5 Properties of Figures

A commercial artist designs logos, symbols, and letterheads that are appealing and easy to recognize.

A logo is a visual representation of a product, company, or organization. Logos remind people of the group or product they represent.

1. Look at the images on this page. Identify any logos that you recognize.
2. What shapes can you identify in the logos?
3. What makes a logo easy to recognize? What makes it pleasing to the eye?

Key Words

regular polygon
equilateral triangle
diagonal
isosceles triangle
obtuse angle
acute angle
isosceles trapezoid
parallelogram
scalene triangle
tessellate
line symmetry
line of symmetry

Career Link

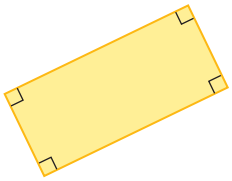
Allison is a commercial artist. She uses her artistic talents and skills to design, promote, display, and advertise companies and other agencies. She designs material for DVDs, web sites, logos, posters, and brochures. Commercial artists also put pictures on T-shirts; create reports, magazines, and newspapers; and design movie sets and furniture. They usually use computer design software to do their work.

5 Get Ready

Polygons

1. Identify each of the polygons.

a)



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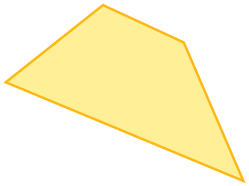
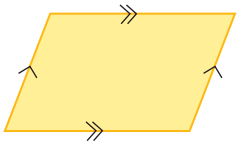
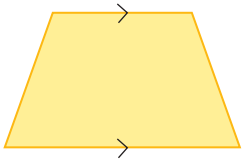


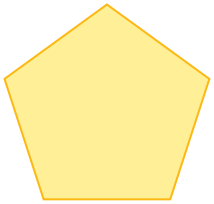
c)



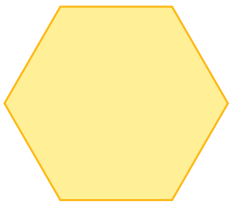
d)



e)



f)



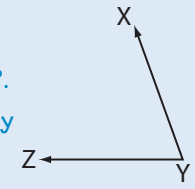
Measuring Angles

2. Estimate the measure of each angle, to the nearest degree. Then, measure each angle.

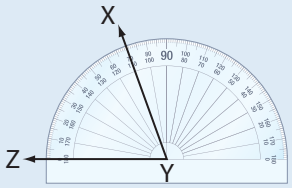
Example:

To estimate, compare the angle to 45° and 90°.

The angle is about halfway between 45° and 90°. So, an estimate of ∠XYZ is about 70°.



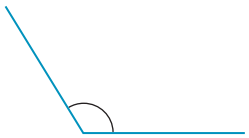
To measure, use a protractor.



The measure of ∠XYZ = 70°.

a)

b)



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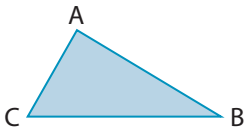

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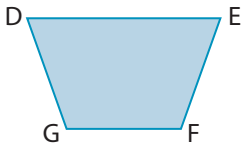


3. Draw an angle with each of the following measures.

a) 90°

b) 25°

c) 155°

d) 180°

Decimal Numbers

4. Determine each sum using mental math.

a) $7.4 + 8.3$

b) $0.11 + 1.1$

c) $5.5 + 3.7$

d) $15 + 9.5 + 6.9$

e) $2 + 0.2 + 0.02$

f) $25.6 + 5.8 + 0.7$

Solving Equations

5. Substitute to solve each equation.

a) $I = Prt$
$P = 1000, r = 0.05, t = 2$

b) $A = \frac{1}{2}bh$
$b = 6, h = 7.8$

c) $v = \frac{d}{t}$
$d = 27, t = 4$

d) $P = lw$
$P = 135, w = 15$

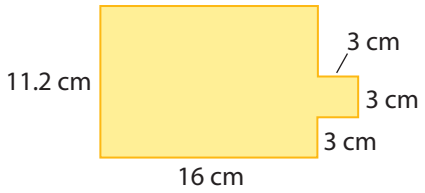
e) $C = \pi d$
$\pi = 3.14, d = 2.3$

f) $V = lwh$
$l = 2, w = 4.2, h = 3.1$

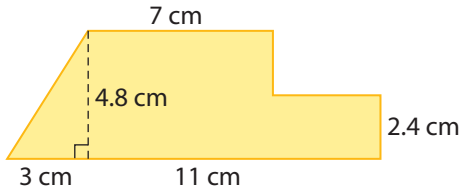
Area of Composite Figures

6. Copy these composite figures. Sketch lines to divide the figures into triangles and/or quadrilaterals. Then, determine the area of each figure.

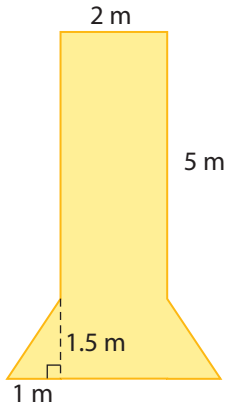
a)



b)



c)



Perimeter

7. Determine the perimeter of each figure in #6.

5.1 Angle Properties of Polygons

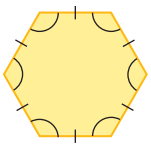
Focus On . . .

- determining the sum and measure of the angles in a polygon
- understanding the types of angles in a polygon
- exploring tessellations of polygons

A textile designer may use regular polygons when designing rug patterns, wallpaper, and tile designs.

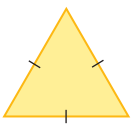
regular polygon

- a closed figure with three or more straight sides and equal side and angle measurements



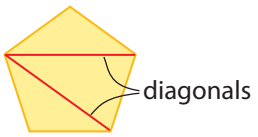
equilateral triangle

- a triangle with equal side lengths



diagonal

- a line segment connecting two non-adjacent vertices in a polygon



Explore Angle Measures of Regular Polygons

The sum of the interior angles in a triangle is 180°. Is there a pattern for the sum of the interior angles for regular polygons?

1. **a)** Draw an **equilateral triangle**.
   b) What is the measure of each interior angle of an equilateral triangle?

An equilateral triangle is a regular polygon.

2. **a)** Draw a square.
   b) Draw a **diagonal** between two non-adjacent corners so the square is divided into triangles.
   c) How many triangles are created?
   d) What is the sum of the interior angles of a square?
   e) What is the measure of each interior angle of a square?

3. Repeat step 2 for a regular pentagon and a regular hexagon.
   Note: You will need to draw more than one diagonal to divide each shape into triangles.

Materials

- ruler, protractor, and compass, or geometry software
- Explore Angle Measures of Regular Polygons BLM
- Triangles BLM, Quadrilaterals BLM, Other Regular Polygons BLM (optional)

4. Copy and complete the table to record your results from steps 1 to 3.

Figure	Number of Sides	Number of Triangles	Sum of Interior Angles	Measure of Each Interior Angle
Equilateral triangle	3	1	180°	
Square				
Regular pentagon				
Regular hexagon				

5. Reflect

a) Do you notice any patterns in the table?

What is the relationship between the number of sides and the sum of the interior angles of a polygon?

b) Without making a sketch, determine the sum of the interior angles of these regular polygons.

- an 8-sided figure
- a 10-sided figure
- a 20-sided figure

c) Determine the measure of each interior angle of these regular polygons.

- an 8-sided figure
- a 10-sided figure
- a 20-sided figure

Strategy

Look for a Pattern

6. Extend Your Understanding

a) Write an equation that relates the sum of the interior angles, S, to the number of sides, n, of a regular polygon.

b) Use your equation to solve for S when $n = 12$.

c) Use your equation in part a) to write an equation for the measure of each interior angle, M, of a regular polygon.

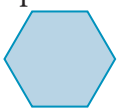
d) Use your equation to solve for M when $n = 16$.

On the Job 1

Determine Angles in Polygons

Tina is building a wooden display frame. She needs to decide what shape to make the frame. She thinks about what kinds of cuts and how many cuts she would have to make. She is condidering one of these shapes:

- a regular hexagon

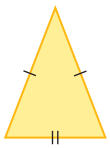


- a square

- an **isosceles triangle**

What kinds of interior angles will Tina need to create for each shape?

isosceles triangle
- a triangle with two equal side lengths



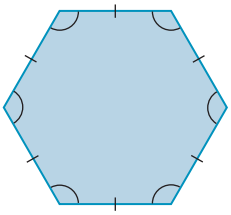
Strategy

Work Backward

Solution

Regular hexagon:

A regular hexagon is a regular polygon with six sides. The formula for the sum of the interior angles of a regular polygon is



$S = 180(n - 2)$, where n is the number of sides.

