6 Transformations

Leanne is planning to install a new floor in her living room based on a design she found on the Internet. She would like to use square and rectangular tiles to create a pattern similar to the one shown.

1. What information should Leanne gather before she begins planning her design?
2. How could Leanne plan her design before she begins working?
3. What other home projects might require planning a design in this way?

Key Words

transformation
image
dilation
scale factor
translation
successive translation
reflection
line of reflection
rotation
centre of rotation

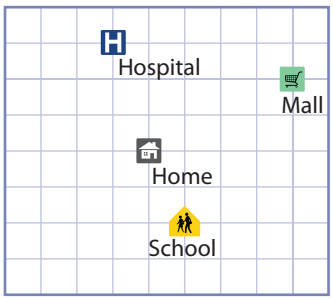
Career Link

Frank owns a landscaping company. He enjoys designing and building unique patterns for walkways and patios. He also likes to experiment with fitting stones of different shapes and colours together.

6 Get Ready

Transformations on a Grid

1. Each square on the grid represents a block on a map. Give directions on how to get to each location from home.



a) school

b) hospital

c) mall

The Coordinate Grid

2. a) Draw a coordinate grid with an x-axis and y-axis that range from -10 to 10. Label the origin and quadrants I through IV.

b) Plot each coordinate on the grid.

i) $(0, 0)$

ii) $(-7, 3)$

iii) $(-2, -8)$

iv) $(4, 9)$

v) $(10, -2)$

vi) $(2, -5)$

vii) $(-2, 8)$

3. Identify the quadrant in which each ordered pair is located.

a) $(15, 12)$

b) $(-13, -14)$

c) $(16, -15)$

d) $(-11, 16)$

e) $(12, 11)$

f) $(19, -11)$

4. If the original point on the coordinate grid is point A, determine the letter of the point that is

a) 5 spaces right

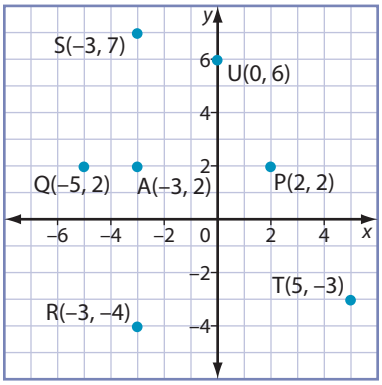
b) 5 spaces up

c) 2 spaces left

d) 6 spaces down

e) 8 spaces right and 5 spaces down

f) 3 spaces right and 4 spaces up



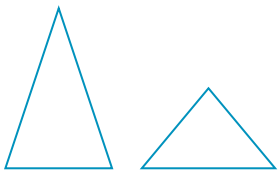
Similar Figures

5. Measure each pair of images using a ruler and a protractor to determine whether they are similar figures.

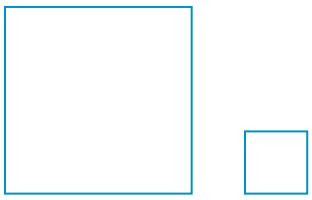
Similar figures have
- the same shape but different size
- equal corresponding angles
- proportional corresponding sides

a)

b)



c)



d)

Drawing Angles

6. Sketch each angle without using a protractor.

a) 30° **b)** 45°

c) 90° **d)** 180°

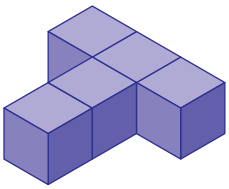
e) 270° **f)** 360°

7. Use a protractor to measure your sketches in #6.

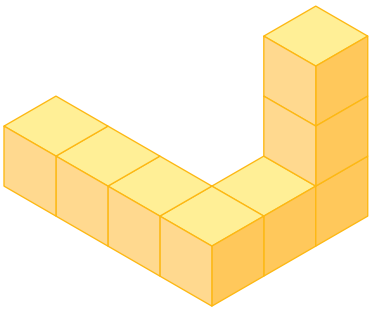
Drawing 3-Dimensional Shapes

8. Draw each figure on isometric dot paper.

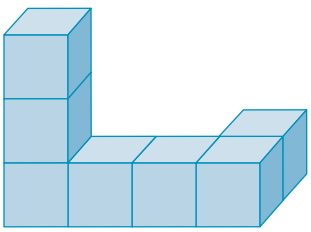
a)



b)



c)



6.1 Dilations

Focus On . . .

- identifying and drawing a dilation of a 2-D and 3-D object
- using similarity to determine and explain dilations

Contractors use transformations to do their work. Blueprints cannot be the same size as the building that is being constructed, so contractors use a smaller image of the building.

transformation

- a change in a figure that results in a new position or size
- Examples: dilations, translations, reflections, rotations

image

- the final shape and/or position of a figure after transformation

dilation

- a transformation in which a figure is enlarged or reduced by a constant factor

Explore Dilations

1. When an object is enlarged or reduced by a constant factor, the change is called **dilation**. The dilated image and the original image are similar.

a) On centimetre grid paper, draw a square that is 8 cm by 8 cm. Label it A.

b) Draw a second square, with the same centre, that measures 4 cm by 4 cm. Label it B.

c) Draw a third square, with the same centre, that measures 16 cm by 16 cm. Label it C.

Using grid paper and a ruler helps you create an accurate drawing.

Materials

- grid paper
- ruler
- Section 6.1 Explore Dilations

scale factor

- the constant factor by which all dimensions of an object are enlarged or reduced
- Example: the dimensions of this rectangle are multiplied by 3, so the scale factor is 3

F.Y.I.

A′ is read "A prime." It is used to label the point that matches point A after a transformation.

2. a) Compare the side length of square A and square B.

i) Is square B an enlargement or a reduction?

ii) By what **scale factor** has the side length changed?

iii) By what factor has the area changed?

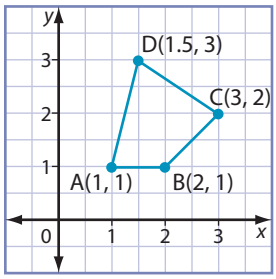
b) Compare the side length of square A and square C.

i) Is square C an enlargement or a reduction?

ii) By what factor has the side length changed?

iii) By what factor has the area changed?

3. a) Sketch polygon ABCD on a coordinate grid as shown.



b) List the coordinates of the vertices of polygon ABCD.

c) To create a dilation with a factor of 3, multiply the x-values and y-values of each ordered pair by 3.

d) Plot the new ordered pairs on the coordinate grid. Join the points to create the polygon A′B′C′D′.

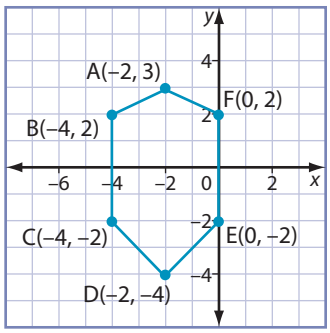
e) What happens to the area?

To create a dilation on a coordinate grid, multiply both the x-value and y-value of an ordered pair by a scale factor.

A(■ × 3, ■ × 3) = A′(■, ■)

4. Reflect In general, how could you use a ruler or a protractor to check that an image is a dilation of an original figure?

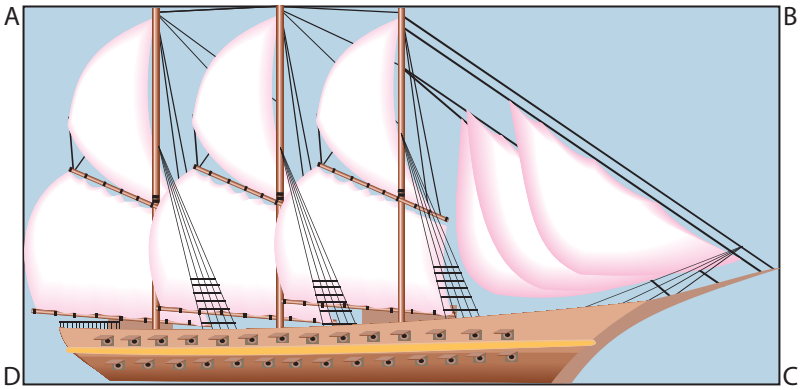
5. Extend Your Understanding Given the polygon, draw an image that is dilated by a factor of $\frac{1}{2}$.



On the Job 1

Identify a Dilation

Tyler wants to insert this image on his blog. It has to fit into rectangle EFGH. Will the image fit if he reduces its size proportionally?



Solution

F.Y.I.

Rectangles and squares have right angles.

The figures are proportional if they are similar. Use a protractor to compare corresponding angles:

$\angle A = 90°$ and $\angle E = 90°$
$\angle B = 90°$ and $\angle F = 90°$
$\angle C = 90°$ and $\angle G = 90°$
$\angle D = 90°$ and $\angle H = 90°$

The corresponding angles are equal.

Use a ruler to compare corresponding sides:

$$\frac{AB}{EF} = \frac{10}{5} = 2 \qquad \frac{BC}{FG} = \frac{5}{2.5} = 2 \qquad \frac{CD}{GH} = \frac{10}{5} = 2 \qquad \frac{AD}{EH} = \frac{5}{2.5} = 2$$

The corresponding sides are proportional.

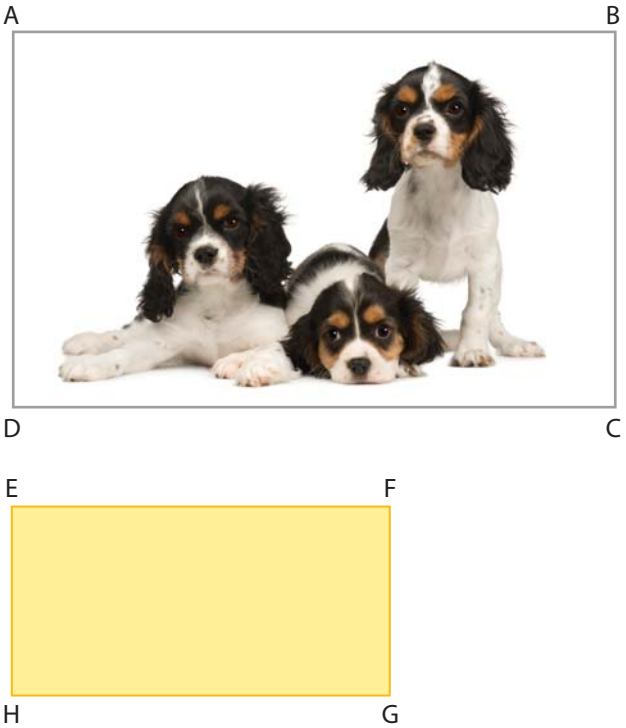
Since the corresponding sides are proportional and the corresponding angles are equal, the image and the rectangle are similar. So, the image will fit if Tyler reduces the size proportionally.

Tools of the Trade

Designers of print and digital products use computer software to make proportional changes to images.

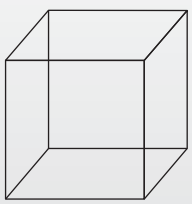
Your Turn

A magazine editor needs to know whether the photograph will fit into rectangle EFGH. Will the image fit if she reduces its size proportionally? Explain using measurements and ratios.

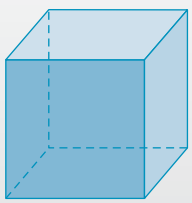


Puzzler

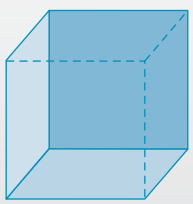
The Necker Cube is an ambiguous line drawing because there are two possible interpretations of it. There are no visual cues to tell you which is the front face of the cube and which is the back face. Both interpretations of the cube are valid. Sketch another example of an ambiguous line drawing.



