3 Linear Relationships

Sandra and her daughter, Tara, want to join a karate dojo. There is a one-time sign-up fee of $120. Children and teens pay $40/month, while adults pay $50/month. There is a special offer this month of no sign-up fee for adults.

The relationship between the total membership cost for Sandra and the number of months that she learns karate is linear.

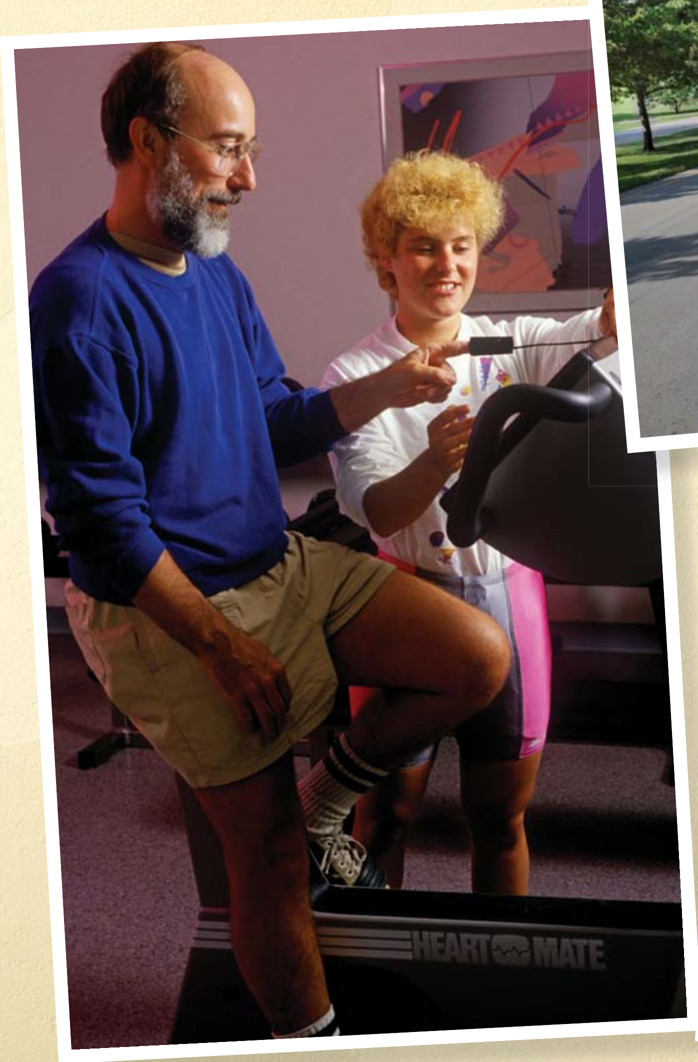
1. What does it mean when a relationship is linear?
2. Do you think the relationship between the membership cost for Tara and the number of months is also linear? Explain your thinking.
3. For Sandra, the total membership cost varies directly with the number of months. For example, the total cost at four months is twice the total cost at two months. Does the total membership cost for Tara vary directly with the number of months? How do you know?

Key Words

line of best fit
linear trend
linear relationship
non-linear relationship
direct variation
initial value
rate of change
partial variation

Career Link

Brian drives a tractor-trailer through eastern Canada. Truck drivers work with linear relationship in many ways. For example, they calculate earnings or fuel costs related to the number of kilometres driven, determine the distance travelled at certain speeds, plan efficient routes, and use the scale of a map to determine the actual distance.

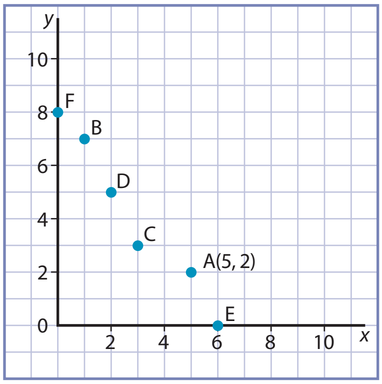


3 Get Ready

Coordinate Graphing

1. Write the coordinates of each point as an ordered pair.

For example, point A is at (5, 2).



2. On grid paper, draw a set of axes like the one in #1. Then, plot and label these points.

To plot (*x*, *y*), start at the origin, (0, 0), and then
- move right *x* units if *x* is positive, or move left *x* units if *x* is negative
- from there, move up *y* units if *y* is positive, or move down *y* units if *y* is negative

P(3, 4)
Q(5, 0)
R(4, 4)
S(0, 6)
T(1, 8)
U(2, 5)

Interpolating and Extrapolating

3. On grid paper, create a scatter plot of the relationship shown in this table of values. Describe any trend that you see in the data.

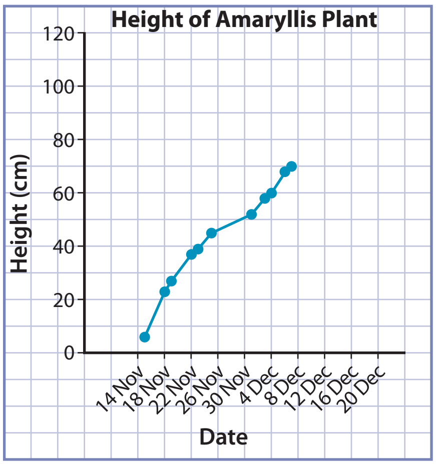
x	*y*
0	2
1	3
2	5
3	7
4	10
5	12

4. This graph shows the growth of a plant over five weeks.

a) Extrapolate to predict the height on December 20.

b) Interpolate to estimate the height on November 28th.

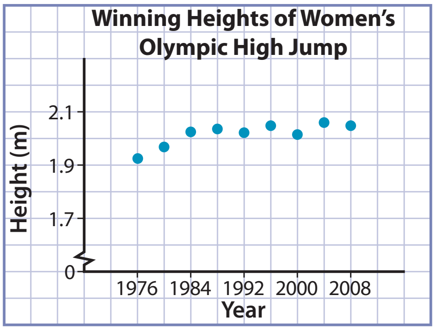
- When you predict values beyond a graph, you are *extrapolating*.
- When you estimate values between plotted points, you are *interpolating*.



5. This graph shows the winning heights for the women's high jump event at the Olympics from 1976 to 2008. The Olympics occur every 4 years

a) Predict the winning height in 2012. Explain your thinking.

b) Why is it impossible to interpolate winning heights for years such as 1987 or 1991?



Slope

6. Calculate the slope of each line.

Slope = $\frac{\text{rise}}{\text{run}}$

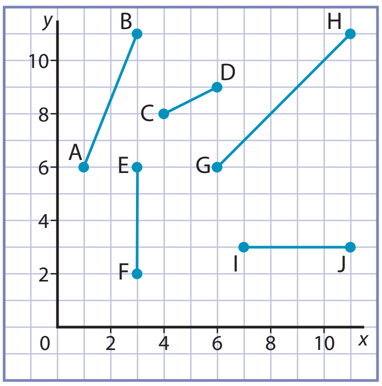
a) $\overline{AB}$

b) $\overline{CD}$

c) $\overline{EF}$

d) $\overline{GH}$

e) $\overline{IJ}$



Describing Number Patterns

7. Describe each number pattern.

For example, the pattern 1, 2, 4, 7, … is an increasing pattern that starts at 1 and increases by 1, then 2, then 3, and so on, increasing by 1 more each time.

a) 2, 4, 6, 8, …

b) 0, 7, 14, 21, …

c) 1, 2, 4, 8, …

d) 200, 100, 50, 25, …

e) 9, 5, 1, −3, …

f) 1, 10, 100, 1000, …

Substituting Values and Solving Equations

8. Substitute the value of 3 for x into each equation to solve for the value of y.

For example,
$y = 2x + 4$
$y = 2(3) + 4$
$y = 6 + 4$
$y = 10$

a) $y = 4x + 5$

b) $y = 10 - 2x$

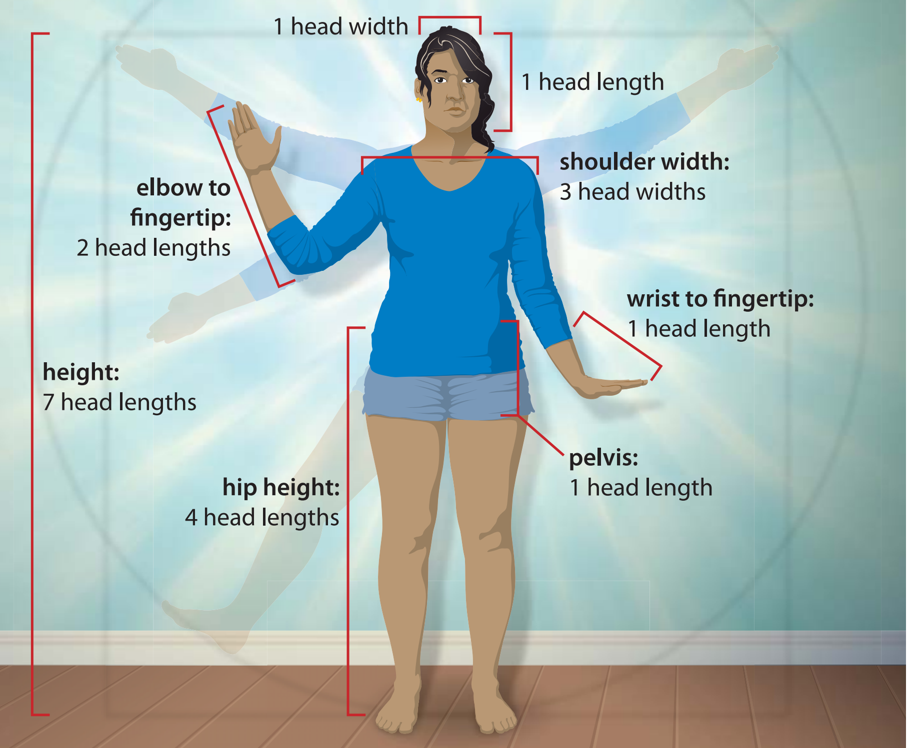
c) $y = x^2$

d) $y = 120 + 60x$

e) $y = 7 - 0.5x$

f) $y = 10\,000 - 1000x$

3.1 Understanding Linear Trends and Relationships



Focus On . . .

- creating tables of values and graphs, including scatter plots with lines of best fit
- identifying and describing linear trends
- determining whether a relationship is linear or non-linear
- solving problems involving linear trends and relationships

A number of patterns and relationships exist in nature, even within our own bodies. For example, there is a relationship between the length of a person's head and his or her height.

Explore a Linear Relationship

Materials

- measuring tape
- grid paper

1. Work with a partner.

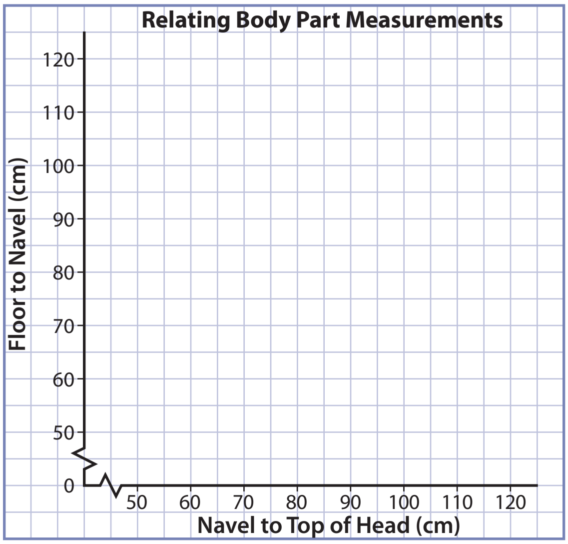
a) Measure the distance from your navel to the top of your head, to the nearest centimetre.

b) Measure the distance from the floor to your navel, to the nearest centimetre.