$S = 180(6 - 2)$

$S = 180(4)$

$S = 720$

The sum of the interior angles of a regular hexagon is 720°.

To determine the measure of each interior angle of a regular hexagon, divide the sum by the number of interior angles in a hexagon.

$\frac{720}{6} = 120$

The formula for the measure of each interior angle of a regular polygon is $M = \frac{180(n - 2)}{n}$.

The measure of each interior angle of a regular hexagon is 120°.

A regular hexagon has six interior angles that are **obtuse angles**.

obtuse angle
- an angle that is greater than 90° and less than 180°

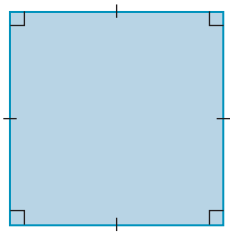
Strategy

Develop Alternative Approaches

You can also measure each interior angle with a protractor. What is the measure of each interior angle?

Square:

A square is a quadrilateral with equal side lengths and equal angle measures.



Use the formula for the measure of each interior angle of a regular polygon:

$$M = \frac{180(n - 2)}{n}$$

$$M = \frac{180(4 - 2)}{4}$$

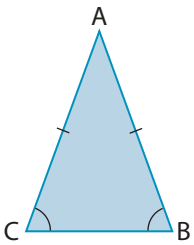
$$M = 90$$

The measure of each interior angle is 90°.

A square has four interior angles that are right angles.

Isosceles triangle:

An isosceles triangle has two equal side lengths and two equal angle measures.



Measure each interior angle with a protractor.

The measure of ∠A is 40°. The measure of ∠B is 70°. The measure of ∠C is 70°.

An isosceles triangle has two equal angles that are **acute angles**. The third angle can be obtuse or acute. In this triangle, it is acute.

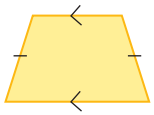
Tina chose an isosceles triangle with these measures. There are many other possibilities that fit the definition of an isosceles triangle.

acute angle

- an angle that is greater than 0° and less than 90°

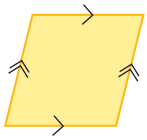
isosceles trapezoid

- a four-sided figure with one set of sides parallel and the other set of sides equal in length



parallelogram

- a four-sided figure with two pairs of parallel sides



Your Turn

Tina thinks about some other shapes she could use for the display frame:

- a regular pentagon
- an **isosceles trapezoid**
- a **parallelogram**

a) Describe the interior angles she might create for each shape.

b) Which angles are equal to each other?

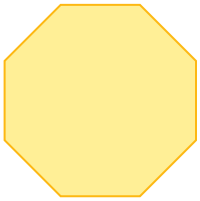
c) Classify each angle as right, obtuse, or acute.

d) How many of each type of angle does each shape have?

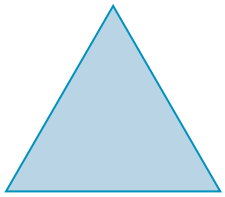
Check Your Understanding

Try It

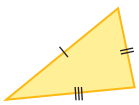
1. What is the measure of each interior angle in a regular octagon?



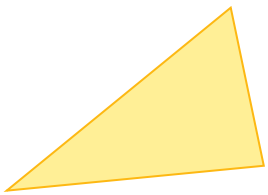
2. What is the measure of each interior angle in an equilateral triangle?



scalene triangle
- a triangle with no equal sides



3. a) What is the measure of each interior angle in this **scalene triangle**?



b) Which angles are equal to each other?

c) Classify each angle as right, obtuse, or acute.

d) How many of each type of angle does the scalene triangle have?

e) Do scalene triangles always have this number of each type of angle? Explain.

Strategy
Eliminate Possibilities

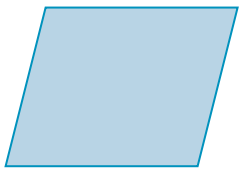
4. a) For each shape in #1 to #3, could you use the formula for determining each interior angle of a regular polygon, $M = \frac{180(n - 2)}{n}$?

b) Name three shapes for which you cannot use this formula.

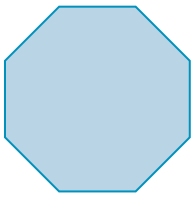
5. What is the sum of the interior angles of the following shapes?

a) isosceles triangle

b) quadrilateral



c) octagon



6. For which shapes in #5 could you use the formula for determining the sum of the interior angles of a regular polygon, $S = 180(n - 2)$?

Apply It

7. Copy and complete the table with the properties related to the angle measures of these polygons.

	Polygon	Number of Interior Angles	Sum of Interior Angles	Measure of Each Interior Angle
a)	Equilateral triangle			
b)	Square			
c)	Rectangle			
d)	Regular pentagon			
e)	Regular hexagon			

F.Y.I.

Many polygon names are formed by using prefixes that indicate a number. For example, *tri* means three, *quad* means four, *penta* means five, and *hexa* means six. To see more polygon names and their prefixes, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

8. Copy and complete the table with the angle properties of these polygons. The first one has been done for you. **Note:** For part f), use a parallelogram that is not a square or a rectangle.

	Polygon	Right Angles	Obtuse Angles	Acute Angles	Equal Angles
a)	Isosceles triangle	possible	possible	possible	2
b)	Scalene triangle				
c)	Equilateral triangle				
d)	Square				
e)	Rectangle				
f)	Parallelogram				
g)	Isosceles trapezoid				
h)	Quadrilateral				
i)	Regular pentagon				
j)	Regular hexagon				

F.Y.I.

Squares and rectangles are types of parallelograms with angles that are all right angles.

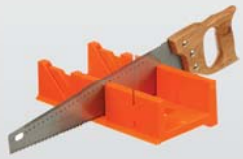
9. Jennifer is making a pillow in the shape of a stop sign for her nephew. She cuts the pieces of fabric for the front and back of the pillow.

a) How many interior angles will each of the two pieces have?

b) What will be the measure of each interior angle?

Tools of the Trade

A mitre box is a woodworking tool used to guide a handsaw to make accurate cuts in a board. A compound mitre saw gives an accurate, and more precise, cut.



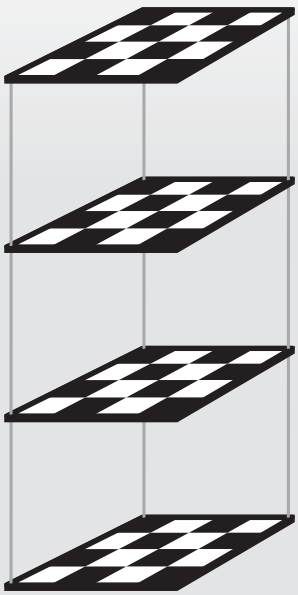
10. Karl is drawing plans for a backyard gazebo. He is making its base a regular pentagon. He needs to plan how he will cut the wooden boards for the base.

a) What will be the measure of the regular pentagon's interior angles?

b) At what angle will he have to cut each board?

Puzzler

Play 3-D Tic Tac Toe. Copy the 3-D board shown below. Get four in a row on one board or across all four boards before your partner. You can play horizontally, vertically, or diagonally.



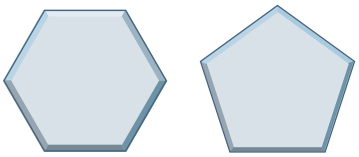
On the Job 2

Tessellate With a Polygon

Jeff is a tile setter. His clients want to use tiles in the shape of a regular hexagon or a regular pentagon for their bathroom floor. Which shape will **tessellate** the area?

tessellate

- to cover an area using the repetition of geometric shapes, with no overlaps and no gaps

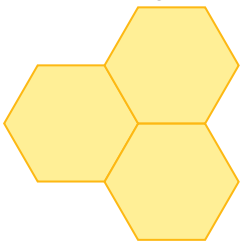


Solution

Regular hexagon:

Method 1: Use Cutout Shapes

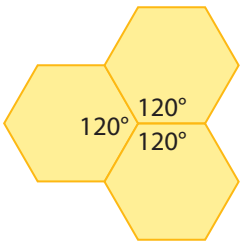
Fit the hexagons together like puzzle pieces.



The hexagons do not overlap or leave gaps. Therefore, a regular hexagon can be used to tile the floor.

Method 2: Measure Interior Angles

Measure the interior angles where the vertices of the hexagons meet.



Each of the interior angles is 120°. The sum of the three angles is 360°.

Therefore, a regular hexagon can be used to tile the floor.

When the interior angles where the vertices meet total exactly 360°, it means the polygon can tessellate an area.

Strategy

Develop Alternative Approaches

Measure the interior angles of the regular hexagon. Does the interior angle measure divide evenly into 360°? How can this help you determine whether the shape will tessellate the area?