Necker Cube



non-ambiguous cube 1

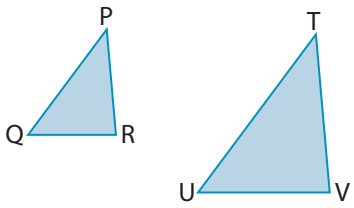


non-ambiguous cube 2

Check Your Understanding

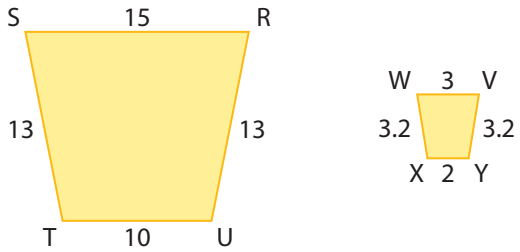
Try It

1. List the corresponding angles and corresponding sides for $\triangle PQR$ and $\triangle TUV$.

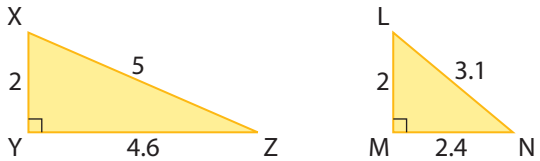


2. Check if each pair of shapes has proportional corresponding sides.

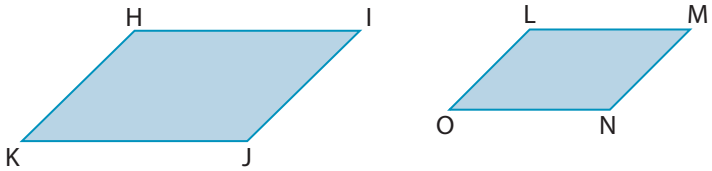
a)



b)

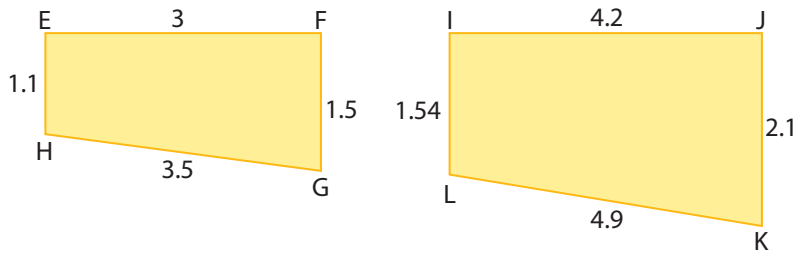


3. Use a ruler to check if the pair of shapes has proportional corresponding sides.

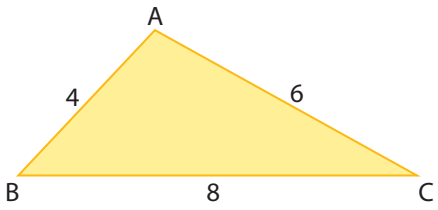


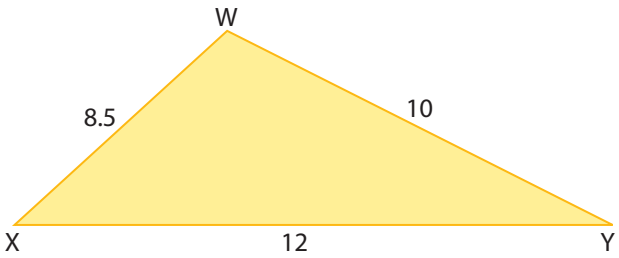
4. Use a protractor to check whether each pair of shapes has equal corresponding angles. State the angle measures.

a)



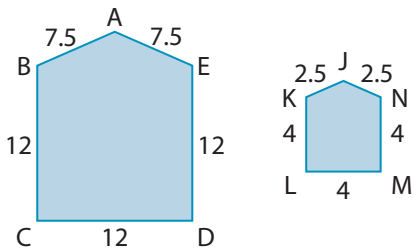
b)



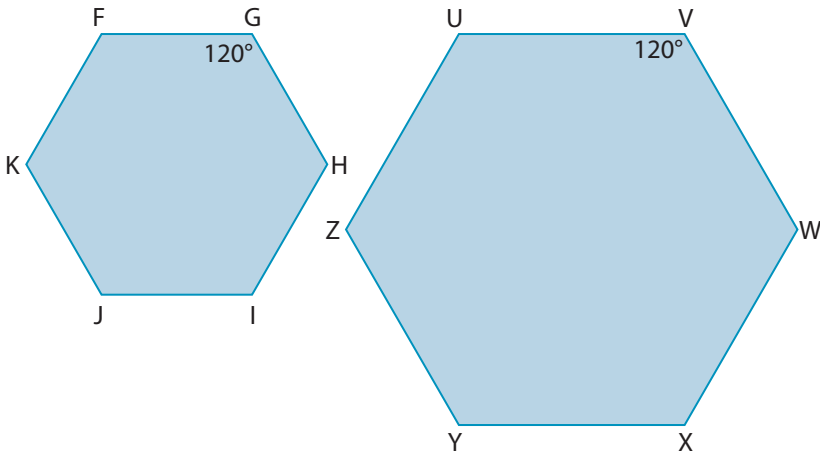


5. Is the smaller polygon a reduction of the larger one? Explain using similarity.

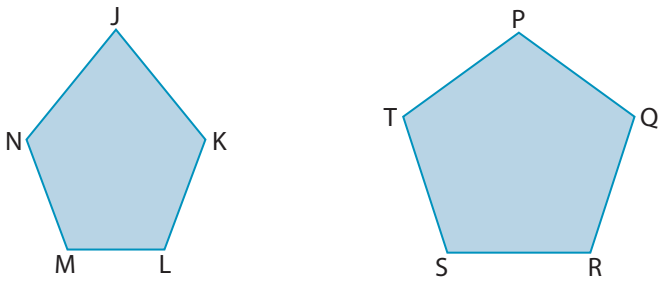
a)



b)

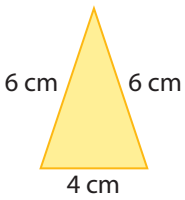


c)

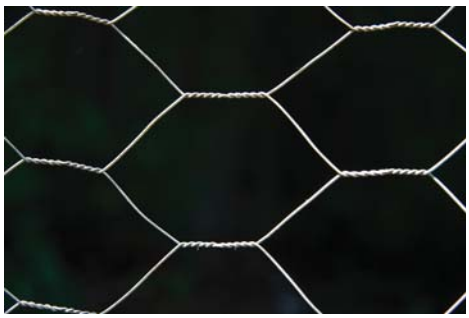


Apply It

6. David is creating a tile border for a kitchen backsplash. He wants to make a pattern using similar isosceles triangles. He creates his first triangle with side lengths of 6 cm, 6 cm, and 4 cm. Determine the dimensions of two other similar triangles that David could use in his pattern.



7. Chicken wire is often used for building fences. It is made of flexible wire with gaps that are shaped like hexagons.



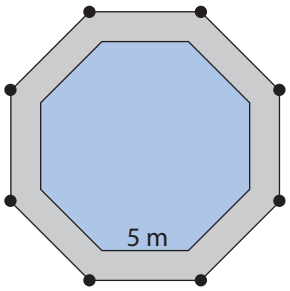
a) Use grid paper to draw and label
- the hexagon shown in the picture
- two hexagons that are dilations of the hexagon shown in the picture
- two hexagons that are not similar to the hexagon shown in the picture

b) For each pair of hexagons, explain how you know if they are similar or not similar.

8. Rachel's family is building a cement deck around their octagonal pool. The deck is the same shape as the pool.

a) What scale factor should Rachel use to find the length of the outer side of the deck?

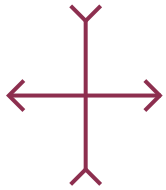
b) How long are the outer sides of the deck?



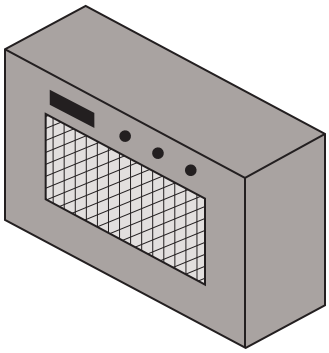
On the Job 2

Draw a Dilation

a) Logan is entering his band, The Arrows, in a Battle of the Bands contest. He is creating a flyer to advertise the contest. He wants to include the band's logo, but he needs to draw it twice as big on the flyer. How can Logan check that his image is proportional?



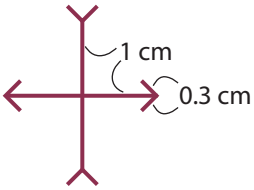
b) Logan also wants to include this 3-dimensional (3-D) drawing of an amp on the flyer. He needs to reduce the dimensions to half their length. How can Logan reduce the dimensions?



Solution

a) ***Method 1: Use a Scale Factor***

Measure the length of each line segment.



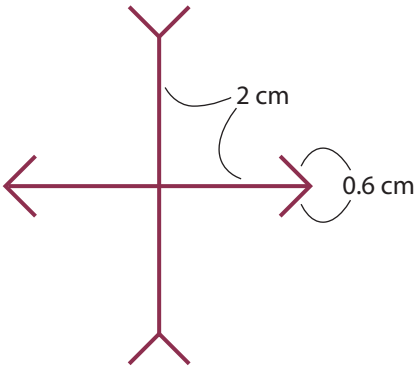
Since Logan needs the logo to be twice as big, multiply each measurement by a scale factor of 2.

$1 \times 2 = 2$

$0.3 \times 2 = 0.6$

The line lengths for the dilation are 2 cm and 0.6 cm.

Use the new line lengths to draw the dilation.



F.Y.I.

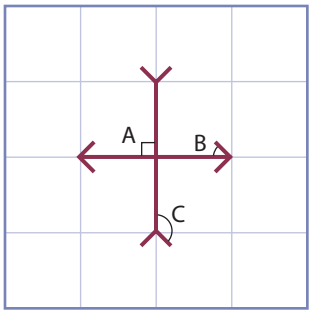
You can use technology to enlarge and reduce images proportionally. For example, use the zoom feature on tablets and smartphones.

Strategy

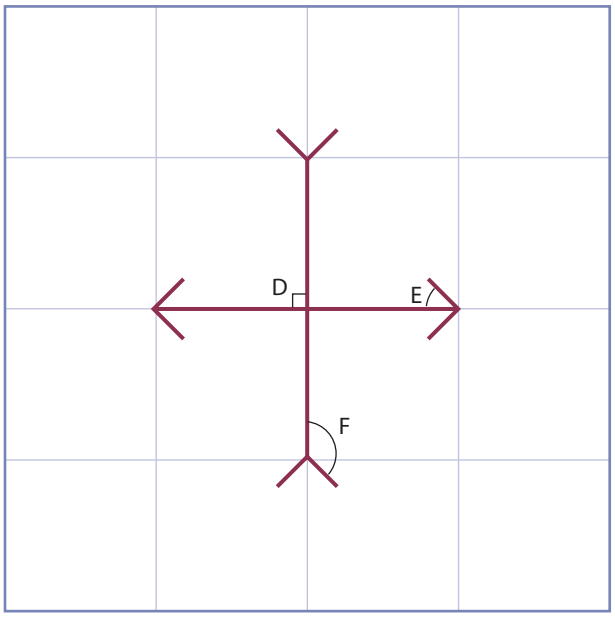
Draw or Model

Method 2: Use Grid Paper

Trace the original logo on centimetre grid paper. Label the angles.



Draw the contents of each grid square into the corresponding region on 2-centimetre grid paper. Label the angles.



To check that the image is proportional, use a protractor to compare the corresponding angles.

$\angle A = 90°$ and $\angle D = 90°$

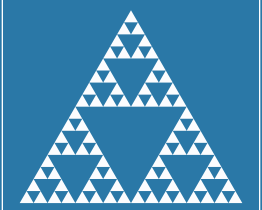
$\angle B = 45°$ and $\angle E = 45°$

$\angle C = 135°$ and $\angle F = 135°$

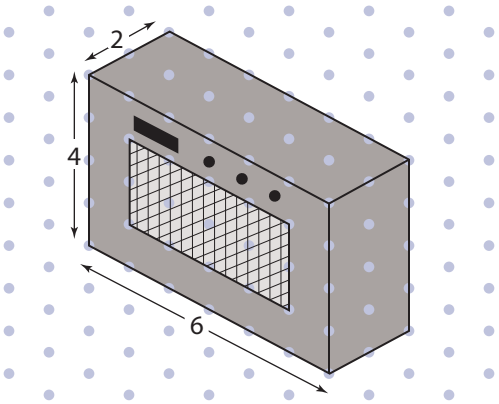
The image is proportional because the corresponding sides have proportional measures and the corresponding angles are equal.

F.Y.I.

Fractals, such as the Sierpinski gasket, are geometric designs created by repeatedly dilating a geometric shape.



b) Use isometric dot paper to draw the 3-D image. Label the lengths of the sides.



Since Logan needs the sides to be half as long, multiply each line length by $\frac{1}{2}$.

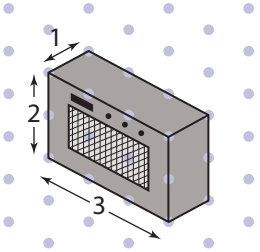
$2 \times \frac{1}{2} = 1$

$1 \times \frac{1}{2} = \frac{1}{2}$

$6 \times \frac{1}{2} = 3$

The line lengths for the dilation are 1, $\frac{1}{2}$, and 3.

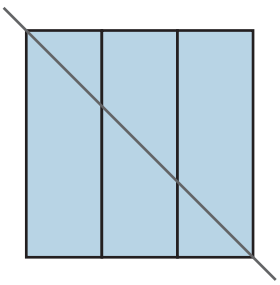
Use the new line lengths to draw the dilation.



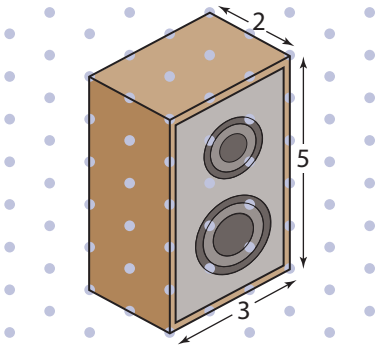
Your Turn

Carla's band, Three Blue, is also entering the contest.

a) Reduce the band's logo to one third of its size.



b) Draw a dilation of the speaker. Use a scale factor of 2. Use isometric dot paper to help you.