2. a) Record the data from each member of the class in a table of values.

Navel to Top of Head (cm)	Floor to Navel (cm)

b) Display the data in the table of values in a scatter plot.



F.Y.I.

Notice that each axis has a break from 0 cm to 50 cm. This is so the graph can start at 0 but also have a reasonable scale of 5 and because there are no points with coordinates less than 50 cm.

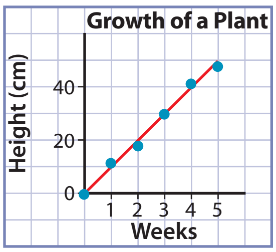
3. Reflect Analyse the pattern in the points on the scatter plot.

a) Draw a **line of best fit** on the graph to represent the pattern of the points. Make sure about half the points are on or above the line, and about half are on or below the line.

b) Use the line of best fit to help you describe the trend in the relationship between the two measurements.

line of best fit

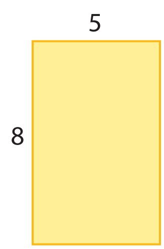
- a straight line that represents a trend in a scatter plot that follows a linear pattern. For example,



4. Extend Your Understanding The ratio of the distance from floor to navel compared to the distance from navel to top of head is an example of the golden ratio.

a) Measure the length and width of several rectangular items in your classroom, such as books, desktops, screens, and posters.

b) Which items have dimensions that are in the same proportion as the golden ratio?



F.Y.I.

The golden ratio is approximately 8 to 5 or 1.6 to 1. It is considered to be naturally pleasing to the eye. The golden ratio exists in nature, art, and design.

On the Job 1

Create a Scatter Plot to Determine a Linear Trend

F.Y.I.

The term *correlation* can be used instead of *trend* to describe a relationship between two variables.

To observe growth or behaviour patterns, scientists measure and tag birds and other animals. Mario measures the height and wingspan of 12 geese. He wonders if there is a trend in the relationship between the two variables that will allow him to make a reasonable prediction of the wingspan when he knows the height.

Height (cm)	77	84	105	95	106	82	88	90	102	90	84	98
Wingspan (cm)	129	140	176	155	176	140	149	151	175	148	138	161

a) Describe how to set up the axes of a graph to display the data. Explain why you have chosen this approach.

b) Create a scatter plot of the data. What do you notice about the pattern in the points?

c) Describe the trend in the relationship between the two variables. Is it positive or negative, or is there no trend?

d) Draw a line of best fit. Describe how well the line represents the trend in the relationship between the variables.

e) Use the graph to predict the wingspan of a goose that is 1 m tall.

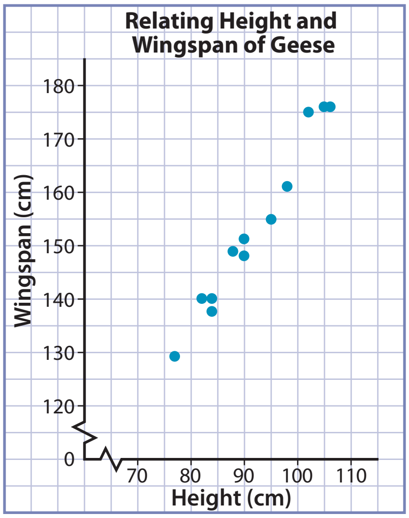
Solution

a) Height is along the horizontal axis since it is the independent variable. A break in the horizontal axis from 0 cm to 70 cm means it can start at 70 cm and go up to 110 cm in intervals of 5 cm.

Height is the independent variable, since Mario is investigating whether he can use height to predict wingspan.

Wingspan, the dependent variable, is along the vertical axis. A break in the axis from 0 cm to 120 cm means it can start at 120 cm and go up to 180 cm in intervals of 5 cm.

b) The points are in an approximate linear pattern that goes upward from left to right.

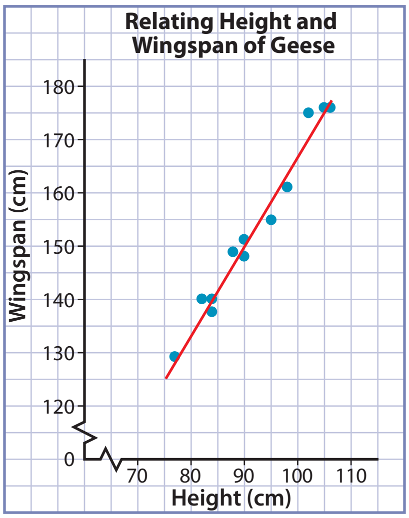


linear trend

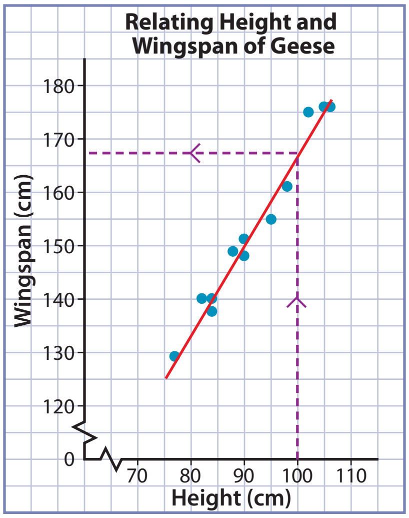
- a trend in which the relationship between two variables follows a linear pattern:
  - The trend is positive when one variable increases as the other variable also increases.
  - The trend is negative when one variable increases as the other variable decreases.

c) There appears to be a positive **linear trend** in the relationship—as the height of a goose increases, so does the wingspan, in an approximate linear pattern.

d) The line of best fit is a good representation of the trend in the relationship, since about half the points are on or above the line and half are on or below the line. As well, most of the points are on or close to the line.



e) The wingspan of a goose that is 1 m, or 100 cm, tall will be about 167 cm.



Even though the graph is set up to predict the wingspan (dependent variable), you can also use the graph to predict the height if you know the wingspan.

Your Turn

Six puffins were measured, tagged, and released to investigate whether there is a trend in the relationship between the wingspan of a puffin and its height. The data were recorded in this table of values.

Height (cm)	29	28	26.5	30.5	32	31
Wingspan (cm)	53	51	50	54	55	54.5

a) Create a scatter plot of the data. What do you notice about the pattern in the points?

b) Describe the trend in the relationship between the variables.

c) Draw a line of best fit. Describe how well the line represents the trend in the relationship.

d) Use the graph to predict the wingspan of a puffin that is about 24 cm tall.

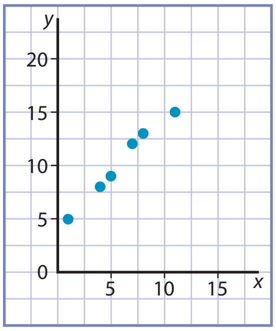
e) Use the graph to predict the height of a puffin that has a wingspan of about 58 cm.

Check Your Understanding

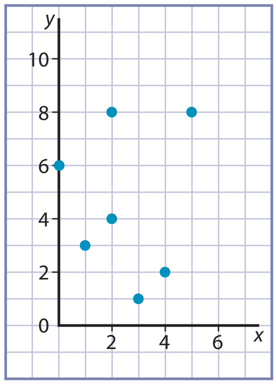
Try It

1. State whether each scatter plot shows a linear trend between the variables. If so, is the trend positive or negative?

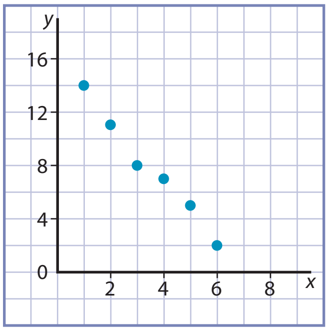
a)



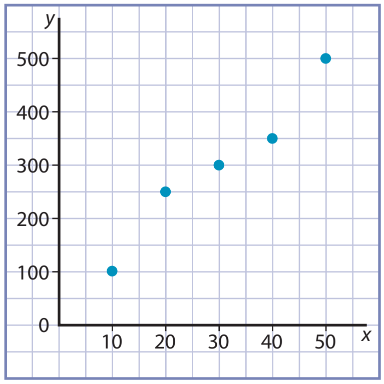
b)



c)



d)



2. Create a scatter plot of each set of data. Then, state whether each scatter plot shows a linear trend between the variables. If so, is the trend positive or negative?

a)

x	0	1	3	4	6	9
y	1	2	3	4	5	6

b)

x	0	2	4	6	8	10
y	10	10	7	4	3	1

c)

x	0	1	2	3	4	9
y	1	7	3	3	8	2

d)

x	10	8	6	4	2	0
y	2	4	5	8	9	10

Tools of the Trade

Personal fitness trainers use tools to measure heart rate, body fat, strength, endurance, and flexibility. They track these measurements to show whether the person's fitness is improving.

To learn more about the tools used by personal trainers, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Apply It

3. One model of fitness training suggests that a person's maximum target heart rate during exercise is related to their age. A trainer collects data from a group of clients with a wide range of ages to decide whether they were exercising at an appropriate level.

Age (years)	Maximum Target Heart Rate (beats/min)
25	195
35	190
45	170
55	170
65	155
75	150

a) Create a scatter plot of the data. What do you notice about the pattern in the points?

b) Describe the trend in the relationship between age and maximum target heart rate.

c) Draw a line of best fit. Describe how well the line represents the trend in the relationship.

d) Assume the graph is a reasonable representation of the recommended maximum target heart rate for the ages shown. Use the graph to predict

i) the maximum target heart rate for someone who is 30 years old

ii) the age of someone with a recommended maximum target heart rate of 175 beats/min

4. Ten books in a bookstore were selected at random to see whether the number of pages is a good predictor of cost. The table below compares the number of pages in the book to its cost, rounded to the nearest dollar.

Number of Pages	255	32	505	454	156	314	402	198	30	224
Cost ($)	16	7	30	26	12	30	22	10	11	19

a) Create a scatter plot of the data in the table. Then, draw a line of best fit.

b) Describe the trend in the relationship between the variables.

c) Use the graph to predict

i) the cost of a 400-page book

ii) the number of pages in a book that costs $25

5. Jason left Port-aux-Basques, NL, and travelled across Newfoundland on the Trans-Canada Highway. He recorded his total distance travelled each hour.

Number of Hours	Distance (km)
1	63
2	144
3	212
4	255
5	320
6	405
7	478

a) Create a scatter plot of the data. Then, draw a line of best fit.

b) Describe the trend in the relationship between the variables.

c) Use the graph to predict

i) Jason's distance from Port-aux-Basques after $\frac{1}{2}$ hour

ii) the number of hours Jason had driven when he was 240 km away from Port-aux-Basques

On the Job 2

Comparing Linear and Non-linear Relationships

When Ry was 13 years old, his parents invested $10 000 for five years to help pay for his post-secondary education. They had the choice of a simple interest account that pays 3% annually or a compound interest account that pays 3% annually. Ry is now 18 and plans to go to college to take a two-year chef program.

a) Create a table of values for each investment. Round all values to the nearest dollar.

b) What do you notice about the number pattern in each column of the simple interest investment? How does this compare to the number patterns in the compound interest investment?

c) Graph each set of data from part a) in a scatter plot.

d) What do you notice about the pattern in the points of each investment? Is there a **linear relationship** between time and the investment value? Explain your reasoning.

e) How does each graph relate to its table of values?

f) Ry's parents chose the compound interest account. Did they choose the right investment? Explain.

linear relationship
- a direct relationship between the *y*-coordinate and the *x*-coordinate
- all the points on a graph of a linear relationship lie along a straight line

Solution

a) and **b)**

For the simple interest investment, time goes up by 1 year, while the value of the investment goes up by $300 each year.

For the compound interest investment, although time also goes up by 1 year, the value of the investment goes up by a different amount each year.