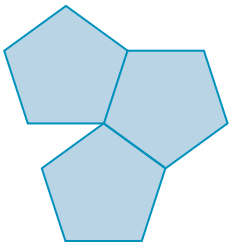
F.Y.I.

Beehives have hexagonal cells because a hexagon creates the maximum amount of space with the minimal amount of material. Many other shapes would create gaps between cells. For more information about the cells of honeycombs, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Regular pentagon:

Method 1: Use Cutout Shapes

Fit the regular pentagons together like puzzle pieces.

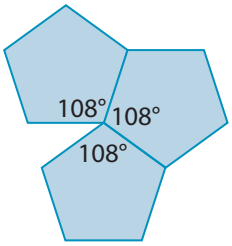


The pentagons leave gaps when you try to fit them together.

Therefore, a regular pentagon cannot be used to tile the floor.

Method 2: Measure Interior Angles

Measure the interior angles where the vertices meet.



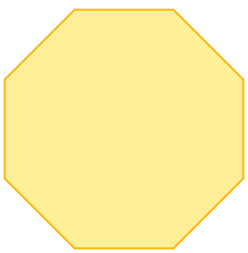
Each of the interior angles is 108°. The sum of three angles is 324°. The sum of four angles is 432°.

Therefore, a regular pentagon cannot be used to tile the floor.

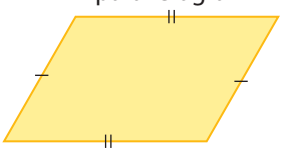
Your Turn

Which of these shapes could Jeff use to tile the bathroom floor? How do you know?

a) regular octagon



b) parallelogram

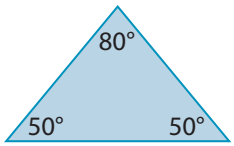


Check Your Understanding

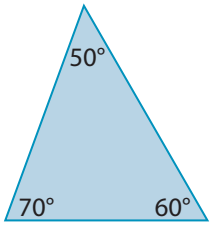
Try It

1. Which of these triangles tessellate? Explain or show the strategy you used.

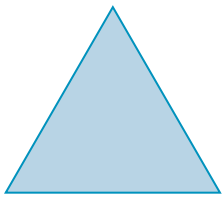
a) isosceles triangle



b) scalene triangle

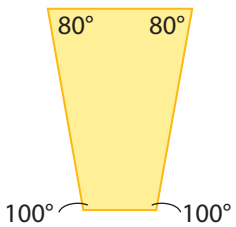


c) equilateral triangle

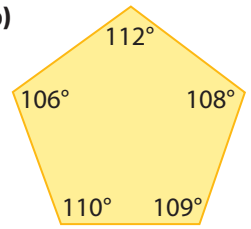


2. Which of these polygons tessellate? Explain or show the strategy you used.

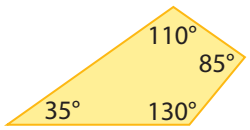
a)



b)



c)



3. Which three types of regular polygons tessellate?

Apply It

4. a) Sarah is designing a pattern to stencil on her wall. She plans to tessellate one quadrilateral for the pattern. Draw a pattern she could use. Label all of the angles on one of the quadrilaterals.

b) Draw another pattern using a different quadrilateral. Label all of the angles on one of the quadrilaterals.

5. Juan wants to use square and rectangular interlocking bricks to create a backyard patio. Can he use these two shapes together to create a tessellating pattern? Sketch a diagram to show if this is possible.

6. Celine is making a tablecloth with tessellating patterns. Both patterns have equilateral triangles and one other polygon. Show two different patterns that she could use.

Work With It

1. Identify the shape(s).

a) It has exactly two acute angles and its interior angles equal 180°.

b) It has four right angles and its interior angles equal 360°.

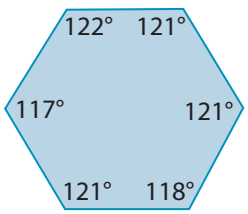
c) It is a regular polygon with more than four angles and it tessellates.

d) It is a regular polygon with fewer than six angles and it does not tessellate.

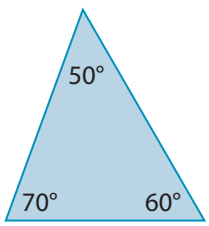
2. Jane is designing trim for a jean jacket. First, she tessellates an isosceles trapezoid in a straight line, as shown.

Then, she draws the following shapes. None of them will tessellate in a straight line. Change the interior angles, as necessary, in each shape so that the shape will tessellate in a straight line.

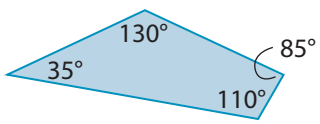
a)



b)



c)



3. Your band is playing at a local venue. You are designing a poster to advertise the concert. You want to include a tessellating pattern for the border of the poster.

a) Draw a tessellating pattern in which all angles are equal at the point where the vertices meet.

b) Draw a tessellating pattern in which all of the angles are *not* equal at the point where the vertices meet.

4. Chelsea wants to use two different regular polygons to create a tessellating pattern for a quilt. Draw a tessellation using two polygons.

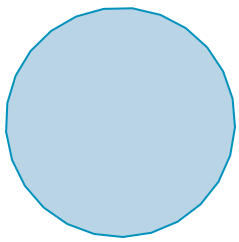
Discuss It

5. Explain how the pattern on a soccer ball is a tessellation.

6. Compare the following angle properties of equilateral, isosceles, and scalene triangles: number of angles, sum of interior angles, measures of interior angles, and types of interior angles.

7. a) What happens to the measure of the interior angles of a regular polygon as the number of sides increases?

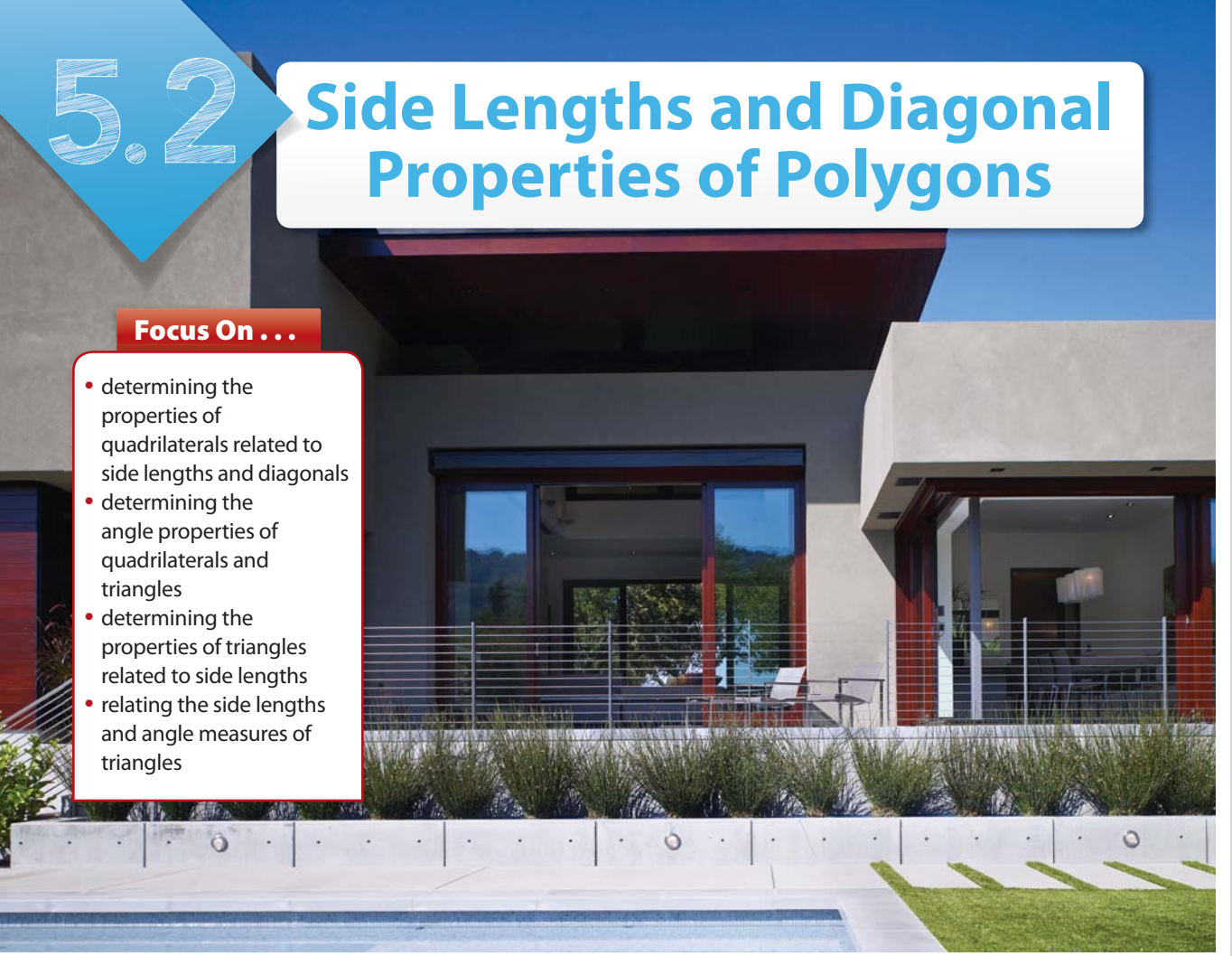
b) If the number of sides of a polygon increases indefinitely, what figure would eventually be formed?