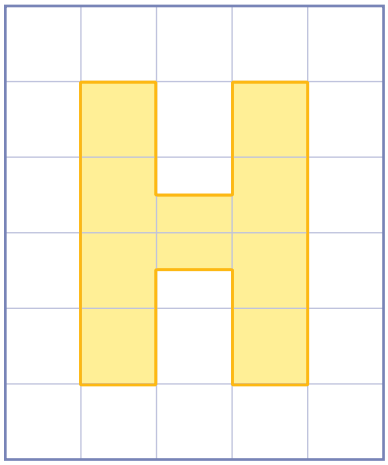


Check Your Understanding

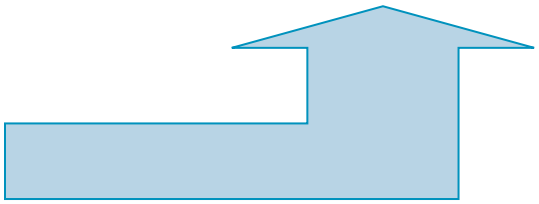
Try It

1. Reduce the size of each figure by half.

a)

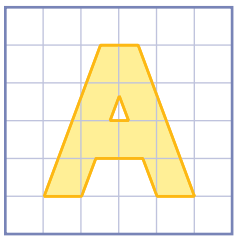


b)

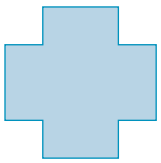


2. Use a scale factor of 2 to enlarge each figure.

a)

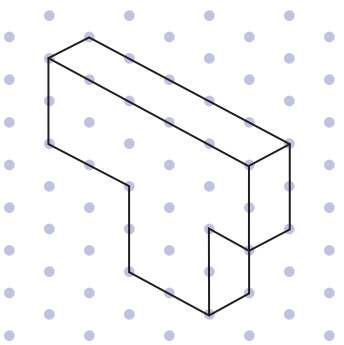


b)

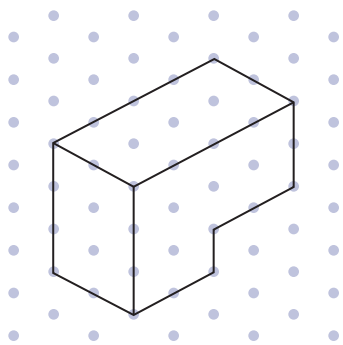


3. Use isometric dot paper to reduce the size of each object. Use a scale factor of 0.5.

a)

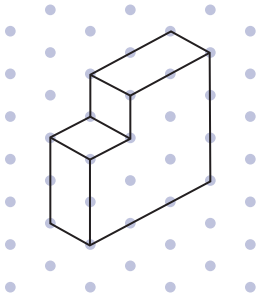


b)

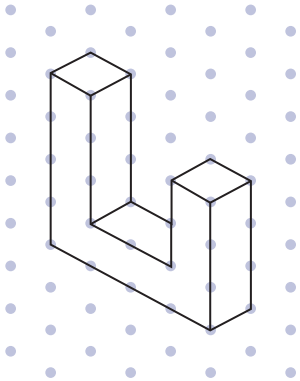


4. Use isometric dot paper to double the dimensions of each object.

a)



b)

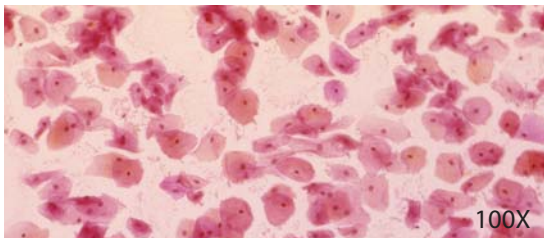


Apply It

5. Melissa is observing a slide of human cheek cells under the microscope.

a) Is this an enlargement or a reduction? Explain your reasoning.

b) What is the scale factor? Explain its meaning.



6. Choose a 3-D object in your classroom.

a) Draw it on isometric dot paper.

b) Draw a dilation of the object using a scale factor of 3.

7. Cape Spear lighthouse is at the easternmost point of North America. Jodi wants to build a scale replica of the lighthouse. If the height of the actual lighthouse is 13.7 m and Jodi wants her model to be 10 cm tall, what scale factor should she use?

Cape Spear, Newfoundland

Strategy

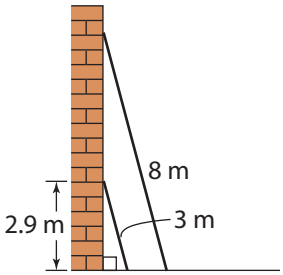
Develop a Strategy

First you need to write the values using the same units. Then, set up a proportion. What units will you use, centimetres or metres?

Work With It

1. An ophthalmologist measures the diameter of Kristen's pupils before and after putting drops in her eyes. Measure each pupil and determine the scale factor of the dilation.

2. Two ladders are leaning at the same angle against a wall.

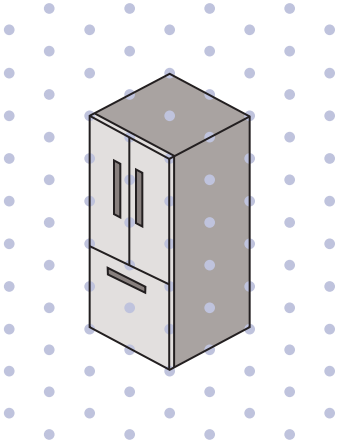


a) Are the two triangles the ladders make with the wall and floor similar? Explain how you know.

b) The 3-m ladder reaches 2.9 metres up the wall. How much farther up the wall does the 8-m ladder reach?

3. An architect draws a floor plan with a scale of 1 : 100. If one room on the floor plan measures 4.7 cm by 8.3 cm, what are the dimensions of the actual room?

4. Cheryl is a kitchen designer. She has made a 3-D drawing of her clients' kitchen layout, but they want to see a larger version. Draw the refrigerator 2 times larger. Use isometric dot paper to help you.



Web Link

Most designers use computers to create 3-D drawings. To practise designing a room, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Discuss It

5. How can you check that the larger image of the airplane is proportional to the dimensions in the original photo? Try your method. Describe your results.

original image

6. Bees make hexagonal-shaped cells in honeycombs as shown here. They build regular hexagons because the formation of these shapes results in the maximum amount of space with the minimum amount of material. If the hexagon was reduced or enlarged by a factor of 2, would it affect the formation? Explain.

Note: In a r*egular figure*, all sides have the same length and all angles are equal.

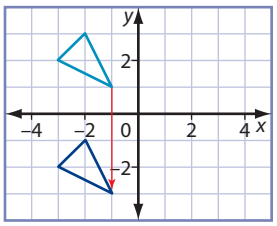
6.2 Translations

Focus On . . .

- identifying and drawing a vertical or horizontal translation
- drawing successive translations
- creating and analysing designs made with translations

translation

- a transformation that slides an object in a straight line without changing its size or orientation



Materials

- grid paper
- ruler
- polygon-shaped block

Mosaics are often made of repeating patterns on tiles. Many patterns are created by translations.

Explore Vertical and Horizontal Translations

1. a) Draw a coordinate grid on grid paper. Label the origin (0, 0).

b) Place a polygon block in Quadrant I with one vertex of the polygon on the origin. Trace the shape. Label the vertex that is touching the origin as A. Label the shape "Original."

c) Slide the shape 4 units left and trace it. Label it "Image 1." Label A′.

d) Return to the original position. This time, slide the shape 5 units down and trace it. Label it "Image 2." Label A″.

e) Return to the original position. Slide the shape 3 units right and 2 units up. Trace and label it "Image 3." Label A‴.

Strategy

Look for a Pattern

Tools of the Trade

General contractors, graphic designers, bricklayers, and landscapers all use translations in their jobs.

2. a) Copy and complete the table. Compare the vertices from each image to the original.

Original	Image 1	Image 2	Image 3
A (0, 0)	A′ (■, ■)	A″ (■, ■)	A‴ (■, ■)

b) What is the relationship between the translation and the change in the ordered pairs?

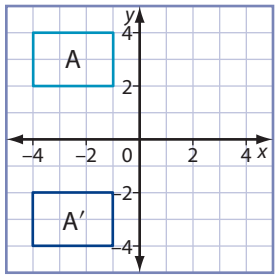
c) Test your conclusion on the other corresponding vertices of the original shape and each image.

3. Reflect What conclusions can you draw about the direction of a translation and the change in the value of the coordinates of the vertices?

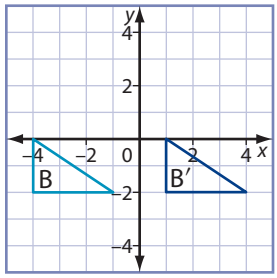
4. Do the images differ in size from the original? How do you know?

5. Extend Your Understanding Use translations to describe the steps for creating each image from the original.

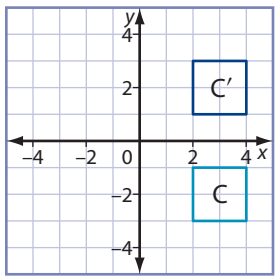
a)



b)



c)



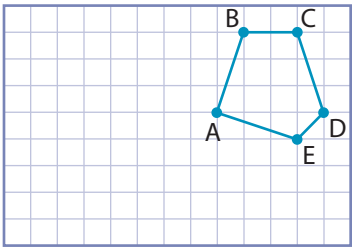
On the Job 1

Draw a Translation

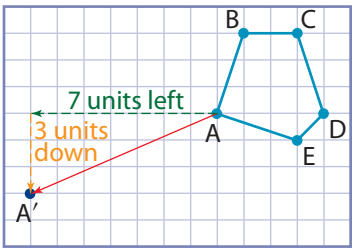
Kara is a cartographer who makes maps of lakes in her province. She made a mistake on one of her maps and put Bear Lake in the wrong location. She needs to move it 7 km west and 3 km south. Show Kara how to translate the figure 7 units to the left and 3 units down.

Solution

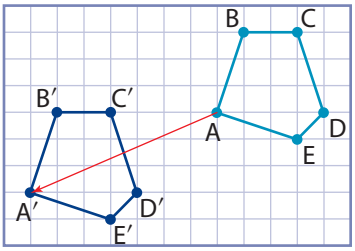
Label the vertices of the figure.



Start at point A. Count 7 units left and 3 units down. Plot the point and call it A′.



Plot points B′, C′, D′, and E′, counting 7 units left and 3 units down for each. Connect the points to form A′B′C′D′E′.

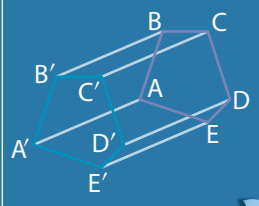


Tools of the Trade

Cartographers use various forms of technology, including computers, GPS, and satellite imagery, to create maps.

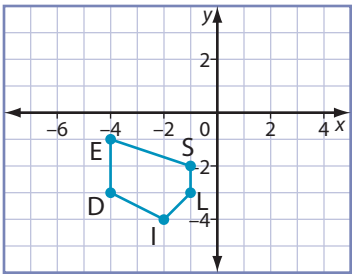
F.Y.I.

You can use translations to make 3-D drawings.



Your Turn

Copy this figure on a coordinate grid. Translate the figure 5 units right and 3 units up.

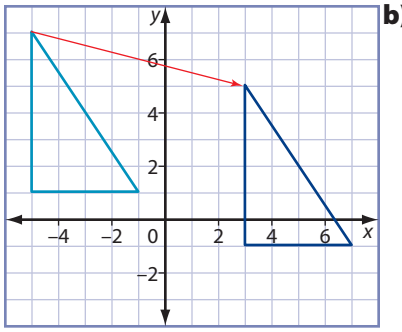


Check Your Understanding

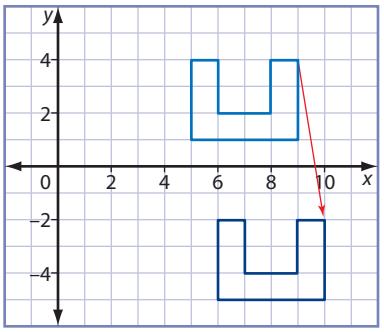
Try It

1. Describe the translation shown in each diagram.

a)



b)



2. Point A is at (4, 2) on a coordinate grid. What are the coordinates of Point A after it is translated

a) 3 units right?

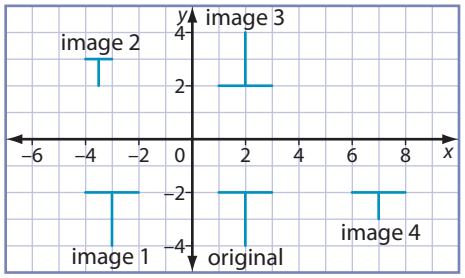
b) 5 units down?

c) 6 units left and 1 unit up?

d) 2 units right and 8 units up?

e) 10 units left and 12 units down?

