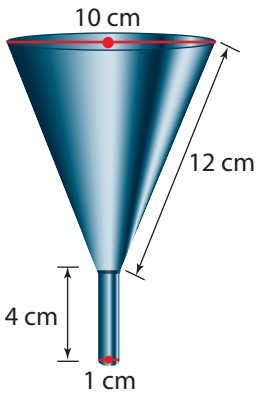

**8.** A souvenir shop in Corner Brook, NL, sells quartz in the shape of square-based pyramids. They have a base length of 50 mm and a slant height of 40 mm. Each pyramid is shrink-wrapped to prevent scratches. Use a formula to calculate the surface area of the wrap for the pyramid shown.

## 1.4 Surface Area of Cones and Spheres, pages 40–49

**9.** A funnel is made up of two shapes: a cone and a cylinder.

**a)** What is the surface area of the stainless steel needed to make the funnel shown? Express your answer to the nearest tenth of a square centimetre.

**b)** Why is this calculation not exact?

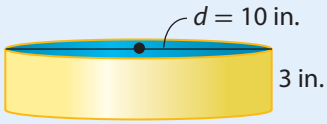



**10.** The outer surface of a baseball is made of two equal-sized pieces of leather that have been stitched together. A baseball is about 3 inches in diameter. What is the surface area of each piece of leather, to the nearest square inch?

# 1 Test Yourself

*For #1 and #2, select the best answer.*

**1.** What is the approximate area of the yellow part of the cylinder below?




**A** 13 in.$^2$ **B** 30 in.$^2$

**C** 90 in.$^2$ **D** 300 in.$^2$

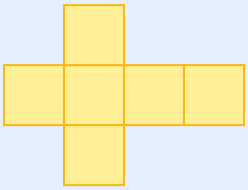
**2.** What is the approximate area of the top of the cylinder in #1?

**A** 10 in.$^2$ **B** 25 in.$^2$

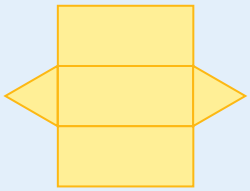
**C** 50 in.$^2$ **D** 75 in.$^2$

**3.** Identify the 3-D object that each net represents.

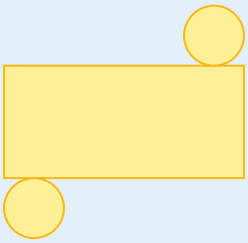
**a)**



**b)**



**c)**

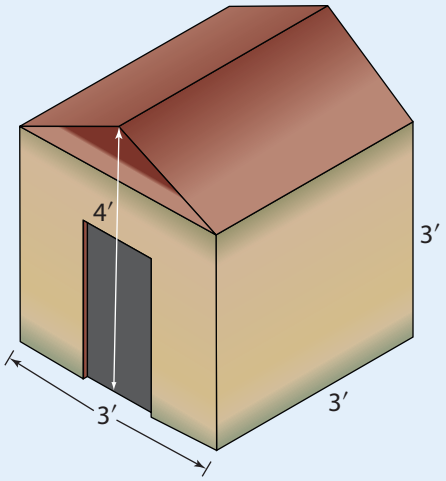


**4.** Part of a playground apparatus has a square-based pyramid shape. The pyramid has a width of about 136 centimetres and a slant height of about 89 centimetres. Determine the surface area of the outer four sides of the pyramid. Do not include the bottom.

**5. a)** Determine the surface area of the cardboard needed to make a pizza box that is 12″ by 12″ by 2″.

**b)** Would the surface area of a pizza box that is 24″ by 12″ by 2″ be double your answer to part a)? Explain.

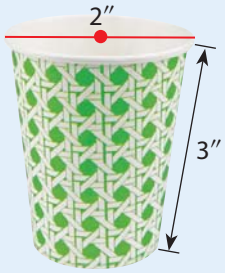
**6.** Andrew wants to re-varnish a doghouse. Calculate the surface area of the exterior of the doghouse.




**7.** John calculated the surface area of the paper cup using the formula for the surface area of a cone. Explain the error in his solution.

$SA = \pi r^2 + \pi rs$
$= \pi + 3\pi$
$= 4\pi$
$= 12.5663...$

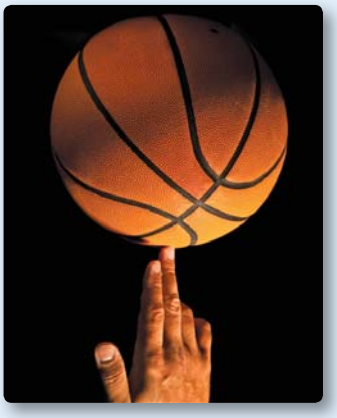
You need about $12\frac{1}{2}$ in.$^2$ of paper to make the cup.




**8.** An NBA basketball has a diameter of about 238 mm. What is the surface area of the leather surface of an NBA basketball?

**a)** Express your answer to the nearest tenth of a square millimetre.

**b)** Express your answer to the nearest tenth of a square centimetre.



# 1 Chapter Project

## Design a Board Game

Most board games involve moving playing pieces around on a board. You are a board game designer. You are going to create a game board and playing pieces for a board game.

### Part A: Build the Game Board

1. Make a cardboard template showing the size and shape of the game board you plan to create.
2. The playing surface will be made of paper that will cover the cardboard. The paper will overlap the edges of the cardboard by 1 cm or $\frac{1}{2}''$. What is the surface area of the paper needed to make the playing surface?

### Part B: Create the Playing Pieces

3. Make sketches of three playing pieces. Use shapes from this chapter.
4. Draw a net of two of these pieces.
5. **a)** Cut out the net of one of the pieces.
   **b)** Determine the area of the net and the surface area of the playing piece it will build.
   **c)** Colour and build the playing piece.

### Part C: Design the Game

6. Make up a board game that requires the players to answer questions related to what you learned in this chapter.

# 1 GAMES AND PUZZLES

## High Roller

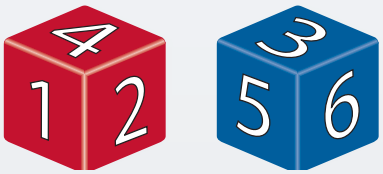
**Materials**
- thick red paper and blue paper
- ruler
- scissors
- glue
- marker

Work with a partner.

1. First, create two dice.
   - **a)** Draw a net of a cube on red paper. Cut out the net and put it together. Write the numbers 1 to 6 on the faces of the cube. The red cube represents positive numbers.
   - **b)** Repeat part a) to make a blue cube. The blue cube represents negative numbers.
2. Play the game with your partner. Roll the two dice. Use the two numbers to calculate an answer that is the greatest possible number. You can add, subtract, multiply, or divide. For example,

   $4 + (-3) = 1, (-3) + 4 = 1$
   $4 - (-3) = 7, (-3) - 4 = -7$
   $4 \times (-3) = -12, (-3) \times 4 = -12$
   $4 \div (-3) = -\frac{4}{3}$ or $-1.3333\ldots$,
   $(-3) \div 4 = -\frac{3}{4}$ or $-0.75$

   In this example, the greatest possible answer is 7.




3. The first player to get the greatest answer gets one point. The first player to get ten points wins.

## Surface Area Dice Challenge

**Materials**
- three dice
- calculator

Work with a partner and three dice.

1. One player rolls the three dice. The numbers are dimensions. Choose two dimensions for a cylinder or cone. Use three for a rectangular prism. The first player to calculate the largest surface area wins a point.
2. Take turns rolling the dice. The first player to get five points is the winner.

*For this game, all the numbers on the dice are positive numbers.*