8. Explain why a regular pentagon does not tessellate but some other pentagons do tessellate.

9. Describe tessellating patterns you have seen in the world around you.

5.2 Side Lengths and Diagonal Properties of Polygons



Focus On . . .

- determining the properties of quadrilaterals related to side lengths and diagonals
- determining the angle properties of quadrilaterals and triangles
- determining the properties of triangles related to side lengths
- relating the side lengths and angle measures of triangles

Tools of the Trade

To learn more about the career of landscape designer, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

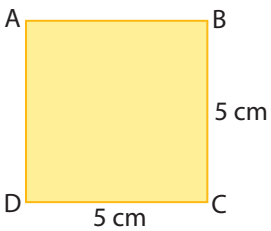
- grid paper
- ruler
- protractor
- Quadrilaterals BLM (optional)

A landscape designer plans the location of buildings, walkways, plants, and trees to combine art and science in creating a landscape. These designers often work with quadrilaterals.

Explore Diagonals of Quadrilaterals

1. On grid paper, sketch a square that measures 5 cm by 5 cm.

2. Label the vertices A, B, C, D.



3. a) Draw the diagonal AC. Measure the length of AC.

b) Draw the other diagonal, BD. Measure the length of BD. What do you notice?

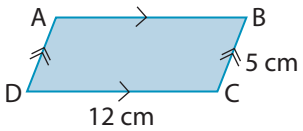
4. a) Label the point where the diagonals intersect as E.

b) Measure the lengths of AE, BE, CE, and DE. What do you notice?

5. a) Measure ∠DEA, ∠AEB, ∠BEC, and ∠CED. What do you notice?

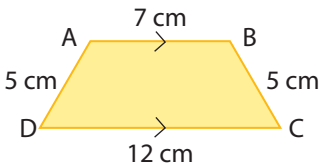
b) What is the sum of the angles where the diagonals intersect?

6. Sketch a parallelogram that is 5 cm by 12 cm, as shown. Repeat steps 2 to 5.



A parallelogram has opposite sides that are equal and parallel.

7. Sketch an isosceles trapezoid with side lengths as shown. Repeat steps 2 to 5.



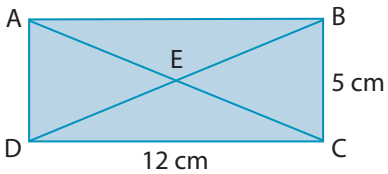
In an isosceles trapezoid, one set of opposite sides is equal and the other set of opposite sides is parallel.

8. Reflect What do you notice about the sum of the angles where the diagonals intersect for all the polygons?

Strategy

Simplify the Original Problem

9. Extend Your Understanding



a) What do you predict about the length of the diagonals, AC and BD in the diagram shown?

b) What do you predict about the lengths of AE, BE, CE, and DE?

c) What do you predict about the sum of the angles where the diagonals intersect?

d) Check your predictions in parts a) to c).

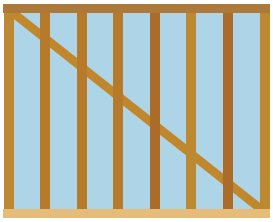
On the Job 1

Determine Side Lengths and Diagonals of Quadrilaterals

F.Y.I.

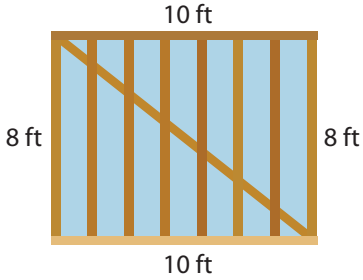
When a frame is square, the boards that make up the outside of the frame all meet at right angles (90°).

Han is framing a wall. He wants to check that the frame is a rectangle. If it is, he will know the frame is "square." How does he know this?



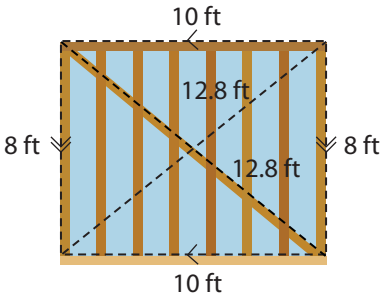
Solution

Han measures the length of all sides.



The opposite sides are equal in length. All rectangles have opposite sides that are equal in length.

Han measures the length of the diagonal board. To determine if the frame is square, he also measures the other diagonal.



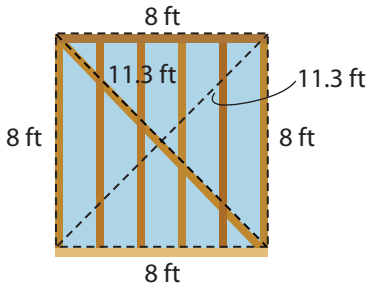
The diagonals are equal in length. All rectangles have diagonals that are equal in length.

The opposite side lengths are equal and the diagonals are equal. That means the opposite sides are parallel. In all rectangles, opposite sides are parallel.

The frame is a rectangle. Since all four interior angles of a rectangle are 90°, Han knows the frame is square.

Your Turn

Han builds another frame in what he thinks is the shape of a square. He measures to check. Is it a square? How do you know?



Tools of the Trade

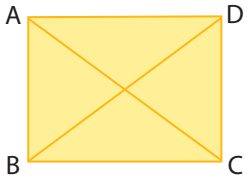
Carpenters use a level and a square to determine if a frame is constructed correctly. To learn how to check a doorframe, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Check Your Understanding

Try It

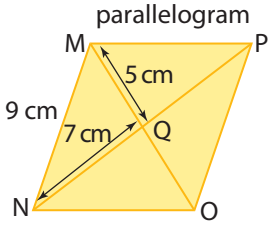
1. Determine the missing measurements.

a) rectangle



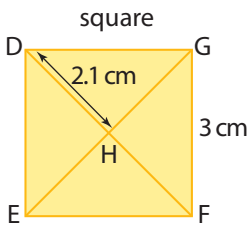
AB = 3 in. AD = 4 in.
AC = 5 in. BC = ■ in.
CD = ■ in. BD = ■ in.

b) parallelogram



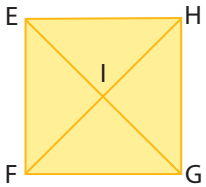
QO = ■ cm MO = ■ cm
PO = ■ cm PQ = ■ cm

c) square



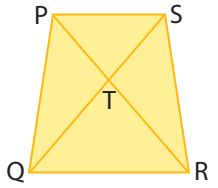
DG = ■ cm HF = ■ cm
GE = ■ cm ∠GHF = ■°

d) square



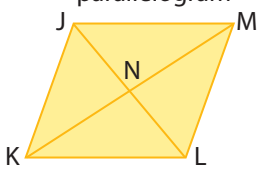
EF = 6.5 cm FG = ■ cm
EG = 9.2 cm FH = ■ cm
∠GFE = ■° ∠EIH = ■°

e) isosceles trapezoid



SR = 4 m PQ = ■ m
∠PTS = 85° ∠QTR = ■°
∠PTQ + ∠QTR + ∠RTS
+ ∠PTS = ■°

f) parallelogram



JM = 6 ft KL = ■ ft
∠JNM = 100° ∠KNL = ■°
∠JNK = ■° ∠LNM = ■°

2. Copy and complete the table. The first one has been done for you. **Note:** For part d), use a parallelogram that is not a square or a rectangle.