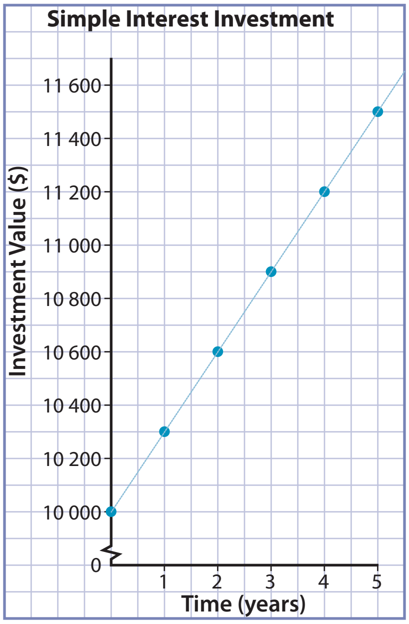
Simple Interest

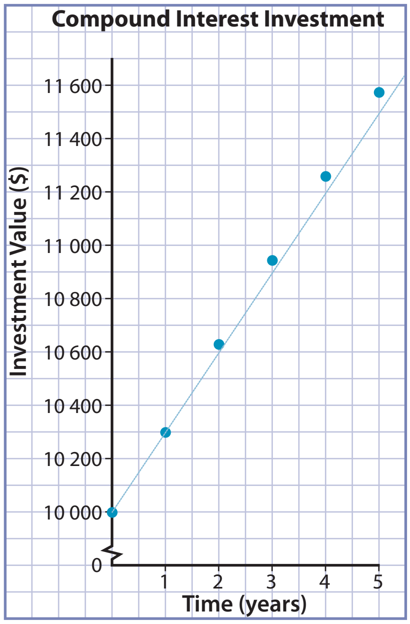
	Time (years)	Value of Investment ($)	
+1	0	10 000	+300
+1	1	10 300	+300
+1	2	10 600	+300
+1	3	10 900	+300
+1	4	11 200	+300
	5	11 500	

Compound Interest

	Time (years)	Value of Investment ($)	
+1	0	10 000	+300
+1	1	10 300	+309
+1	2	10 609	+318
+1	3	10 927	+328
+1	4	11 255	+338
	5	11 593	

c)





A straight line goes through every point.

A straight line does not go through every point.

d) For the simple interest investment, the points show a positive linear relationship between the variables, time and value. Since all the points lie exactly in a straight line, we can say there is a linear relationship.

For the compound interest investment, the points show an approximate linear pattern but they do not show a linear relationship between time and value, since the points do not lie exactly in a straight line. We can describe it as a **non-linear relationship**.

non-linear relationship

- no direct relationship between the *y*-coordinate and the *x*-coordinate
- the points on a graph of a non-linear relationship do not lie along a straight line

e) The graph of the linear relationship (the simple interest investment) goes up by a constant amount each year, just like the number pattern in the table. That is why the points lie in a straight line.

The graph of the non-linear relationship (the compound interest investment) goes up by a different and increasing amount each year, just like the number pattern in the table. That is why the points do not lie exactly along a straight line.

f) Yes, they chose the better investment. Even though the interest rates were the same, the annual compounding resulted in a greater investment after five years.

Your Turn

Ferial is taking an automotive technology course at college. She is doing a project to investigate how gas usage is affected by driving patterns; for factors such as stopping and starting at stop signs and traffic lights; traffic; and driving at various speeds. She drove her family's car the same distance and for the same amount of time along two different routes. She discovered that route 2 used more gas than route 1.

Route 1

Time (min)	Distance (km)
0	0
1	1.3
2	2.6
3	3.9
4	5.2
5	6.5

Route 2

Time (min)	Distance (km)
0	0
1	2.2
2	3.0
3	3.0
4	3.1
5	6.5

a) What do you notice about the number pattern in each column of the table for route 1? How does this compare to the number patterns in the table for route 2?

b) Graph each set of data from part a) in a scatter plot.

c) Is there a linear relationship between time and distance for each route? Explain your reasoning.

d) How does each graph relate to its table of values?

e) Route 2 used more gas than route 1. Why do you think that happened, even though the same car was used and the overall distance and time were the same?

Check Your Understanding

Try It

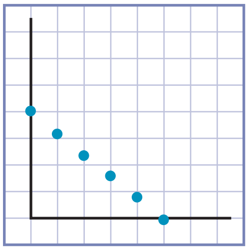
Strategy

Develop a Strategy

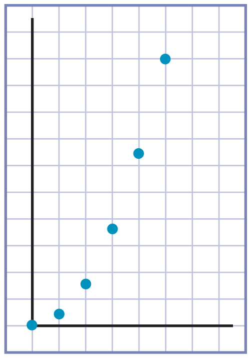
What strategy can you use to determine whether the relationship in a scatter plot is linear? Compare your strategy with a classmate's.

1. State whether the relationship shown in each scatter plot is a linear relationship or a non-linear relationship. Explain your reasoning.

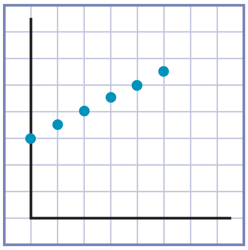
a)



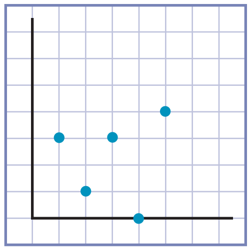
b)



c)



d)



2. Choose a graph from #1 that could match each situation. Explain your reasoning.

a) the value of a simple interest investment over time

b) the area of a square as the side length increases

c) the balance of a savings account after regular withdrawals of the same amount

d) the number of people walking along a trail each day for five days

3. Calculate the difference between successive values in each row of each table of values. State whether each table represents a linear relationship or a non-linear relationship.

a)

x	0	1	2	3
y	2	3	4	5

b)

x	0	1	2	3
y	0	1	4	9

c)

x	0	1	2	3
y	12	9	6	3

d)

x	0	1	3	6
y	20	50	30	35

Apply It

4. A skydiver jumps from an airplane. The table shows the distance that she falls during each second of her skydive. The values are rounded to the nearest metre.

Time (s)	Distance (m)
0	0
1	5
2	19
3	42
4	74
5	115

a) Do the data show a linear relationship or a non-linear relationship? Explain your reasoning.

b) Graph the data. Does the graph support your answer to part a)?

5. Jordin earns minimum wage at her part-time job at the concession stand in an arena.

a) Create a table of values to show how much she earns altogether for up to 5 h on a game night. Then, graph your data.

b) Is the relationship between earnings and number of hours worked linear or non-linear?

c) How does the table support your answer in part b)?

d) How does the graph support your answer in part b)?

6. Computers and other electronics can depreciate by 50% or more of the original value each year.

a) A new laptop sells for $1600. Create a table of values for the first four years of the laptop's life, assuming a depreciation rate of 50% each year.

b) Is the relationship between the value of the laptop and its age linear or non-linear? How does the table support your answer?

F.Y.I.

When skydivers leave the plane, their speed increases over time. They accelerate downward due to gravity, even though air resistance acts against this acceleration.

Web Link

To find out what the minimum wage is in your province, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

F.Y.I.

For a depreciation rate of 50%/year, if an item cost $100 new, it would be worth 50% less, or $50, after 1 year and 50% less, or $25, after 2 years, and so on.

Work With It

1. A coffee shop sells coffee in four different-sized cups for the prices shown.

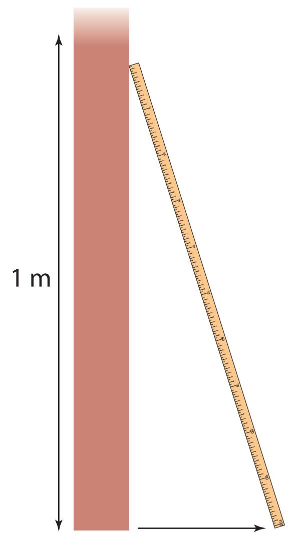
Cup Size (fl oz)	Price ($)
10	1.25
14	1.50
20	1.60
24	2.00

   a) Graph the data in a scatter plot.

   b) Is there a linear trend in the relationship between price and cup size? Explain your reasoning.

Materials
- 3 metre sticks
- grid paper

2. **MINI LAB** The top of a metre stick standing up against a wall reaches 1 m up the wall. As the bottom end of the metre stick is moved away from the wall, the height of the top end against the wall decreases.



STEP 1

Work with a partner. Stand a metre stick upright against a wall.

STEP 2

a) Create a table of values like the one shown to relate the height of the top of the metre stick to the distance between the bottom of the stick and the wall.

My Predictions

Distance Between Bottom of Stick and Wall (cm)	0	10	20	30	40	50	60	70	80	90	100
Height of Top of Stick (cm)	100										0

b) Predict if the relationship between the two variables will be linear or non-linear. Explain your prediction.

c) Complete the bottom row of the table with your predictions.

STEP 3

a) Starting with the metre stick upright against the wall, move the bottom end 10 cm away from the wall. Measure the height of the top of the metre stick.

b) Move the bottom end 10 cm farther away from the wall. Measure the height of the top of the metre stick. Repeat until the metre stick is lying on the floor.

c) Create a table of values like the table in step 2 to record your measurements.

STEP 4

a) Graph the results from step 3.

b) Does the graph support your predictions in step 2?

F.Y.I.

The formula $A = \pi r^2$ shows how the radius of a circle is related to its area.

Diameter is $2 \times$ radius ($d = 2r$).

3. This table of values shows the relationship between the area of a circle (to the nearest square inch) and its diameter.

Diameter (in.)	Area (in.^{2})
1	1
2	3
3	7
4	13
5	20

a) Graph the data in the table.

b) Is the relationship between area and diameter linear or non-linear?

c) Explain how the graph supports your answer to b).

d) Explain how the table of values supports your answer.

Discuss It

F.Y.I.

A depreciation rate of 10%/year means that the item retains 90% of its value each year. So, if the item is $100 new, after 1 year it will be worth $90, which is 10% less ($10 less) than $100. After 2 years, it will be worth $81, which is 10% less ($9 less) than $90.

4. Suppose you calculated the differences between successive values of the variables in a table of values, such as the one in the Mini Lab. How might that help you decide whether the relationship between the two variables is linear?

5. Item A is $100 new and depreciates by $10 each year. Item B is $100 new and depreciates at a rate of 10% each year. Suppose you were to graph the value of each item over 10 years. How will the graphs be the same? How will they be different?

6. All linear relationships are linear trends but not all linear trends are linear relationships. Explain what this means. Use examples to help you explain.

7. All graphs of linear relationships are the same in one way—the points are all in a straight line. Why is it not possible to describe the graphs of all non-linear relationships in one way?

3.2 Direct Variation

Focus On . . .

- understanding why a relationship is a direct variation
- modelling a direct variation relationship with a table of values, a graph, or an equation
- solving problems involving direct variation relationships

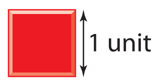
If you are paid by the hour, there is a direct relationship between the number of hours you work and your gross pay. For example, if you work twice the number of hours, you get twice the amount of gross pay.

Materials

- square tiles in 2 colours, or grid paper

Explore a Direct Variation Relationship

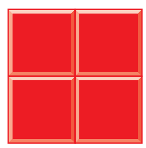
1. a) Place one square tile in front of you.



b) The side length of one square is 1 unit. Calculate the perimeter of the tile in units.

c) What is the value of the perimeter divided by the side length?

2. a) Place four tiles in front of you in a 2-by-2 square.



b) What is the length of one side of the 2-by-2 square in units?

c) Calculate the perimeter of the tile in units.

d) What is the value of the perimeter divided by the side length?

3. a) Continue in this manner for squares up to a 5-by-5 square.

b) Record your results in a table like the one shown.