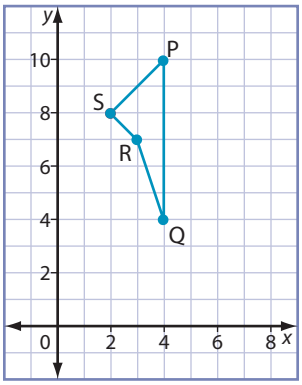
3. Which image represents a translation of the original shape?



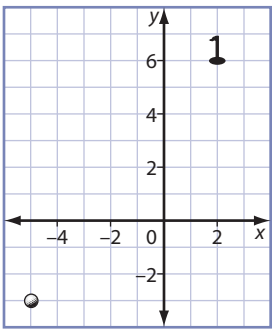
4. Copy figure PQRS onto a coordinate grid.

a) Translate figure PQRS 3 units right and 6 units down.

b) What are the coordinates of the translated image?



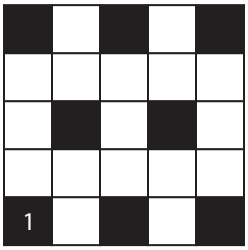
5. Describe the translation that would allow the ball to fall into the hole.



Apply It

Strategy
Look for a Pattern

6. James is using black and white tiles to create a pattern for the walls of his shower. The first black tile he places is in the bottom left corner. Describe, using translations, the location of the other black tiles in the pattern in relation to the first tile.

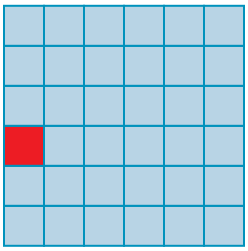


7. Kendra is making a mosaic using coloured pieces of glass. She lays the glass tiles on a grid to help her organize her pattern.

She places the first red tile in the grid. She wants to arrange the other red tiles in the following pattern:

- 2 spaces up and 3 spaces right
- 1 space down and 4 spaces right
- 3 spaces up and 5 spaces right
- 2 spaces down and 2 spaces right

Copy the grid and colour the other four red tiles.



8. The plans for a new park are drawn on a coordinate grid. The climbing structure is placed at coordinates M(−4, 4), N(−3, 5), R(−2, 4), P(−2, 2), and Q(−4, 2). The park manager wants to move the structure to Quadrant IV. She wants points M and Q to lie on the *y*-axis of the grid, and point N to lie on the *x*-axis.

a) Plot MNRPQ on a coordinate grid.

b) Draw MNRPQ in its new position.

c) Describe the translation.

On the Job 2

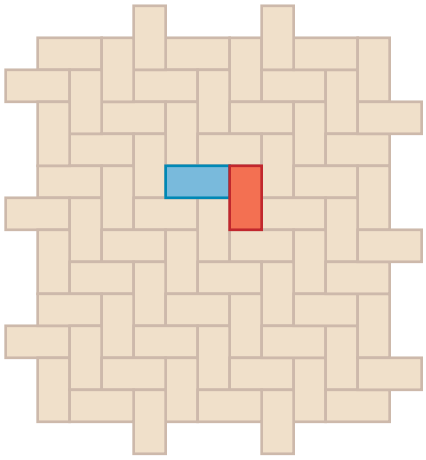
Successive Translations

successive translation

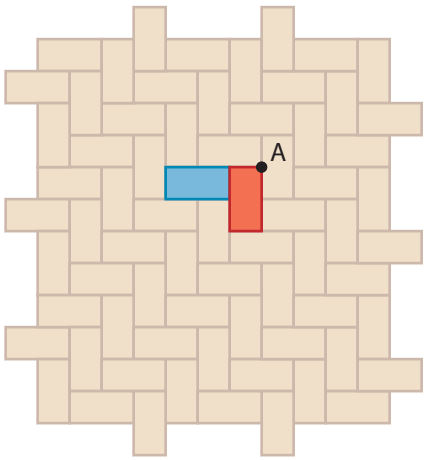
- a pattern created by translating a figure multiple times using the same translation

Patterns for quilts, tiles, stone pathways, and many other things are often made by **successive translations**.

a) You can make patios from interlocking bricks. Describe the successive translations used to make this patio. Each stone is 2 by 1.

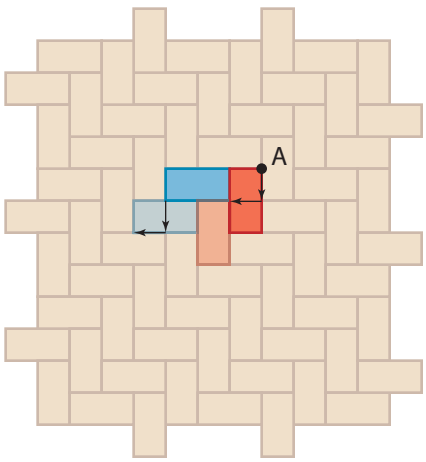


b) Draw the successive translations on a grid. Plot point A at the origin. Repeat the pattern three times.

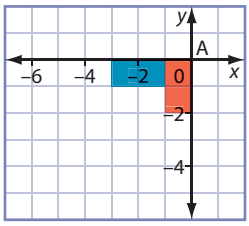


Solution

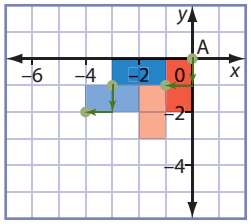
a) Each stone is translated 1 unit down and 1 unit left.



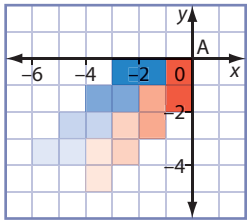
b) Draw the pattern, and label point A.



Translate A 1 unit down and 1 unit left to $(-1, -1)$. Draw the stone. Do the same for the blue stone.

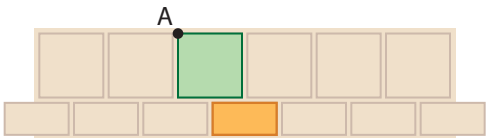


Repeat the pattern two more times.



Your Turn

a) Describe the successive translations used to make this pattern. The square stone is 2 by 2. The rectangular stone is 2 by 1.

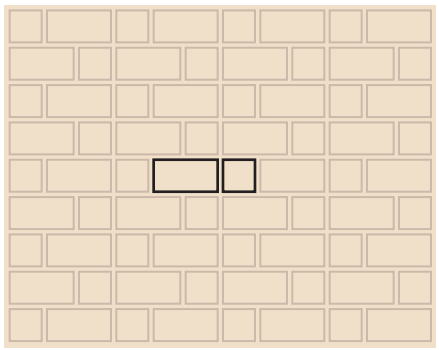


b) Draw three successions of the pattern on a coordinate grid. Plot point A at the origin.

Check Your Understanding

Try It

1. Describe the successive translations shown in the diagram.



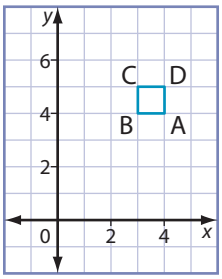
2. Plot each point on a coordinate grid. Then, plot the translation described. Repeat the translation three times.

a) A (−7, 12) 4 units down, 3 units right

b) B (8, −10) 2 units left, 1 unit up

c) C (3, 9) 1 unit right, 5 units down

3. Perform three successive translations by sliding the square 2 units down and 3 units left. Use grid paper to help you.

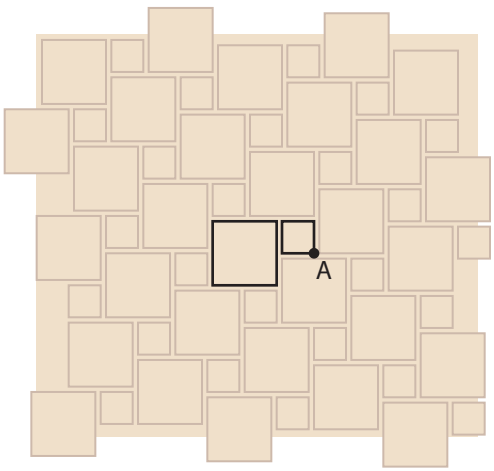


Apply It

4. a) Describe the successive translation for this brick pattern.

b) Draw the pattern on a grid, plotting point A at (5, 5).

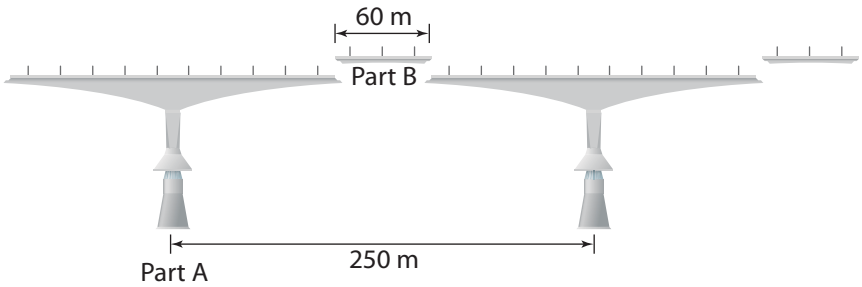
c) Repeat the translation three times.



5. Sketch a polygon of your choice. Create a pattern by performing a successive translation. Describe your translation.

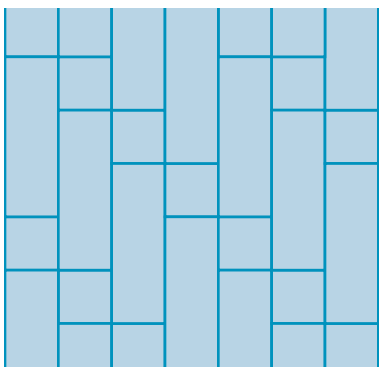
6. Confederation Bridge is 12.9 km long and connects New Brunswick to Prince Edward Island. It is the world's longest bridge over ice-covered water.

The bridge was built using T-shaped piers that are connected by straight pieces. Describe the translations to Part A and Part B needed to build the bridge.



7. a) Describe the successive translations shown in the tiling pattern.

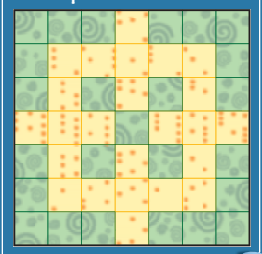
b) Does the order in which you do the translations matter? Explain your answer.



Work With It

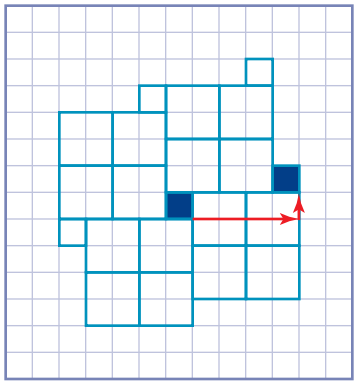
F.Y.I.

A quilt is made by sewing together quilt blocks. The blocks are usually geometric patterns. This is an example of one block.



1. Use a 10 by 10 grid to design a quilting block pattern that shows successive translations.

2. A tiler draws a diagram of a tile pattern on grid paper.

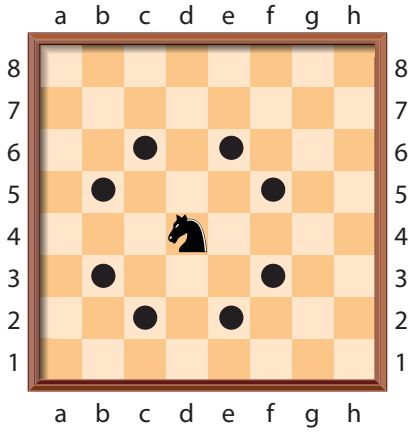


a) What is the translation indicated by the red arrows on the grid?

b) The scale is 1 : 20. What would the full-size translation be?

Discuss It

3. The black dots on the diagram show all the places the black knight can move on the chess board. Describe how to move the knight from d4 to h7 using successive translations.



4. Research and explain how successive translations can be used in animation to show movement.

6.3 Reflections

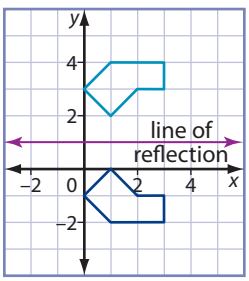
Focus On . . .

- identifying a reflection and drawing the image of a shape that is reflected
- drawing shapes that are reflected and translated
- creating and analysing designs made with reflections

Kenojuak Ashevak is considered a pioneer of modern Inuit art. Born on the southern coast of Baffin Island in 1927, she has been drawing and painting since the late 1950s. One of her favourite subjects is birds. In this print,* Resplendent Owls, *there is almost an exact reflection. Where is the line of reflection in this example?

reflection

- a transformation in which an object is shown as its mirror image over a line of reflection

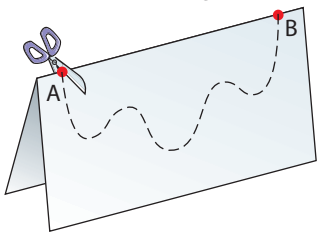


line of reflection

- a line that an object is reflected over
- the corresponding points on both sides of the line are the same distance away from the line

Explore Reflections

1. Fold a piece of paper in half. Mark two points, A and B, on the fold. Draw a wavy line between the points on one side of the paper. Cut along the line and then unfold your cutout figure.



a) How does the fold affect the shape of the cutout?

b) Explain why it makes sense to refer to your fold line as a line of reflection.