	Polygon	Number of Equal Side Lengths	Number of Sets of Parallel Sides	Number of Equal Diagonals
a)	Rectangle	2 sets of 2	2 sets of 2	2
b)	Square			
c)	Isosceles trapezoid			
d)	Parallelogram			

3. List all the quadrilaterals that could fit each description.

a) has at least one set of parallel sides

b) has four equal side lengths

c) has two equal diagonals

4. Sketch and name a quadrilateral that fits each description.

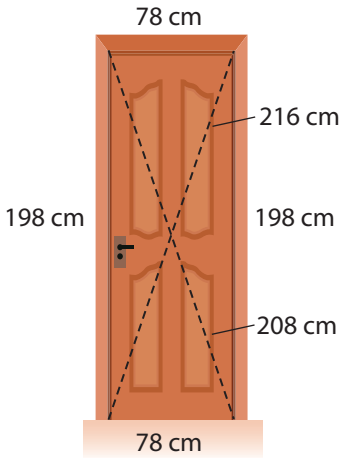
a) The diagonals are equal, but the sides are not all equal.

b) The diagonals are equal, and all the sides are equal.

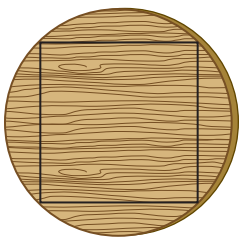
c) The diagonals are not equal, and no two sides are equal.

Apply It

5. Jane has built a door frame. She measures to check it. Is Jane's door frame a rectangle? Explain.



6. Graham works at a store that sells office furniture and supplies. He needs to attach a frame to a circular tabletop. He uses the edge of a level to draw a square to the edges of the circle as shown. How can he use this square to find the centre of the circle?

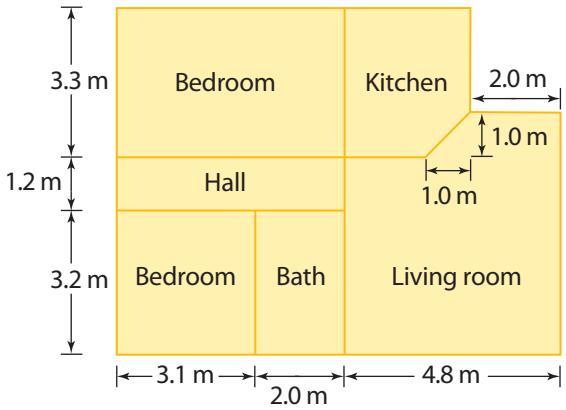


7. Larry and Teresa are building their own cabin.

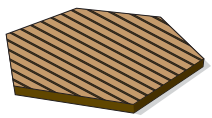
a) They are going to carpet the two bedrooms with the same carpeting. What is the total area of carpeting they need?

b) They want to check that they built the hallway square. How can they do this?

c) They are going to tile the bathroom. How can they locate the centre of the room?



8. Matthew is building a deck. The shape of the foundation is a regular hexagon with a perimeter of 9.6 m. Will a bench of length 1.5 m fit on one side of the deck? Explain.



9. Emma's greenhouse is in the shape of a geodesic dome. The dome is made up of equilateral triangles that form regular pentagons and regular hexagons. Each equilateral triangle has a base of 75 cm and a height of 65 cm. The structure consists of 6 pentagons and 15 hexagons. What is the total surface area of the structure in square metres?

F.Y.I.

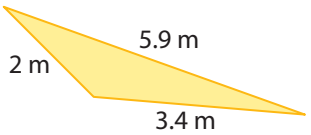
A geodesic dome is a structure formed from equilateral triangles. The triangles make the structure strong and distribute the stress equally across the structure. To learn more about geodesic domes, go to www.mcgrawhill.ca/books/mathatwork12.

On the Job 2

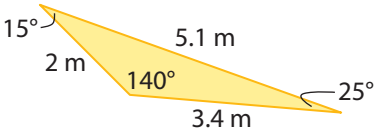
Determine the Relationship Between Angles and Side Lengths in Triangles

Cathy is a landscaper. She is creating a design for a client's backyard.

a) The client gives Cathy the measurements for the existing triangular garden. How does Cathy know the measurements are incorrect?



b) Cathy asks the client for correct measurements, including angle measures. How does Cathy know that the client made another error?

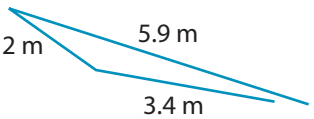


Tools of the Trade

Landscapers often use software programs to draw plans.

Solution

a) Cathy draws a scale diagram of the triangular garden using the client's measurements.

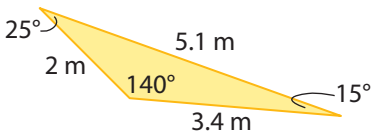


It is not possible to draw a triangle with the provided measurements. The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Strategy

Simplify the Original Problem

b) Cathy measures the angles on the client's scale diagram.



In a triangle,

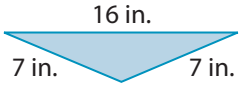
- the side opposite the greatest angle is always the longest side
- the side opposite the smallest angle is always the smallest side
- the side opposite the midsize angle is always the midsize side

So, Cathy knows the client's drawing is incorrect because the client labelled the angle opposite the smallest side with the second smallest angle.

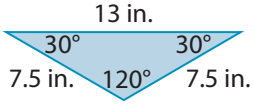
Your Turn

Are these triangle measures possible?

a)



b)



Web Link

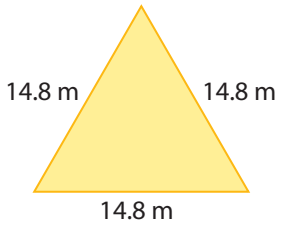
To explore the relationship between side lengths and angles of triangles, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Check Your Understanding

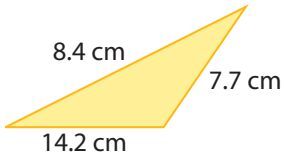
Try It

1. Which of the triangle side lengths are labelled correctly?

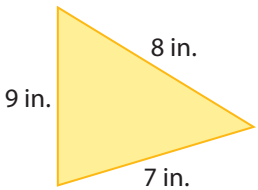
a)



b)

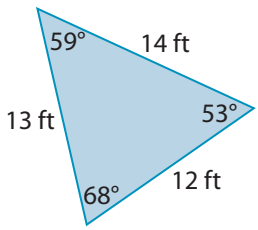


c)

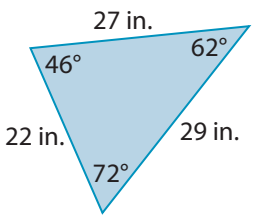


2. Which of the triangle angle measures are possible?

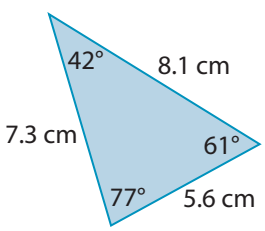
a)



b)



c)



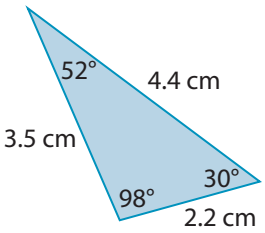
3. For each incorrect triangle in #1 and #2, copy the triangle and rearrange the labels so that they are correct.

Apply It

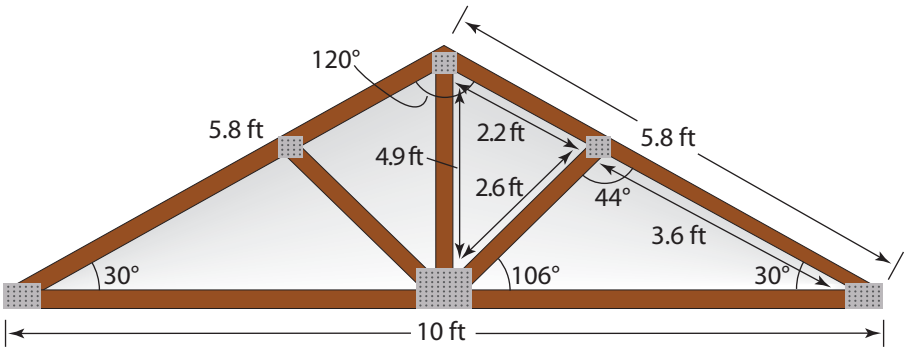
4. **a)** Draw a triangle labelled with side lengths that are not possible.

b) Exchange triangles with a partner. Correct the labels on your partner's triangle so that it is possible.

5. Karen draws a triangle that she plans to use to create a wallpaper border. What mistake has she made?



6. Julia draws plans for a roof truss for a garage. Will they work? Explain.



Work With It

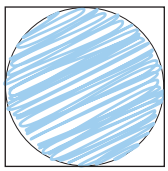
1. How many diagonals does each regular polygon have?

a) pentagon

b) hexagon

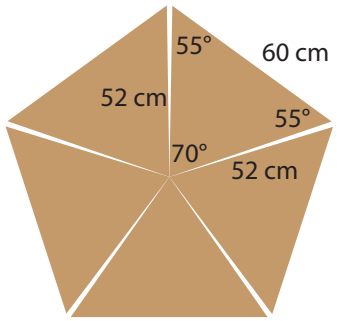
c) octagon

2. A landscaper draws plans for a circular fountain. The water pipe for the fountain will be in the middle of the circle. The landscaper draws a square around the edge of the circle, as shown. How can the landscaper use the square to find the centre of the circle?