	Side Length, *s* (units)	Perimeter, *P* (units)	$P \div s$
0-by-0 square			
1-by-1 square			
2-by-2 square			
3-by-3 square			

Strategy
Look for a Pattern

4. Suppose you were to graph the relationship between side length and perimeter. Predict the slope and y-intercept of the graph. Explain your prediction.

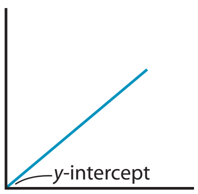
5. a) Create a scatter plot of the data in the table.

b) Draw a line through the points.

c) Determine the slope and y-intercept of the line. Were your predictions correct?

$\text{Slope} = \frac{\text{rise}}{\text{run}}$
To calculate the slope of a line, determine how much the line rises or drops (the vertical distance) for each run (horizontal distance) of 1 unit from left to right.

direct variation
- a linear relationship in which one variable is always a fixed multiple of the other variable, e.g., the y-value is always 3 times the x-value
- in a graph of a direct variation relationship, the slope of the line is the fixed multiple, and the y-intercept is always zero



6. **Reflect** Analyse the table of values and the graph.

a) How does the table of values show that the relationship is linear?

b) How does the table show that the relationship is a **direct variation**?

7. **Extend Your Understanding** A square is a four-sided regular polygon. Apply what you have learned to other regular polygons.

a) What is the perimeter of a regular pentagon with a side length of $2\frac{1}{2}$ in.?

b) What is the side length of a regular hexagon with a perimeter of 24 in.?

F.Y.I.

In a regular polygon, all sides are the same length and all angles have the same measure. A pentagon has five sides. A hexagon has six sides.

initial value

- the value of the dependent variable when the independent variable is zero
- in a direct variation relationship, the initial value is always zero

rate of change

- the amount by which the dependent variable changes when the independent variable increases by 1 unit
- in a direct variation relationship, the rate of change is constant

F.Y.I.

The official symbol used for hours, according to the Canadian Standards Association, is h, but you will often see hrs or hr. used informally.

On the Job 1

Model a Direct Variation Relationship With a Table of Values

The dispatcher of a school bus company is monitoring the local weather forecast to decide whether to cancel school buses for the next day. At 9:00 p.m., there is no snow on the ground but it has started snowing. It is predicted that 2 cm of snow will fall each hour through the night and into the next morning.

a) What is the **initial value**, that is, the amount of snow on the ground at 9:00 p.m. (0 h)?

b) What is the expected **rate of change** in the depth of snow each hour?

c) Create a table of values for 0 h to 10 h of snowfall. How does your table show the initial value and rate of change for the relationship?

d) How does it show that the relationship has direct variation?

e) Predict what a graph of the table of values would look like.

f) What time will it be when there are 12 cm of snow on the ground?

g) The decision to cancel buses must be made by 6:00 a.m. How much snow will be on the ground at that time?

Solution

a) The initial value is 0 cm of snow at 9:00 p.m., or 0 h.

b) The rate of change is 2 cm each hour.

c)

Time (h)	Depth of Snow (cm)
0	0
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18
10	20

initial value

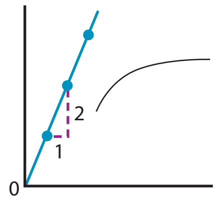
rate of change

+1 ... +2

d) In a direct variation relationship, the initial value is zero, and one variable is a fixed multiple of the other because the variables vary directly.

- The table shows that the initial value is zero: the depth is 0 cm at 0 h.
- The table shows that the depth of the snow is a fixed multiple of the time. For example, at 2 h, there is 2 times as much snow as at 1 h and at 6 h, there is 2 times as much snow as at 3 h.

e) The graph would be points in a line going upward starting at zero on the y-axis, the initial value, and the slope would be the rate of change per hour, which is 2.



$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$$

f) There should be about 12 cm of snow 6 h after the snow started, which is 3:00 a.m.

g) 6:00 a.m. is 9 h after it started snowing. There should be about 18 cm of snow.

Your Turn

In another school district, the snow does not begin to fall until 11:00 p.m., and 3 cm of snow is predicted to fall each hour. There is no snow on the ground before 11 p.m.

a) What is the initial value of the depth of snow at 11:00 p.m. (0 h)?

b) What is the expected rate of change in the depth of snow?

c) Create a table of values for the first 10 h. Explain how your table shows the initial value and the rate of change.

d) Explain how it shows that the relationship is a direct variation.

e) Predict what a graph of the table of values would look like.

f) What time will it be when there are 12 cm of snow on the ground?

g) The decision to cancel buses must be made by 6:00 a.m. How much snow will be on the ground at that time?

Strategy

Develop a Strategy

What strategy can you use to determine if a relationship in a table of values is linear? What strategy can you use to determine if it is a direct variation? Compare strategies with a classmate's.

Check Your Understanding

Try It

1. Which tables of values represents a direct variation relationship? How do you know?

A

Time (h)	Cost ($)
0	0
1	15
2	30
3	45
4	60

B

Time (min)	Degrees (°C)
0	8
1	10
2	12
3	14
4	16

C

x	*y*
0	0
1	1
2	4
3	9
4	16

D

Time (days)	Earnings ($)
1	20
2	40
3	60
4	80
5	100

2. Each table of values represents a linear relationship with direct variation. Determine the missing values.

a)

Time (s)	Distance (km)
0	0
1	80
2	160
3	■
4	320

b)

Number of Games	Number of Points
0	0
1	■
2	8
3	■
4	16

c)

Time (min)	Number of Words
0	■
1	■
2	70
3	105
4	■

d)

Time (years)	Interest ($)
0	■
1	7.50
2	■
3	22.50
4	■

3. What are the initial value and rate of change for each relationship in #2?

Apply It

4. At a restaurant, toppings on a pizza cost \$1.75 each. Create a table of values that shows the total cost of 0 to 5 toppings. How do you know the relationship between cost and number of toppings is a direct variation?

Web Link

In the 2010–2011 season, the highest NHL salaries belonged to Vincent Lecavalier and Roberto Luongo. Each earned an average of \$10 million that season.

For more data on NHL player salaries, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

5. From the start of the 1990–1991 season to the end of the 1992–1993 season, Wayne Gretzky earned \$3 million per year.

a) Predict whether the relationship between his cumulative earnings over the three years and the number of years is a direct variation. Explain your prediction.

b) Create a table of values that shows his cumulative earnings each year for the first three years of the contract. Does your table support your prediction in part a)?

c) Gretzky's contract was extended through to the end of the 1993–1994 season. What were his cumulative earnings over the life of the four-year contract?

6. A flooring retailer sells carpet runners for \$12.99 per metre. The store manager wants to post a chart in the store for customers to refer to. Create a table of values that shows the cost of runners in full metres from 1 m to 10 m.

7. Jake pays a \$20 sign-up fee and then \$25 at the end of each month for a gym membership. Create a table of values to show the cumulative amount Jake will pay for his membership each month for the first six months. How do you know that this is not a direct variation relationship?

On the Job 2

Model a Direct Variation Relationship With a Graph

Kelly works part-time at an ice cream shop. She writes down her hours and her earnings for each of her first four weeks on the job.

	Number of Hours	Money Earned
Week 1	12	$132
Week 2	8	$88
Week 3	17	$187
Week 4	15	$165

a) What is Kelly's hourly rate of pay?

b) Create a scatter plot of the data. Describe the pattern of the points.

c) Do you think it is reasonable to draw a straight line through the points to connect them and to represent the relationship? Why? If it is reasonable, draw the line.

d) What are the slope and *y*-intercept of the graph?

e) Use the graph to estimate how many hours Kelly would have to work to earn $200.

f) Explain how the graph shows that the relationship between number of hours worked and earnings is a direct variation.

Solution

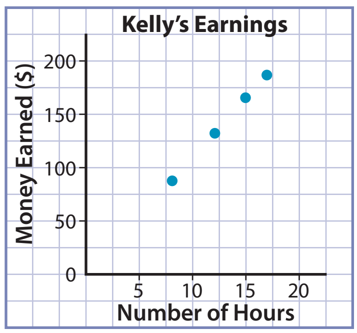
F.Y.I.

If a different rate of pay was calculated for any of the weeks, it might show that Kelly worked overtime at a higher rate of pay, or that her rate of pay changed over the four weeks.

a)

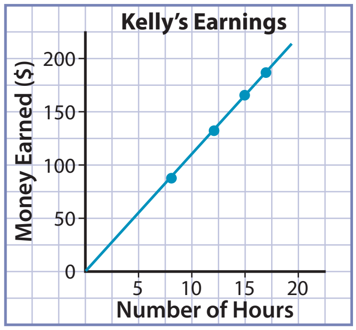
	Number of Hours	Money Earned	Rate of Pay (money earned ÷ number of hours)
Week 1	12	$132	$11/h
Week 2	8	$88	$11/h
Week 3	17	$187	$11/h
Week 4	15	$165	$11/h

b) The points follow a straight line that shows that as the hours increase, so does the money earned.



> **Strategy**
> **Develop a Strategy**
> What strategy can you use to determine if data are discrete or continuous? Compare your strategy with a classmate's.

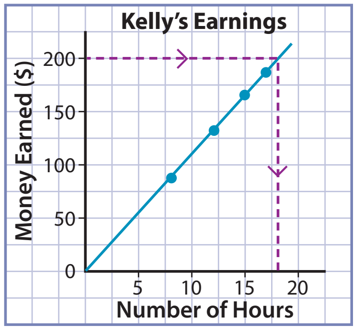
c) Yes, it is reasonable to connect the points with a solid straight line and extend it down to 0 h and beyond 17 h. The solid line shows that the relationship is continuous. It represents the relationship between Kelly's earnings and *any* number of hours or partial hours worked, including partial hours such as $15\frac{1}{4}$ h or $15\frac{1}{2}$ h.



> **Strategy**
> **Develop a Strategy**
> What strategy can you use to determine the slope? Compare your strategy with a classmate's.

d) Each hour, the graph goes up by \$11, so the slope is 11. The y-intercept is 0, at the origin.

e) Kelly would have to work about 18 h to earn \$200.



f) The graph shows that the relationship is a direct variation.

- The graph is linear because the slope, which is Kelly's hourly pay rate, is fixed at \$11/h.
- The graph intercepts the y-axis at 0, since the dependent variable is \$0 when the independent variable is 0 h.
- The y-value is always 11 times the x-value.

Your Turn

After three months at the ice cream shop, Kelly receives a pay raise to \$11.50/h. However, from now on, all employees will be scheduled to work for only whole numbers of hours.

a) Create a scatter plot using Kelly's new rate of pay for 0 h to 17 h of work.

b) Explain why you cannot draw a solid or dashed line through the points.

c) What are the slope and y-intercept of the graph?

d) Use the graph to estimate how many hours Kelly would have to work to earn \$200.

e) Compare the graphs of Kelly's two pay rates. How are they the same and different?