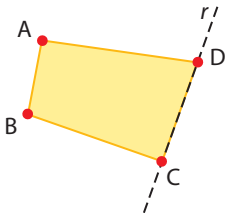
Materials

- scissors

2. a) How could you fold a piece of paper so that a cutout shape is reflected twice? Use your method to create a cutout with two reflections.

b) Fold and cut a piece of paper to make a design with four reflections.

3. The diagram shows half of a shape. Line r is the line of reflection for the shape. Copy the diagram. Then, complete the shape.



4. Reflect What are some ways to complete the shape in step 3? Describe one way to a partner. See if your partner can follow your instructions.

5. Extend Your Understanding A rectangle is reflected over the x-axis. Is there another transformation that might create the same image? Explain.

6. Consider the Taj Mahal. Use the photo to explain how a line of reflection and a line of symmetry are the same.

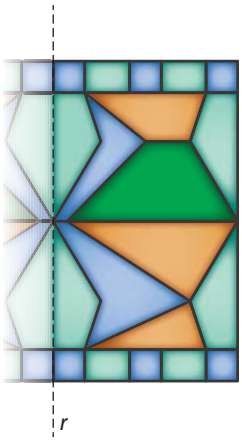
F.Y.I.

The Taj Mahal is a tomb in Agra, India. It took 22 years to build and was finished in 1653.

On the Job 1

Draw a Reflection

Joe is creating a stained-glass window. He wants the left side to be a reflection of the right side. One of the shapes in the window is the dark green quadrilateral shown. Draw the reflection of the dark green quadrilateral over the line of reflection, r.

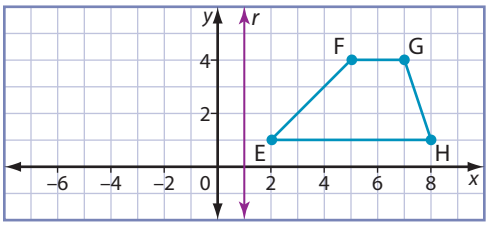


Strategy

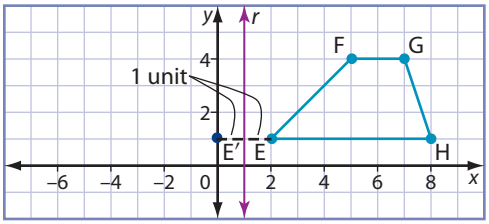
Draw or Model

Solution

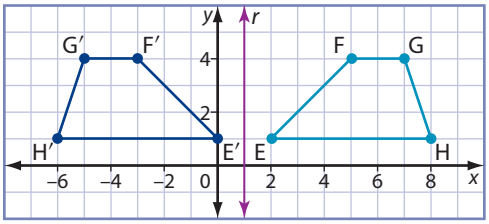
Draw the quadrilateral and line of reflection on a coordinate grid.



Count how many spaces each point is from line of reflection r. Point E is 1 unit away from the line of reflection. Point E′ will be 1 unit away on the opposite side of the line of reflection. Plot point E′.



Plot points F′, G′, and H′. Connect the points to form E′F′G′H′.



F.Y.I.

A line of reflection can also be considered a line of symmetry for the original figure and its image.

A reflection is a mirror image of the original figure. The image "flips" over the line of reflection.

Your Turn

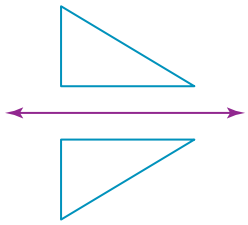
Copy the original polygon above on a coordinate grid. Reflect the figure using the x-axis as the line of reflection.

Check Your Understanding

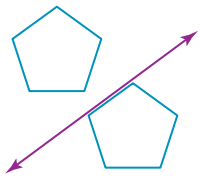
Try It

1. Which of these transformations represent a reflection?

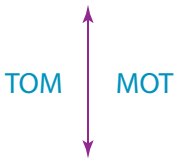
a)



b)

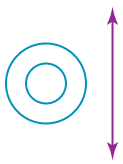


c)



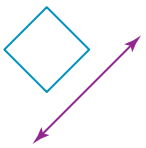
2. Copy the following shapes and lines of reflection. Reflect each shape.

a)



b)

c)

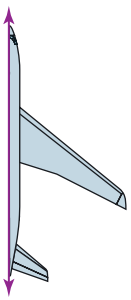


3. Copy the figures. Complete the pattern by drawing the reflection along the line of reflection.

a)

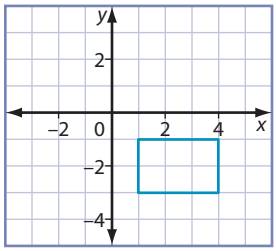
b)

c)



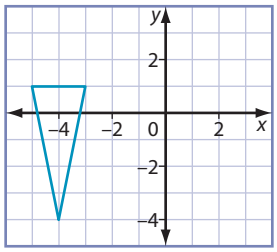
4. Copy each figure on a coordinate grid. Draw the line of reflection and then reflect the figure over the line of reflection.

a)



Line of reflection at $y = 1$

b)



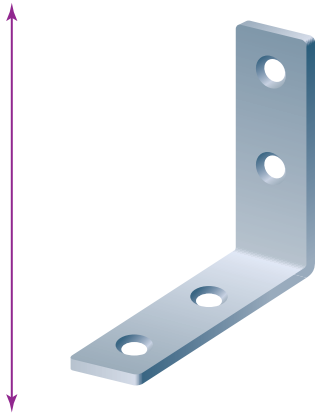
Line of reflection at $x = -2$

Apply It

Strategy

Draw or Model

5. A metal fabrication plant makes metal posts with hangers for shrimp trawlers. The posts are used on both the port and starboard sides of the boat, so they must be mirror images of each other. Sketch the bracket. Then, draw a reflection of it over the line of reflection.



6. When you look at something in a mirror, the reflection you see is the reverse of the object. For example, when you lift your left hand, it looks like your right hand in the mirror. Ambulances reflect the word *ambulance* on the hood so it can be read properly in a rearview mirror. Draw a reflection of your name and check that it reads properly in a mirror.

7. An artist sculpting a human face wants to check that the face is symmetrical.

a) Give a possible method the artist could use to check for symmetry.

b) Explain how doing this involves a reflection.

On the Job 2

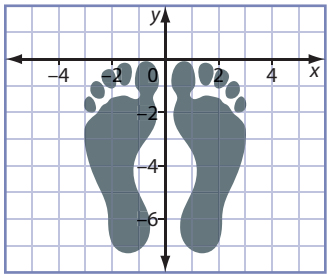
Reflections and Translations

Jeanine designs stationery. She draws a border that is an image of footprints left by someone walking in the sand. She wants the footprints to be exactly the same size and shape, so she creates a stencil of one footprint. What transformations can she use to make sure the trail of footprints is evenly spaced? Draw the transformations to show Jeanine how to create the design.

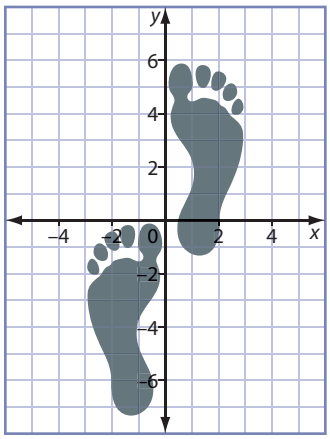
Solution

Create the trail of footprints by reflecting and then translating the new reflected image.

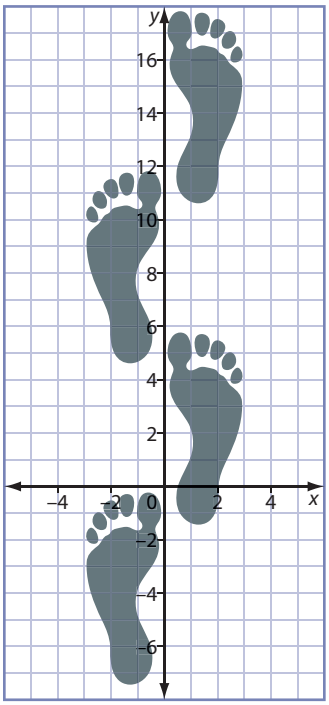
First, draw the footprint on a grid. Then, reflect it over the y-axis.



Next, translate the reflected image 6 units up.



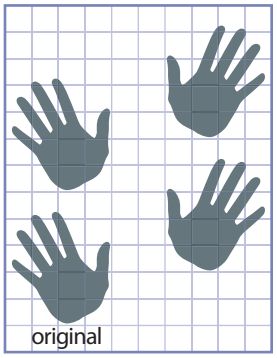
Repeat the reflections and translations to get a trail of footprints.



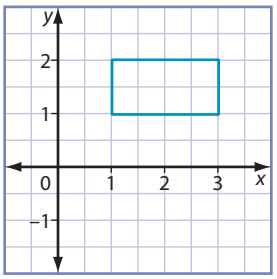
Your Turn

Charles is making a design using handprints.

a) Describe the reflections and translations Charles used to create his pattern.



b) Copy the figure and reflect it over the x-axis. Then, translate it 3 units left. Repeat the pattern three times.

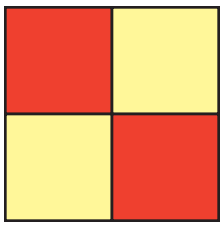


Check Your Understanding

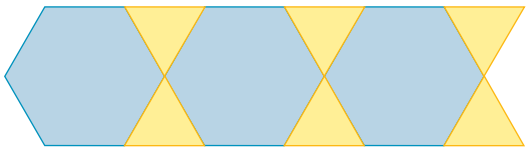
Try It

1. Which patterns use transformations that combine a reflection and a translation?

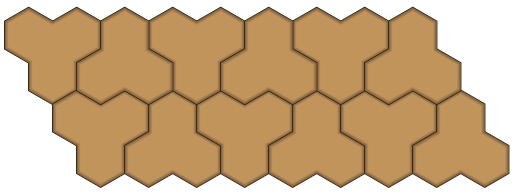
a)



b)

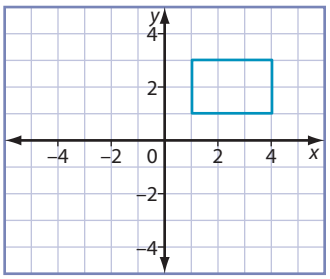


c)



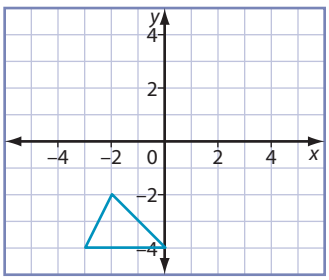
2. a) Copy the rectangle on a coordinate grid. Draw a vertical line of reflection at $x = -2$. Reflect the rectangle over the line of reflection.

b) Translate the reflected image 3 units down.



3. a) Copy the triangle on a coordinate grid. Draw a horizontal line of reflection at $y = -2$. Reflect the triangle over the line of reflection.

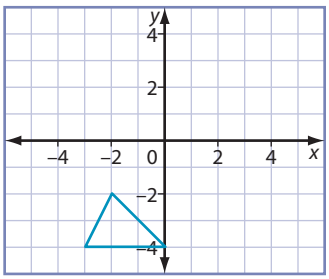
b) Translate the reflected image 2 units up.



4. a) Copy the triangle on a coordinate grid.

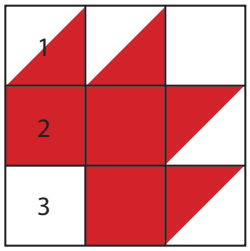
b) Draw a horizontal line of reflection at $y = 1$. Reflect the triangle over the line of reflection.

c) Is your translated image the same as in #3b)?

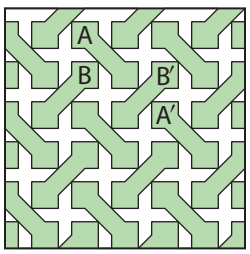


Apply It

5. Describe the transformations each numbered block undergoes to create the quilt pattern.



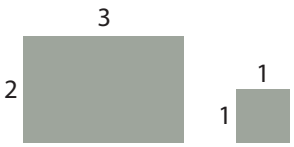
6. a) Describe the transformations used to create the pattern. How does A become A′ and B become B′?



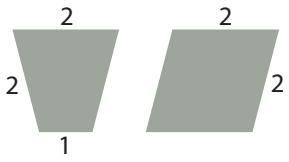
b) Using the same arrow, create a new pattern.

7. Use the paving stones to create a pattern that covers a 10-by-10 grid.

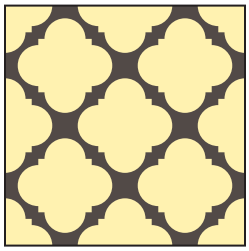
a)



b)



8. Describe two different ways to create the pattern.



Strategy

Develop Alternative Approaches

What combinations of translations and reflections do you see?