3. Karen wants to use a triangle to tessellate a pattern for her wallpaper border. Two of the side lengths are 6 cm and the other is 2.5 cm. Two of the angle measures are 24° and the other is 78°. Is it possible to tessellate this triangle? Explain.

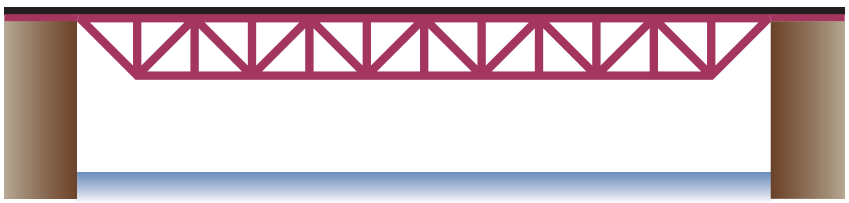
4. George is building a deck. He has designed a deck medallion in the shape of a regular pentagon made up of five identical triangles. What is wrong with George's design?



5. On grid paper, create a design for a decorative window using one or more polygons. For each polygon, label the length of each side and the measure of each angle.

Discuss It

6. Describe how you would determine whether the squares in this bridge structure are square.



7. A rectangle is a type of parallelogram that has right angles. Why is this the case? Explain using what you know about the properties of rectangles and parallelograms.

8. The following property applies to all equilateral, isosceles, and scalene triangles: The sum of two side lengths is greater than the third side length. Why is it easiest to check for this property in an equilateral triangle?

5.3 Symmetry

Focus On . . .

- relating the property of symmetry to the classification of triangles, quadrilaterals, and other polygons
- determining which polygons have lines of symmetry
- determining the number of lines of symmetry in polygons

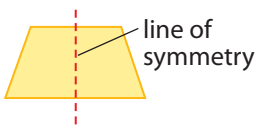
Housing designers often use symmetry when designing new housing developments. Symmetry is not only pleasing to the eye, it can also be functional. For example, construction is easier if the styles of the homes are similar.

line symmetry

- a type of symmetry in which an image or object can be divided into two identical halves by a line of symmetry

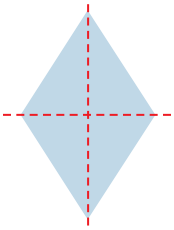
line of symmetry

- a line that divides a figure into two identical halves
- sometimes called a line of reflection or axis of symmetry



Explore Symmetry

Some shapes have **line symmetry** and some do not. You can determine which ones have symmetry in a number of ways.



Think of each red, broken line drawn through the kite as a mirror. The part of the kite on the left side of the vertical mirror has a corresponding part on the right side of the mirror. Each part of the kite below the horizontal mirror has a corresponding part above the mirror. The "mirror line" is called a **line of symmetry**.

Materials

- scissors
- grid paper
- ruler
- coloured pencils
- Quadrilaterals BLM and Other Regular Polygons BLM (optional)

Strategy

Draw or Model

1. Cut two pieces of grid paper into four pieces each.
2. On one piece, draw a square.
3. Draw any lines of symmetry on the square.
4. Check by folding along the lines of symmetry you drew. Is each half of the polygon the same on either side of the fold line?

5. Repeat steps 2 to 4 with a rectangle, parallelogram, isosceles trapezoid, regular pentagon, regular hexagon, and regular octagon.

Use a parallelogram that is not a square or a rectangle.

6. Copy and complete the table.

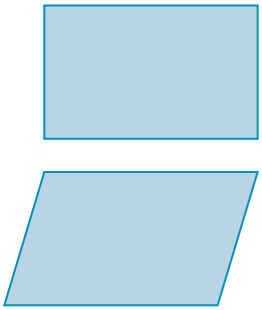
	Figure	Regular Polygon? (Yes or No)	Number of Sides	Number of Lines of Symmetry
a)	Square			
b)	Rectangle			
c)	Parallelogram			
d)	Isosceles trapezoid			
e)	Regular pentagon			
f)	Regular hexagon			
g)	Regular octagon			

7. **Reflect**
   - a) Which polygons from step 6 have lines of symmetry?
   - b) What do you notice about the number of lines of symmetry and the number of sides in a regular polygon?
8. **Extend Your Understanding**
   - a) On each polygon in step 5, use a different coloured pencil to draw the diagonals.
   - b) Do the lines of symmetry correspond with the diagonals? Describe what you observe.

On the Job 1

Determine Symmetry in Quadrilaterals

Jerome is building an inlaid tabletop. He knows that symmetry in designs is pleasing to the eye. Which shape shown will provide the greater symmetry, the rectangle or the parallelogram?



Tools of the Trade

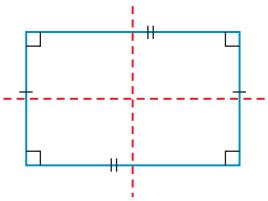
To create wood inlay, it is best to first create a template of the design. You can then use a router to make the cuts. For more information on wood inlay, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Solution

Rectangle:

A rectangle has two sets of two equal side lengths. It also has four angles that are equal in measure.

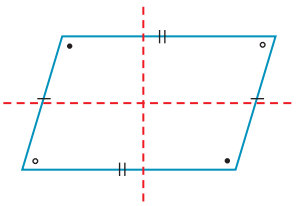
A rectangle has two lines of symmetry.



Parallelogram:

A parallelogram is a four-sided polygon with opposite sides that are parallel and opposite angles that are equal in measure.

The parallelogram shown has no lines of symmetry.

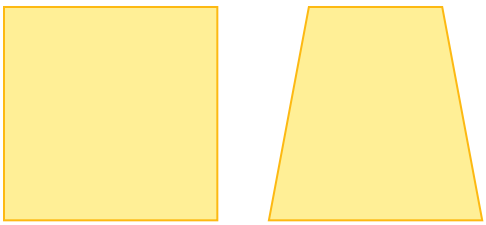


A rectangle will provide the most symmetry for Jerome's tabletop design.

Are there parallelograms that *do* have lines of symmetry?

Your Turn

Jerome is considering other quadrilaterals to use for his tabletop design. Which shape will provide the greater symmetry, the square or the isosceles trapezoid? Explain.

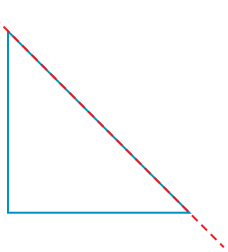


Check Your Understanding

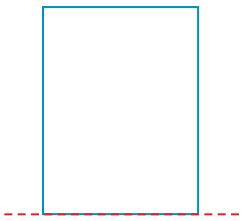
Try It

1. Each diagram shows one half of a quadrilateral. The red line is a line of symmetry. Identify each type of quadrilateral.

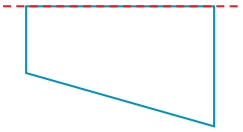
a)



b)



c)

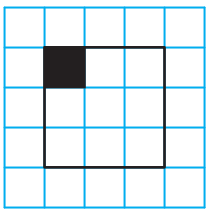


d)

2. For each quadrilateral in #1, copy the diagram. Then, draw the whole shape and add all other lines of symmetry.

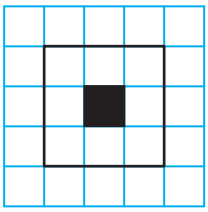
3. Copy each 3-by-3 square onto grid paper. Shade grid squares so that each large square has the number of lines of symmetry indicated.

a)



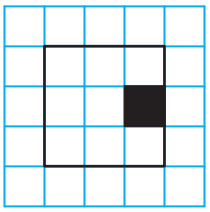
1 line of symmetry

b)



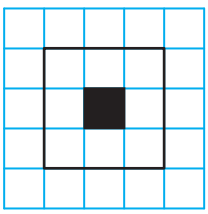
4 lines of symmetry

c)



2 lines of symmetry

d)



0 lines of symmetry

Apply It

4. Sketch a copy of the road sign shown.

a) Draw any lines of symmetry within the entire road sign.

b) What geometric shapes can you identify in the road sign?

c) Draw any lines of symmetry within each shape on the road sign.

d) How many lines of symmetry are there in total?

e) What other road signs have symmetry? Draw each one, and then draw the lines of symmetry.

f) What road signs do not have any symmetry within the shapes in the sign?