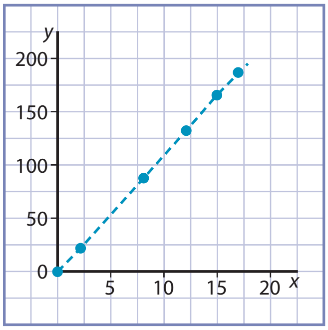
> **F.Y.I.**
> *For discrete data, a dashed line can be drawn through the points if some of the values between the plotted points are valid. If no values between plotted points are valid, the points are not connected with a line.*

Check Your Understanding

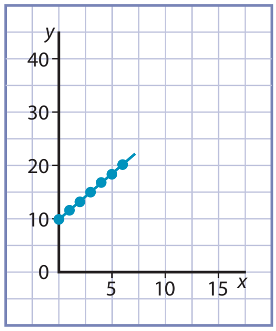
Try It

1. Which graphs represent a direct variation relationship? How do you know?

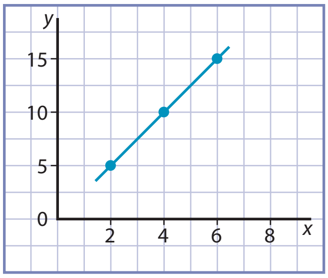
A



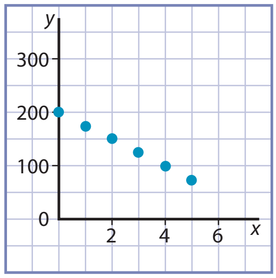
B



C



D



Strategy

Develop a Strategy

What strategy can you use to determine if a graph represents a relationship that has direct variation? Compare your strategy with a classmate's.

2. Sketch two different graphs for each.
   - **a)** a direct variation relationship
   - **b)** a relationship that does not have direct variation

Apply It

3. Sam earns \$8/h babysitting.
   - **a)** Create a table of values to show his total earnings for 0 h to 5 h of babysitting.
   - **b)** What is the rate of change?
   - **c)** Graph the relationship between Sam's total earnings and the number of hours of babysitting.
   - **d)** Is it reasonable to draw a solid line from the origin to connect the points? Explain your thinking.
   - **e)** What are the slope and y-intercept of the graph?
   - **f)** Use the graph to estimate how many hours Sam has to babysit to earn \$20.
   - **g)** Suppose he earned \$9/h. How would the graph compare to the one in part c)?

4. Each minute of a song in MP3 format takes up about 1.4 MB of storage space.
   a) Create a table of values to show the amount of storage space needed for each minute of music, for 0 min to 10 min.
   b) Graph the results from part a) in a scatter plot.
   c) Connect the points with a solid or dashed line, and explain your choice.
   d) What are the slope and y-intercept of the graph?

5. Darryl is an occasional driver of his parents' car. The monthly cost of insurance for Darryl for the next year will be $115, provided Darryl maintains a good driving record.

Month	Cumulative Insurance Costs ($)
1	115
2	230
3	

   a) Create a table of values that models his cumulative monthly insurance costs over one year.
   b) Graph the results from part a) in a scatter plot. Connect the points with a solid or dashed line, and explain your choice.
   c) Meagan, Darryl's sister, is also an occasional driver, but she pays only $60 a month. On the same grid, graph Meagan's cumulative monthly insurance costs for one year.
   d) Compare the two graphs. How are they the same? How are they different, and why?

6. Jenna repairs appliances in people's homes. She charges a $50 service call fee and then $25/h labour.
   a) Suppose you graphed the amount Jenna would charge altogether for 0 h, 1 h, 2 h, 3 h, and 4 h. Predict what the graph will look like.
   b) Is this a direct variation relationship? Explain your reasoning.

On the Job 3

Model a Direct Variation Relationship With an Equation

Brian owns a repair garage. He has four cars scheduled for service today. Brian charges $72 an hour for labour. He estimates how long each job will take using an online automotive database. He estimates that the four jobs will take 1.3 h, 2.0 h, 1.8 h, and 2.5 h.

a) Write an equation to model the relationship between Brian's labour charge and the number of hours the job takes.

b) Use the equation to determine

  i) the labour cost for each of the four jobs

  ii) the total labour charged for the day

c) Predict what a graph of the relationship would look like.

d) How does the graph relate to the equation?

Solution

a) The relationship between Brian's labour charge and the number of hours a job takes can be modelled with the equation $L = 72t$, where L is the total labour cost, in dollars, and t is the time required to do the job, in hours.

> The equation $L = 72t$ makes sense, because
> Total labour cost
> $= \$72 \times$ total number of hours the job takes

F.Y.I.

An equation that represents a mathematical relationship is also called a formula.

b) **i)** Labour cost for each job:

Job 1: $L = 72t$
$L = 72 \times 1.3$
$L = 93.6$

The labour cost for the 1.3-h job is $93.60.

Job 2: $L = 72t$
$L = 72 \times 2.0$
$L = 144$

The labour cost for the 2-h job is $144.00.

Job 3: $L = 72t$
$L = 72 \times 1.8$
$L = 129.6$

The labour cost for the 1.8-h job is $129.60.

Job 4: $L = 72t$
$L = 72 \times 2.5$
$L = 180$

The labour cost for the 2.5-h job is $180.00.

Strategy

Develop a Strategy

A relationship can be represented by an equation, a table of values, or a graph. Sometimes one model is better than another for solving a particular problem. When might an equation be the best model for solving a problem?

ii) Total labour charged for the day:

$L = 72t$

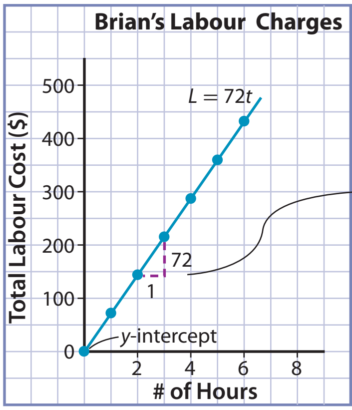
$L = 72 \times (1.3 + 2.0 + 1.8 + 2.5)$

$L = 72 \times 7.6$

$L = 547.2$

The total cost for labour for the day (7.6 h) is \$547.20.

c) The graph would be points in a straight line going upward with a solid line through them. The line would meet the y-axis at 0 and have a slope of 72.



$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{72}{1} = 72$$

d) The slope is 72, which is the number that you multiply the variable t (number of hours) by in the equation to get L (the total labour cost).

Your Turn

Brian plans to raise his labour rate to \$75 an hour.

a) Write an equation to model the new relationship between Brian's increased labour charge and the number of hours a job takes.

b) Use the equation to determine

i) the labour cost for each job (1.3 h, 2.0 h, 1.8 h, and 2.5 h)

ii) the total labour cost for the day

c) Predict how the graph of the \$75/h relationship will compare to the graph of the \$72/h relationship.

d) How does the new graph relate to the new equation?

e) How much more would the total labour charge be for the day at \$75/h compared to \$72/h?

Check Your Understanding

Try It

1. a) Copy and complete the table of values.

t	*C* ($C = 11t$)
0	
1	
2	
3	

b) What do you notice about the number patterns in each column?

c) Does the equation $C = 11t$ model a relationship with direct variation? How do you know?

2. a) Substitute the values $x = 0, 1, 2,$ and 3 into the equation $y = 4x$ and solve for y.

b) Does the equation $y = 4x$ model a relationship with direct variation? Explain why or why not.

c) Predict what a graph of the relationship between x and y in the equation $y = 4x$ would look like.

Strategy

Develop a Strategy

What strategy can you use to determine if an equation represents a direct variation relationship? Compare your strategy with a classmate's.

3. a) Substitute the values $s = 0, 1, 2,$ and 3 into the equation $A = s^2$ and solve for A.

b) Does the equation model a relationship with direct variation? Explain how you know.

4. a) Substitute the values $t = 0, 1, 2,$ and 3 into the equation $d = 50t + 5$ and solve for d.

b) Does the equation model a relationship with direct variation? Explain how you know.

5. Which equations represent a relationship with direct variation?

A $P = 4s$

B $d = 80t$

C $y = 3x$

D $y = 2$

E $C = 70 + 65t$

F $y = \frac{1}{2}x - 3$

G $E = 12.5h$

H $y = x$

Apply It

6. A common price for downloadable apps is 99¢.

a) Write an equation that can be used to determine the total cost of buying apps, *I*, if you know the number of downloads, *d*.

b) Use your equation to calculate the total cost for 0, 1, 2, and 3 downloads.

c) Does the equation model a direct variation relationship? Explain.

d) Use your equation to determine the total cost for

i) 10 000 downloads

ii) 250 000 downloads

iii) 1.45 million downloads

7. The cost to a customer at a bulk food store depends on how much the customer buys and the unit cost of the item purchased. One type of almond snack sells for $2.67/100 g.

F.Y.I.

1 kg = 1000 g

1 g = 0.001 kg

a) Express the cost of the almond snack per kilogram.

b) Write a formula that might be programmed into the store's cash register to calculate the cost of the almond snack if you enter the number of kilograms purchased.

c) Determine the cost of 312 grams of the snack.

d) Predict what a graph of the equation would look like. Explain your thinking.

8. The amount of money raised for a 10-km walkathon varies directly with the number of kilometres walked. Renee collected a total of $86 for each whole kilometre that she walks.

a) Write an equation that Renee can use to calculate how much money she has raised altogether, once she knows how many kilometres she has walked.

b) Use the equation to calculate how much she would raise if she walked 8 km.

c) Renee ended up collecting $774. Use the equation to determine how far she walked.

9. For each problem in #6, #7, and #8, why is the equation a better model for solving the problems than a table of values or a graph?

Work With It

1. Depending on your size and speed, cross-country skiing burns around 500 calories per hour.
   a) Create a table of values to show the total calories burned from 0 h to 4 h.
   b) Use the table to determine the calories burned in $3\frac{1}{2}$ h of skiing.
   c) How many calories would be burned in 5 h of skiing? How do you know?
   d) How do you know the relationship between the calories burned and the amount of time skiing is a direct variation?

2. An electricity provider charges 10¢ per kWh (cents per kilowatt-hour) of energy used during the peak time, which is between 7 a.m. and 7 p.m. Monday to Friday.
   a) Create a table and a graph to show the total cost for 0 kWh to 10 kWh.
   b) During off-peak time, the cost is 5.9¢/kWh. On the same grid, create a graph to show the total cost for 0 kWh to 10 kWh during off-peak time.
   c) Use the graphs to estimate the amount of money saved when using 5 kWh of electricity during off-peak time compared to peak time.

3. Moham works in a clothing store and earns 8% commission. This means that he earns 8¢ for every dollar of merchandise he sells.
   a) Write a formula that relates his commission earnings, *E*, to his sales in number of dollars, *d*.
   b) Predict what a graph of the relationship would look like.
   c) Use the formula to determine his earnings each week in April.

Week Starting Monday	Total Sales ($)
April 3	8267
April 10	7330
April 17	4915
April 24	5084

Tools of the Trade

Police officers use radar guns to measure the speed of a vehicle. The radar speed gun was invented in 1954. To learn more about radar guns, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

F.Y.I.

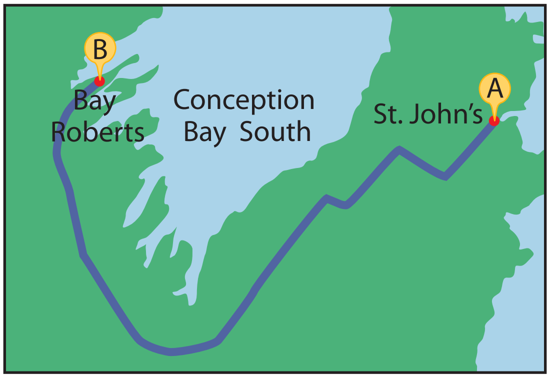
The formula $C = \pi d$ shows how the diameter of a circle is related to its circumference.

The formula $A = \pi r^2$ shows how the radius of a circle is related to its area.

Diameter is $2 \times$ radius ($d = 2r$).

Discuss It

4. Suppose you graphed the relationship between the distance travelled over time for a drive at a steady speed from downtown St. John's to a cabin near Bay Roberts. Would it be an example of a direct variation relationship? How do you know?



5. Suppose you measured the diameters of a number of different-sized circles and then calculated their circumferences and their areas.

a) Would the relationship between diameter and circumference be a direct variation? How do you know?

b) Would the relationship between the diameter and area be a direct variation? How do you know?