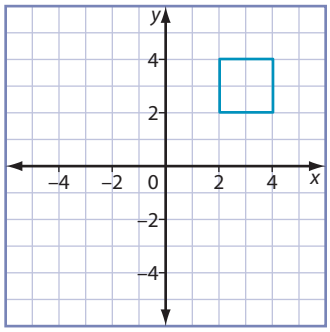
Work With It

1. Samantha is designing a kitchen backsplash that uses two different tiles shaped like polygons. She uses two translations to create a pattern for the backsplash. Use grid paper to show an example of Samantha's backsplash. Show your pattern.

2. Use a 3-by-3 grid to create a pattern using two different shapes. Reflect and translate the shapes to make the pattern.

Discuss It

3. Max reflects the square over the y-axis. He then translates the original square 8 units left. Max determines that all images created by reflections can also be created by translations. Do you agree with him? Explain your answer.



4. When you look in a mirror, why is your image reflected left to right and not upside down?

Strategy

Look for a Pattern

5. **a)** Plot the ordered pairs on a coordinate grid. Connect the points.

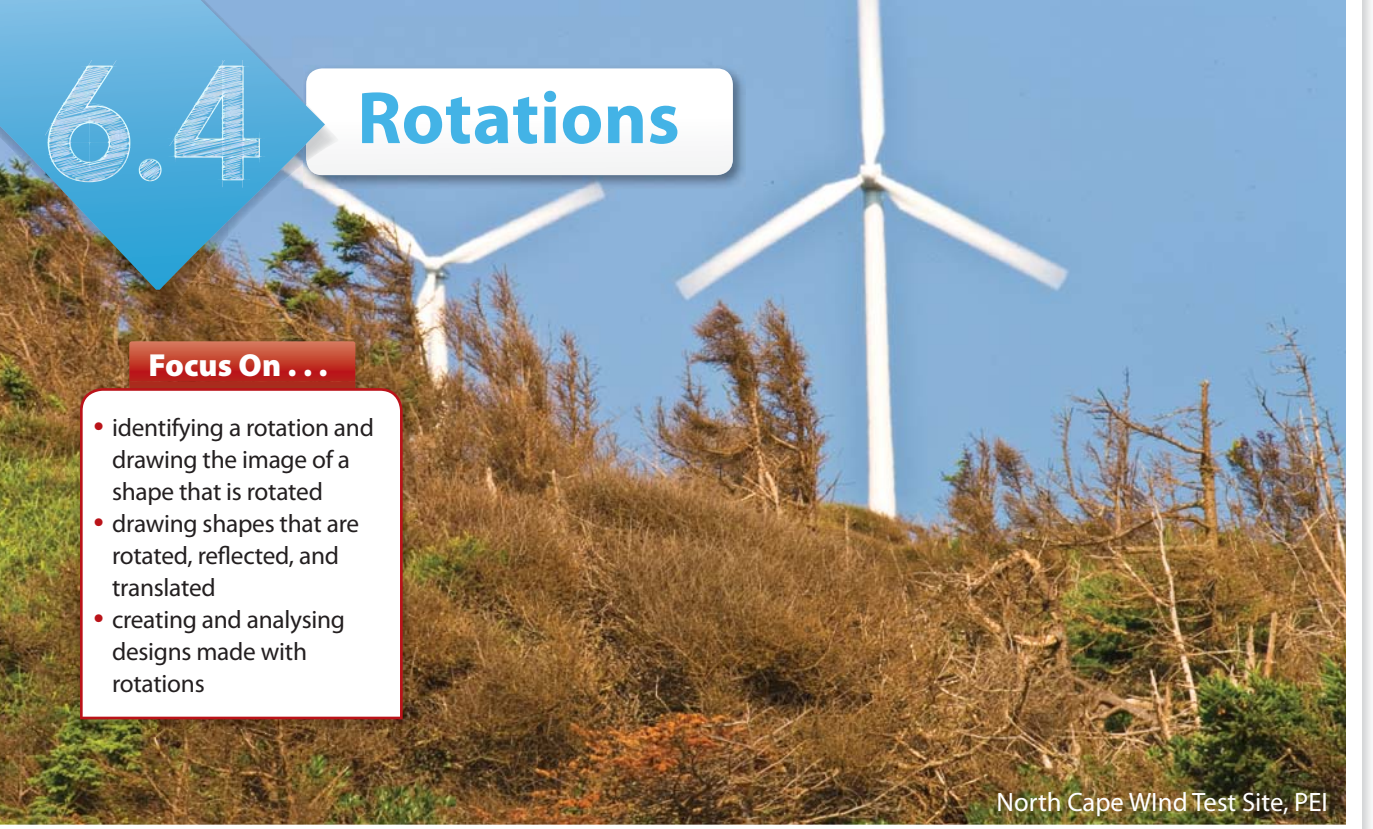
   A(5, −4) B(4, 2) C(−3, 7) D(−8, −6)

   b) Reflect each point over the x-axis. Write the coordinates of points A′, B′, C′, and D′. What do you notice?

   c) Reflect each original point over the y-axis. Write the coordinates of points A″, B″, C″, and D″. What do you notice?

   d) The line $y = x$ can also be a line of reflection. Reflect the original points over the line $y = x$. Write the coordinates of points A‴, B‴, C‴, and D‴. What do you notice?

6.4 Rotations


North Cape Wind Test Site, PEI

Focus On . . .

- identifying a rotation and drawing the image of a shape that is rotated
- drawing shapes that are rotated, reflected, and translated
- creating and analysing designs made with rotations

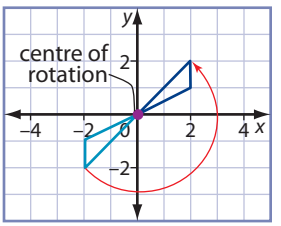
Materials

- grid paper
- ruler
- compass
- protractor

Windmills turn the power of wind into energy. A windmill has blades that are the same shape and size and that rotate around a central section. The blades can turn clockwise or counterclockwise.

rotation

- a transformation that moves an object around a fixed point that is called the centre of rotation



centre of rotation

- the point about which an object is rotated

Explore Rotations

1. Perform a **rotation** of the point (2, 3). Rotate the point 90° clockwise using the origin as the **centre of rotation**.

a) Draw a four-quadrant coordinate grid on grid paper. Plot point (2, 3).

b) Place the metal tip of the compass at the centre of rotation, (0, 0).

c) Use a compass to draw an arc from (2, 3) in the direction of the rotation (clockwise).

d) Place the centre of the protractor on the centre of rotation and measure 90° from (2, 3). Draw a dot. This is the rotated point. Label its coordinates.

2. On the same grid, rotate the point (2, 3) 180° using the origin as the centre of rotation. Place the centre of the protractor on the centre of rotation and measure 180° from (2, 3). Draw a dot. This is the rotated point. Label its coordinates.

3. a) Copy and complete the table.

Degree and Direction of Rotation	Original Coordinates	New Coordinates
90° clockwise	(2, 3)	
180°	(2, 3)	

b) What general rule can you make about the new coordinates of a point rotated 90° clockwise when the centre of rotation is the origin?

c) What general rule can you make about the new coordinates of a point rotated 180° when the centre of rotation is the origin?

Strategy

Look for a Pattern

4. Reflect

a) Other than 0°, what rotational angle will return an object to its original position?

b) Rotate a point 90° clockwise. What other rotation could you use to get the point to the same position?

5. Do you need to indicate the direction of a 180° rotation? Explain your answer.

6. Extend Your Understanding

a) What do you think the coordinates of (2, 3) rotated 90° counterclockwise will be?

b) Check your answer to part a). Were you right?

c) What general rule can you make about the new coordinates of a point rotated 90° counterclockwise around the origin?

Puzzler

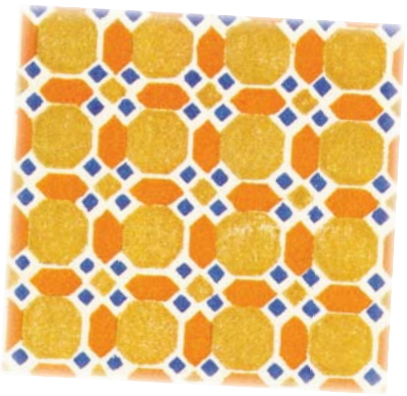
Create a valid mathematical expression by replacing each letter with a number. Use the same number for the same letter.

SEND
+MORE
MONEY

On the Job 1

Draw a Rotation

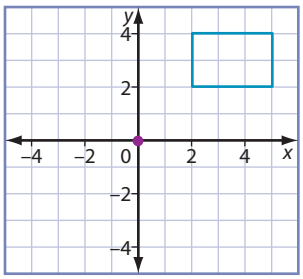
Mosaics are often made using repeating patterns of tiles. The orange hexagon in this mosaic is rotated to create a tiling pattern.



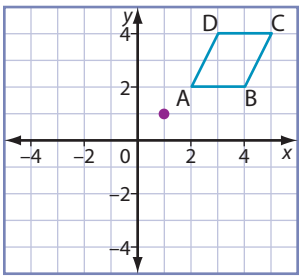
F.Y.I.

A pattern that covers an area without leaving gaps or overlapping is called a tessellation.

a) Rotate the rectangle 90° clockwise, 90° counterclockwise, and 180° using the origin as the centre of rotation.



b) Rotate the parallelogram 90° clockwise and 90° counterclockwise using (1, 1) as the centre of rotation.

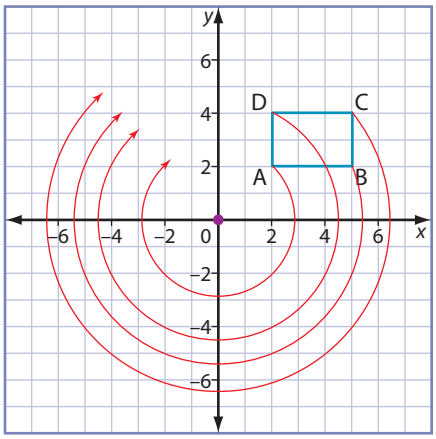


Solution

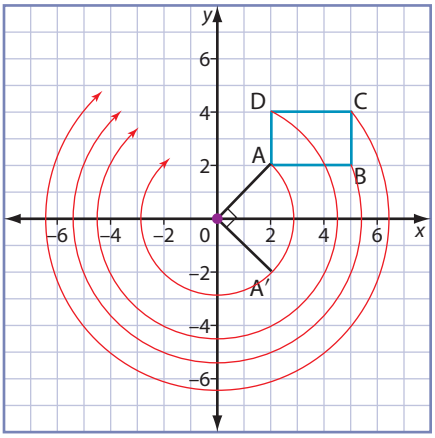
a) ***Method 1: Use a Compass and Protractor***

90° clockwise:

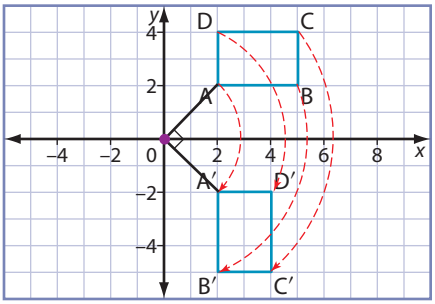
Use a compass to draw an arc from each vertex in the direction of the rotation.



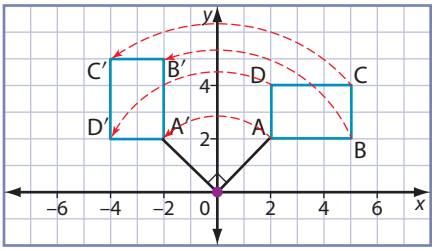
Place the centre of the protractor on (0, 0) and measure 90° clockwise from each vertex to the corresponding arc.



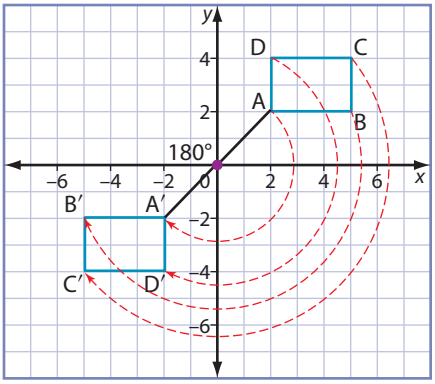
Mark each intersection with a point, and join these points to form the rotated image of the rectangle.



90° counterclockwise:



180°:



You could also draw the arcs counterclockwise.

Method 2: Use The Rotation Rule

You can apply general rules to rotate figures 90° clockwise, 90° counterclockwise, and 180° around the origin.