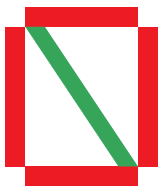
5. Jackson is a graphic artist. He is studying symmetry in logos. For each logo shown,

- identify the geometric shapes
- identify which shapes have symmetry
- identify which shapes do not have symmetry
- copy each shape and draw the lines of symmetry

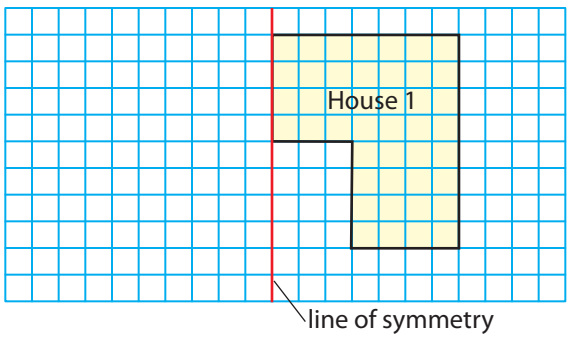
a)

b)

c)



6. Janice is working on the plans for a semidetached house. She is going to make one house the reflection of the other. She uses the property line between the lots as a line of symmetry. Copy the plan for the house shown and draw its reflection.



On the Job 2

Determine Symmetry in Polygons

Gillian is making a quilt out of triangular pieces of cloth. She wants to know which type of triangle will give her the most lines of symmetry.

a) The perimeter of the triangles will be 30 cm and the side measurements will be whole numbers. Can the triangular pieces be equilateral triangles? isosceles triangles? scalene triangles?

b) Which type of triangle has the greatest symmetry?

Strategy

Make a Systematic List

Solution

a) Scalene triangle:

A scalene triangle has no equal side lengths.

There are many possible configurations. List some possible side measurements.

When listing the possible side lengths, be sure that the sum of the lengths of any two sides is greater than the length of the third side.

Perimeter (cm)	Side 1 (cm)	Side 2 (cm)	Side 3 (cm)
30	12	10	8
30	12	4	14
30	11	9	10
30	8	9	13

Isosceles triangle:

An isosceles triangle has two equal side lengths. There are many possible configurations. List some possible side measurements.

Perimeter (cm)	Two Equal Sides (cm)	Third Side (cm)
30	$8 + 8$	$30 - 16 = 14$
30	$9 + 9$	$30 - 18 = 12$
30	$10 + 10$	$30 - 20 = 10$
30	$11 + 11$	$30 - 22 = 8$
30	$12 + 12$	$30 - 24 = 6$
30	$13 + 13$	$30 - 26 = 4$
30	$14 + 14$	$30 - 28 = 2$

Equilateral triangle:

An equilateral triangle has three equal side lengths.

Since the perimeter is 30 cm, divide 30 by 3.

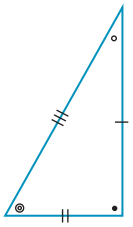
$\frac{30}{3} = 10$

An equilateral triangle would have side lengths of 10 cm.

The triangles could be scalene, isosceles, or equilateral.

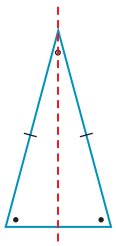
b) Scalene triangle:

Since a scalene triangle has no equal side lengths and no equal angle measures, there are no lines of symmetry.



Isosceles triangle:

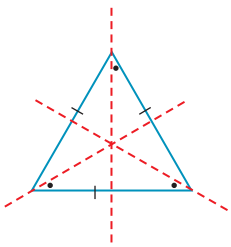
Since an isosceles triangle has two equal side lengths and two equal angle measures, there is one line of symmetry.



Equilateral triangle:

Since an equilateral triangle has three equal side lengths and three equal angle measures, there are three lines of symmetry.

An equilateral triangle has the greatest symmetry.



Your Turn

Gillian is making another quilt using a regular pentagon or a regular hexagon in the design.

a) What is the number of equal side lengths in each of these two regular polygons?

b) What is the number of equal angle measures?

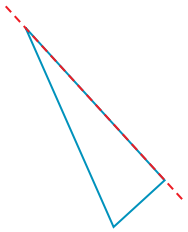
c) Which of these two regular polygons has the greatest number of lines of symmetry?

Check Your Understanding

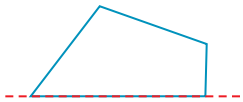
Try It

1. Each diagram shows one half of a polygon. The red line is a line of symmetry. Identify each type of polygon.

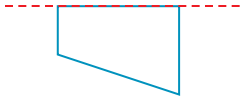
a)



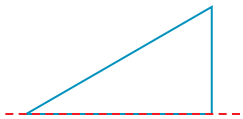
b)



c)



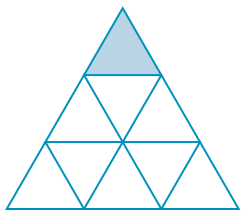
d)



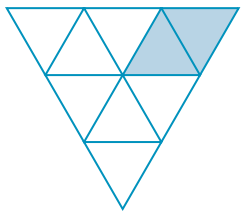
2. For each polygon in #1, copy the diagram. Then, draw the whole shape and add all other lines of symmetry.

3. Copy each large equilateral triangular tile onto isometric dot paper. Add shading so that each large triangle has three lines of symmetry.

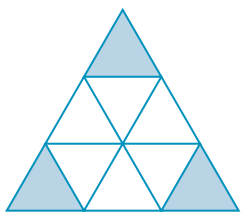
a)



b)



c)



Apply It

F.Y.I.

An octagon-shaped ring is used for mixed martial arts because it is safer and fairer than a square-shaped ring. For more information about the octagonal rings, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

4. Tim is a fitness trainer. He is setting up the ring used for mixed martial arts. It is a regular octagon with a side length of 12.25 ft. How many lines of symmetry does a regular octagon have? Explain where they are located or sketch the lines on a regular octagon.

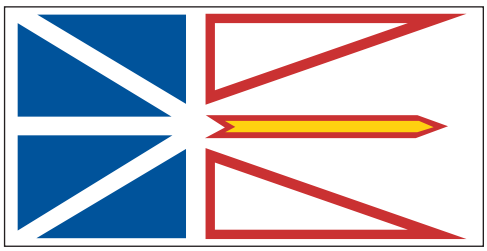
5. List 2-D objects you have seen in your classroom, in your home, or elsewhere that have

a) exactly one line of symmetry

b) exactly two lines of symmetry

c) an infinite number of lines of symmetry

6. The flag of Newfoundland and Labrador includes triangles.



a) Identify the type (s) of triangles used in the design of the flag.

b) Does the flag contain one or more lines of symmetry? If yes, sketch the flag and draw the line(s).

c) Look up the flags of other provinces and countries. Sketch two or three that include triangles. Classify the triangles. If any of the flags have lines of symmetry, draw the lines.

7. Start with four rectangular sheets of paper. Fold each sheet into an airplane that has the following:

a) one line of symmetry

b) equilateral triangles

c) isosceles triangles

d) scalene triangles

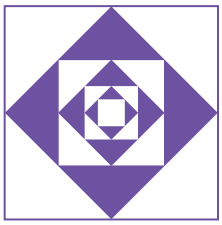
Compare your paper airplanes to those made by your classmates.

Tools of the Trade

A fitness trainer coaches groups or individuals in exercise activities and sports. A trainer must explain safety rules and regulations about sports, recreational activities, and exercise equipment. For more information about a career in fitness training, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Work With It

1. Draw the following, if possible.
   - **a)** a triangle with exactly three lines of symmetry
   - **b)** a triangle with exactly two lines of symmetry
   - **c)** a triangle with one line of symmetry
   - **d)** an isosceles triangle with perimeter of 15 cm
   - **e)** an equilateral triangle with perimeter of 21 cm
   - **f)** a triangle with three equal angles
   - **g)** a triangle with two equal angles
2. Identify each type of triangle that you drew in #1.
3. What is the angle sum of each triangle you drew in #1?
4. A sign builder cuts material for stop signs. Does a stop sign have one or more lines of symmetry? Sketch or describe the location of the line(s).
5. Examine the diagram.
   - **a)** Identify the shapes and the number of each type of shape in the diagram.
   - **b)** How many lines of symmetry does the diagram have?