6. Bryan answered #5 by saying,

"It is a direct variation relationship because a circle with a 0-cm diameter has a 0-cm circumference."

Do you agree with Bryan? Explain your reasoning.

7. Jana graphed a direct variation relationship.

a) What do you know about the graph? What do you not know?

b) What do you know about the y-intercept?

c) Can you be sure about whether a line can be drawn through the points? Why?

8. Do you agree or disagree with this statement? Explain your reasoning.

"All direct variation graphs are linear, and all linear graphs are direct variations."

3.3 Partial Variation

Focus On . . .

- understanding why a relationship is a partial variation
- modelling a partial variation relationship with a table of values, a graph, or an equation
- solving problems involving partial variation relationships
- comparing direct and partial variation relationships

Many taxis charge an initial fixed amount plus a variable amount that depends on the length of the trip. This means that if trip A is twice as long as trip B, the cost of trip A is twice as much as the cost of trip B, plus the initial amount.

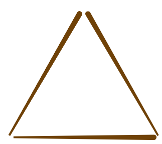
Materials

- toothpicks
- grid paper

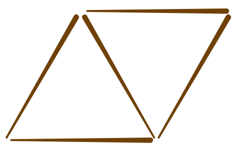
Explore a Partial Variation Relationship

1. Create a toothpick pattern.

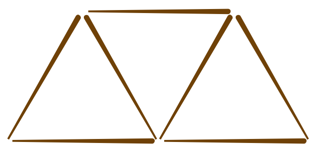
a) Arrange toothpicks to form a triangle. Count the toothpicks.



b) Add toothpicks to build a second triangle that shares a side with the first. Count the toothpicks.



c) Add toothpicks to build a third triangle. Count the toothpicks.



2. a) Write the number pattern formed by the number of toothpicks. What do you notice?

   b) Predict the number of toothpicks needed for 4, 5, and 6 triangles.

3. a) Create a table of values that relates the number of triangles to the number of toothpicks.

   b) Continue adding toothpicks to complete the table.

Number of Triangles	Number of Toothpicks
1	3
2	
3	
4	
5	
6	

Strategy
Look for a Pattern

4. a) Create a scatter plot of the data in the table.

   b) Would you draw a line, dashed or solid, to connect the points? Explain.

Recall that a dashed line shows discrete data that have values between the plotted points that are valid. If there are no valid values between plotted points, no line is drawn. A solid line shows continuous data.

   c) What are the slope and *y*-intercept of the line?

5. **Reflect** Analyse the table of values and graph.

   a) How does the table of values show that the relationship is linear?

   b) How does the table show that the relationship is a **partial variation**?

   c) How could you have predicted the slope and *y*-intercept of the graph from the table?

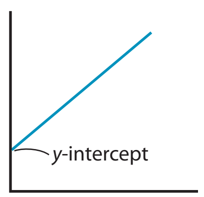
6. **Extend Your Understanding** Use what you know about the relationship to answer each question.

   a) How many toothpicks would you need to build 7 triangles? Explain.

   b) How many toothpicks would you need to build 100 triangles? Explain.

partial variation

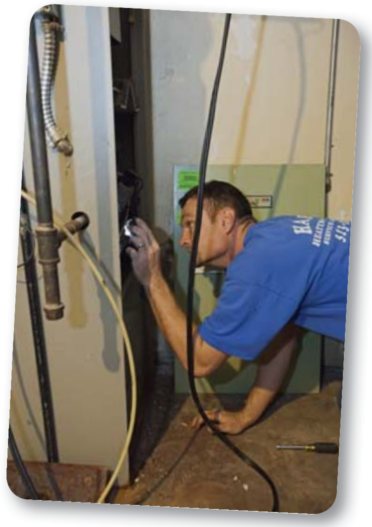
- one variable in a linear relationship is a fixed multiple of the independent variable plus a constant amount, e.g., the *y*-value is always 3 times the *x*-value plus 2
- in a graph of a partial variation relationship, the constant amount is the *y*-intercept and the fixed multiple is the slope of the line



On the Job 1

Model a Partial Variation Relationship With a Table of Values

Colton repairs heating and air-conditioning systems. When he is out on a service call, he charges a service call fee of $75 plus $65 per hour for his labour.



a) What are the initial value and the rate of change?

b) Create a table of values for the cost of 0 h to 5 h of Colton's services. How does your table show the initial value and the rate of change?

c) Predict what a graph of the table of values would look like.

d) What does Colton charge for a job that takes 1.5 h? Show your work.

e) Explain why the cost of a 2-h repair is not double the cost of a 1-h repair.

Tools of the Trade

Having the right tools is essential to repairing a furnace. Furnace repair requires tools such as a variety of screwdrivers, a drill, tin snips, duct tape, and a shop vacuum to clean up metal shavings and dust.

To learn more about the tools used furnace for repair, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Solution

a) The initial value charged is $75.

The rate of change is the hourly labour charge, $65/h.

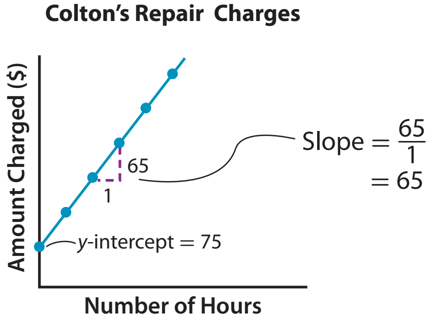
Strategy

Look for a Pattern

b)

	Time (h)	Total Amount Charged ($)	
	0	75 (initial value)	
+1			+65 (rate of change)
	1	140	
+1			+65
	2	205	
+1			+65
	3	270	
+1			+65
	4	335	
+1			+65
	5	400	

c) The graph would be points in a line going upward from left to right, with a solid line through them because the data are continuous. The line would start at 75 on the y-axis, the initial value, and the slope would be the rate of change per hour, 65.



d) The initial fixed charge is \$75, and the charge for labour is \$65/h $\times$ 1.5 h.

The total amount is $75 + (65 \times 1.5) = 172.5$.

Colton charges \$172.50 for a service call that lasts 1.5 h.

Strategy

Develop a Strategy

How could you use the table of values to determine the charge for a 1.5-h service call?

e) Since this is a partial variation relationship, one variable is not a multiple of the other. The amount charged for the labour is a multiple of the number of hours, plus the service-call fee.

1 h is $75 + (65 \times 1) = 140$

2 h is $75 + (65 \times 2) = 205$

Even though 2 h is double 1 h, \$205 is not double \$140.

Your Turn

A taxi company charges a \$5 fixed fee plus a variable charge of \$2 per kilometre.

a) What are the initial value and the rate of change?

b) Create a table of values for the first 10 km of a taxi ride.

c) Predict what a graph of the table of values would look like.

d) What is the total cost for a 20-km ride?

Check Your Understanding

Try It

Strategy

Develop a Strategy

What strategy can you use to determine if the relationship in a table of values is a partial variation? Compare your strategy with a classmate's.

1. Which tables of values represent a partial variation relationship? How do you know?

A

Time (h)	Cost ($)
0	0
1	20
2	40
3	60
4	80

B

Time (h)	Cost ($)
0	8
1	12
2	16
3	20
4	24

C

x	y
0	10
1	11
2	13
3	16
4	20

D

Sales ($)	Earnings ($)
0	200
1000	400
2000	600
3000	800
4000	1000

Strategy

Look for a Pattern

2. Each table of values represents a linear relationship with partial variation. Determine the missing values.

a)

Time (weeks)	Earnings ($)
0	400
1	440
2	480
3	■
4	■

b)

Time (h)	Depth of Snow (cm)
0	30
1	■
2	38
3	■
4	■

c)

Time (h)	Cost ($)
0	■
1	40
2	■
3	64
4	■

d)

Time (years)	Value ($)
0	1000
1	■
2	■
3	1075
4	■

3. State the rate of change and the constant or fixed amount for each relationship in #2.

Apply It

4. A large pizza with cheese and sauce is $11.50. Each additional topping is $1.50. Create a table of values showing the total cost of a pizza for 0 to 6 additional toppings. How do you know that the relationship between cost and number of toppings is a partial variation?

5. A women's size 5 shoe is usually about 9 in. long. Each next size up or down is about $\frac{1}{4}$ in. longer or shorter. Create a table of values that compares a women's shoe size to its length for sizes 4 through 10. How do you know this is an example of a partial variation?

6. A phone company offers customers a cell phone for $99 if the customer agrees to a three-year contract at $49 per month, paid at the end of each month.

a) What are the initial value and the rate of change?

b) What is the total cost to the customer at the end of the first month?

c) Create a table of values that shows the relationship between the number of months and the total cost for the first year.

d) Predict what a graph of the table of values would look like.

7. Hannah has a tree pruning business. She charges $20/h for labour, plus the $30 it costs her to rent equipment.

a) Create a table of values that Hannah can use to estimate what she will charge for up to 8 h of pruning.

b) Use the table to estimate how much Hannah will charge for a job that takes $3\frac{1}{2}$ h.

c) How would the amount Hannah charges for a $3\frac{1}{2}$ h job change in each situation?

i) She decreases the amount she charges for renting equipment to $25.

ii) She increases her hourly rate to $25.

On the Job 2

Model a Partial Variation Relationship With a Graph

Taneeka borrowed $400 from her parents to help pay for a driver training course. She needs to have a driver's licence to apply for a job that she wants after she graduates. She has an after-school job and agrees to pay her parents back at a rate of $50 at the end of each week. Taneeka is keeping track of how much she owes her parents on a weekly basis.

Week	What I Owe
0	$400
1	$350
2	$300
3	$250

a) What are the initial value and the rate of change?

b) Create a scatter plot of the data.

c) Decide whether to draw a line, dashed or solid, through the points. Explain your decision.

d) How does your graph show the initial value and the rate of change?

e) How do you know that the graph represents a partial variation relationship?

f) Suppose she paid back $80/week. How would the graph compare to the one in part b)?

Solution

a) The initial value is the \$400 that Taneeka owes to her parents.

The rate of change is −50, or −\$50/week. It is negative because the amount owing is decreasing.

Week	What I Owe
0	\$400
1	\$350
2	\$300
3	\$250

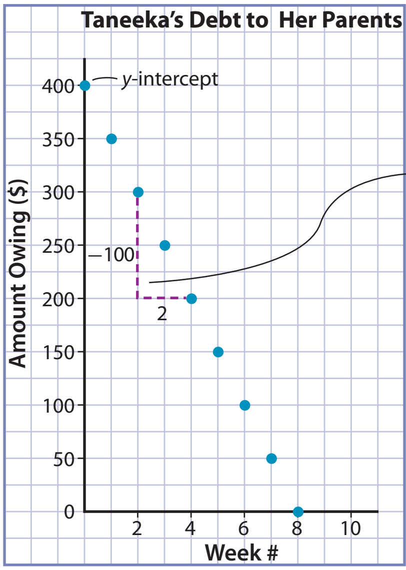
initial value

$\}$ −50 rate of change

$\}$ −50

$\}$ −50

b)



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-100}{2}$$
$$= -50$$

A negative slope means that the graph goes downward from left to right—the change in the rise is negative, while the run is positive.

c) There should be no line drawn through the points, since there are no valid values between the plotted points.