Degree and Direction of Rotation	Original Coordinates Pattern	New Coordinates Pattern
90° clockwise	(x, y)	$(y, -x)$
90° counterclockwise	(x, y)	$(-y, x)$
180°	(x, y)	$(-x, -y)$

So, the coordinates of the vertices for a 90° clockwise rotation will change as follows:

$A(2, 2) \rightarrow A'(2, -2)$
$B(5, 2) \rightarrow B'(2, -5)$
$C(5, 4) \rightarrow C'(4, -5)$
$D(2, 4) \rightarrow D'(4, -2)$

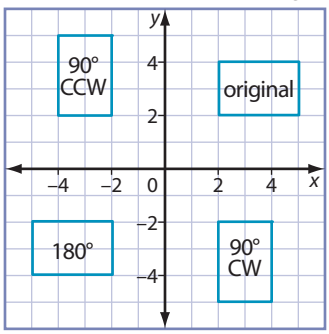
The coordinates of the vertices for a 90° counterclockwise rotation will change as follows:

$A(2, 2) \rightarrow A'(-2, 2)$
$B(5, 2) \rightarrow B'(-2, 5)$
$C(5, 4) \rightarrow C'(-4, 5)$
$D(2, 4) \rightarrow D'(-4, 2)$

The coordinates of the vertices for a 180° rotation will change as follows:

$A(2, 2) \rightarrow A'(-2, -2)$
$B(5, 2) \rightarrow B'(-5, -2)$
$C(5, 4) \rightarrow C'(-5, -4)$
$D(2, 4) \rightarrow D'(-2, -4)$

Plot the rotated rectangles on the grid.



CW stands for clockwise; CCW stands for counterclockwise.

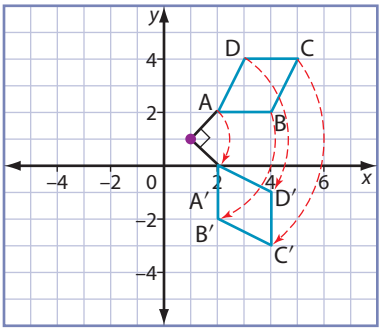
F.Y.I.

You cannot use the rotation rule if the centre of rotation is not the origin.

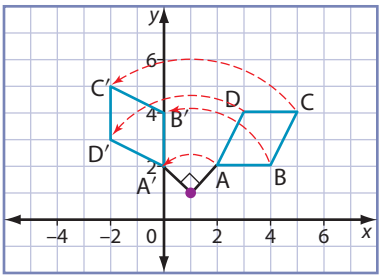
b) The centre of rotation is not the origin, so use a compass and protractor.

90° clockwise:

Use a compass to draw an arc from each vertex in the direction of the rotation. Place the centre of the protractor on (1, 1) and measure 90° clockwise from each vertex to the arc. Mark each intersection with a point, and join these points to form the rotated image of the parallelogram.

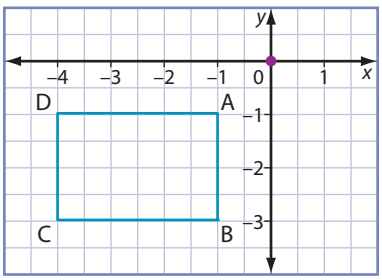


90° counterclockwise:



Your Turn

Rotate the rectangle 90° clockwise, 90° counterclockwise, and 180° using the origin as the centre of rotation.



Check Your Understanding

Try It

1. Use a compass and protractor to rotate each point according to the instructions.

a) (3, 5) 90° clockwise about the origin

b) (−2, −4) 90° counterclockwise about centre of rotation (−1, 3)

2. Use the rotation rule to rotate each point according to the instructions.

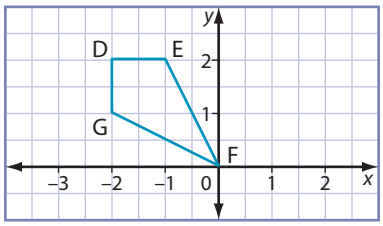
a) (−5, 10) 270° clockwise about the origin

b) (4, −2) 180° about the origin

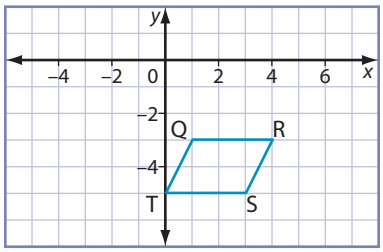
270° clockwise is the same as 90° counterclockwise.

3. Use the rotation rule to rotate each shape according to the instructions.

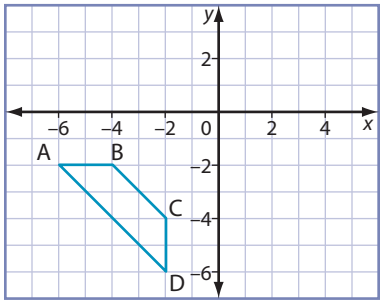
a) 180° clockwise about the origin



b) 90° counterclockwise about the origin

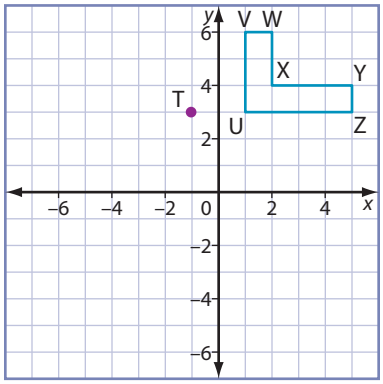


c) 90° clockwise about the origin

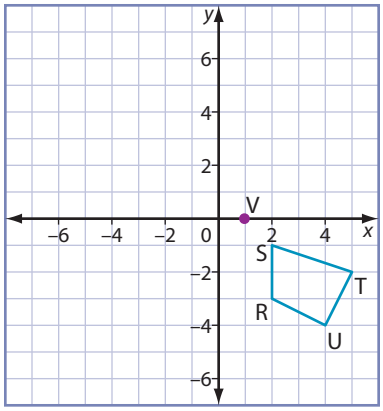


4. Use a compass and protractor to rotate each shape according to the instructions.

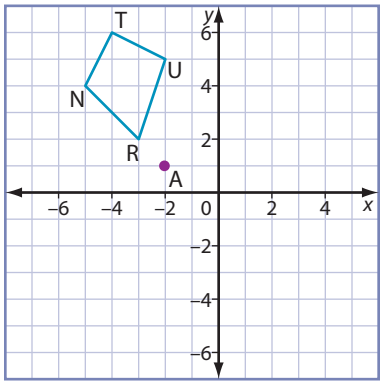
a) 90° counterclockwise about the centre of rotation T



b) 45° clockwise about the centre of rotation V



c) 270° clockwise about the centre of rotation A



Apply It

5. If a square is rotated 90° around its centre, the original shape and the image look the same. Name another shape and rotation that will have this result.

6. The flag of Hong Kong contains a white five-petal flower called the *Bauhinia blakeana*.

a) Sketch the flag and label the centre of rotation.

b) Use a protractor to determine the angle of rotation of one petal.

7. Draw a figure on a coordinate grid in Quadrant I. First, rotate the figure 90° clockwise about the origin. Then, rotate the figure 90° counterclockwise. Describe the results using the coordinates of the vertices.

Strategy

Develop Alternative Approaches

8. The point B′(8, −2) is the image of a point that was rotated 180° about the origin. Determine the location of the original point.

9. Marsha is designing a ceiling fan light fixture.

a) Describe how she could use rotations to design the fan.

b) Sketch your own design of a fan. Label the centre of rotation and state the angle of rotation between the fan blades.

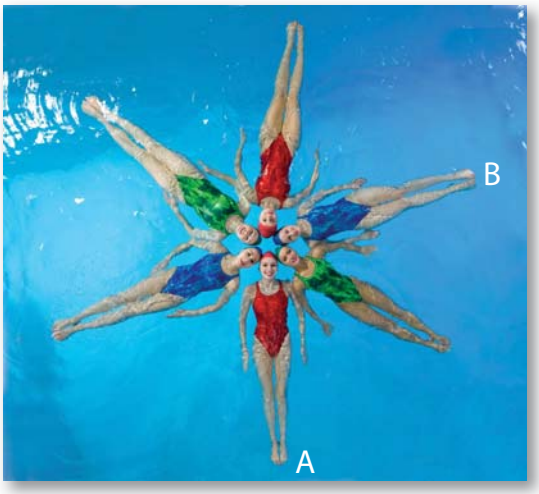
On the Job 2

Rotation, Translation, and Reflection

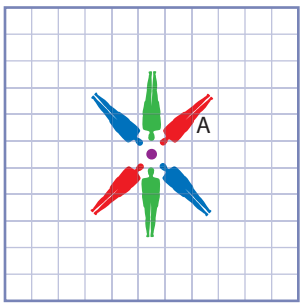
a) Sherry is a coach for a synchronized swimming team. She is teaching her team to hold and rotate the pattern shown. It is important for the swimmers to keep the pattern symmetrical as they rotate. If the pattern is symmetrical, it will look like a reflection. Swimmer A rotates clockwise to the position of swimmer B. Describe the rotation, stating the

- centre of rotation
- angle of rotation

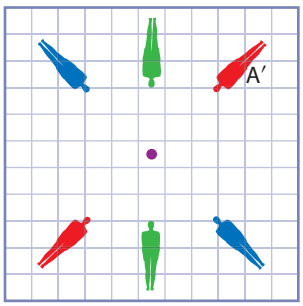
Then, show the lines of reflection.



b) For the next move, Sherry needs to show the swimmers how to make the pattern look like the star is bursting as the swimmers move apart. She sketches the swimmers on a grid. First, she draws their original position. Then, she draws their position after they move apart. Describe the translation of swimmer A.



Position 1



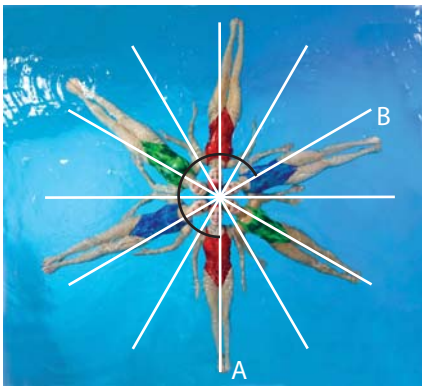
Position 2

Solution

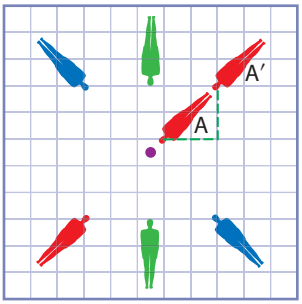
a) The centre of rotation is the middle of the pattern where the swimmers' heads meet. Draw lines from A and B to the centre of rotation. Then, measure the angle.

The angle of rotation is about 240°.

The photo shows six lines of reflection.



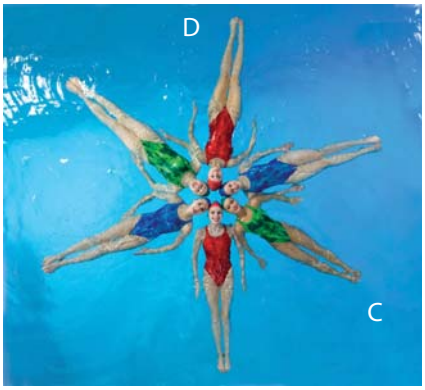
b) From the diagram, swimmer A moves 2 units right and 2 units up.



Your Turn

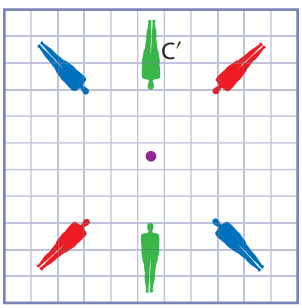
a) Swimmer C rotates counterclockwise to the position of swimmer D. Describe the rotation, stating the

- centre of rotation
- angle of rotation



b) Describe the translation of swimmer C when the swimmers move apart as shown.

c) How many lines of reflection are there after the translation?



Check Your Understanding

Try It

1. On a coordinate grid, plot, rotate, and reflect each point as indicated.

a) P(−3, 5) Rotate 180° about the origin, and reflect over the y-axis.

b) G(8, 2) Rotate 90° counterclockwise about (1, 3), and reflect over the x-axis.

2. On a coordinate grid, plot, rotate, and translate each point as indicated.

a) A(−2, −6) Rotate 90° counterclockwise about the origin, and translate 7 units up.

b) H(8, −5) Rotate 90° clockwise about (1, 3), and translate 4 units left.

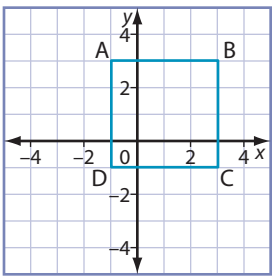
3. On a coordinate grid, plot, rotate, reflect, and translate each point as indicated.

a) Q(2, −4) Rotate 45° clockwise about (4, −6), reflect over $y = 2$, and translate 2 units right.

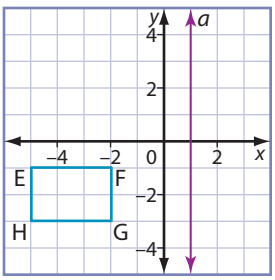
b) I(−4, 4) Rotate 90° clockwise about (−2, −2), reflect over $x = 1$, and translate 6 units down.