Tools of the Trade

A sign builder makes signs and welds iron and steel riders and other material to form structures and frameworks. For more information about the career of sign builder, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Discuss It

6. **a)** Find two or three company logos that contain triangles and quadrilaterals. Sketch them, or copy and paste them on paper. Draw in the lines of symmetry.
   - **b)** Discuss the following statements. Do you agree or disagree with each statement? Why?
     - **A** Logos that contain lines of symmetry are more pleasing to the eye than logos without lines of symmetry.
     - **B** Logos that contain lines of symmetry are easier to remember than logos without lines of symmetry.
7. Find six or seven pictures of buildings, monuments, bridges, and other structures. Sketch or copy them. Do any of these structures have lines of symmetry? Explain why there are or are not lines of symmetry.

British Columbia Parliament Buildings, Victoria, BC

5 Skill Check

What You Need to Know

Section	After this section, I know how to . . .
5.1	■ determine the sum and measure of the angles in a polygon ■ identify the types of angles in a polygon ■ explore tessellation of polygons
5.2	■ determine the properties of quadrilaterals related to side lengths and diagonals ■ determine the angle properties of quadrilaterals and triangles ■ determine the properties of triangles related to side lengths ■ relate the side lengths and angle measures of triangles
5.3	■ relate the property of symmetry to the classification of triangles, quadrilaterals, and other polygons ■ determine which polygons have lines of symmetry ■ determine the number of lines of symmetry in polygons

If you are unsure about any of these questions, review the appropriate section or sections of this chapter.

5.1 Angle Properties of Polygons, pages 226–237

1. Determine the sum of the angles in each regular polygon. Then, state the measure of each interior angle.

a) pentagon

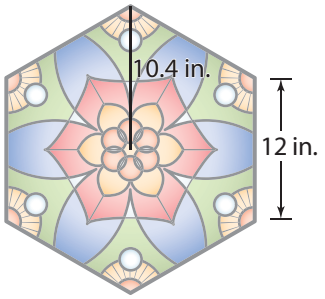
b) octagon

c) 10-sided figure

d) 11-sided figure

2. Jason is designing a mosaic top for a patio table using an isosceles triangle. Can he tessellate this shape to create a pattern? Show whether this is possible. State the measure of each interior angle in your design.

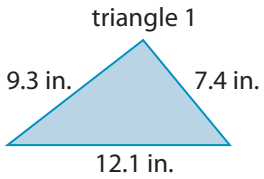
3. Mary is placing a hexagonal stained-glass window above her front door. What is the total area of glass she needs to purchase?



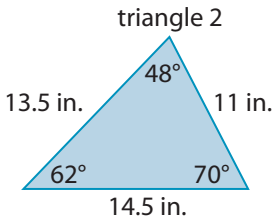
5.2 Side Lengths and Diagonal Properties of Polygons, pages 238–247

4. Raoul is designing two inlaid triangular designs for the headboard of a bed.

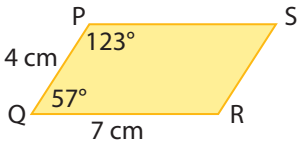
a) Are the measurements for the side lengths possible in triangle 1?



b) Are the angle measures possible in triangle 2?

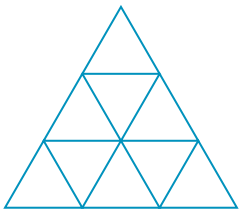


5. Determine the missing side lengths and angle measures of the parallelogram.



5.3 Symmetry, pages 248–257

6. The large equilateral triangle shown is divided into nine smaller equilateral triangles. Draw three copies of the large triangle. Shade one or more of the smaller triangles, but not all nine, to match each description.



a) The large triangle has exactly one line of symmetry.

b) The large triangle has exactly three lines of symmetry.

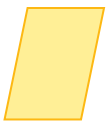
c) The large triangle has no lines of symmetry.

7. How many ways are there for Mary to position the stained-glass window in #3 so that one of the lines of symmetry is a vertical line?

8. Meredith is designing a logo for her art club using quadrilaterals. State the name of each quadrilateral and the number of lines of symmetry it has.

a) 

b) 

c) 

d) 

5 Test Yourself

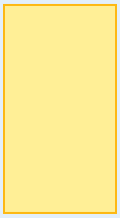
For #1 to #5, select the best answer.

1. A triangle has a perimeter of 28 cm. Which three measurements, in centimetres, could *not* be the measure of the side lengths?

A 5, 10, 13 **B** 7, 10, 11

C 8, 10, 10 **D** 4, 4, 20

2. How many lines of symmetry does this rectangle have?



A 0 **B** 1

C 2 **D** 3

3. Which quadrilateral has two equal sides, no right angles, and two equal diagonals?

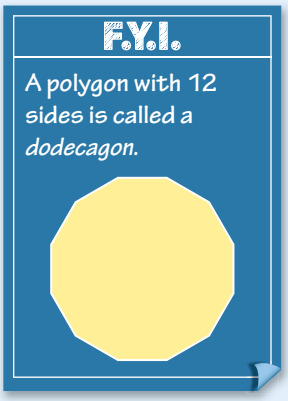
A square **B** rectangle

C parallelogram **D** isosceles trapezoid

4. How many lines of symmetry are there in a regular polygon with 12 sides?

A 4 **B** 6

C 12 **D** 24



5. What is the sum of the interior angles of a hexagon?

A 180° **B** 360°

C 540° **D** 720°

6. Sketch and label quadrilaterals with the following properties:

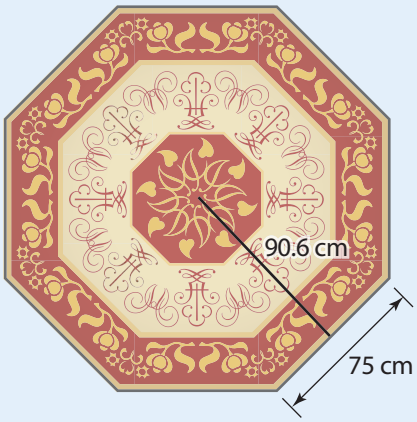
a) Only the opposite side lengths are equal and the diagonals are equal.

b) There is only one pair of parallel sides.

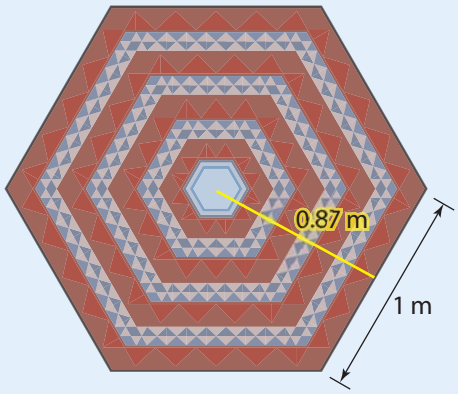
c) All side lengths are equal and only opposite angles are equal.

7. An interior designer wants to buy the largest rug he can find in the shape of a regular polygon. The designer found these two rugs.

rug 1



rug 2



a) What is the area of each rug?

b) How many lines of symmetry does each rug have?

c) Which rug is the largest?

8. Choose two of the regular polygons you have studied in this chapter. Copy the table and fill in the information.

Number of Sides		
Name of Regular Polygon		
Number of Lines of Symmetry		
Number of Diagonals		
Sum of Interior Angles		
Measure of Each Interior Angle		

9. Sandra is a textile designer. She is creating a design for placemats. She wants to use a single shape to tessellate a pattern. Show how she can do this. State the measure of each interior angle in the shape you recommend.

5 Chapter Project

Logo Pro

You are a commercial artist. Your assignment is to create a logo for a company. Follow these guidelines when creating your logo.

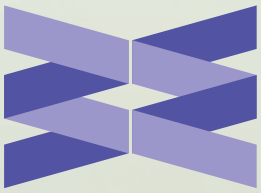
Materials

- computer with Internet access
- grid paper
- coloured pencils
- geometry software (optional)

1. Find examples of company or product logos. Brainstorm with classmates to make a list of qualities that effective logos have.
2. Choose an existing product, company, or organization, or make up your own.
3. Write a brief description of the product, company, or organization.
4. Use grid paper or geometry software to draw the logo. Incorporate topics and shapes that you studied in this chapter: diagonals, lines of symmetry, tessellations, triangles, quadrilaterals, regular polygons, and so on. Use colour and shading.

   When designing the logo, consider the qualities that you listed in #1.

   - Is it pleasing to the eye?
   - Is it easily identifiable?
   - Does it convey a message associated with the product, company, or organization?
   - Is it clear and simple without unnecessary letters or details?
5. Make the logo big enough that it could be used as a crest on a T-shirt or jacket.



5 GAMES AND PUZZLES

Shape Search

Materials
- Shape Search Grid BLM

Find and outline each of the figures on the triangular grid:
- equilateral triangle
- isosceles triangle
- right triangle
- scalene triangle
- parallelogram that is not a rectangle
- rectangle
- isosceles trapezoid
- pentagon
- hexagon
- octagon