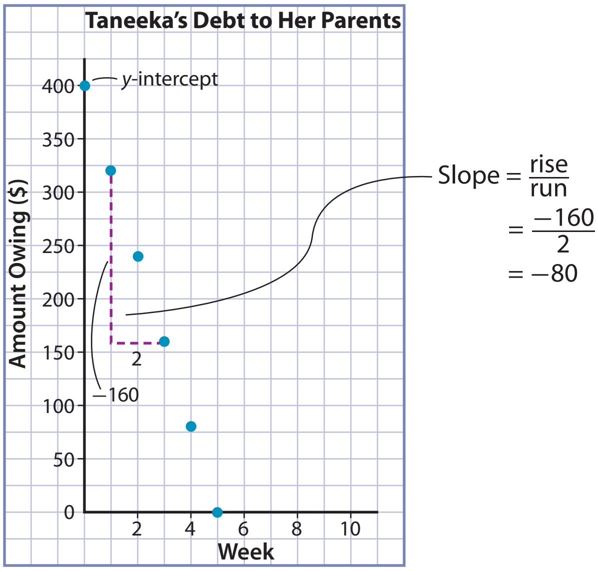
d) The initial value is the y-intercept, or the amount owing at 0 weeks, which is \$400.

The rate of change is the slope of the line, which is −50.

e) The graph shows the relationship is a partial variation.

- The graph is linear because the rate of change (the slope, or the rate at which Taneeka is paying off her debt each week) is fixed at −\$50/week.
- The graph does not intercept the y-axis at zero, since the dependent variable is \$400 when the independent variable is 0 weeks.

f) The graph would still be linear, there would not be a line through the points, it would still start at \$400 on the y-axis, and the slope would still be negative. However, the slope would be steeper, because the rate of change is now –\$80/week.



Your Turn

Bailey wants to save \$500 for a new computer. He has \$200 in a bank account. He has a part-time job and deposits \$60 each week.

a) What are the initial value and the rate of change?

b) Create a table of values that shows Bailey's balance each week for five weeks.

c) Create a scatter plot of the data.

d) Decide whether to draw a line, dashed or solid, through the points. Explain your decision.

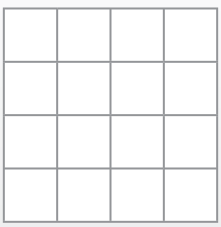
e) How does your graph show the initial value and the rate of change?

f) How do you know the graph represents a partial variation relationship?

g) Suppose Bailey starts with \$100 in his account. How would the graph compare to the one in part c)?

Puzzler

Draw a 4-by-4 grid.



Colour each square red, green, blue, or yellow.

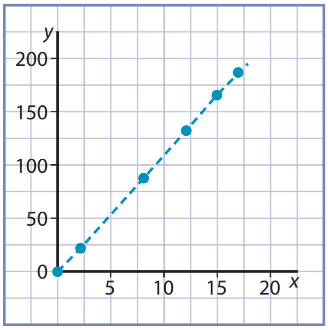
The same colour cannot appear in any row, column, or diagonal.

Check Your Understanding

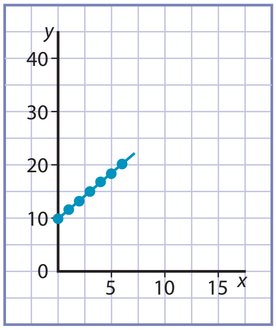
Try It

1. Which graphs represent a partial variation relationship? How do you know?

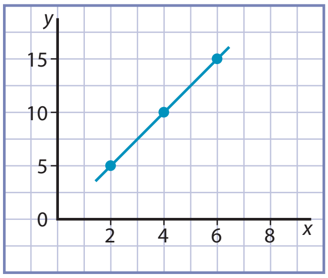
A



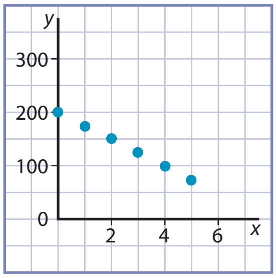
B



C



D



2. Sketch two different graphs for each situation.
   - **a)** a partial variation relationship
   - **b)** a relationship that does not have partial variation

Apply It

3. A $200 savings bond earns interest at a rate of 2% simple interest per year for five years. Interest is paid out at the end of each year.
   - **a)** How much interest will the bond earn in one year?
   - **b)** Create a table of values to show the total value of the bond plus interest at the end of each of the five years.
   - **c)** Graph the data. Explain why you would not draw a dashed or solid line through the points.
   - **d)** Suppose the interest earned was 3%. How would the graph of this situation compare to the graph in part c)?

Strategy

Develop a Strategy

What strategy can you use to determine if a relationship in a graph is a partial variation? Compare your strategy with a classmate's.

F.Y.I.

Simple interest is paid only on the initial amount invested, or principal.

Web Link

For more information on snow making, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

4. A ski resort has a 50-cm base of snow. They are making new snow at a rate of 5 cm per hour. The machines are running continuously.

a) Create a table of values to show the total depth of snow for the first 5 h of snow making.

b) Graph the data from part a). Draw a solid line through the points. Why is it reasonable to connect the points with a solid line?

c) What are the slope and y-intercept of the graph?

d) Explain why the depth of snow after 4 h is not double the depth after 2 h.

e) Use the graph to estimate how many hours it will take for the snow to be 62 cm deep.

5. A plane flying at an altitude of 1000 m begins to climb at a rate of 10 m per second.

a) Graph the altitude of the plane for the first 10 s of its ascent.

b) Draw a solid line through the points. Why is it reasonable to connect the points with a solid line?

c) Use the graph to estimate when the plane will reach an altitude of 1045 m.

d) How would the graph change if the plane started its ascent at 500 m and climbed at 15 m per second?

6. A major league baseball pitcher earns a base salary of $6 million plus a $500 000 bonus for each win during the season.

a) Create a graph of the total earnings for up to 10 wins. Draw a dashed or solid line, and explain your choice.

b) Suppose you graph only the amount he earns in bonuses for up to 10 wins. How would the graph compare to the graph in part a)?

On the Job 3

Model a Partial Variation Relationship With an Equation

Cassie decides to lease a car for her new job. She makes a down payment of \$1700 and payments at the end of each month of \$233 for three years.

a) Write an equation to model the relationship between the total amount spent on the lease and time.

b) Use the equation to determine
- what Cassie will have paid by the end of the first year
- the total amount she will spend over the three-year lease

What do you notice?

c) Predict and draw what a graph of the equation would look like. What do you notice?

d) Write an equation to model only the amount spent in monthly payments over three years. How do you know this relationship is no longer a partial variation?

Solution

a) Model the total amount spent with the equation $A = 1700 + 233m$, where A is the total amount spent, in dollars, and m is the number of months.

The equation $A = 1700 + 233m$ makes sense, because
Total amount
$= \$1700$ down payment $+ \$233 \times$ number of months

b) The total cost after one year (12 months) is

$A = 1700 + 233m$
$A = 1700 + (233 \times 12)$
$A = 4496$

The total amount Cassie spends in the first year is \$4496.00.

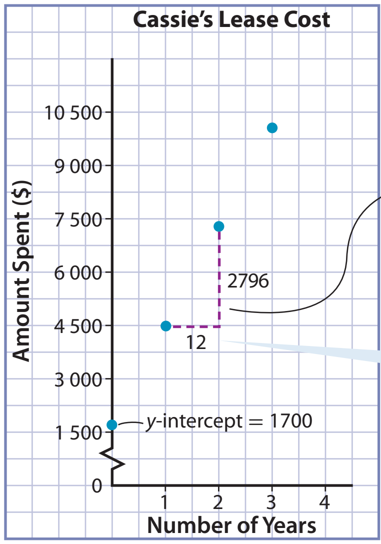
The total cost over three years (36 months) is

$A = 1700 + 233m$
$A = 1700 + (233 \times 36)$
$A = 10\,088$

Over the three years of the lease, Cassie will pay a total of \$10 088.00.

The total amount Cassie spends after three years is not triple the amount she spends after one year.

c) The graph would have points in a straight line going upward with a dashed line through them. The line is dashed because she pays monthly but the plotted points only show what she has paid at the end of each year. The line would start at 1700 on the y-axis and have a slope of 233. The slope and y-intercept of the graph are parts of the equation.



$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{2796}{12} = 233$$

1 year = 12 months

d) $A = 233m$, which is a direct variation relationship because the amount at 0 months would be \$0, and one variable is a multiple of the other.

In a graph of the new equation $A = 233m$, the rate of change or slope would be the same, 233, but the y-intercept would change to zero.

Your Turn

Another dealership offers Cassie the same car but with a \$1000 down payment and monthly payments of \$272 for three years.

a) Write an equation to model the relationship between the total amount spent for the car and time with an equation.

b) Use the equation to determine
- what she would pay by the end the first year
- the total amount she would spend over the three-year lease

c) Predict what a graph of the relationship would look like.

d) Which lease should Cassie choose? Why?

Check Your Understanding

Try It

1. **a)** Copy and complete the table of values.

b) Does the equation model a relationship with partial variation? How do you know?

s	E ($E = 200 + 10s$)
0	
1	
2	
3	

2. **a)** Substitute the values $x = 0$, 1, 2, and 3 into the equation $y = 3x + 1$ and solve for y. Record your answers in a table of values.

b) Does the equation model a relationship with partial variation? How do you know?

c) Predict what a graph of the relationship would look like.

Strategy

Develop a Strategy

What strategy can you use to determine if an equation represents a partial variation relationship? Compare your strategy with a classmate's.

3. **a)** Substitute the values $s = 0$, 1, 2, and 3 into the equation $P = 4s$ and solve for P. Record your answers in a table of values.

b) Does the equation model a relationship with partial variation? How do you know?

4. Which equations represent a relationship with partial variation?

A $P = 4s$ **B** $d = 80t$

C $y = 3x$ **D** $A = l \times w$

E $C = 70 + 65t$ **F** $y = -6x + 2$

G $E = 12.5h$ **H** $y = x$

5. For each partial variation equation in #4, determine the slope and y-intercept of its graph.

Apply It

6. Jim drives a truck through eastern Canada and the eastern United States. His average speed is 90 km/h. When he is 1500 km from home, he uses the equation $y = 1500 - 90x$ to determine his distance from home, y, after x hours of driving.

a) What part of the equation is constant or fixed? What does this represent?

b) What part of the equation varies over time? What does this represent?

Strategy

Guess and Check

c) Determine Jim's distance from home after 7 h.

d) Predict how many hours it will take him to drive all the way home. Use your equation to test your prediction. Keep guessing and testing until you solve the problem.

7. A car enters a highway on ramp going 50 km/h. Then, the driver accelerates to highway speed. The speed of the car on the ramp can be modelled with the equation $s = 50 + 10t$, where s is the speed of the car, in kilometres per hour, and t is time, in seconds.

a) What part of the equation is constant or fixed? What does this represent?

b) What part of the equation varies over time? What does this represent?

c) Determine the speed of the car after 2 seconds.

d) How long will it take the car to reach a speed of 90 km/h?

e) Explain why using this equation to determine the speed of the car after 20 seconds is unreasonable.

f) Describe what a graph of the equation would look like.

8. The cost of renting a snowboard is \$20/day. On the first day, you pay a fitting fee of \$35. A display of rental costs in the rental shop is shown. The equation $C = 35 + 20d$, where C is the total cost and d is the number of days, can be used to model the situation.

Number of Days	Total Cost (\$)
1	55
2	75
3	95
4	115

a) What part of the equation is constant or fixed? What does this represent?

b) What part of the equation varies over time? What does this represent?

c) Use the equation to determine the total cost for a 7-day rental.

Work With It

1. The cost of renting a banquet hall is posted below. The amounts are shown for increments of 5 people, but the total amount for any number of guests can be calculated on a per person basis.