4. Transform each shape as instructed.

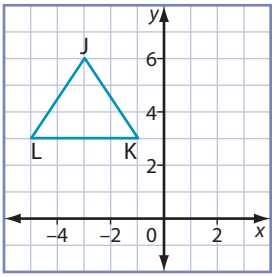
a) Rotate 180° about (−2, −2). Then, translate 3 units up.



b) Rotate 270° counterclockwise about the origin. Then, reflect over line of reflection a.



c) Rotate 45° clockwise about the origin. Then, reflect over the x-axis and translate 4 units right and 2 units down.



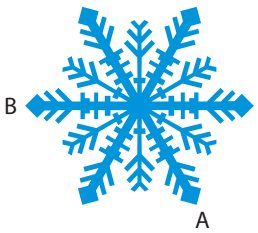
Strategy
Draw or Model

5. The point P(8, 1) is rotated 90° counterclockwise about the origin and then translated 5 units left and 2 units down. What are the coordinates of the transformed point?

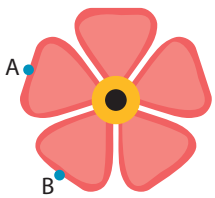
Apply It

6. Describe the centre of rotation and angle of rotation if point A rotates clockwise to point B for each figure. How many lines of symmetry does each figure have?

a)

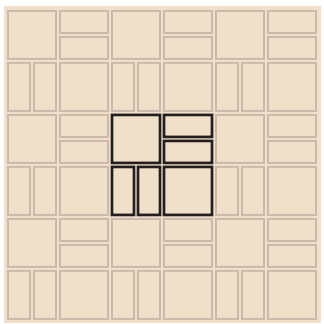


b)

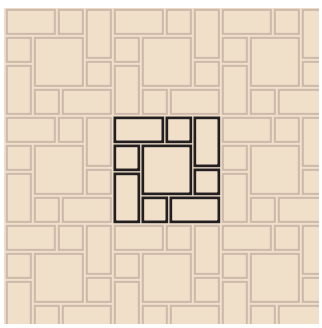


7. Describe how each pattern is created using rotations, translations, and/or reflections.

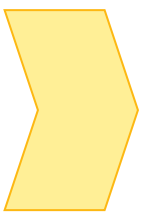
a)



b)



8. Create a wallpaper pattern by rotating and translating the shape.

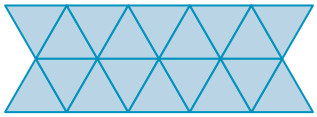


Tools of the Trade

Woodworkers use routers to cut, trim, and carve pieces of wood. Routers can be hand-held or machine-driven. Routers are often used to create finished edges or shapes.

9. On a coordinate grid, use rotations, reflections, and translations to make a four-point star design. Describe your process.

10. Brian is using a router to carve this design as a border on the top of a coffee table. Describe how he can make the pattern using transformations.



Work With It

1. Draw a square in Quadrant II.

a) Reflect the square over the line $y = x$.

b) Rotate the original square 270° clockwise about the origin.

c) What do you notice?

d) Create another series of transformations that has the same result.

2. Logos often use symmetry.

a) For each logo, describe how rotation and/or reflection are used.

i)

ii)

b) Find other logos that use rotation, reflection, or both. Sketch them and show how the transformations are used.

Discuss It

3. a) What transformation is used to create the king of hearts playing card? Describe the transformation.

b) Why do you think cards are designed this way?

c) Which cards in a standard deck are not designed this way?

4. In what ways are translations similar to rotations? In what ways are they different?

6 Skill Check

What You Need to Know

Section	After this section, I know how to . . .
6.1	■ identify and draw a dilation of a 2-D and 3-D object ■ use similarity to determine and explain dilations
6.2	■ identify and draw a vertical or horizontal translation ■ draw successive translations ■ create and analyse designs made with translations
6.3	■ identify a reflection and draw the image of a shape that is reflected ■ draw shapes that are reflected and translated ■ create and analyse designs made with reflections
6.4	■ identify a rotation and draw the image of a shape that is rotated ■ draw shapes that are rotated, reflected, and translated ■ create and analyse designs made with rotations

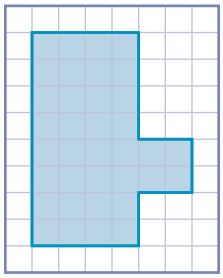
If you are unsure about any of these questions, review the appropriate section or sections of this chapter.

6.1 Dilations, pages 268–281

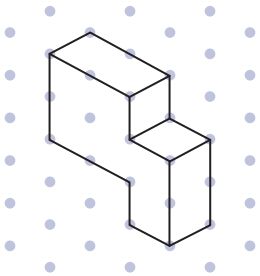
1. a) Is the smaller polygon a reduction of the larger one? Explain using similarity.

b) Use a ruler to help you determine the scale factor of the reduction.

2. a) Draw a dilation of the figure with a scale factor of $\frac{1}{4}$.

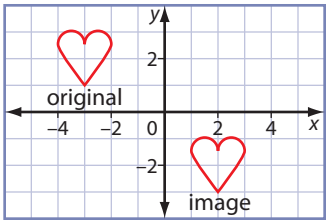


b) Draw a dilation of the figure with a scale factor of 3.



6.2 Translations, pages 282–291

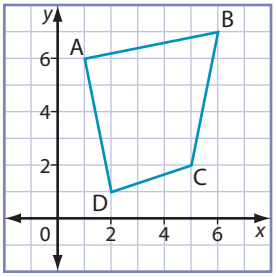
3. Describe the translation of the figure.



4. Draw a rectangle with coordinates at (−6, 6), (−4, 6), (−6, 7), and (−4, 7) on a coordinate grid. Create a tile pattern by translating the rectangle 1 unit down and 1 unit right. Repeat the successive translation 4 times.

6.3 Reflections, pages 292–301

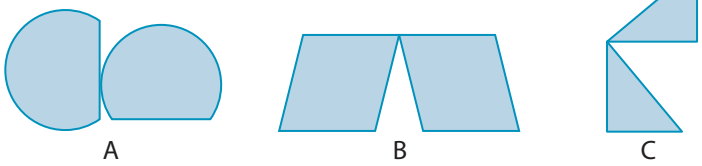
5. Copy the quadrilateral on a grid and reflect it over the y-axis.



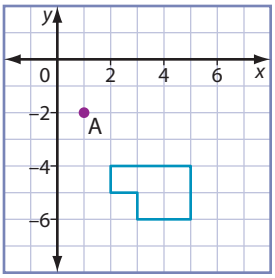
6. Give three examples of real-life reflections.

6.4 Rotations, pages 302–315

7. Which of the following transformations represent(s) a rotation?



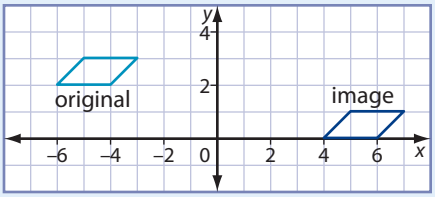
8. Rotate the figure 90° clockwise around centre of rotation A(1, −2). Then, reflect the figure over $x = 1$.



6 Test Yourself

For #1 to #5, select the best answer.

1. Which pair of transformations was performed on the parallelogram?



A translated 10 units right and 2 units down

B rotated 180° about the origin

C reflected over the x-axis and y-axis

D reflected over the y-axis and translated 2 units down

2. Point P(5, 2) is reflected over the x-axis and then translated 5 units down. What are the coordinates of the image?

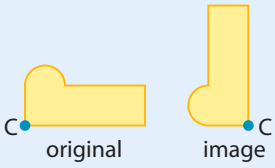
A $(-5, 7)$

B $(5, 7)$

C $(-5, -7)$

D $(5, -7)$

3. Which rotation describes how the figure has been transformed? C is the centre of rotation.



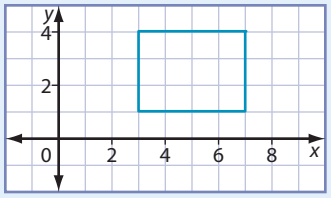
A clockwise 270°

B clockwise 45°

C counterclockwise 270°

D 180°

4. Suppose the rectangle is reflected over the y-axis. Which transformation will create the same image?



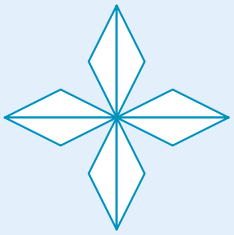
A rotation of 90° clockwise

B translation of 10 units left

C translation of 10 units right

D rotation of 90° counterclockwise

5. Which transformations were used to create the pattern?



A rotation and reflection

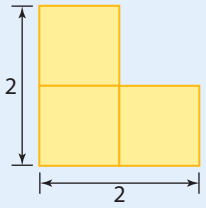
B reflection and translation

C reflection and dilation

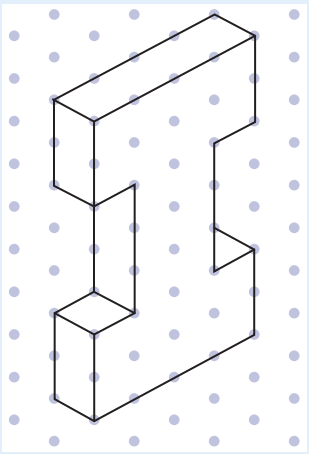
D rotation and dilation

6. The point P(13, 9) is rotated clockwise 90° about the origin, reflected over the y-axis, and translated 2 units left and 10 units up. Determine the coordinates of the transformed image.

7. a) Using a scale factor of 3, draw a dilation of the figure.



b) Using isometric dot paper, reduce the figure by a factor of $\frac{1}{2}$.



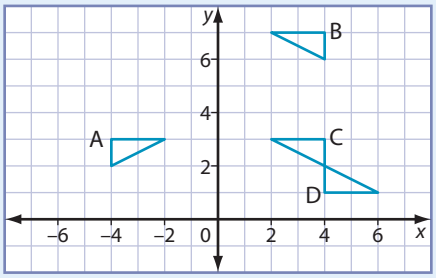
8. Describe the transformation(s) that create

a) A from C

b) C from B

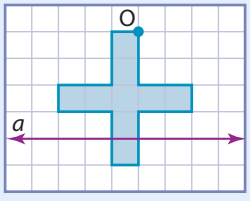
c) D from C

d) B from A

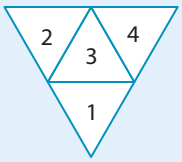


9. Perform the following transformations, in order, using the diagram.

- translate the shape 2 units left and 3 units up
- rotate 90° counterclockwise about vertex O
- reflect over line a



10. The net of a triangular pyramid is made up of four triangles. Describe the transformations that can be applied to triangle 1 to form the net.



Chapter Project

Make an Animation

Before cartoons and animations were made with computers, illustrators drew images of each movement of each character.

Create an animation flip pad using transformations to show an image moving over a coordinate plane. Use at least three different transformations in your animation. Choose from

- dilation
- translation
- reflection
- rotation

Draw each step in your animation on a separate piece of paper.

The more sheets of paper you use with smaller changes in movement between each, the more effective your animation will be.

Staple the pages of your animation together in order.

Web Link

You can also use computer programs to create animations. For a tutorial on creating animations in Microsoft® PowerPoint, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

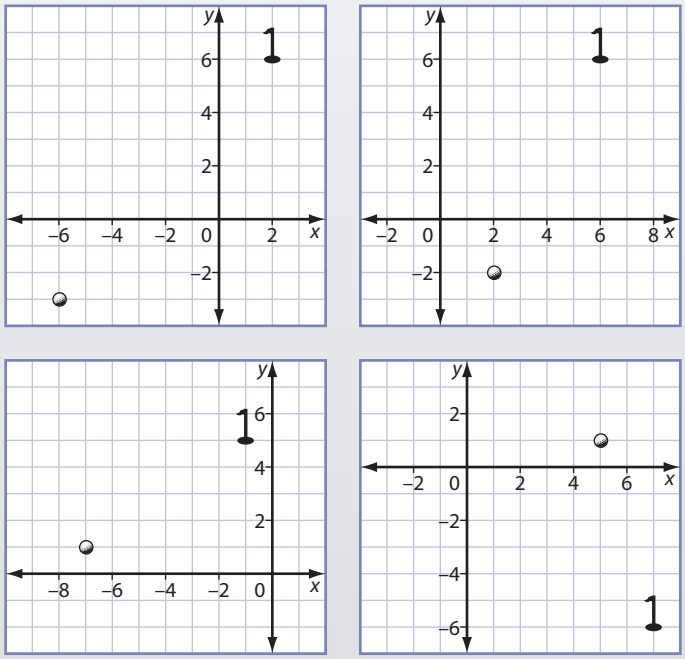
6 GAMES AND PUZZLES

Transformation Golf

Materials
- Transformation Golf
- grid paper
- ruler
- protractor
- compass

1. Get the ball in the hole in the fewest strokes possible.
- You can translate the ball left, right, up, and down. Each square equals 1 stroke.
- You can rotate the ball 90° clockwise or counterclockwise about the origin. Each rotation equals 1 stroke.
- You can reflect the ball over the x-axis or y-axis. Each reflection equals 1 stroke.

Compete with a partner to see who can get the lowest score.



2. Create three of your own Transformation Golf games. Exchange with your partner and try each other's golf games.