Number of People	Cost ($)
0	500
5	650
10	800
15	950
20	1100
25	1250
30	1400
35	1550

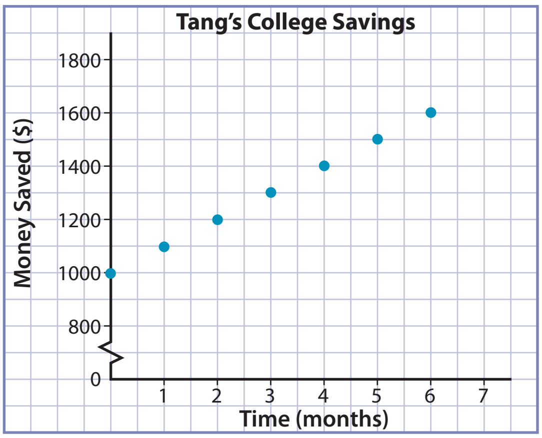
a) What is the fixed cost of renting the hall?

b) What is the variable amount per group of 5 people? per person?

c) How did you determine the answers to part b)?

d) Determine the cost of renting the hall for a banquet with 33 guests. Show your work.

2. A graph of Tang's monthly college savings fund is shown.



a) Would you draw a line, dashed or solid, through the points? Why?

b) What does the y-intercept of the graph represent?

c) What does the slope of the graph represent?

d) How many months will it take for his savings to double?

e) Write an equation to model how his savings will grow over time.

3. A conversion formula for degrees Celsius to degrees Fahrenheit is $F = 32 + 1.8C$.

   a) Explain how you know that this formula represents a partial variation relationship.

   b) Predict what a graph of this equation would look like.

   c) Use the formula to determine the equivalent of 25 °C on the Fahrenheit scale.

   d) What is the equivalent of −40 °C in Fahrenheit?

4. A resort rents bicycles. For tandem bikes, there is a flat $5 fee plus $3 for each whole or partial hour. For example, a 2 hour and 15 minute rental would be charged for 3 hours.

   a) Create a table of values to show the total rental cost for 0 h to 8 h.

   b) Is the relationship between rental cost and time linear or non-linear? Explain how you know.

   c) Is this relationship a direct or partial variation? Explain how you know.

   d) Graph the data in the table. Draw a solid or dashed line through the points. Explain your choice.

   e) What are the slope and y-intercept of the graph?

   f) Write an equation to model the relationship between total rental cost, C, and number of hours rented, n.

   g) Olli and his sister rent a tandem bike for the day and use it for $6\frac{3}{4}$ h. Determine the total rental cost. Which model did you use to solve the problem—the table, the graph, or the equation? Why?

Discuss It

5. How can you tell if each of the following models represents a partial variation relationship?

   a) table of values b) graph c) equation

6. How can you tell the initial value and the rate of change from looking at a graph of a partial variation relationship?

7. How can you tell the initial value and the rate of change from looking at an equation of a partial variation relationship?

8. How are the graphs of a partial variation relationship and a direct variation relationship similar? How are they different?

3 Skill Check

What You Need to Know

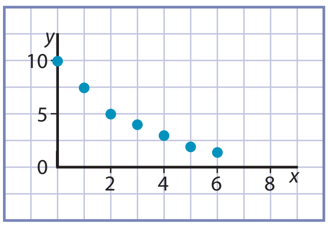
Section	After this section, I know how to . . .
3.1	■ create tables of values and graphs, including scatter plots with lines of best fit ■ identify and describe linear trends ■ determine whether a relationship is linear or non-linear ■ solve problems involving linear trends and relationships
3.2	■ understand why a relationship is a direct variation ■ model a direct variation relationship with a table of values, a graph, or an equation ■ solve problems involving direct variation relationships
3.3	■ understand why a relationship is a partial variation ■ model a partial variation relationship with a table of values, a graph, or an equation ■ solve problems involving partial variation relationships ■ compare direct and partial variation relationships

If you are unsure about any of these questions, review the appropriate section or sections of this chapter.

3.1 Understanding Linear Trends and Relationships, pages 112–126

1. State whether each graph shows a positive linear trend, a negative linear trend, or no linear trend. Also state whether there is a linear relationship.

a)



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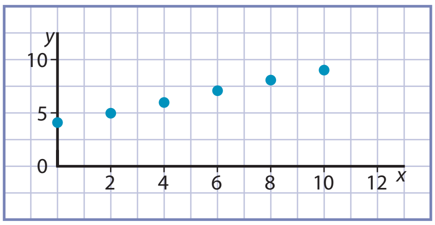

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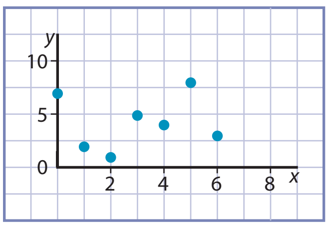

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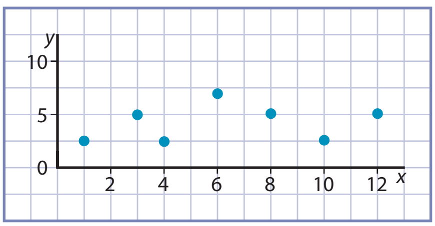


2. Which tables of values represent a linear relationship? Explain.

A

x	1	1	4	6	9	10
y	1	2	3	4	5	6

B

x	0	2	4	6	8	10
y	12	10	8	6	4	2

C

x	0	1	2	3	4	5
y	1	7	13	19	25	31

D

x	10	9	8	7	6	5
y	2	4	6	8	10	12

3.2 Direct Variation, pages 127–142

3. Kathi's keyboarding speed is a steady 45 words per minute.

a) Create a table of values to show the relationship between the number of words typed and the number of minutes for 0 min to 10 min.

b) Write an equation (or formula) that models the relationship between the number of words, w, and the number of minutes typing, m.

c) Predict what a graph of the relationship would look like.

d) How many words can Kathi type in 7.5 min? Explain.

e) How many words can she type in 27.25 min? Explain

3.3 Partial Variation, pages 143–159

4. The cost of running an advertisement on a radio station is a $350 set-up fee plus $400 a week.

a) What are the fixed and variable costs of running an advertisement?

b) Write an equation that can be used to calculate the total cost of advertising if you know the number of weeks the ad will run.

c) Predict what a graph of the equation would look like.

d) Determine the cost of running an ad for four weeks.

e) Determine the cost for eight weeks.

f) Explain why the answer to part e) is not double the answer to part d).

3 Test Yourself

For #1 to #3, select the best answer.

1. Marissa's pay varies directly with the number of hours she works. She earned $72 for a 6-h shift. Which equation models the relationship between Marissa's earnings, E, and the number of hours worked, h?

A $E = 6h$ **B** $E = 72$ **C** $E = 12h$ **D** $E = 72 + 6h$

2. Which equation represents a non-linear relationship?

A $x = 5$ **B** $A = \pi r^2$ **C** $E = 14.50h$ **D** $C = 75 + 40t$

3. A taxi charges an initial $3.50 plus $1.00 per kilometre. What is the cost of a 10-km ride?

A $35.00 **B** $13.50 **C** $4.50 **D** $10.00

4. a) Which table of values represents a direct variation relationship? How do you know?

b) Which shows a partial variation relationship? How do you know?

A

x	0	1	2	3	4	5
y	1	2	3	4	5	6

B

x	0	1	2	3	4	5
y	0	2	4	6	8	10

C

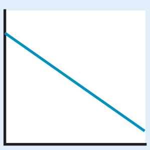
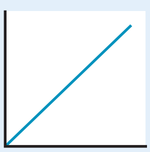
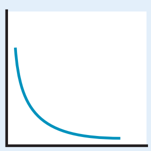
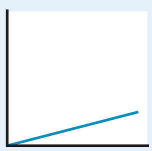
x	0	1	2	3	4	5
y	1	2	4	8	16	32

D

x	0	1	2	3	4	5
y	0	200	250	350	400	500

5. a) Which graphs represent a direct variation relationship? How do you know?

b) Which shows a partial variation relationship? How do you know?

A

B

C

D


6. **a)** Which equation in #2 represents a direct variation relationship? How do you know?

   b) Which equation in #2 represents a partial variation relationship? How do you know?

7. What are the slope and y-intercept of the graph of each equation?

   a) $y = 7 + 4x$ **b)** $y = 4x$ **c)** $y = 1.5x + 7$

8. Janice works at a local store. She recorded her hours and earnings for five weeks.

	Number of Hours Worked	Money Earned ($)
Week 1	22	286
Week 2	20	260
Week 3	17	221
Week 4	27	351
Week 5	15	195

   a) What is Janice's rate of pay?

   b) Create a scatter plot of the data in the table.

   c) Would you draw a line, dashed or solid, through the points? Explain your decision.

   d) What are the slope and y-intercept of the graph?

   e) Is the relationship a partial or direct variation? Explain.

   f) Write an equation to model the relationship between Janice's earnings, E, and the number of hours she works, n. Use the equation to calculate her earnings for 36 h of work.

9. Drew is an automotive service technician. He charges customers $64/hour for labour. He uses a computer database to estimate the time needed to complete each type of job.

   a) Is the relationship between the total amount that Drew charges for his labour and the length of time a job takes linear or non-linear? Explain your reasoning.

   b) Is it a direct or partial variation? Explain how you know.

   c) Write an equation to model the relationship between total labour cost and time.

   d) Use your equation to determine the labour cost for a 2.3-h job.

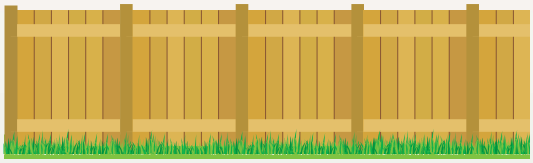
3 Chapter Project

Build a Fence

You have a summer job building fences. Your boss asks you to create a booklet to show customers the materials and costs involved in building fences of different designs and lengths.

Materials
- grid paper

For one of the popular fence designs, the first 8-ft section of fence uses 2 posts. Each 8-ft section after that uses 1 more post. For each section, your boss charges $125.



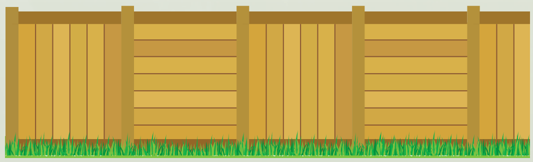
1. a) Create a table of values and a graph that show the cost of a fence for 1 to 5 sections.

b) Write an equation that could be used to calculate the cost when you know the number of sections.

2. a) Create a table of values and a graph that show the number of posts required for a fence with 1 to 5 sections.

b) Write an equation that could be used to calculate the number of posts when you know the number of sections.

3. Another fence design alternates between vertical and horizontal boards. The vertical sections use 6 boards and the horizontal sections use 7 boards. Describe what a graph of the number of boards required for a fence with 1 to 5 sections would look like.



4. How do the tables, graphs, and equations that you have included in the booklet show what you have learned about linear relationships?

3 GAMES AND PUZZLES

Sudoku®

> **Materials**
> - grid paper or Sudoku® Puzzle BLM

Sudoku® is a logic puzzle. Logic puzzles require "*Since* …, *then* …" thinking to solve them. To solve a Sudoku®, the digits 1 through 9 must appear in every row, in every column, and in every 3-by-3 box exactly once.

In the example below,
- *since* the only digits missing from column 1 are 7 and 8, and *since* there is a 7 in row 2 already, *then* a 7 belongs in the **red square**
- *since* the bottom row contains an 8, and *since* the second from the bottom row contains an 8, *then* the only place that an 8 can be in the bottom-left 3-by-3 box is in the **blue square**

6		5	2					3
	3	9		5			7	2
		2				5		4
4			9		3			1
9			8		1			7
1			5		7			9
3		7				2		
2	9			8		7	3	
5					2	1		8

1. Copy the puzzle above onto grid paper, or use the blackline master. Use "*Since* … , *then* … " logic to solve the puzzle. You can work with a partner or on your own.
2. Choose one answer that you recorded in the puzzle. Explain the logic you used to determine which digit belonged in that square.
3. How would your logic change if there were no 3-by-3 boxes in the puzzle and you had to consider only rows and columns?